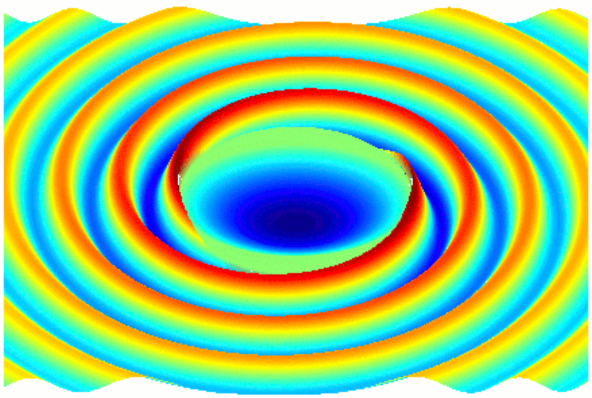
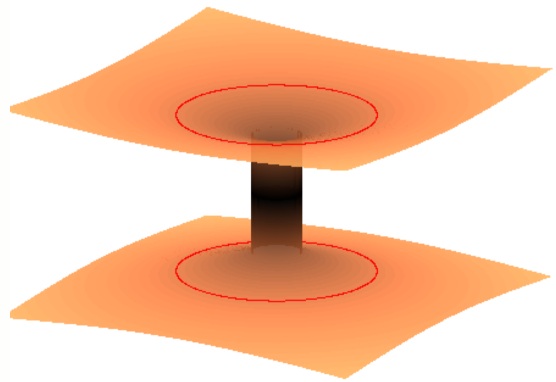
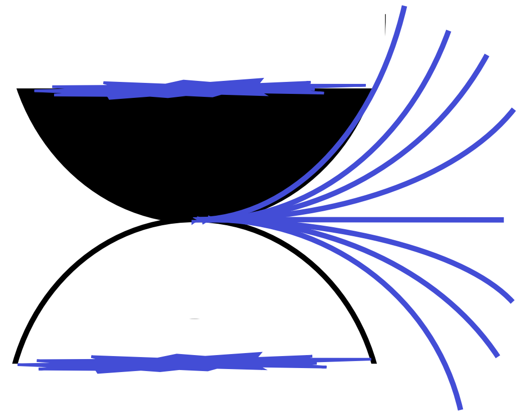
Relativity: The Fun Stuff

Time Dilation, E=mc^2, Black Holes, GWs & More

The Laser Interferometer Gravitational-wave Observatory: a Caltech/MIT collaboration supported by the National Science Foundation

Gregory Mendell

LIGO Hanford Observatory

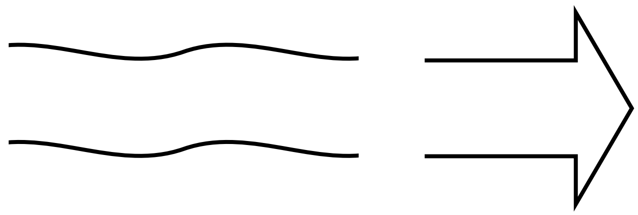
LIGO-G1200075

Einstein Wondered

- Einstein is famous for his thought experiments.
- In 1895, around age 16, he wondered, can we catch light?
- If yes, your image in a mirror would disappear. You would know your speed independent of any outside frame of reference. This would violate Galilean relativity.
- Einstein decides we cannot catch light; nothing can go faster than light.

Einstein Wondered:

Photo: *Albert Einstein at the first Solvay Conference, 1911; Public Domain*



Can we catch light?

Mirror

Special Relativity and Time Dilation

- Einstein published the postulates of special relativity in 1905. Nothing can go faster than light and the speed of light is the same in all reference frames.
- On the next slide is a thought experiment that illustrates time dilation. Imagine Einstein and Bohr, with each holding a stopwatch that they start simultaneously. They then slowly walk to the positions shown on the next slide. They have also arranged for two motorcycles to travel left to right at speed v. The rider at the top also has a stopwatch, and is to start it when crossing the "starting line". Simultaneously with this, the rider at the bottom flashes a light towards the top rider, and Bohr stops his stopwatch. Einstein is positioned to see the flash reach the top rider, and stops his stopwatch when the flash arrives, simultaneous with the top rider.
- The motorcycle riders see the light travel just the vertical straight line distance between them, at speed c for time ΔT, giving the distance of the left side of the triangle $c\Delta T$. Einstein and Bohr see the light travel along the hypotenuse of the triangle (also at speed c!) and find the time difference between when they stopped their stopwatches is Δt, for a distance of $c\Delta t$. During this same time, the motorcycles travelled a distance $v\Delta t$.
- Using the Pythagorean Theorem, we see the time ΔT is less than Δt by the relativistic time dilation factor .
- Note that the motorcycle drivers saw Bohr and Einstein approach them at speed v, but pass them in time ΔT. Thus, they saw the length between Bohr and Einstein contracted by this same factor.

