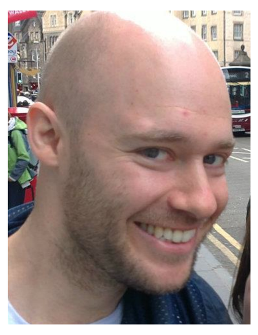

Astrophysical event rates in the detection era

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▸ Introduction

Distinguishing signals from noise is often aided by setting thresholds on the value of the detection statistic. In the fortunate case of having detected *multiple events* above this threshold (e.g. in the advanced detector era) we will be able to perform inference on the physical parameters governing the *population of sources*. We expand upon existing work on astrophysical rates estimates for single detections [1, 2] by developing a *fully Bayesian approach*. This allows us to naturally incorporate multiple events and informs us of the strong influence of the detection statistic threshold. In general, incorrect handling of this threshold can significantly bias inference on global parameters.

▸ Formalism

We will create a simple model of a typical search process by making the following assumptions:

- We have a detection statistic x.
- x is described by a set of models $\{\mathcal{H}\}$ (typically including the noise and signal+noise models).
- There is a set of *global parameters* $\boldsymbol{\gamma}$ governing the population of sources.
- Each source has its own local model-dependent parameter set $\boldsymbol{\theta}_{\mathcal{H}}$.
- During the search N independent measurements of the data were made (i.e. number of templates)
- A *trigger* is produced when the detection statistic exceeds some *threshold value* x_{th}.
- When there is no trigger x is *not* recorded.

From the experiment we learn the information $\{D\}$: the set of all n triggers $\{x\}$ and also the fact that in all $N-n$ *non-detections* $\{D^-\}$ the detection statistic was $x<x_{\text{th}}$. We wish to update our prior knowledge of the global parameters $p(\boldsymbol{\gamma}|I)$ using a Bayesian analysis. The likelihood (conditional on the global parameters) of each trigger is given by

$$p(x_j|\boldsymbol{\gamma},I)=\sum_{\mathcal{H}}\int p(x_j,\boldsymbol{\theta}_{\mathcal{H}}|\boldsymbol{\gamma}_{\mathcal{H}},\mathcal{H},I)P(\mathcal{H}|\boldsymbol{\gamma},I)d\boldsymbol{\theta}_{\mathcal{H}}$$

where we apply a sum over the different models. The corresponding, and usually explicitly ignored, component to our final result is the likelihood associated with each non-detection given by

$$P(D^-|\boldsymbol{\gamma},I)=\sum_{\mathcal{H}}\int P(D^-,\boldsymbol{\theta}_{\mathcal{H}}|\boldsymbol{\gamma},\mathcal{H},I)p(\mathcal{H}|\boldsymbol{\gamma},I)d\boldsymbol{\theta}_{\mathcal{H}}.$$

Under the assumption that each measurement is statistically independent then the posterior probability on the global parameters is given by

$$p(\boldsymbol{\gamma}|\{D\},I)=\frac{p(\boldsymbol{\gamma}|I)}{p(\{D\}|I)}p(D^-|\boldsymbol{\gamma},I)^{N-n}\prod_{j=1}^{n}p(D_j^+|\boldsymbol{\gamma},I)$$

▸ An astrophysical rates example

In practice the calculation of the component probabilities in the above equation is complicated by *un-modeled (non-Gaussian) noise* and the statistical dependence of *correlated templates*. In the following example we use *timeslides* and populations of *signal injections* to *empirically* determine these quantities for all models. The results of this analysis are shown in Fig. 1 where a templated search over signal arrival time was conducted for a simple Sine-Gaussian signal and compared to a theoretical analysis. We find that our analysis produces unbiased results on the rate R, *even when the threshold is low enough for false alarms to pass as triggers*, and that the lowering of the threshold significantly increases the precision of estimation.

