LIGO-G1200375-v2

Interfacing analytical- and numerical relativity for gravitational-wave astronomy

Semi-analytical gravitational waveforms for binary black holes and their applications

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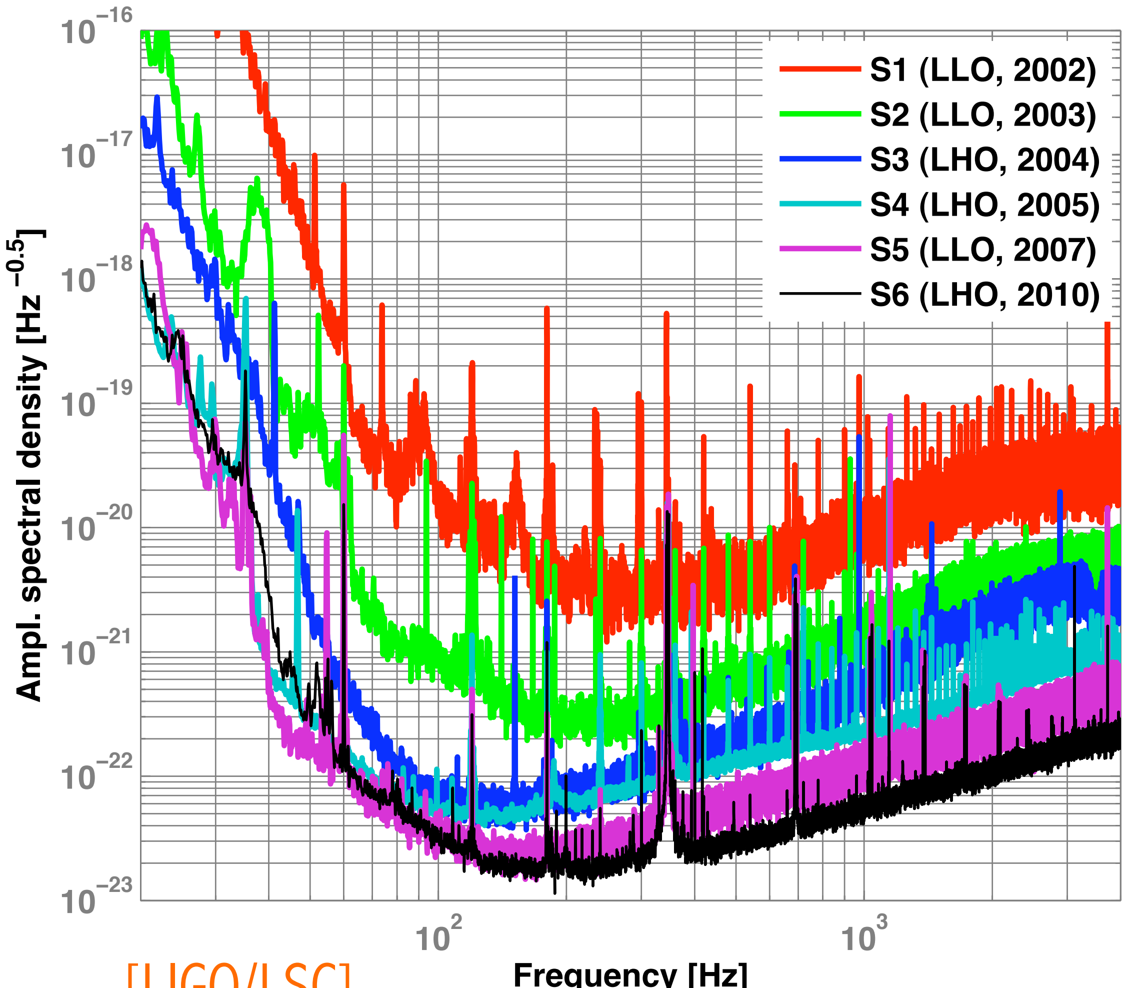
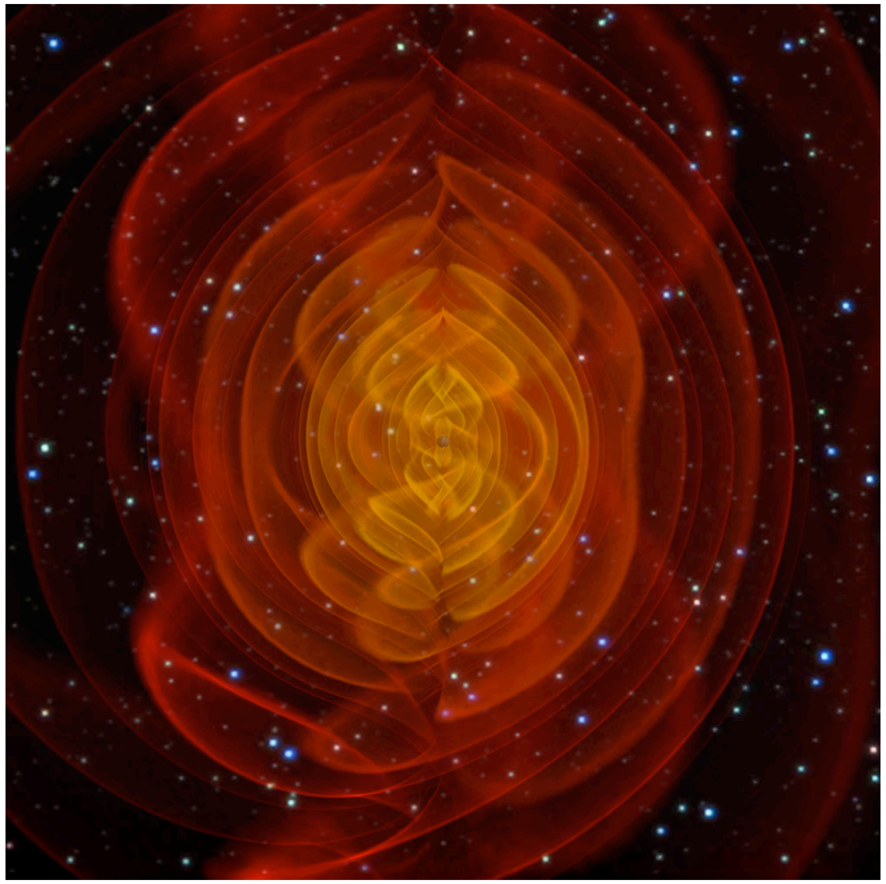
APS April Meeting, Atlanta 31 March 2012

The big picture

- Significant progress in the experimental effort for the first direct detection of GWs.
- Great breakthroughs in analytical & numerical relativity.



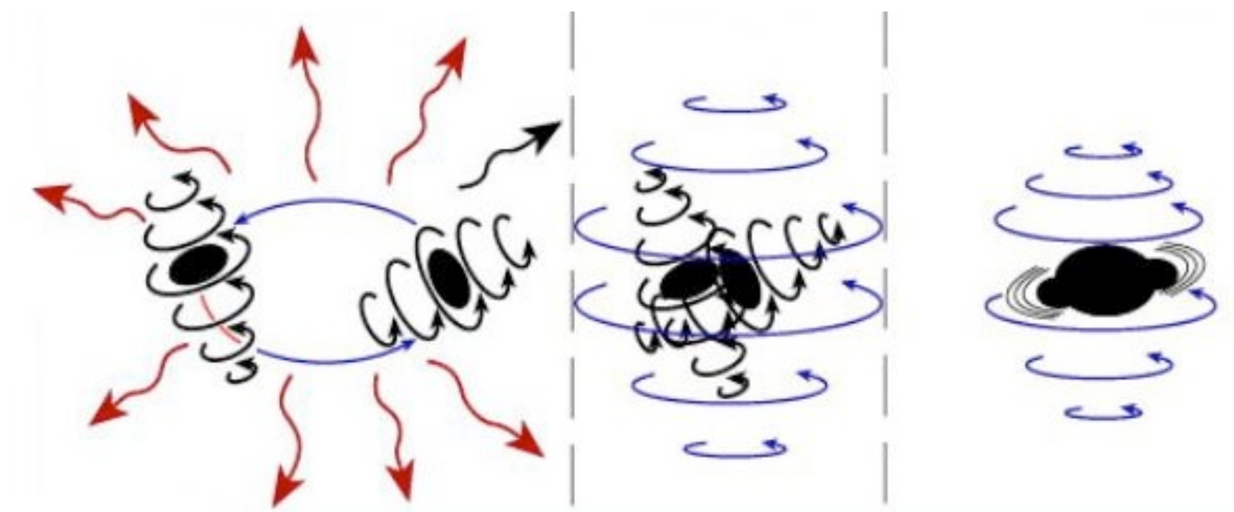
[LIGO/LSC]



[NASA/GSFC]

Coalescence of compact binaries: The most promising sources

Inspiral Merger Ring down

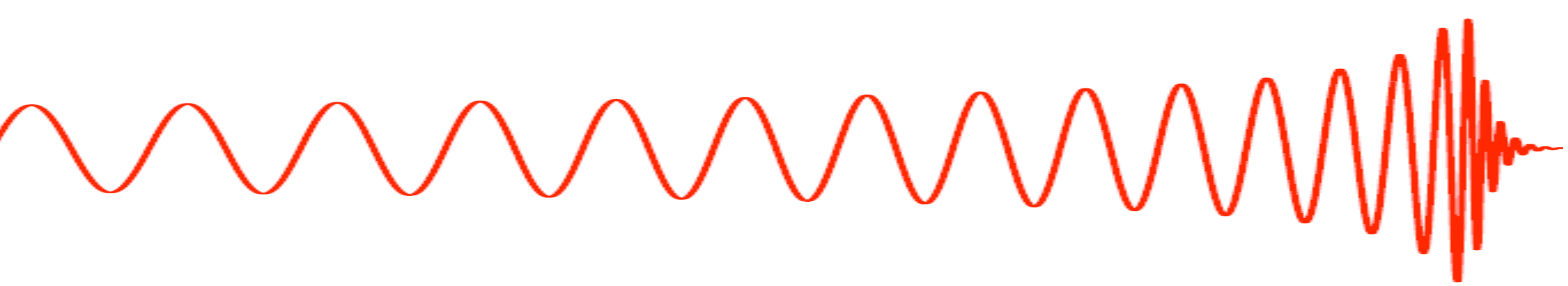


Post-Newtonian theory

Numerical Relativity

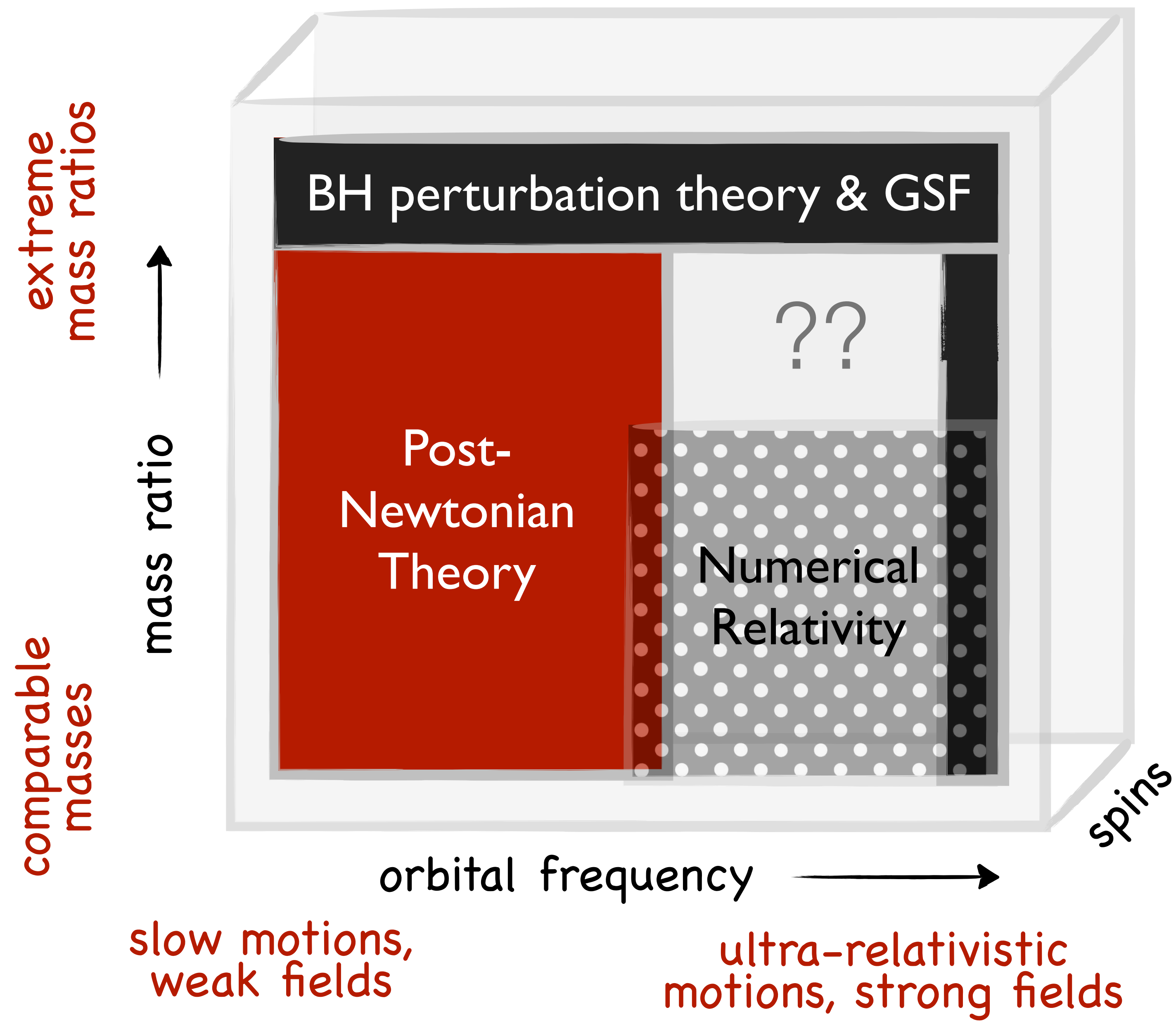
BH perturbation theory

(Pic. K. Thorne)



$h(t)$

Binary-black-hole parameter space



[Talk by A. Le Tiec in this session]

Constructing inspiral-merger-ringdown waveforms for binary black holes

Numerical-relativity (NR) simulations are computationally expensive. Practically impossible to generate sufficient number of NR simulations finely covering the entire parameter space.