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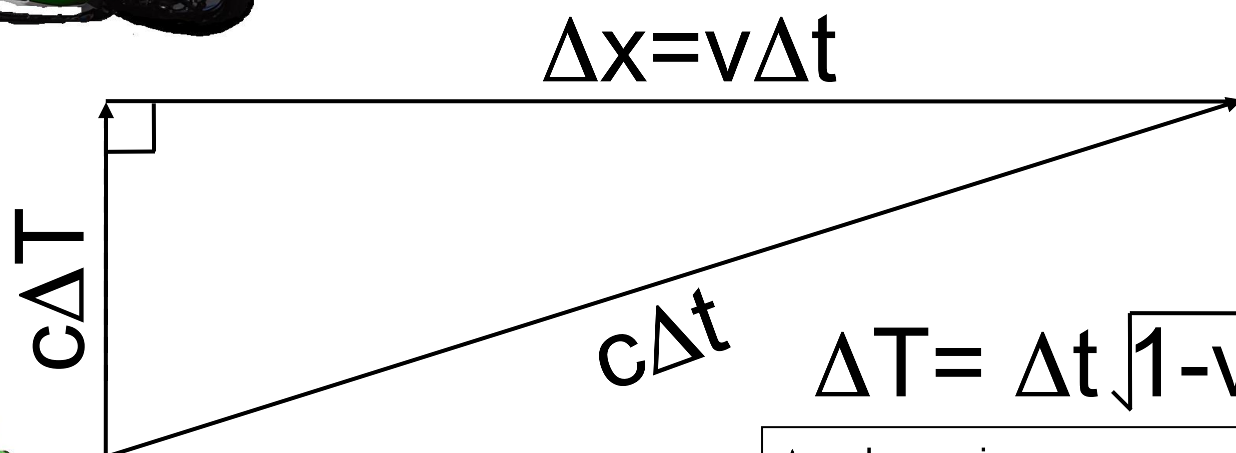
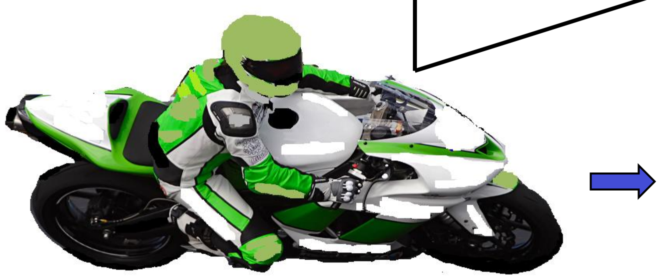
Niels Bohr

Time Dilation

Albert Einstein



$$\Delta T = \Delta t \sqrt{1 - v^2/c^2}$$



Δ = change in

T = time measured by motorcycle riders

t = time measured by observer at “rest”

v = speed of motercycles

c = speed of light

Start

Warning: thought experiment only; do not try this at home.

Motorcycle: http://en.wikipedia.org/wiki/Motorcycle_racing

The Pythagorean Theorem Of Spacetime

$$c^2\Delta T^2 + v^2\Delta t^2 = c^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - v^2\Delta t^2$$