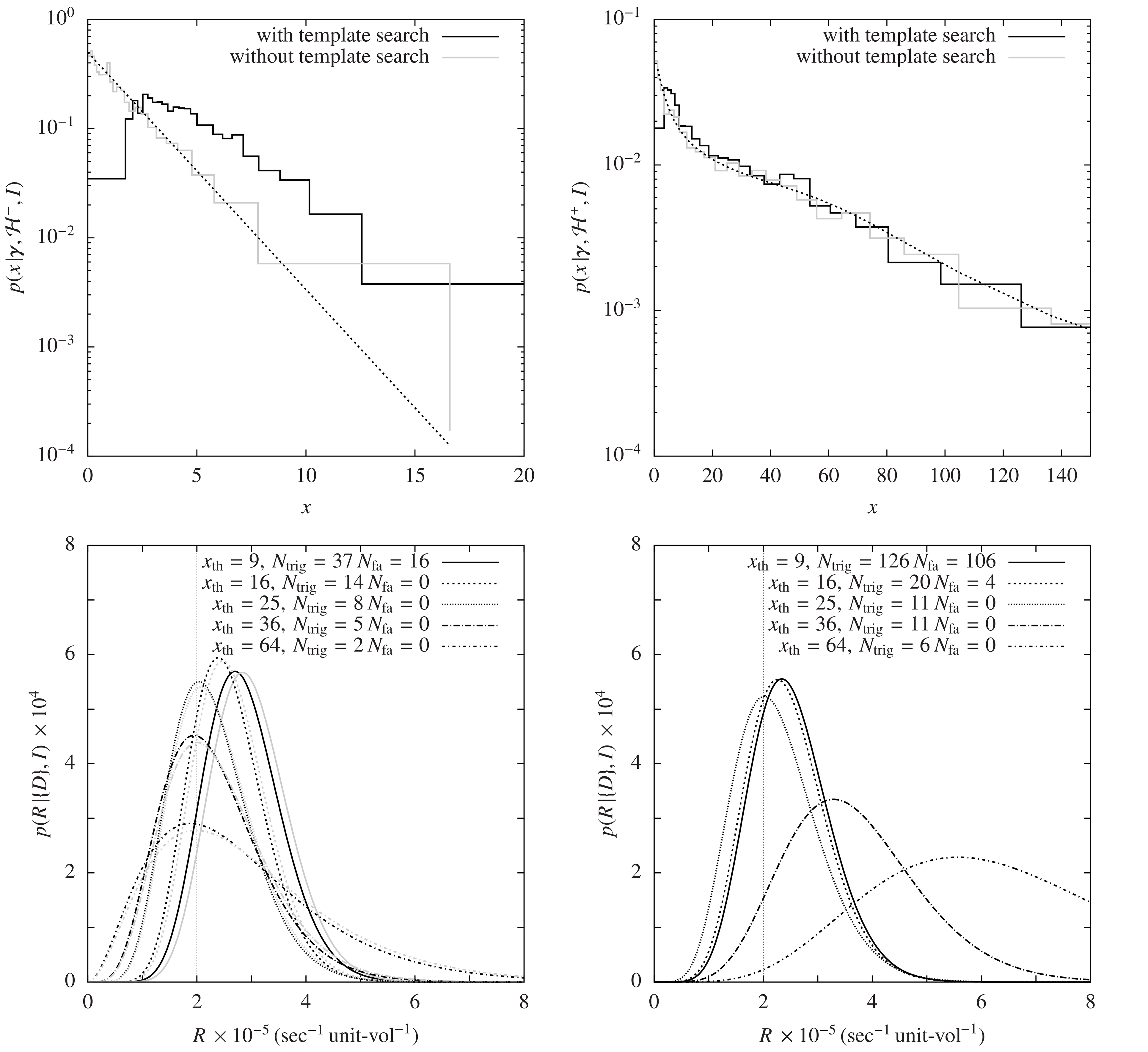

FIG. 1: (Upper plots) Empirically estimated binned probability distributions of the detection statistic x derived from 2×10^4 timeslides (left) and injections into timeslides (right). Grey curves and dotted curves correspond to cases where no search over templates was performed and the corresponding expected theoretical distribution respectively. (Lower plots) Posterior pdfs on the rate of events per unit time per unit volume for a variety of x_{th} values showing the improvement in precision at lower thresholds. Curves corresponding to non-numerically maximised results (left) have corresponding theoretically computed results (grey). The vertical grey dotted line indicates the true simulated rate.

▸ Conclusion

We have shown that there exists a *general Bayesian* formalism that can perform inference on astrophysical rates with natural application to *multiple events*. We used it to show an *improvement in precision of rate estimation* when the threshold is lowered, using information from *both triggers and non-triggers* without biasing the estimate.

▸ References

[1] Rahul Biswas, Patrick R. Brady, Jolien D. E. Creighton, and Stephen Fairhurst. The loudest event statistic: general formulation, properties and applications. *Classical and Quantum Gravity*, 26(1):5009, September 2009.

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