Solution: Analytical waveforms calibrated against NR simulations

- Effective-one-body-based approaches
- Phenomenological approaches

The effective one-body approach

[Buonanno & Damour]

- Map the two-body dynamics into one-body dynamics in the presence of an effective metric.

$$ds^2_{\rm eff} = -A(r)\, c^2\, dt^2 + \frac{D(r)}{A(r)}\, dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

"deformed" Schwarzschild metric

The effective one-body approach

- Map the two-body dynamics into one-body dynamics in the presence of an effective metric.
- Introduce adjustable parameters. Propose some ansatz for the mass-ratio (and spin) dependence of these parameters. Tune them against NR.

[Buonanno & Damour]

$$ds_{\text{eff}}^2 = -A(r)\,c^2\,dt^2 + \frac{D(r)}{A(r)}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

$$A_k(r) = \sum_{i=0}^{k+1} \frac{a_i(\nu)}{r^i}, \qquad D_k(r) = \sum_{i=0}^{k} \frac{d_i(\nu)}{r^i}$$

Up to k=3, known from PN; terms linear in ν known at all orders from perturbation theory

The effective one-body approach

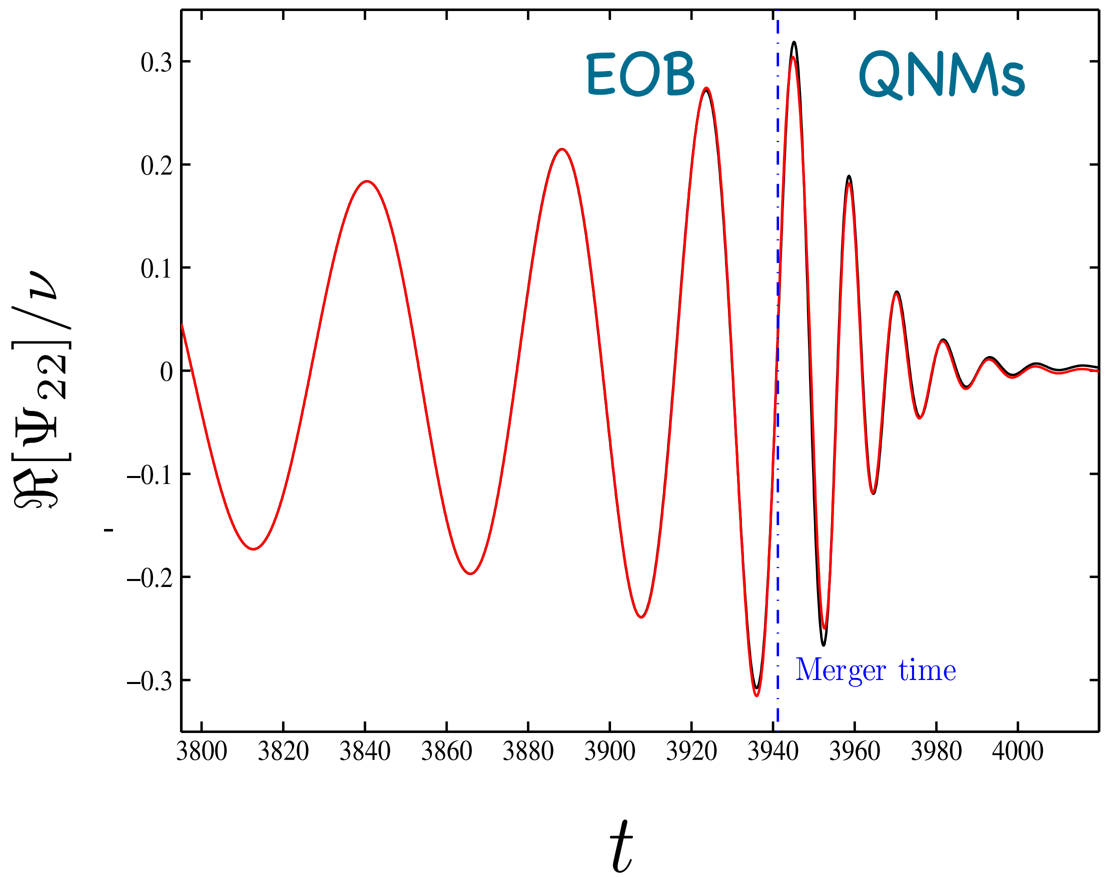
- Map the two-body dynamics into one-body dynamics in the presence of an effective metric.

- Introduce adjustable parameters. Propose some ansatz for the mass-ratio (and spin) dependence of these parameters. Tune them against NR.

- Compute the EOB Hamiltonian. During inspiral-plunge, evolve the trajectories using Hamilton's equations.

- Match the calibrated EOB inspiral-plunge waveforms with several QNM modes.

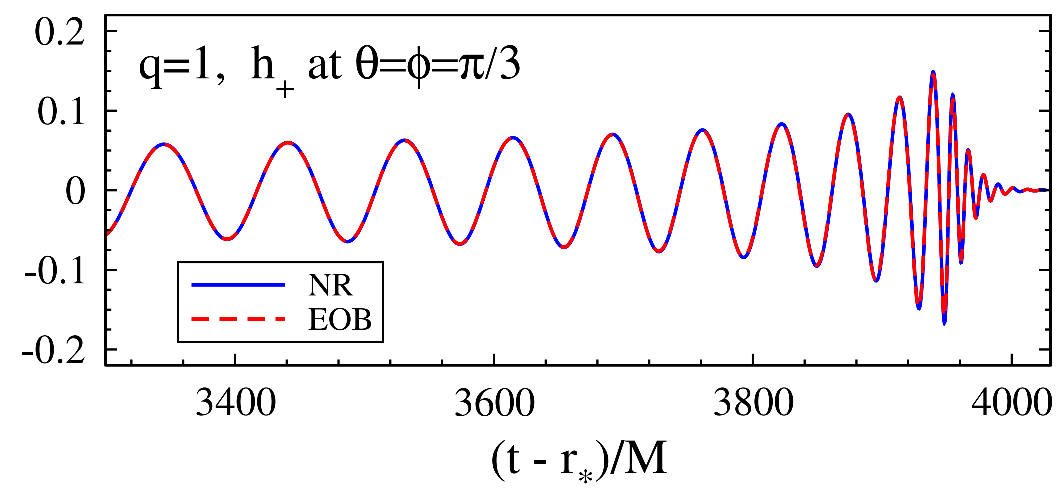
[Damour & Nagar 2009]

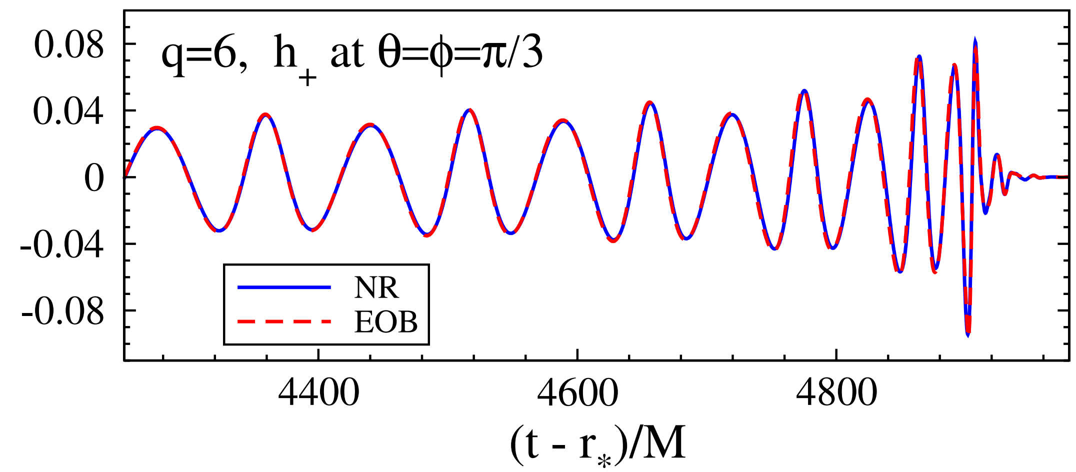


Status of the EOB approach

- **Non-spinning EOB** Calibrated 5 dominant modes to 5 SpEC NR simulations (q = 1,2,3,4,6). [Damour & Nagar 2009, Barausse et al. (2011), Pan et al (2011)]

[Pan et al (2011)]





Modeling errors are comparable to NR numerical errors at calibration points. With no further calibration, compare well with Teukolsky/RWZ waveforms in the EMR ($q \to \infty$) limit. [Bernuzzi et al (2011), Pan et al (2011)]

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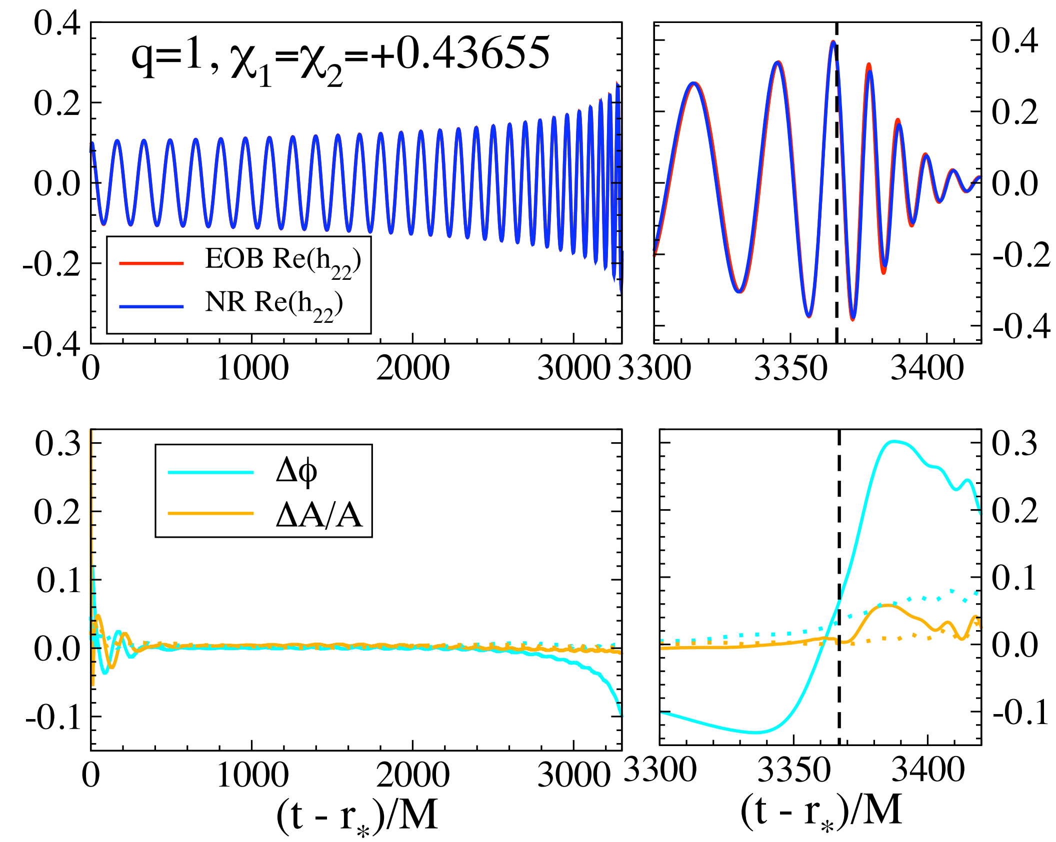
- **Aligned-spins** Calibrated the dominant 22 mode to 7 SpEC NR simulations (5 non-spinning + 2 equal-mass, equal, non-precessing spins ~ 0.44) [Taracchini et al. (2012), Barausse et al. (2011)]

[Taracchini et al. (2012)]



Reasonable waveforms when extrapolated to any mass ratio with spins $-1 < \chi < 0.7$

Status of the EOB approach

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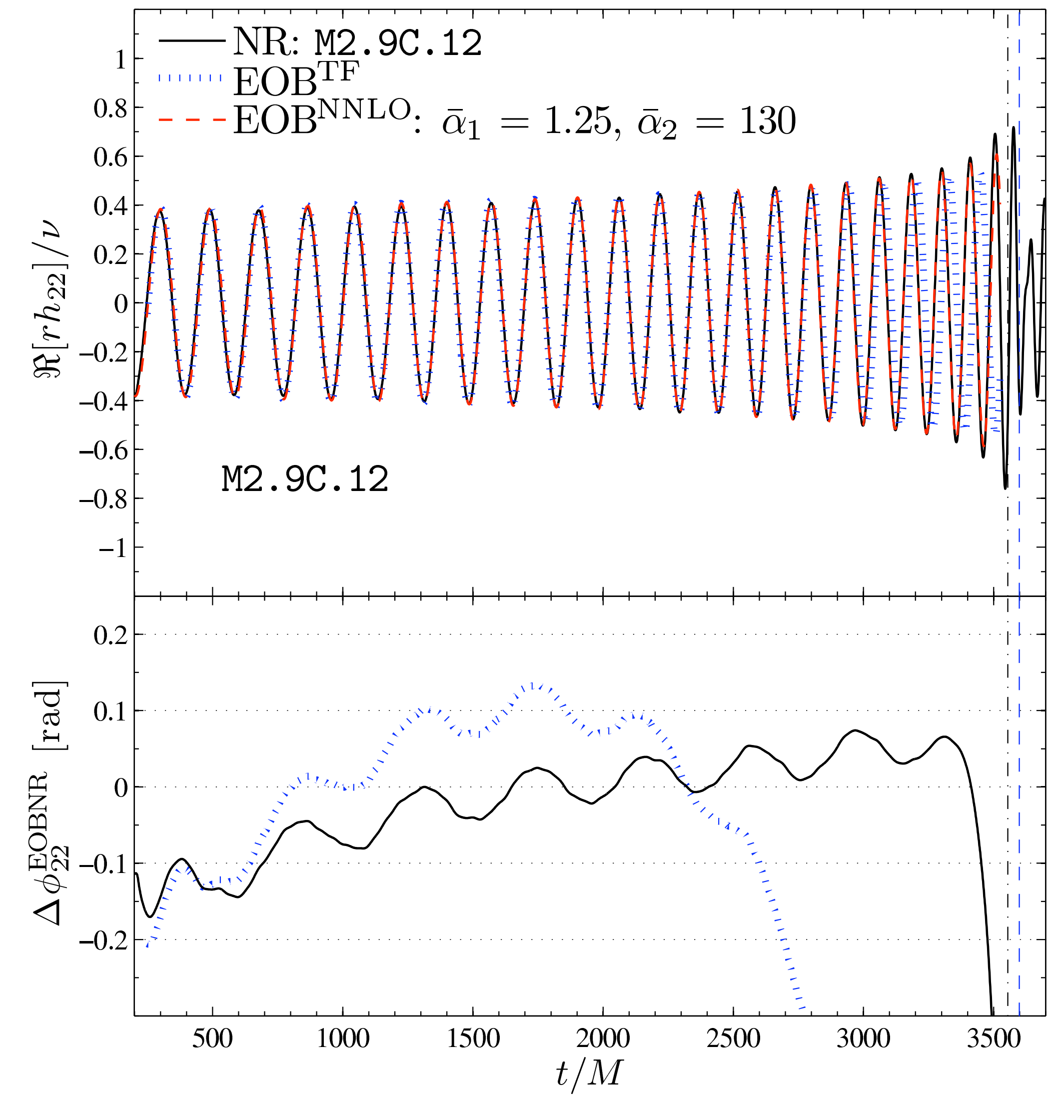
- **Tidal effects** Progress in incorporating neutron-star tidal effects in EOB dynamics & waveforms. [Damour & Nagar (2010), Baiotti et al. (2010)]