$$c^2\Delta T^2 = c^2\Delta t^2 - \Delta x^2$$

c = 1 light-year/year

$$\boxed{\Delta T^2 = \Delta t^2 - \Delta x^2}$$

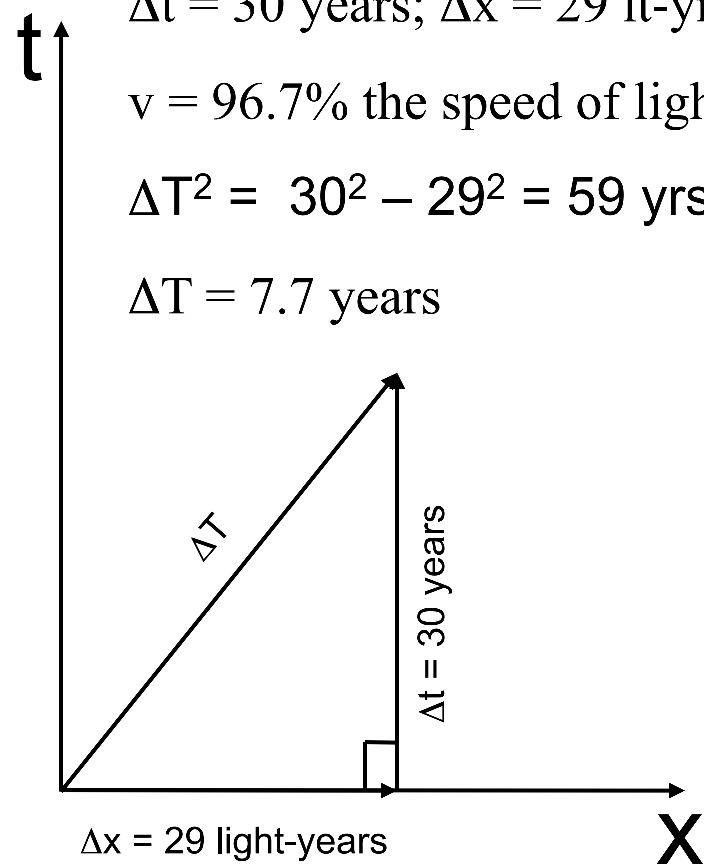
Pythagorean Thm. of Spacetime

Δt = 30 years; Δx = 29 lt-yrs.

v = 96.7% the speed of light

$\Delta T^2 = 30^2 - 29^2 = 59\ \text{yrs}^2$

ΔT = 7.7 years



Spacetime

The Speed of Light

c = 186,000 miles/s = 670,000,000 miles/hr

	v	Δt	ΔT
Car	60 mph	1 day	1 day - .35 nanoseconds
Plane	600 mph	1 day	1 day – 35 nanoseconds
Shuttle	17,000 mph	1 day	1 day – 28 microseconds
Voyager	38,000 mph	1 day	1 day – 140 microseconds
Andromeda	300,000 mph	1 day	1 day – 8.7 milliseconds
Electrons	99% c	1 day	3.4 hours

Photo: Stanford Linear Accelerator Center (SLAC); Public Domain

The faster you go the slower time goes!

Nothing can go faster than light!

The Twin Paradox

- If all motion is relative, then who sees less time go by? This is resolved by first noting that the Pythagorean Theorem of spacetime, given previously, works in inertial frames. Thus, chose an inertial frame and use it to find the time on the clocks, and with care one will get the correct answer.
- On the next slide is an illustration of the Twin Paradox. This is covered in all text books on relativity, but here are a few notes. Since the Earth's speed is much less than c, and its escape velocity is much less than c, we can treat the Earth as an inertial frame at rest. Thus Bob ages 60 years, while Betty ages only 15.4 years. If you read up on this you will find from Betty's point of view that she sees most of Bob's aging occur while she turns around.

Spacetime Diagram

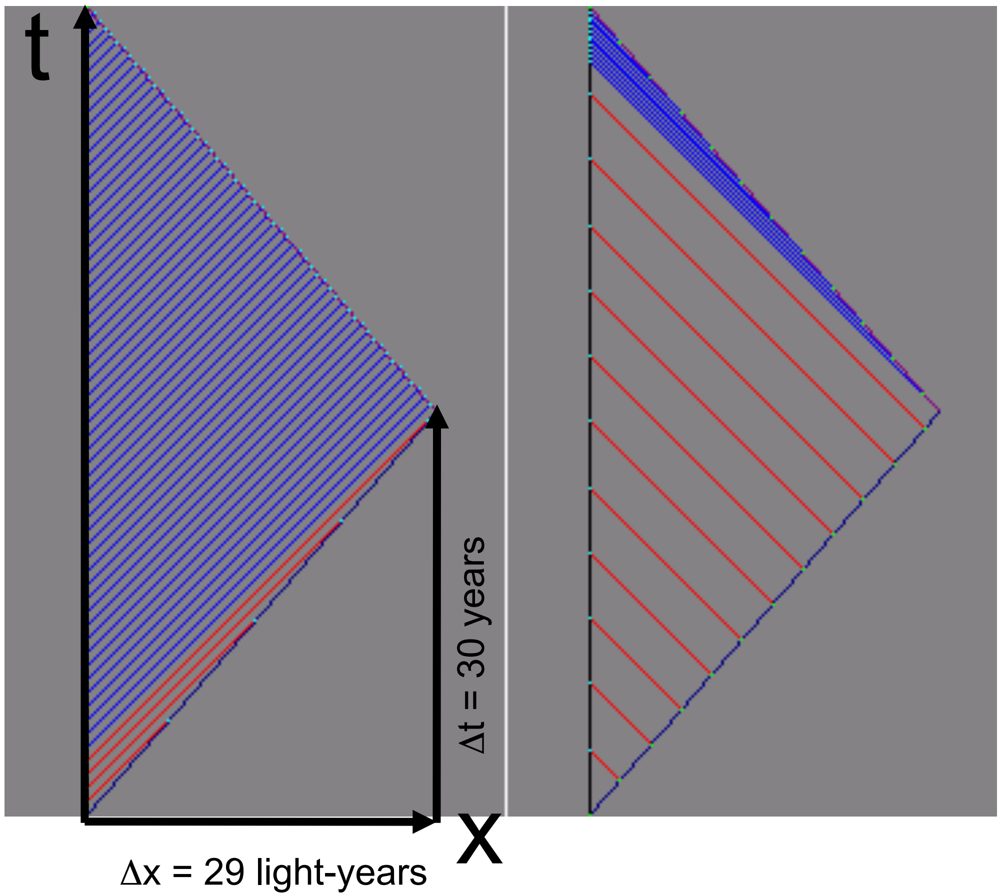
The Twin Paradox

•Imagine twins, Betty and Bob, separated 1 year after birth.

Baby Betty & Bob: ☺ ☺

•Betty takes a rocket travelling at 96.67% the speed of light and travels 29 lt-yrs from Earth and back.

•When Betty returns she is sweet 16, and Bob is 61 years old!!!



$$\Delta T = 30 \text{ yrs}\sqrt{1-(.9667)^2} = 7.7 \text{ yrs}$$

Figure: http://en.wikipedia.org/wiki/Twin_paradox

$E=mc^2$

- The next 3 slides show 3 ways to derive $E=mc^2$.
- In the first of these, the Pythagorean Theorem of spacetime is multiplied by mc^2 and then the ratio $\Delta t/\Delta T$ from the time dilation formula is used. After some Taylor expansions, terms that look related to the Newtonian energy and momentum appear, and by conjecture one arrives at the relativistic energy-momentum relationship, and finally $E=mc^2$.
- The next derivation is based on conservation of energy and is exactly that given by Einstein in 1905, except for a slight simplification of the test case and a few changes in the notation to put the final result in the familiar form.
- The third derivation uses conservation of momentum, rather than conservation of energy. Note that no relativistic formula are used at all. It seems that $E=mc^2$ is a consequence of light having momentum $P = E/c$, which is a consequence of Maxwell's equations and is known to exist e.g., via measurements radiation pressure. That the momentum depends on the frequency of the light means it transforms via the same blue/red shift factors, and only the nonrelativistic form of these is needed here. Thus, I think this means $E=mc^2$ would remain valid, even if it had turned out Special Relativity was not.

$E=mc^2$

$c^2\Delta t^2 = c^2\Delta T^2 + v^2\Delta t^2$

$m^2c^4\Delta t^2 = m^2c^4\Delta T^2 + m^2c^2v^2\Delta t^2$

$m^2c^4\Delta t^2/\Delta T^2 = m^2c^4 + m^2c^2v^2\Delta t^2/\Delta T^2$

$[mc^2/(1-v^2/c^2)^{1/2}]^2 = [mc^2]^2 + [mv/(1-v^2/c^2)^{1/2}]^2c^2$

Newtonian Momentum

$[mc^2 + 1/2mv^2]^2 = E^2 = [mc^2]^2 + p^2c^2$

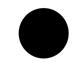
Approximate to order $v^{2/}c^2$ == Newtonian Kinetic Energy

For $v = 0$: $E = mc^2$

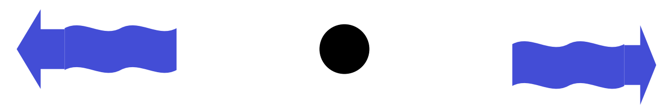
$E=mc^2$: What Einstein Said

"Does The Inertia Of A Body Depend Upon Its Energy-Content?" by A. Einstein, in "The Principle Of Relativity" translated by W. Perrett & G. B. Jeffery (Dover: 1952) from A. Einstein, Annalen der Physik, 17, 1905.