[Baiotti et al. (2010)]

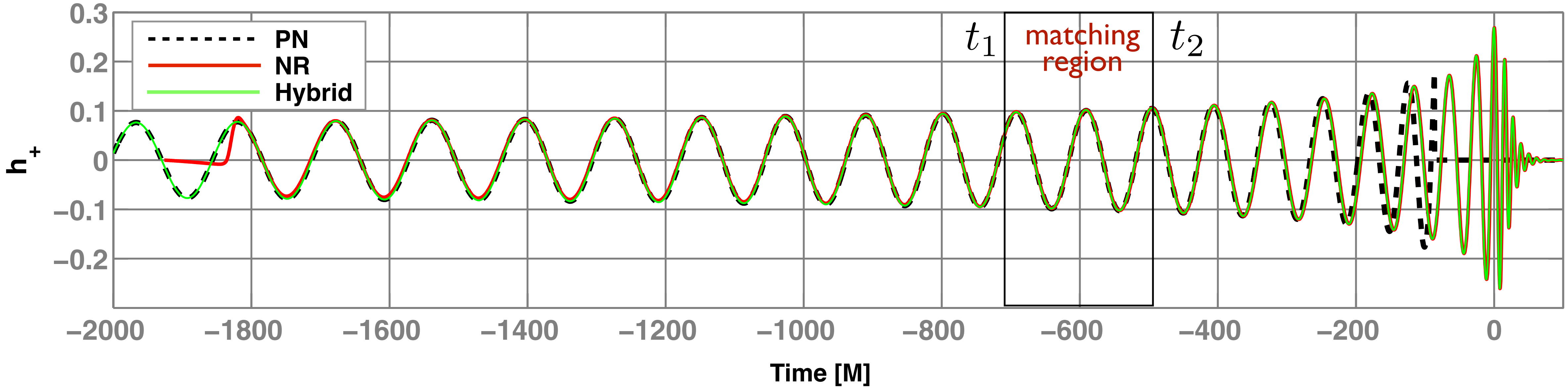


Comparison of BNS NR simulations and EOB phasing (with and without tidal effects)

Phenomenological approach

- Match PN and NR waveforms in a region where both calculations are expected to be valid.

[Ajith et al (2008)]

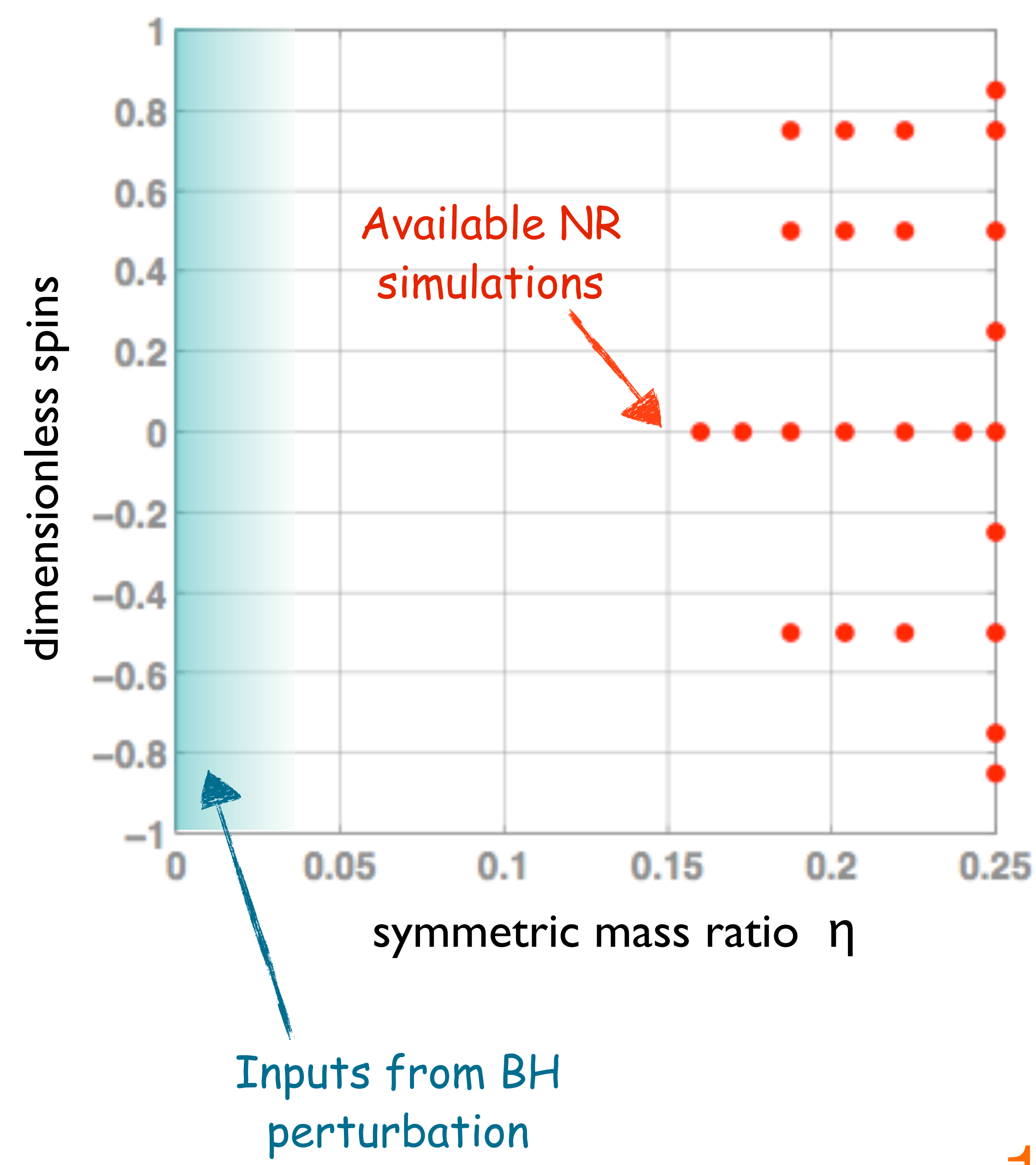


good agreement between PN and NR

Phenomenological approach

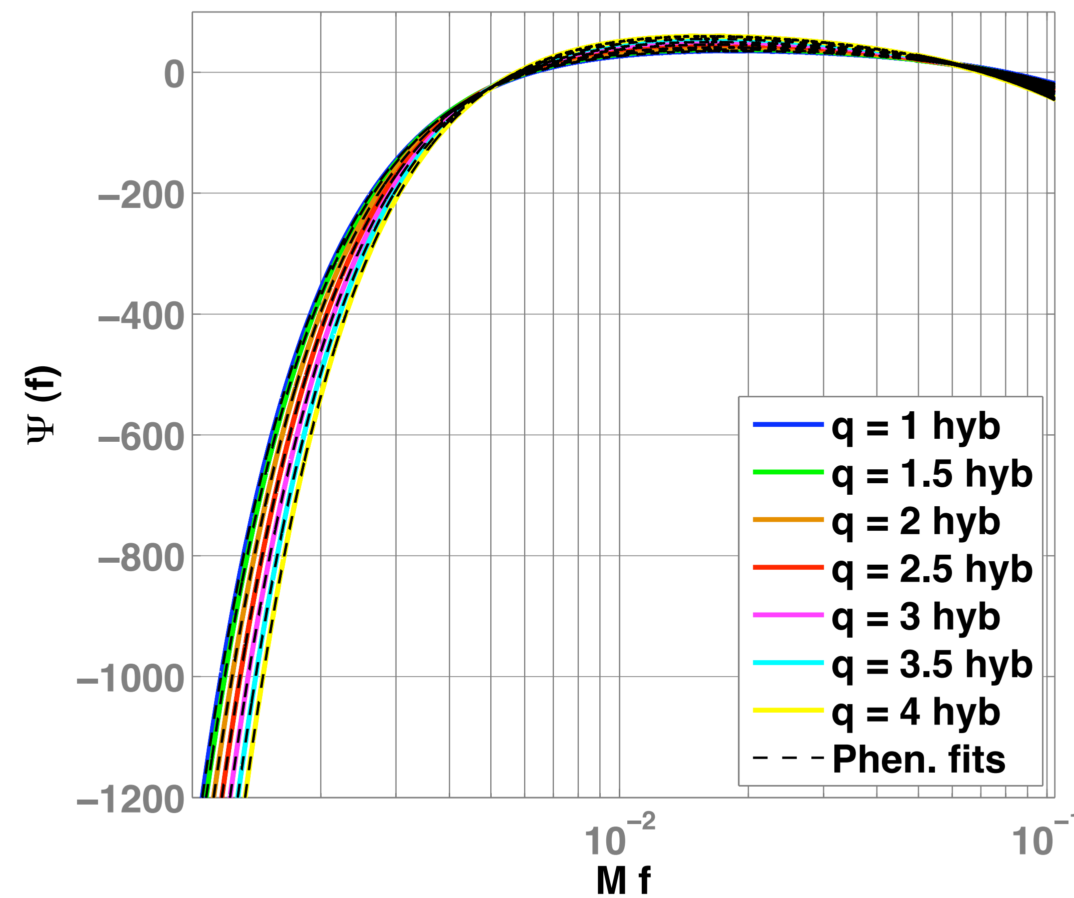
- Match PN and NR waveforms in a region where both calculations are expected to be valid.
- Construct NR+PN hybrid waveforms in a small number of discrete points in the parameter space.

[Ajith et al (2011)]



Phenomenological approach

- Match PN and NR waveforms in a region where both calculations are expected to be valid.
- Construct NR+PN hybrid waveforms in a small number of discrete points in the parameter space.
- Interpolate hybrid waveforms over the parameter space using interpolation functions motivated from perturbative approaches.



Amplitude: $$A(f) \propto \begin{cases} f'^{-7/6}\,(1+\sum_{i=2}^{3}\alpha_i\, v^i) & : \text{inspiral} \\ w_m\, f'^{-2/3}\,(1+\sum_{i=1}^{2}\epsilon_i\, v^i) & : \text{merger} \\ w_r\, \mathcal{L}(f, f_2, \sigma) & : \text{ringdown} \end{cases}$$

given by PN

from NR

BH perturbation + tuned to NR

Phase: $$\Psi(f) \equiv 2\pi f t_0 + \varphi_0 + \frac{3}{128\,\eta\, v^5}(1+\sum_{k=2}^{7} v^k\, \psi_k).$$

Newtonian term

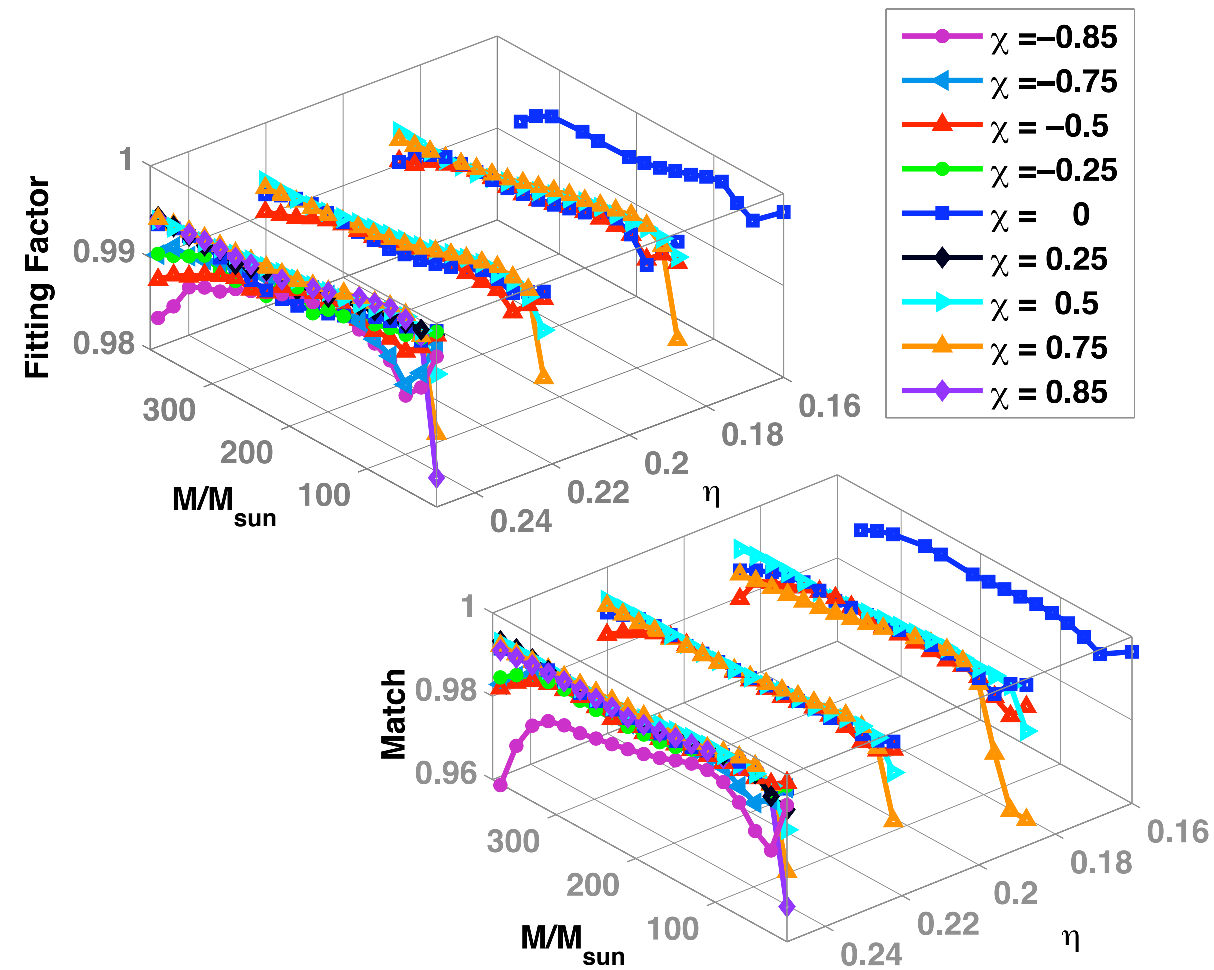
estimated from hybrids

[Ajith et al (2011)]

Phenomenological approach

- Match PN and NR waveforms in a region where both calculations are expected to be valid.
- Construct NR+PN hybrid waveforms in a small number of discrete points in the parameter space.
- Interpolate hybrid waveforms over the parameter space using interpolation functions motivated from perturbative approaches.
- Test the interpolated (analytical) waveform family against different sets of hybrid waveforms.