Consider a particle with energy U.

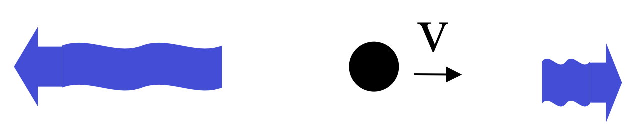


After it emits pulses of light with energy 0.5 E in opposite directions, the particles energy is H. Note that v does not change.



In a moving frame its energy is U + K1.

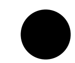
After it emits pulses of light with energy 0.5 γ E (1 +/- v/c) in opposite directions the particle's energy is H + K2. Note the relativistic blue/red shift factor is used.



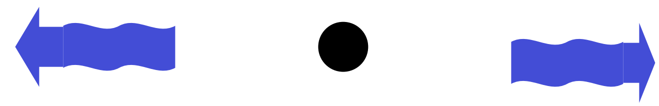
By conservation of energy U = H + E and U + K1 = H + K2 + γ E. Thus, $(\gamma-1)E = K1 - K2 = \Delta K$ so $\frac{1}{2}(v^2/c^2)E = \frac{1}{2}(\Delta m)v^2$ to lowest order. The particle lost mass Δm. For Δm max. equal to m: $E = mc^2$.

$E=mc^2$: Using Momentum

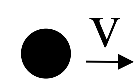
Consider a particle at rest.



It emits pulses of light with momentum 0.5 E/c in opposite directions. Note that v does not change.



In a moving frame the particle's moment is P1.



After it emits pulses of light with momentum 0.5 E/c(1 +/- v/c) in opposite directions the particle's momentum is P2. Note the nonrelativistic blue/red shift factor is used.

Thus by conservation of momentum P1 = P2 + 0.5 E/c(1 + v/c) - 0.5 E/c(1 - v/c). Thus, $(v/c^2)E = P1 - P2 = \Delta P$, so $(v/c^2)E = (\Delta m)v$. The particle lost mass Δm. For Δm maximum equal to m: $E = mc^2$.