[Ajith et al (2011)]

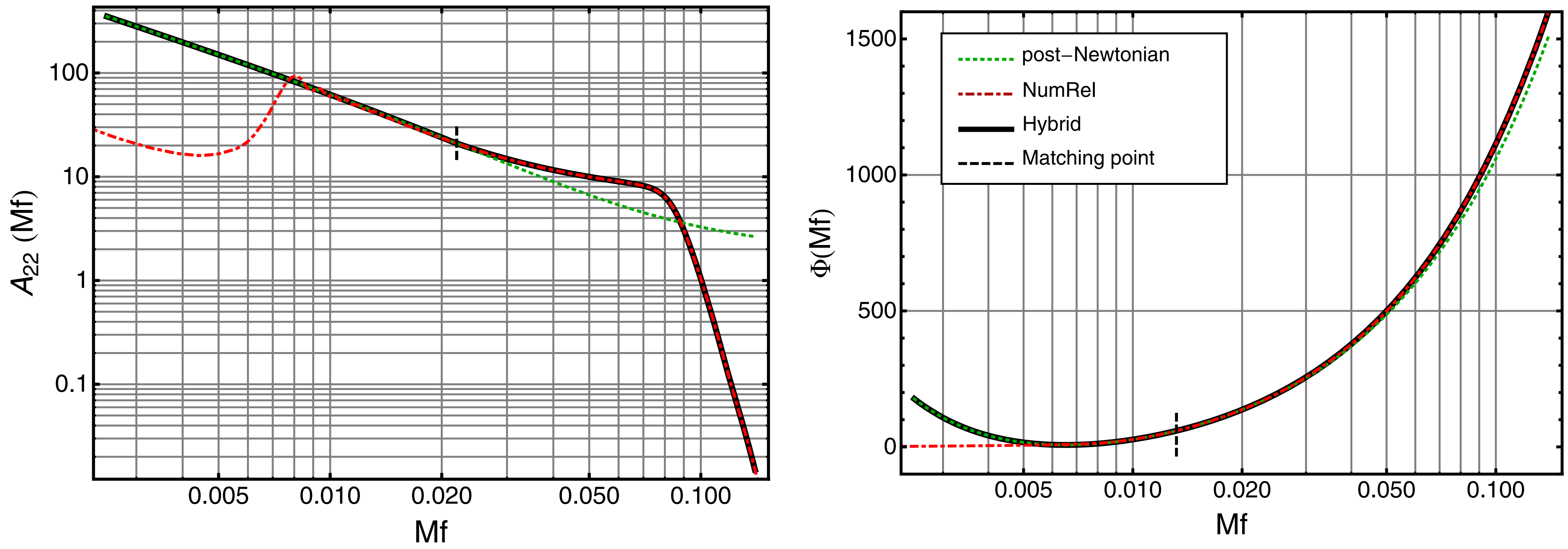


Fitting factor and match of the phenomenological IMR templates with the hybrid waveforms (Ini LIGO).

Different variants of the phenomenological approach

- PN-NR matching can be done in frequency domain also (provided "long-enough" NR waveforms are available so that FFTs of NR waveforms can be calculated).

[Santamaria et al (2010)]



Different variants of the phenomenological approach

- PN-NR matching can be done in frequency domain also (provided "long-enough" NR waveforms are available so that FFTs of NR waveforms can be calculated).
- Phenomenological fits to the amplitude and phase ensuring that the PN waveforms are retrieved in the low-mass limit.

inspiral (PN) merger (NR)

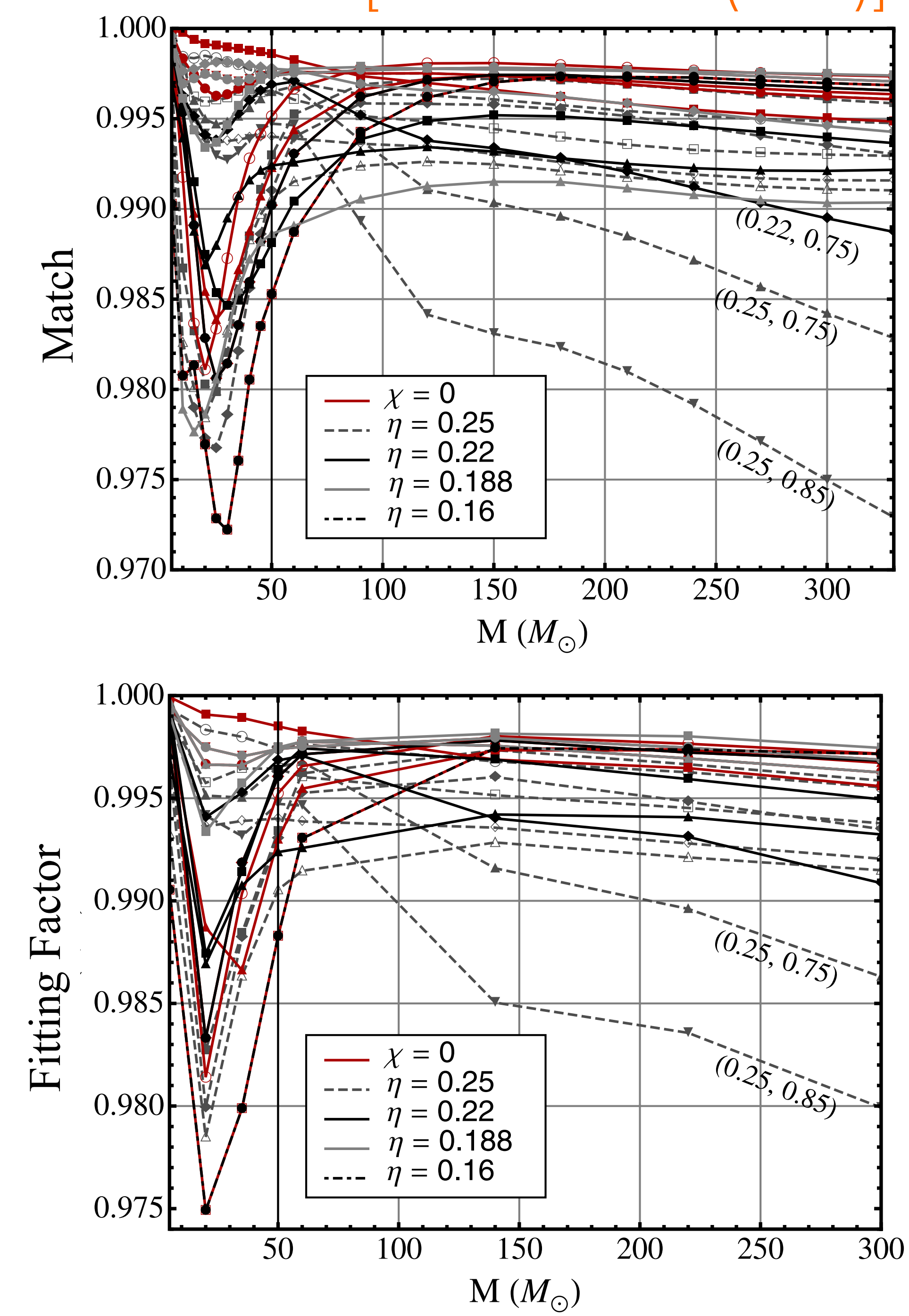
$$\Phi_{\rm phen}(f) = \psi^{22}_{\rm SPA} w^-_{f_1} + \psi^{22}_{\rm PM} w^+_{f_1} w^-_{f_2} + \psi^{22}_{\rm RD} w^+_{f_2}$$

$$\tilde{A}_{\rm phen}(f) = \tilde{A}_{\rm PM}(f) w^-_{f_0} + \tilde{A}_{\rm RD}(f) w^+_{f_0}$$

inspiral+merger (PN+NR)

ringdown (BH pert.)

[Santamaria et al (2010)]



FF and match in Adv LIGO

Phenomenological waveforms with generic spins: first efforts

[Sturani et al (2010)]

- Waveforms are computed using time-domain adiabatic PN evolution (TaylorT4) until the onset of “merger”.

Evolution of frequency: $$\frac{dv}{dt} = -\frac{F(v)}{dE/dv}$$

Evolution of spins: $$\dot{\mathbf{S}}_{\mathbf{1,2}} = \boldsymbol{\Omega}_{1,2} \times \mathbf{S}_{1,2}$$

Evolution of ang. momentum: $$\dot{\hat{\mathbf{L}}} = -\frac{\nu}{v}\left(\dot{\mathbf{S}}_{\mathbf{1}} + \dot{\mathbf{S}}_{\mathbf{2}}\right)$$

Phenomenological waveforms with generic spins: first efforts

[Sturani et al (2010)]

- Waveforms are computed using time-domain adiabatic PN evolution (TaylorT4) until the onset of "merger".
- A phenomenological ansatz is proposed for the merger.

Evolution of frequency:

$$\omega_{orb}(t) = \frac{\omega_1}{1 - t/T_A} + \omega_0$$

Evolution of ang. momentum:

$$\frac{d\alpha}{dt} = \frac{\dot{\alpha}_1}{1 - t/T_A} + \dot{\alpha}_0$$

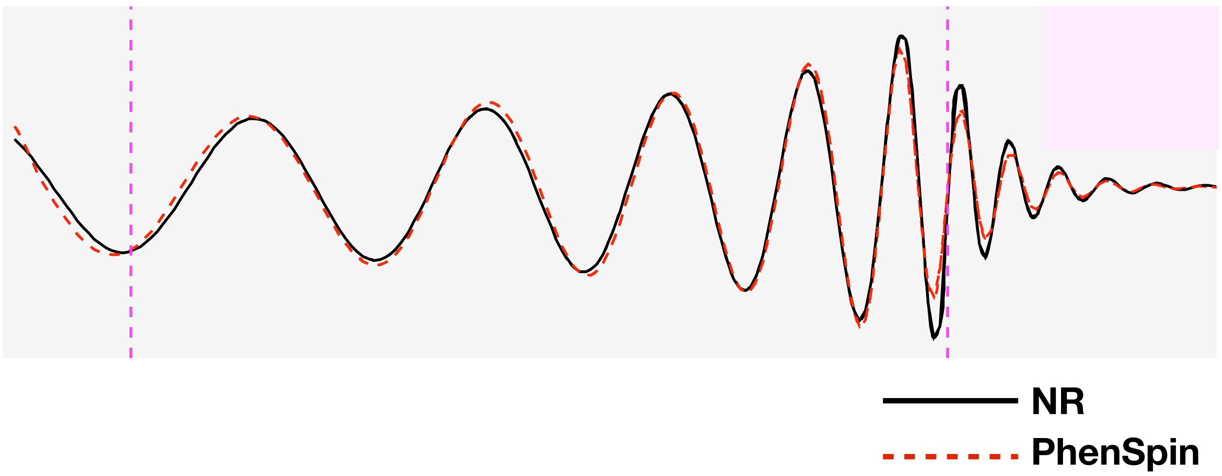
$$\frac{d\iota}{dt} = 0$$

angles describing the orientation of the ang. momentum

Phenomenological waveforms with generic spins: first efforts

- Waveforms are computed using time-domain adiabatic PN evolution (TaylorT4) until the onset of “merger”.
- A phenomenological ansatz is proposed for the merger.
- The inspiral-merger waveform is matched to a spectrum of quasi-normal modes (similar to EOBNR matching).

[Sturani et al (2010)]



$$h_{2,2}^{(rd)}(t) \;=\; \sum_n e^{-t/\tau_n} A_n e^{i\omega_{rd\,n} t}$$

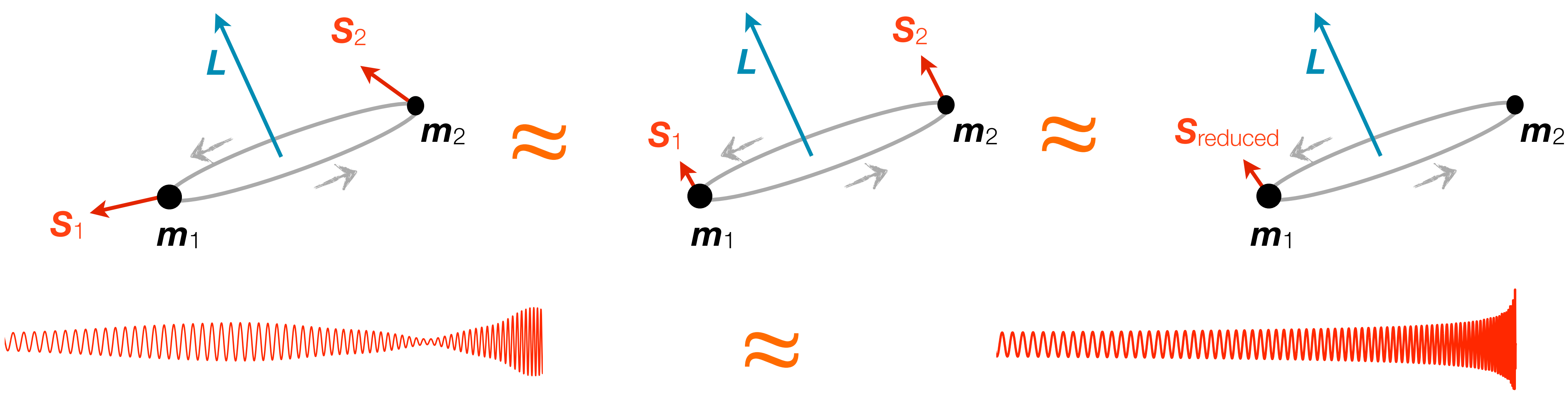
Much harder problem!
Will require hundreds of generic spinning NR waveforms for accurate calibration.
Currently only a handful is available (and used).

"Detection templates" for generic spinning binaries

- For the purpose of GW detection, it is possible to reduce the dimensionality of the problem. For comparable-mass binaries, dominant spin effects can be described by one parameter:

$$\chi \equiv \frac{1}{m_1 + m_2} \left(\frac{S_1}{m_1} + \frac{S_2}{m_2} \right)$$

(dominant term in the spin-orbit coupling)

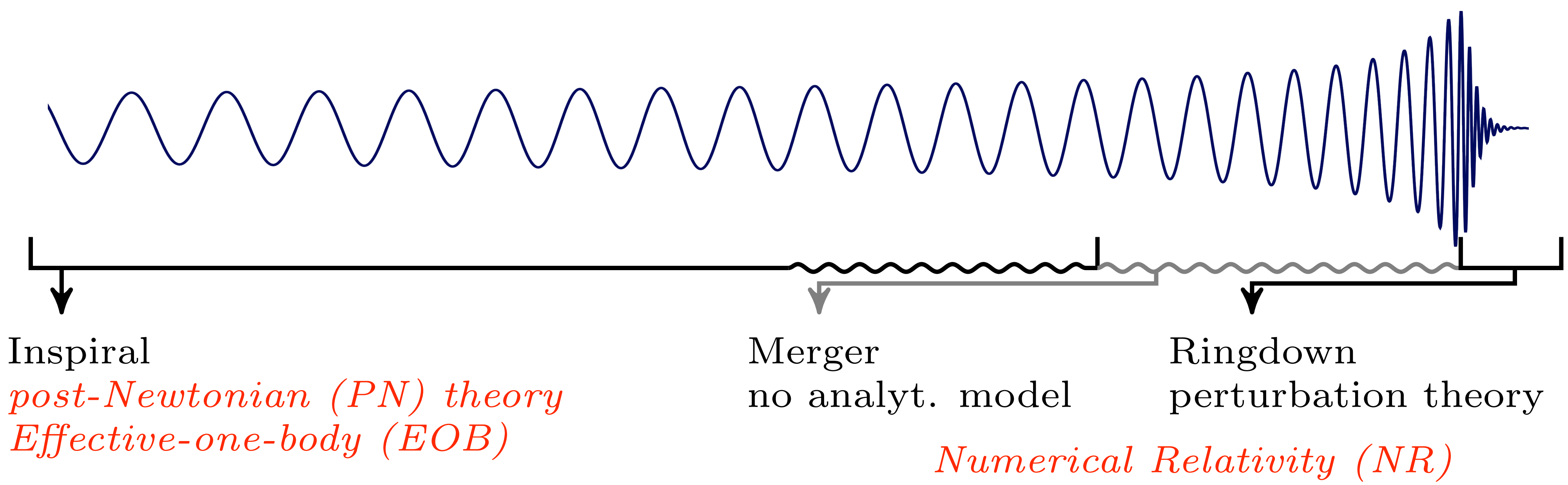


[Ajith et al (2011), Ajith (2011)]

[Prev. work: Apostolatos et al 1994, Buonanno et al 2004, Pan et al 2004]

Length requirements on NR waveforms

- In hybrid PN+NR waveforms, the long PN inspiral dominates the error budget, unless NR simulations cover hundreds of orbits. [Hannam et al (2010),]



[Ohme (2011)]

Length requirements on NR waveforms

- In hybrid PN+NR waveforms, the long PN inspiral dominates the error budget, unless NR simulations cover hundreds of orbits. [Hannam et al (2010),]
- **Pragmatic view:** Current NR simulations are long enough for detection templates (moderate mass ratios & nonprecessing-spins). Systematic errors in estimating parameters are ~few % (~statistical errors). [Ohme et al (2011)]

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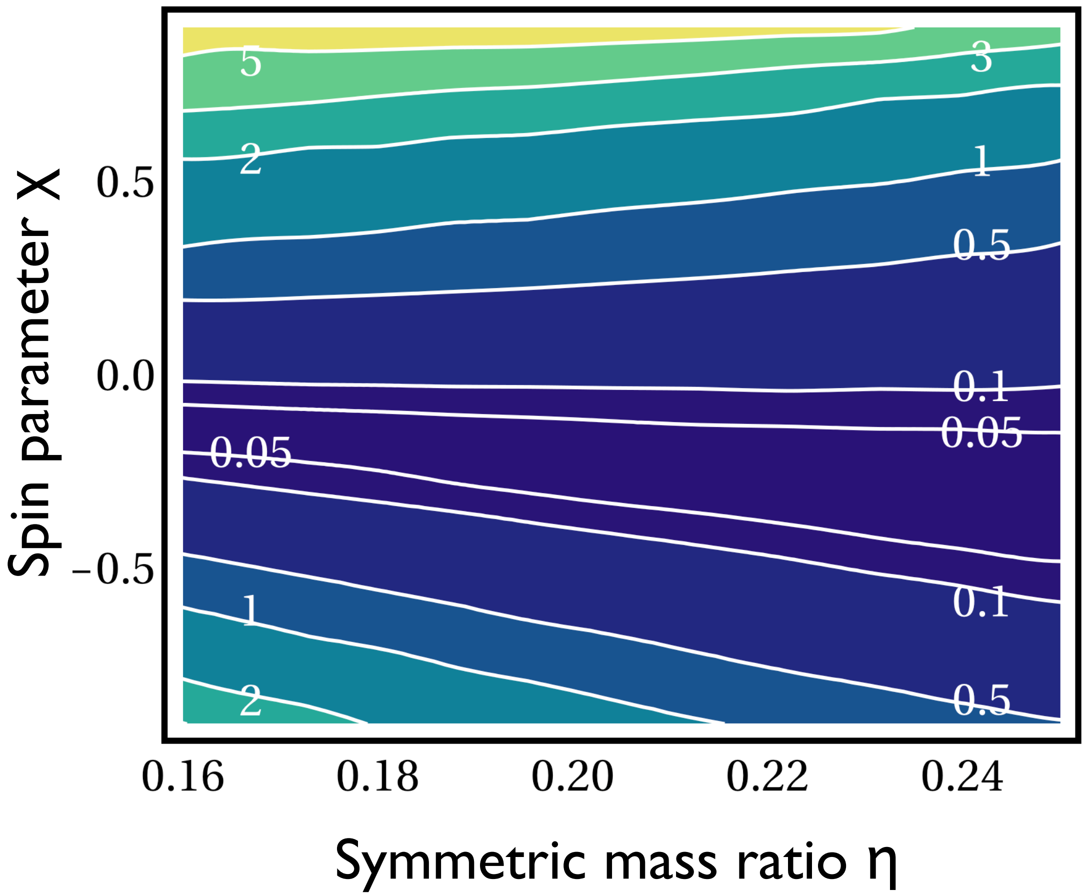

Mismatch contours (in %) between TaylorT1/F2 hybrids ($M\omega_{\mathrm{match}} = 0.06$).

Length requirements on NR waveforms

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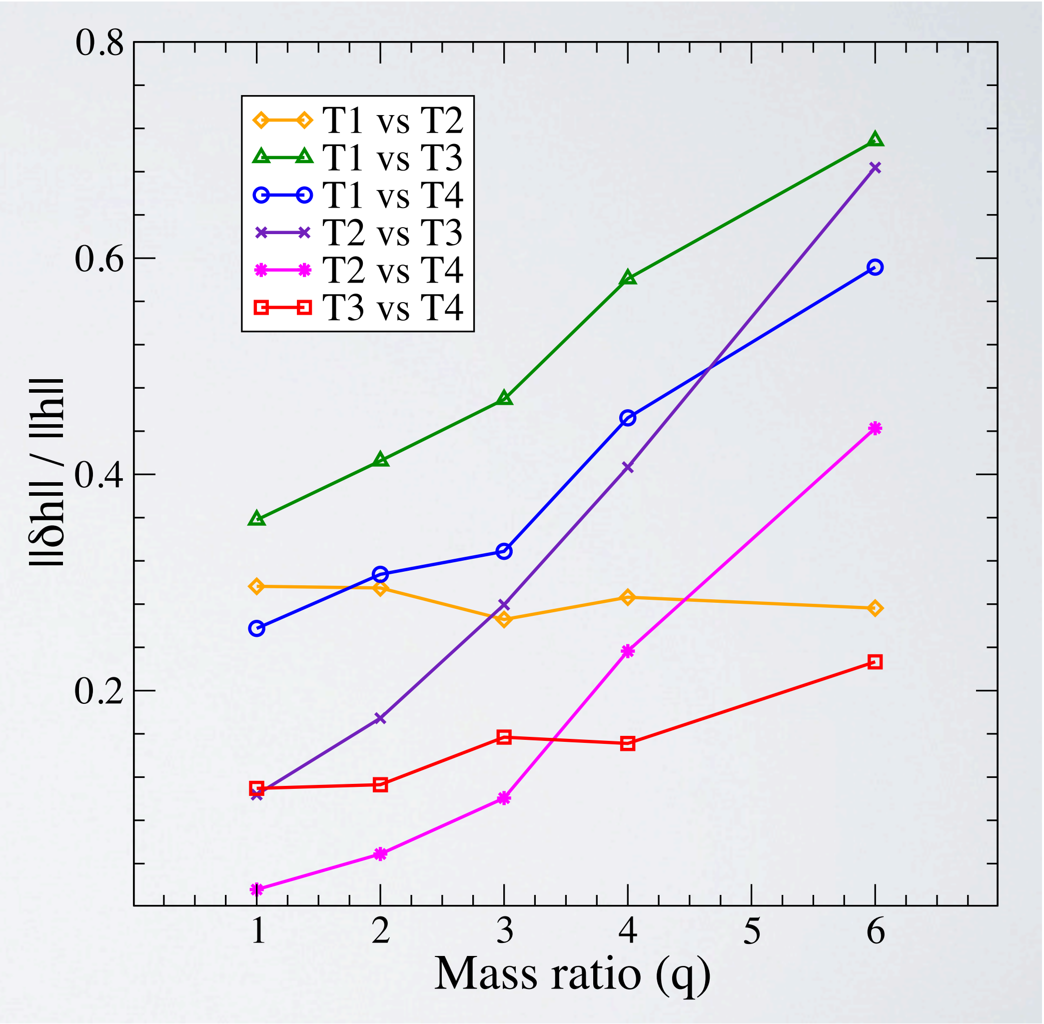
- **Pragmatic view:** Current NR simulations are long enough for detection templates (moderate mass ratios & nonprecessing-spins). Systematic errors in estimating parameters are ~few % (~statistical errors). [Ohme et al (2011)]
- **Ambitious view:** Much longer (>10x) NR simulations will be needed for accurate parameter estimation (including tests of GR) in advanced detector era. [MacDonald et al (2011); Boyle et al (2011)].

[Talk by I. MacDonald in session C8]

[MacDonald et al (in prep)]

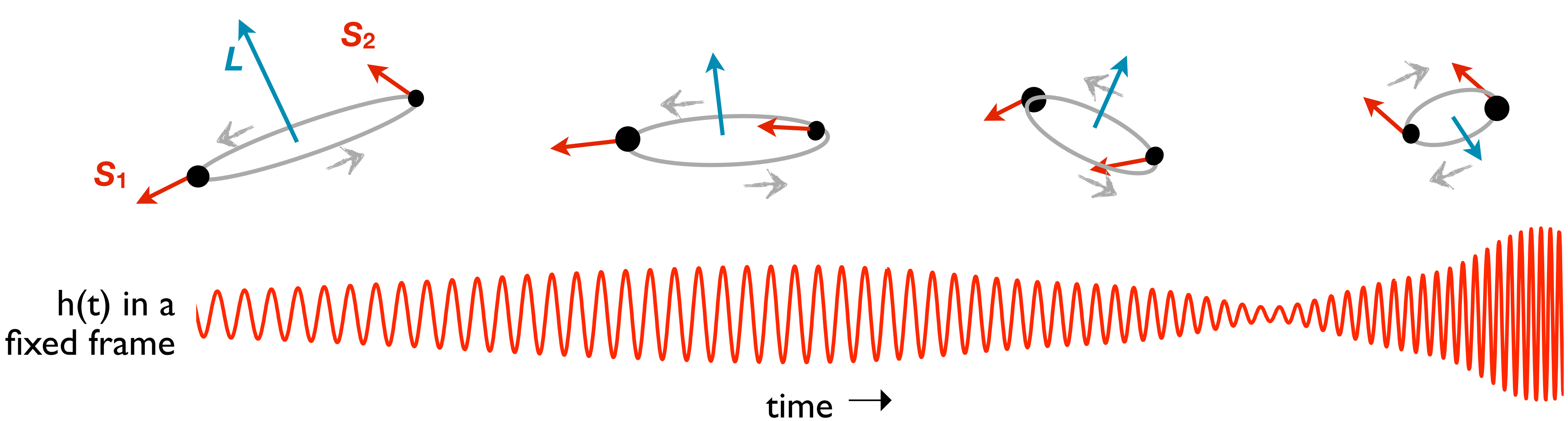


"Inaccuracy" of TaylorT1/T2/T3/T4 hybrids ($M\omega_{\text{match}} = 0.046$)

Requirement: $$\frac{||\delta h||}{||h||} < \frac{1}{\rho}$$

Issue of the preferred frame

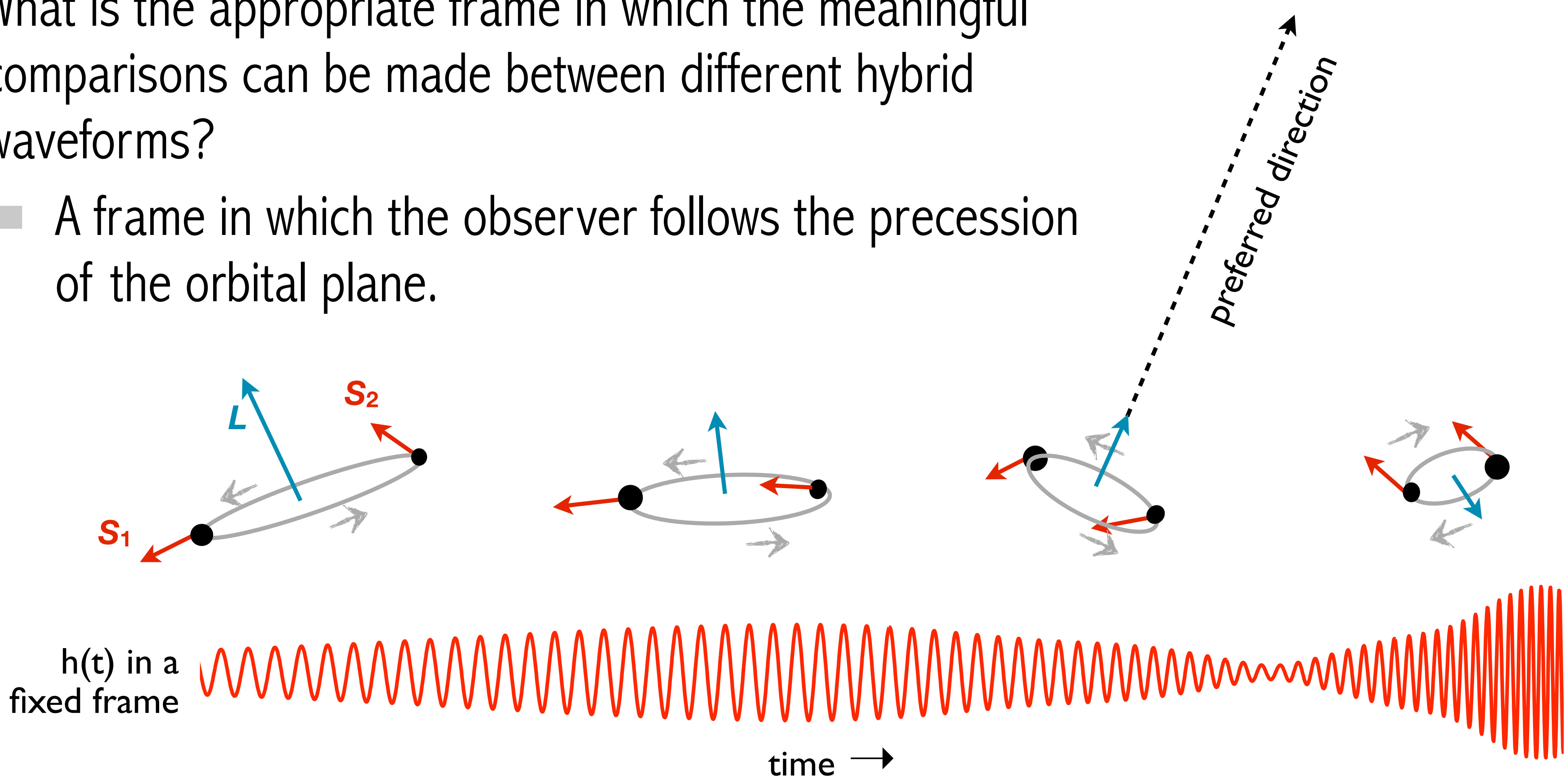
- Misaligned spins will cause precession. Waveform observed by a fixed detector will contain amplitude & phase modulations (changing the multipolar structure of the waveform).
- What is the appropriate frame in which the meaningful comparisons can be made between different hybrid waveforms?



Issue of the preferred frame

- Misaligned spins will cause precession. Waveform observed by a fixed detector will contain amplitude & phase modulations (changing the multipolar structure of the waveform).

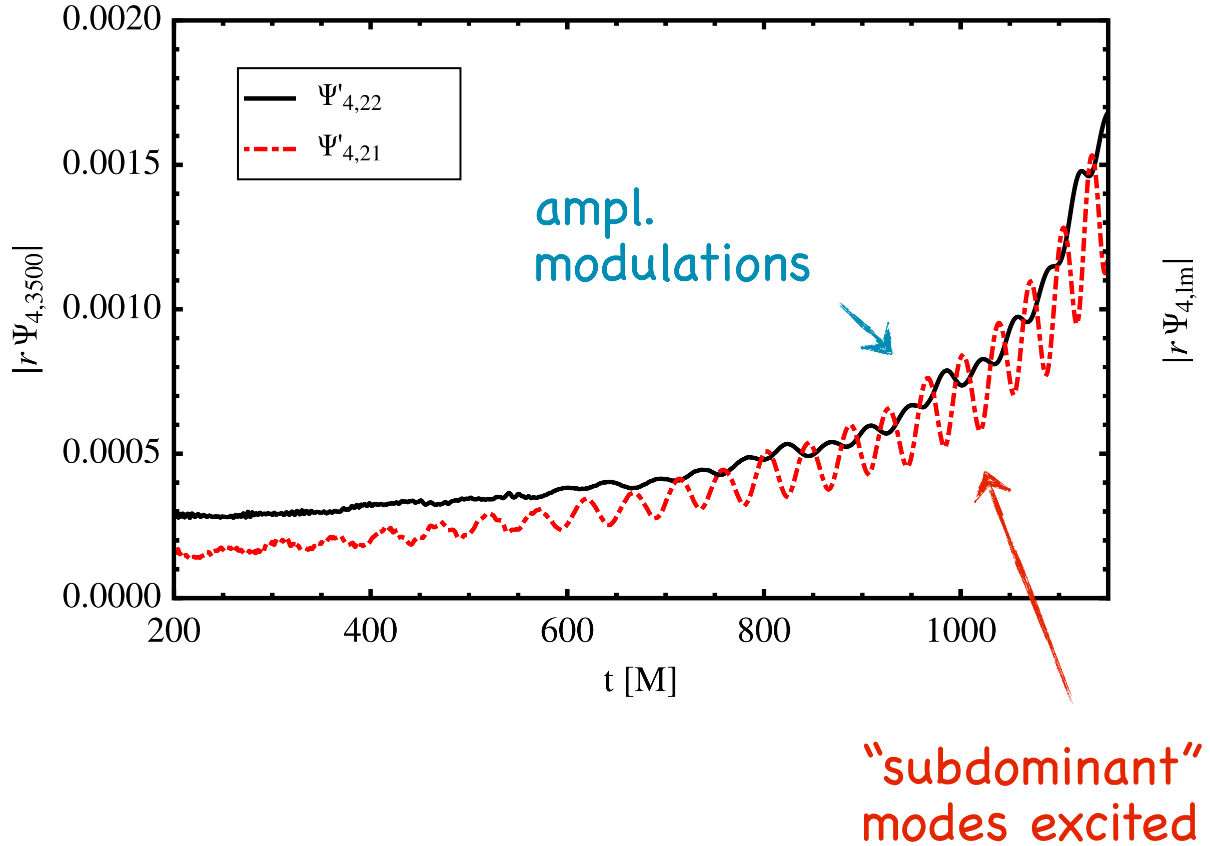
- What is the appropriate frame in which the meaningful comparisons can be made between different hybrid waveforms?
  - A frame in which the observer follows the precession of the orbital plane.



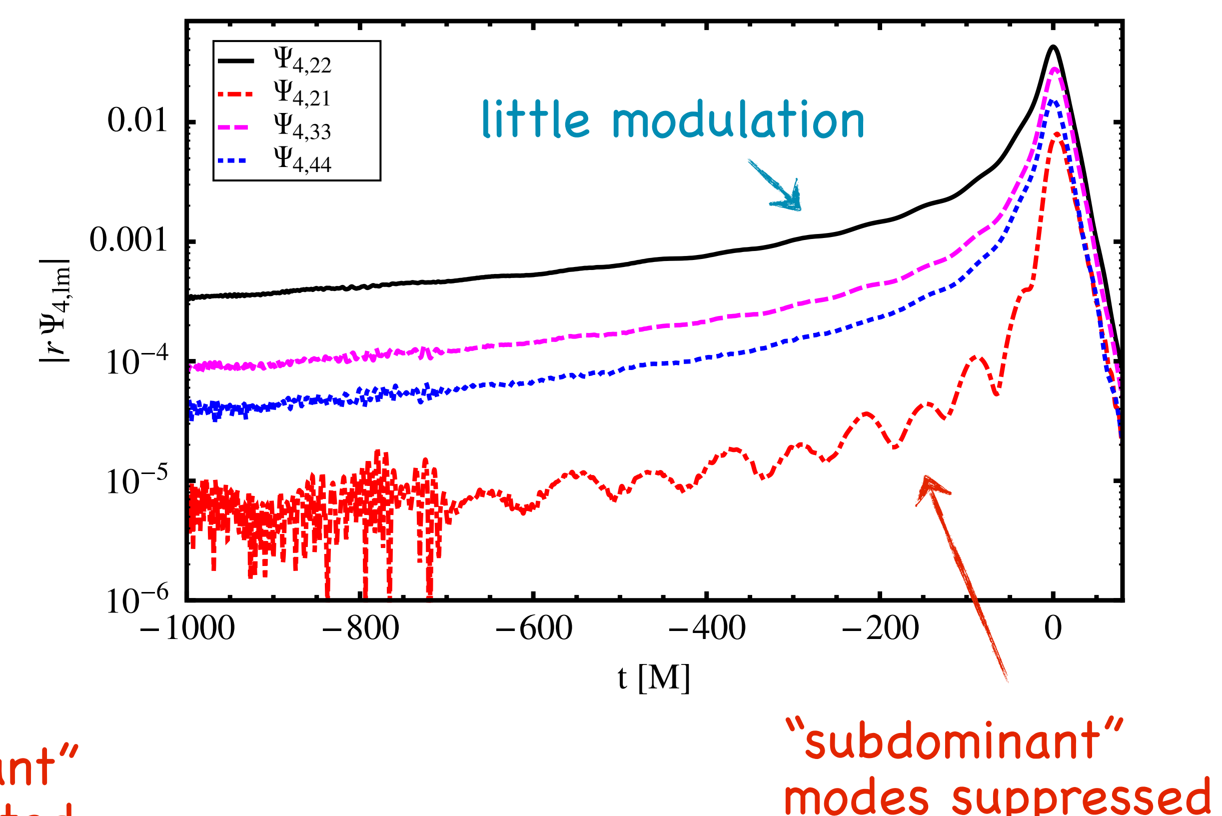
Finding the preferred frame

- **Quadrupole-aligned frame** Determine the time-dependent direction that maximizes the $l = 2$, $m = \pm 2$ modes. [Schmidt et al (2011)]

GW modes in L_{ini} frame



GW modes in QA frame

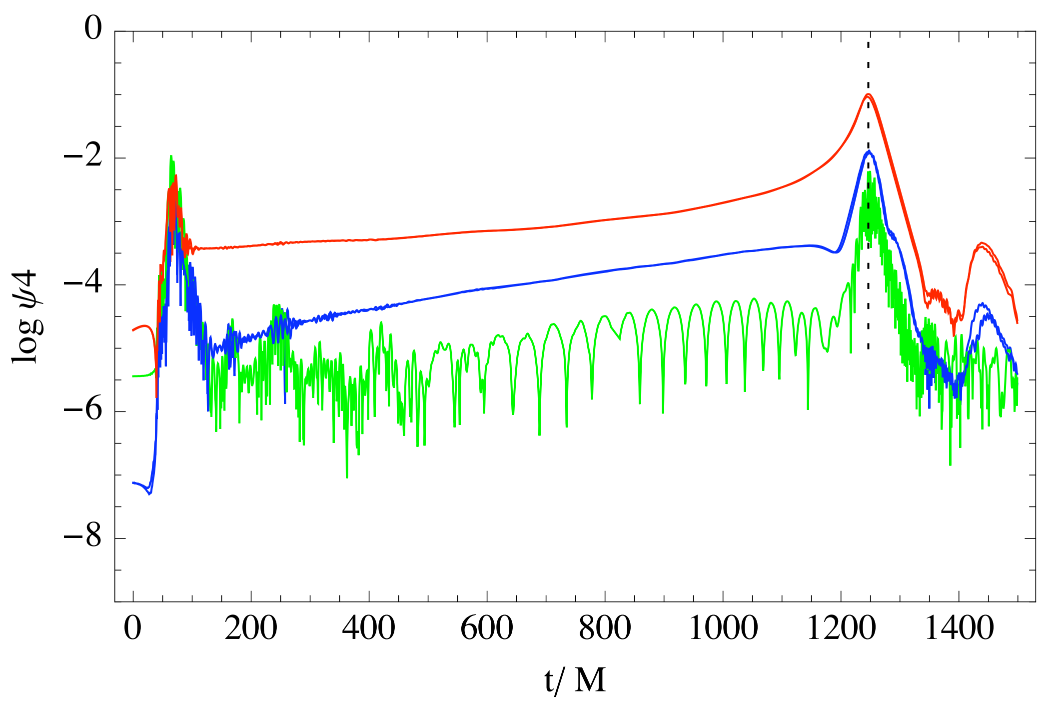


Finding the preferred frame

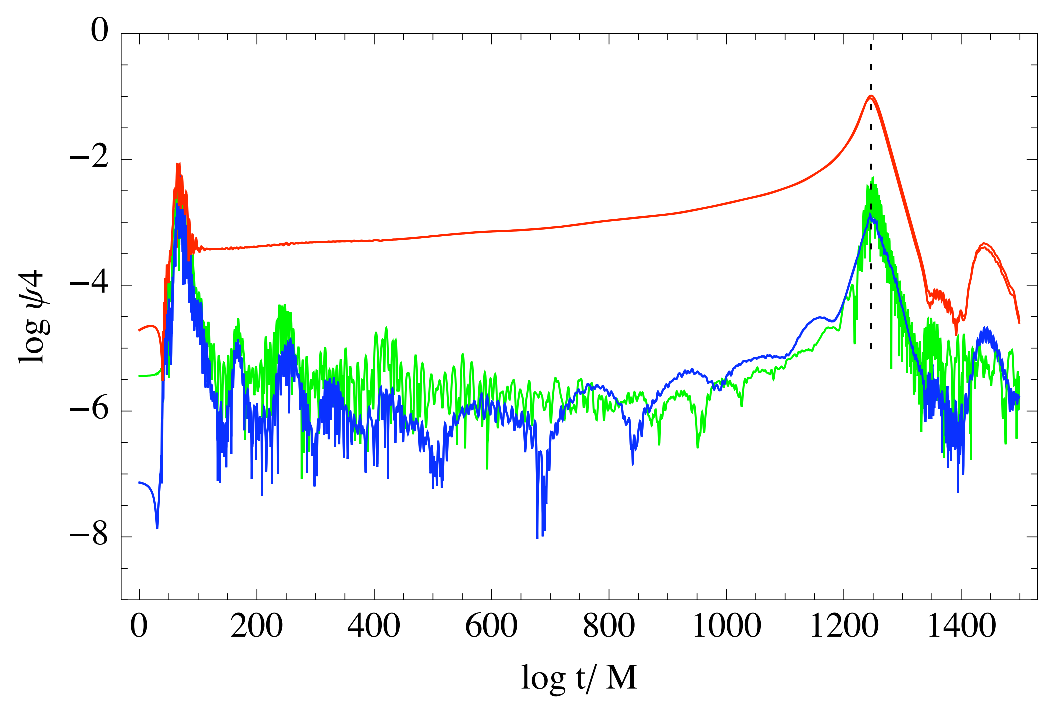
- **Quadrupole-aligned frame** Determine the time-dependent direction that maximizes the $l = 2$, $m = \pm 2$ modes. [Schmidt et al (2011)]
- **Corotating frame** Efficient algebraic method for estimating the instantaneous/average emission direction. [O'Shaughnessy et al (2011)]

[Talk by R. O'Shaughnessy in session Q7]

GW modes in L_{ini} frame



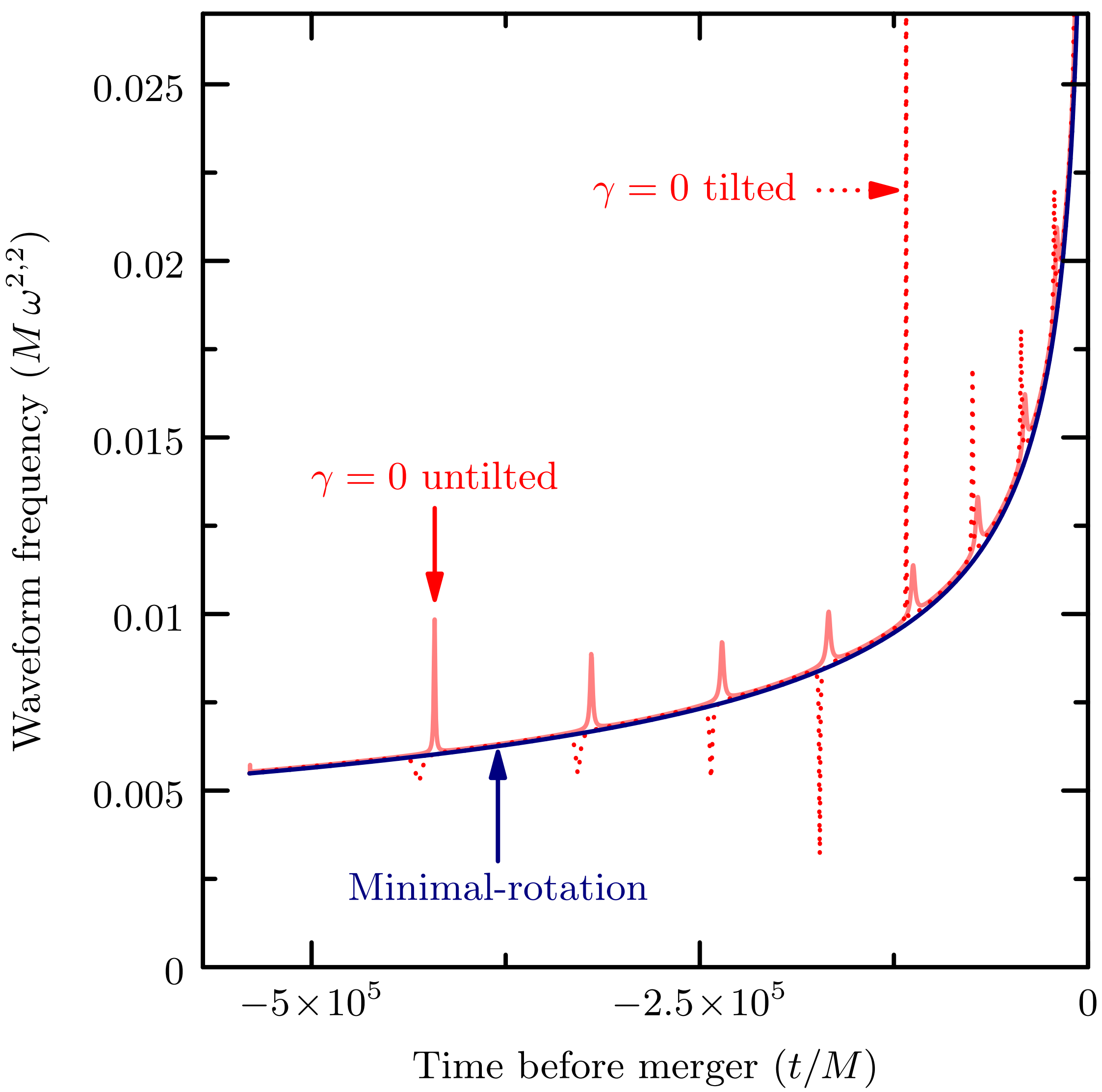
GW modes in Corotating frame



Finding the preferred frame

- **Quadrupole-aligned frame** Determine the time-dependent direction that maximizes the $l = 2, m = \pm 2$ modes. [Schmidt et al (2011)]
- **Corotating frame** Efficient algebraic method for estimating the instantaneous/average emission direction. [O'Shaughnessy et al (2011)]
- **Minimal rotation frame** Make use of the third (time-dependent) Euler angle also to maximize the modes. This "minimal-rotation" frame gets rid of the unphysical "phase jumps". [Boyle et al (2011)]

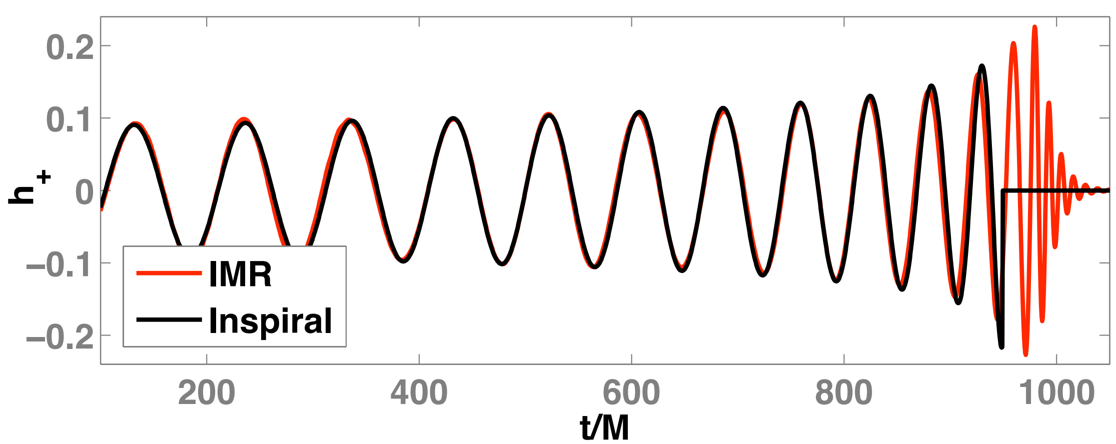
[Boyle et al (2011)]



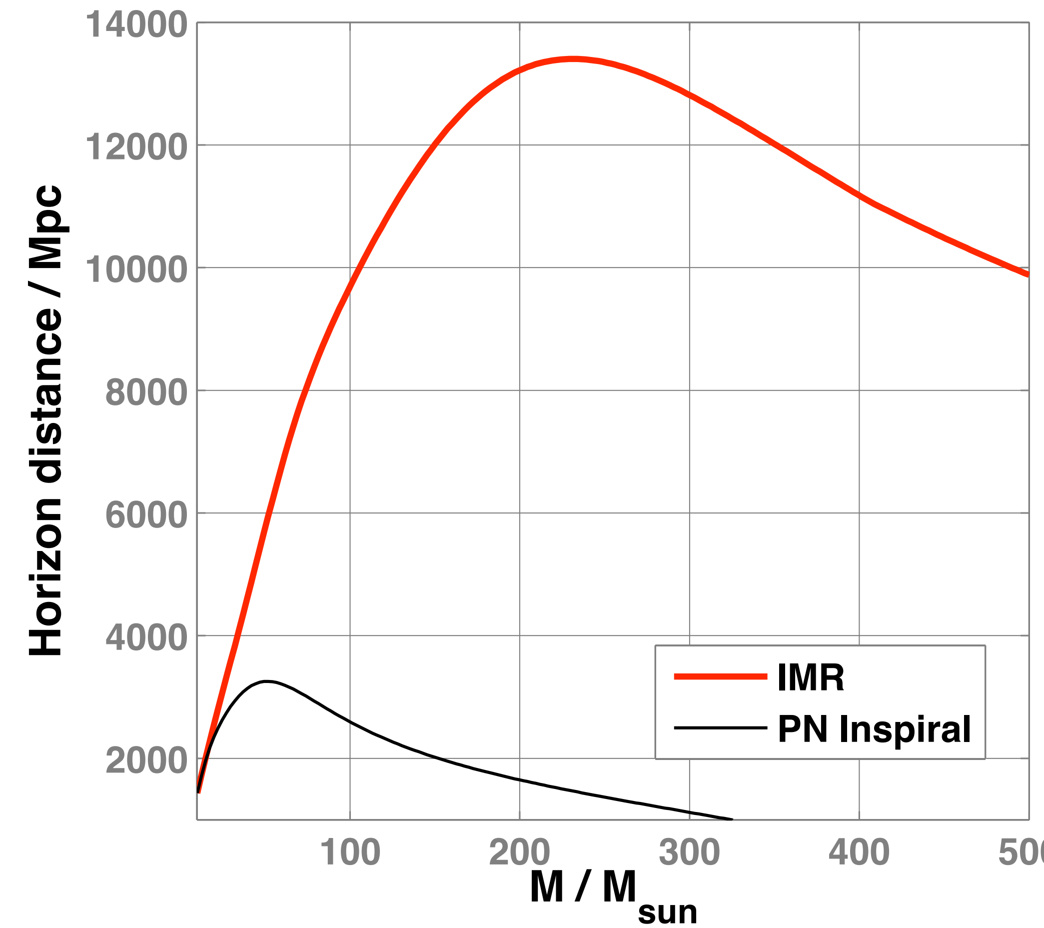
[Talk by R. Owen in session C8]

Implications in GW data analysis

- **Distance reach** Significant improvement in the "distance reach" for "high-mass" binaries.



[Ajith et al (2008)]

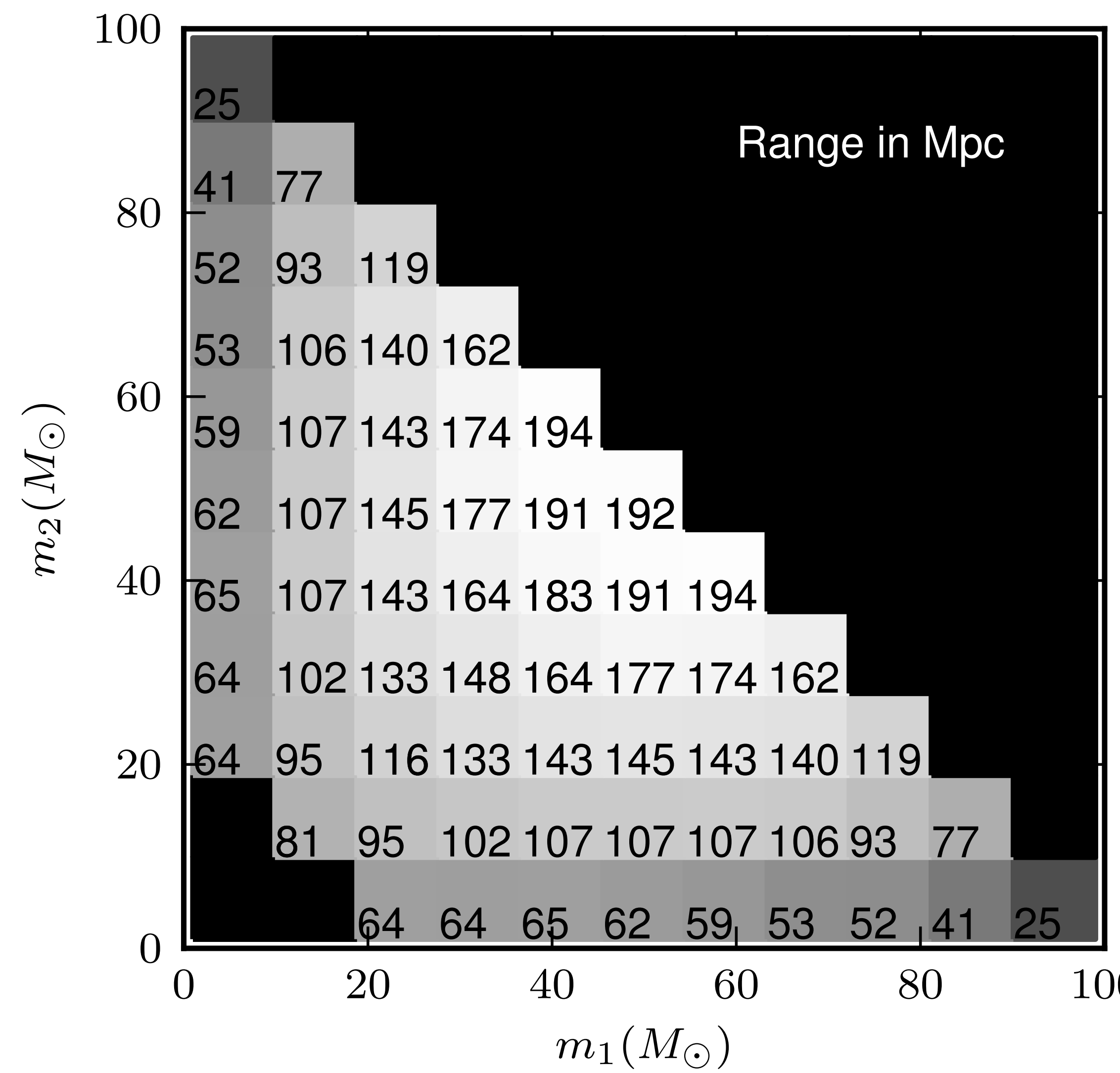


Distance to optimally-oriented equal-mass binaries producing SNR = 8 in Adv LIGO.

Implications in GW data analysis

- **Distance reach** Significant improvement in the "distance reach" for "high-mass" binaries.
  - Improved upper limits. Significant enhancement in the detection probability in advanced detector era.

[Abadie et al arXiv:1102.3781]

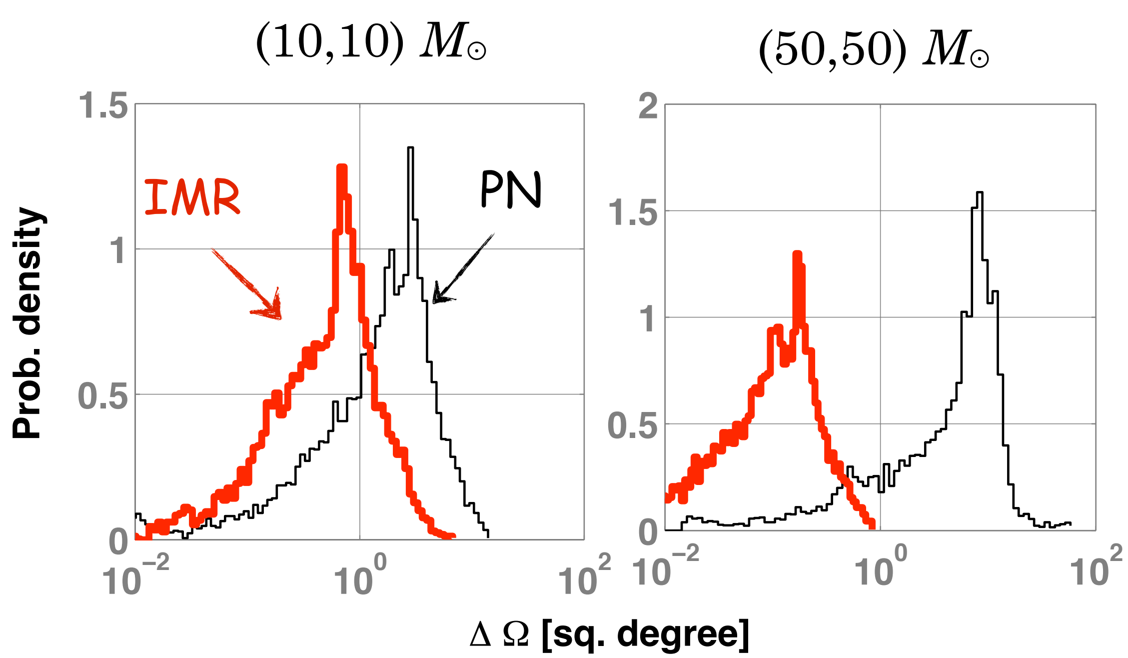


Distance reach (averaged) of the search for "high-mass" CBCs using LIGO S5 data.

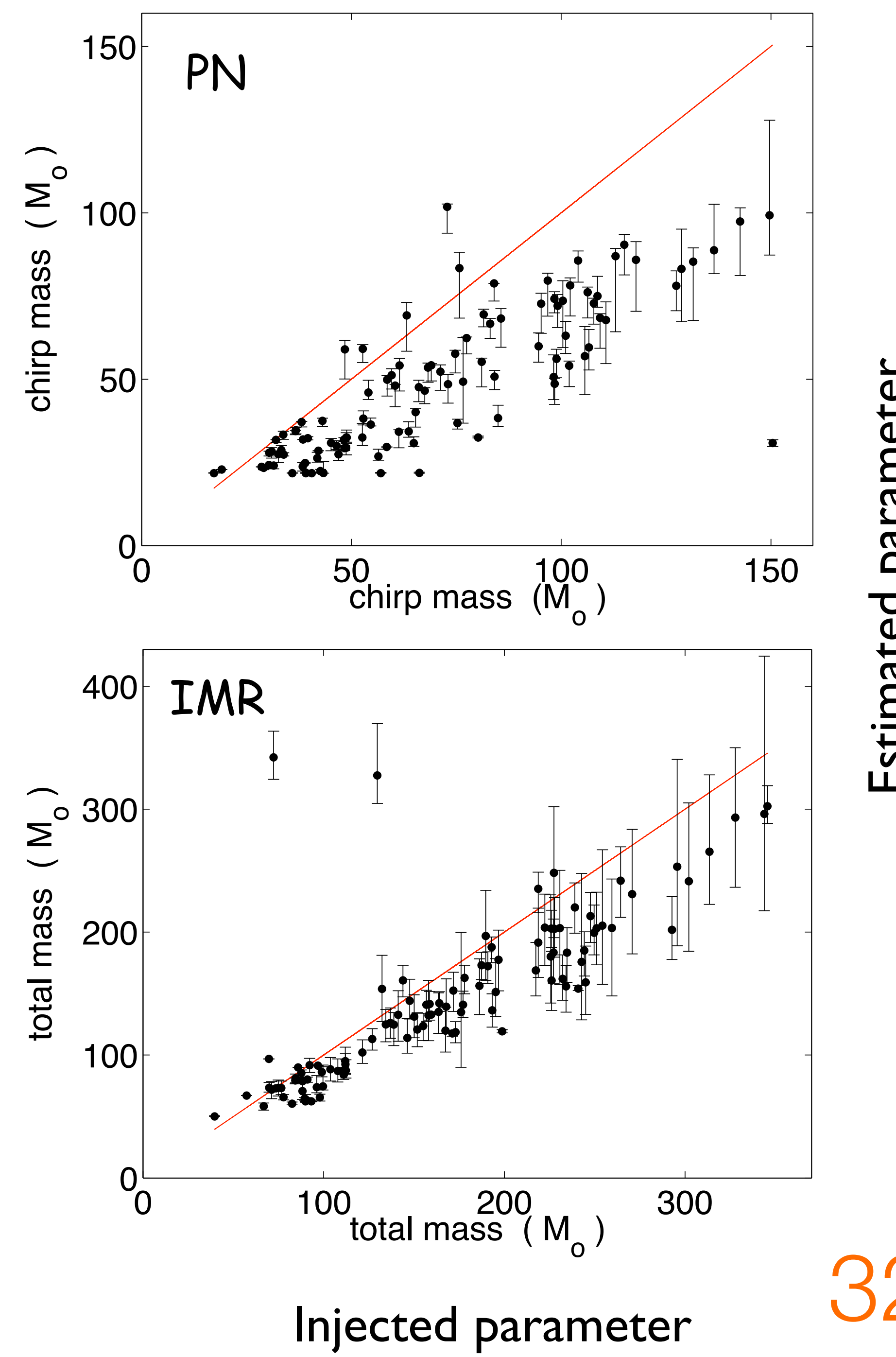
Implications in GW data analysis

- **Parameter estimation** Helps to disentangle the correlation between different parameters.

[Aylott et al (NINJA), 2009]



Expected errors on sky-position (equal-mass binaries at 1Gpc), detected by Adv LIGO-Virgo. [Ajith & Bose (2009)]



Improved ability for astronomy, tests of GR...

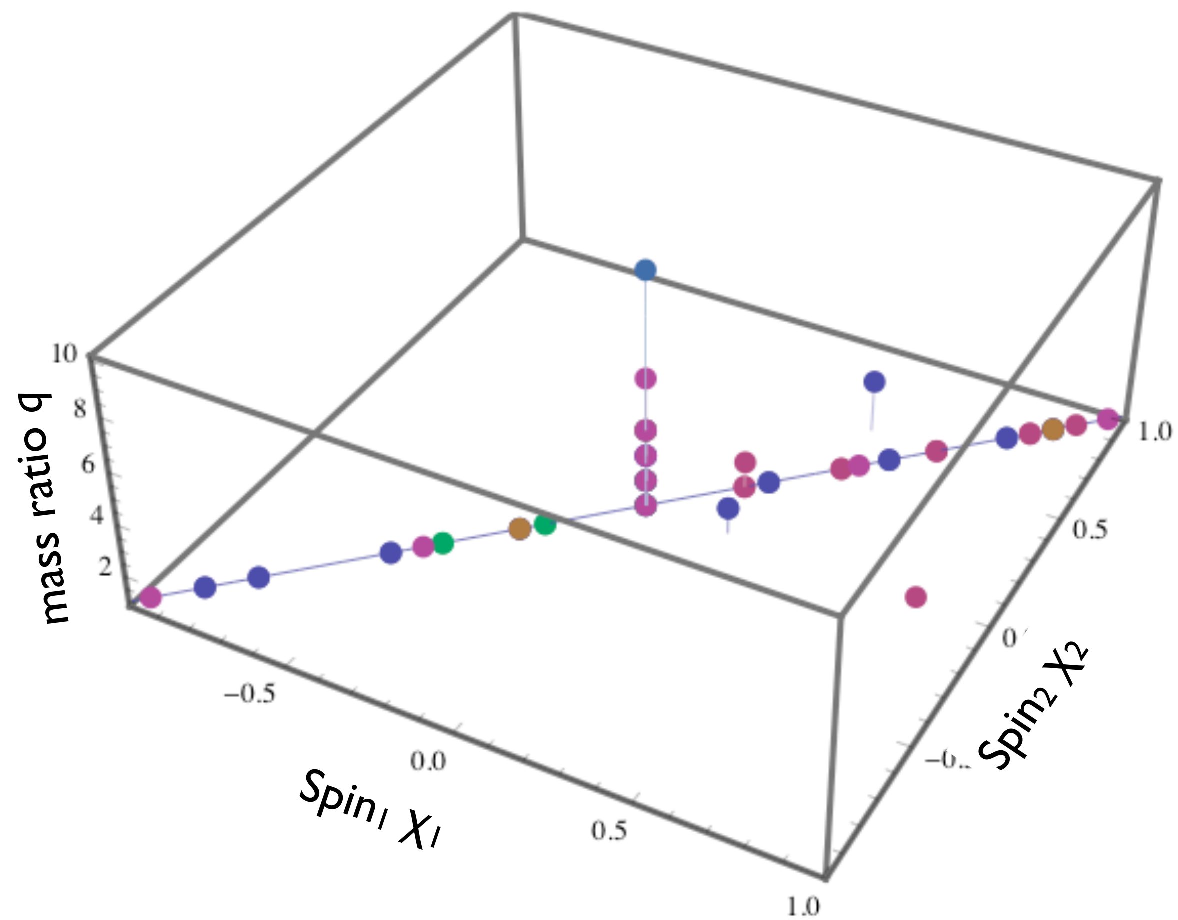
[Talk by R. Lang in this session]

Collaborative efforts between NR, AR and GW communities

- **NINJA** Numerical-Relativity Injection-Analysis
  - Use NR simulations to study the efficiency of GW detection/parameter estimation pipelines in real LIGO/Virgo data.
  - Formal collaboration with LIGO-Virgo. More than 100 members from ~30 institutions.

https://www.ninja-project.org

[Talk by L. Pekowsky in session D8]



Hybrid waveforms in Ninja-2 catalogue.
[arXiv:1201.5319]

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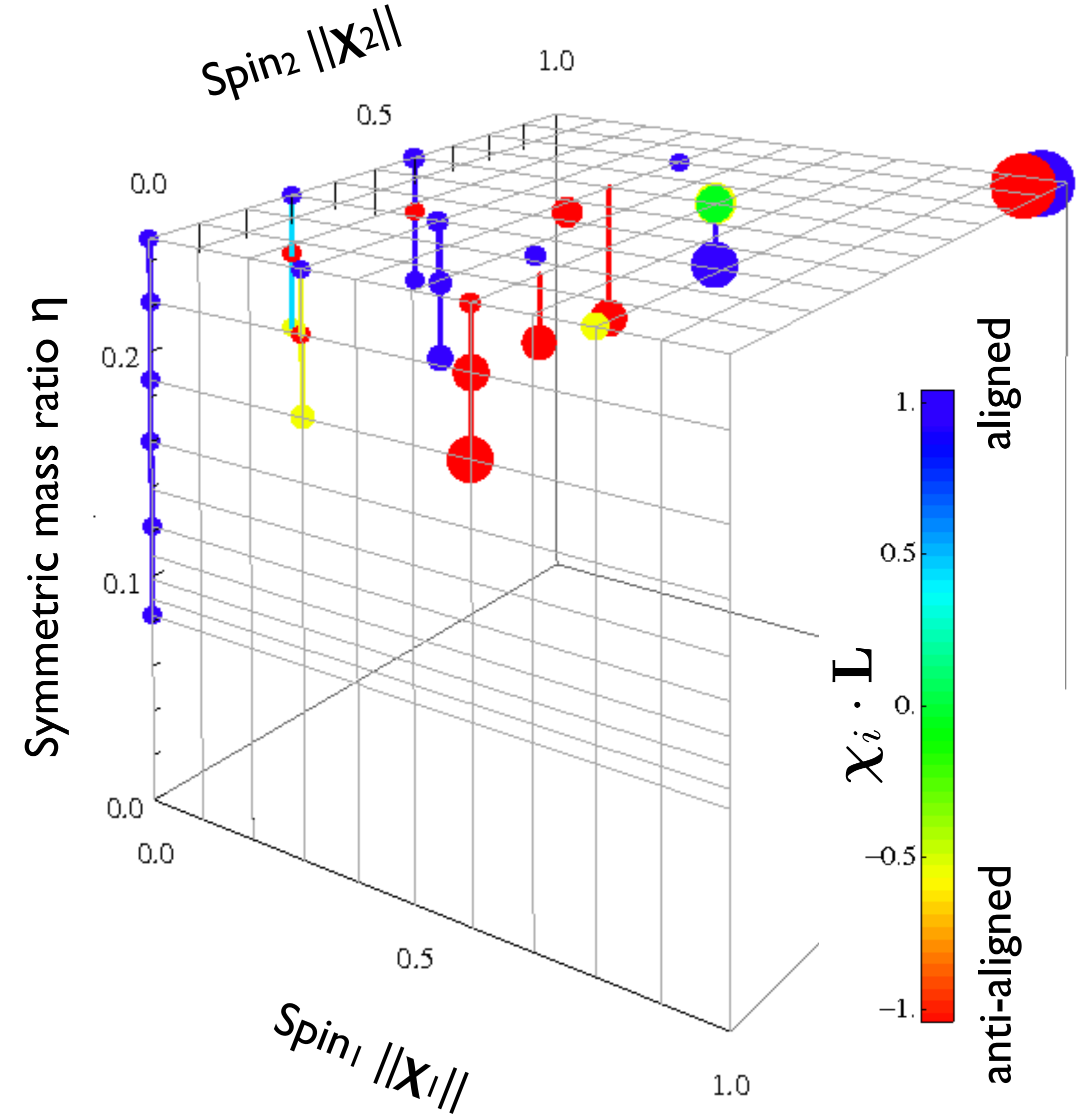
  - Formal collaboration with LIGO-Virgo. More than 100 members from ~30 institutions.
- **NR-AR Collaboration**
  - Produce a catalogue of NR simulations covering a large parameter space; develop analytical template families calibrated to NR; make them available for GW searches.
  - NSF has made available 11M CPU hours in supercomputer Kraken (all used!).
  - Currently building the first NR repository (29+30 waveforms).



Completed/ongoing NR simulations.
[Pic. Y. Pan]

Summary

- Significant progress in the experimental efforts for the first direct detection of GWs. Similar breakthroughs in the modelling of GW sources.
- Numerical-relativity in combination with perturbative calculations enable us to model the coalescence of compact binaries accurately. Important impacts on the expected detection rates and parameter estimation accuracies.
- Analytical inspiral-merger-ringdown templates (calibrated to NR) are already mature for the purpose of GW detection for some fraction of the parameter space (non-spinning & non-precessing BBHs with comparable masses).
- Community-lead efforts under way to expand the parameter space (mass ratios, spins, length, accuracy, tidal effects, ...) of NR simulations, and to use these simulations for the construction of IMR templates + GW data analysis.