Einstein's Happiest Thought: Gravity Disappears When You Free Fall

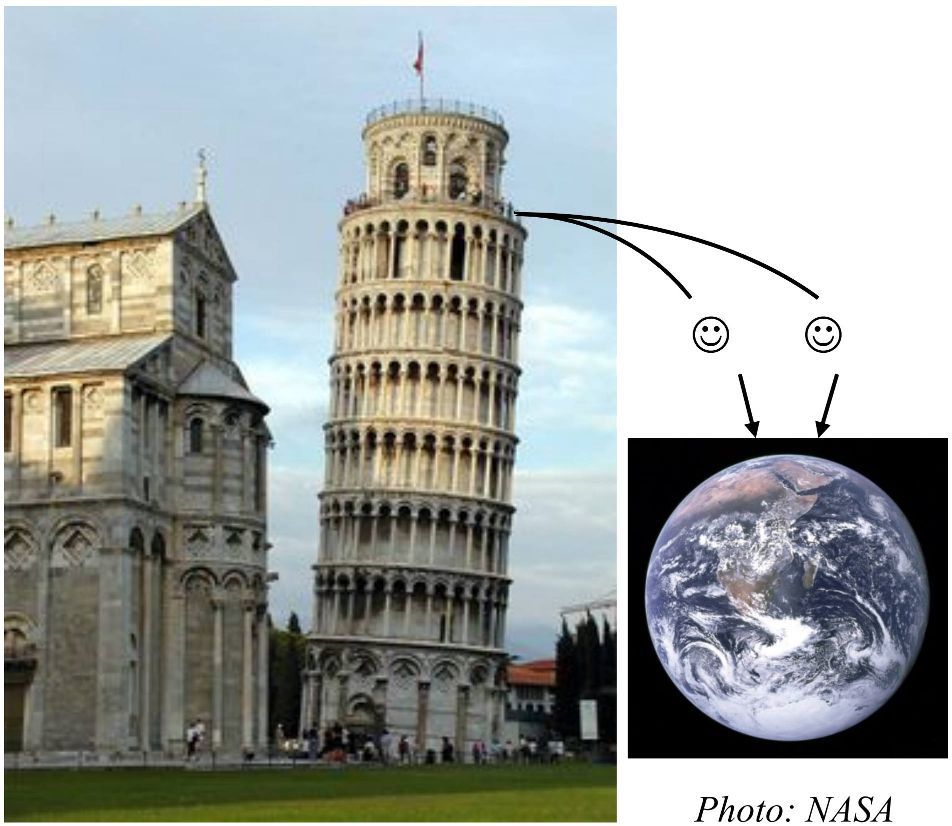


Photo: NASA

http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa

Einstein had this thought in 1907. This lead to the idea that gravity is the curvature of spacetime. Here I paraphrase a thought experiment I first heard from Kip Thorne. Suppose two friends jump parallel to each other off the Leaning Tower of Pisa. For the friends, gravity has disappeared, and they believe they are in empty space. Strangely though, they find their parallel paths converging at the center of the Earth. That can happen in empty space only if that space is not flat but curved. Einstein thought about the geometry of rotating objects, and other things, and after 8 more years produced General Relativity, which is a theory of gravity and spacetime. He had help from a mathematician, Marcel Grossmann.

Warning: thought experiment only; do not try this at home.

Newtonian Gravity

$$\vec{g} = -G\int \frac{\rho}{r^2}\hat{e}_r d^3x$$ Gravitational Field

$$\Phi = -G\int \frac{\rho}{r} d^3x$$ Gravitational Potential

$$\vec{\nabla} \times \vec{g} = 0$$

Conservative Force

$$\vec{g} = -\vec{\nabla}\Phi$$

$$\oint \vec{g} \cdot \hat{n} d^2x = -4\pi G M_{enclosed}$$ Gauss' s Law

$$\int \vec{\nabla} \cdot \vec{g} d^3x = -4\pi G \int \rho d^3x$$ Applying the Divergence Thm.

$$\nabla^2 \Phi = 4\pi G \rho$$ Poisson Equation for Φ

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Pythagorean Theorem and Einstein's General Theory of Relativity

$\Delta \rightarrow d$ = infinitesimal change

$dT^2 = g_{tt}dt^2 + g_{xx}dx^2$

$dT^2 = g_{\mu\nu}dx^{\mu}dx^{\nu}$

In GR the components of a 4x4 symmetric matrix called the metric tensor define the curvature of spacetime.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}; R = g^{\mu\nu} R_{\mu\nu}$$

$$R^{\alpha}{}_{\mu\beta\nu} = \partial_{\beta}\Gamma^{\alpha}{}_{\mu\nu} - \partial_{\nu}\Gamma^{\alpha}{}_{\mu\beta} + \Gamma^{\alpha}{}_{\beta\gamma}\Gamma^{\gamma}{}_{\mu\nu} - \Gamma^{\alpha}{}_{\gamma\nu}\Gamma^{\gamma}{}_{\mu\beta}$$

$$\Gamma^{\alpha}{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} (\partial_{\nu} g_{\mu\beta} + \partial_{\mu} g_{\beta\nu} - \partial_{\beta} g_{\mu\nu})$$

Einstein's Field Equations

$$\frac{dx^{\alpha}}{dT} = U^{\alpha}; \quad U_{\alpha} = g_{\alpha\beta} U^{\beta}$$

U = 4-Vel.; T = Proper Time

$$\frac{dU_{\alpha}}{dT} = \frac{1}{2} \partial_{\alpha} g_{\beta\gamma} U^{\beta} U^{\gamma}$$

Geodesic Equation

Black Holes History

- Dark Stars, John Michell 1784 (Also Pierre-Simon Laplace, 1796)
- General Relativity, Einstein, 1915
- Spherically Symmetric Solution, Karl Schwarzschild, 1916
- Einstein-Rosen Bridge, 1935

Schwarzschild Black Hole

- Einstein's Field Equations are complicated. However, once you have a solution, what you have is the arc length formula for spacetime. You can use it to find the proper distance between two points by integrating ds^2, or take $dT^2 = -1\ ds^2$ and integrate to get the proper time on a clock that travels between two points in spacetime.
- After General Relativity came out, Schwarzschild looked for a static, spherical symmetric solution and found it, as given on the next slide. It looks "relatively" simple, but it took decades to understand that it contains a black hole, a worm hole attaching our universe to perhaps another universe, and a white hole.
- Note this is a mathematical solution. Black holes formed by stellar collapse to not contain worm holes or white holes.

Schwarzschild Black Hole

$$c^2 dT^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

$$c^2 dT^2 = \left(1 - \frac{v_{esc}^2}{c^2}\right) c^2 dt^2 - \frac{1}{\left(1 - \frac{v_{esc}^2}{c^2}\right)} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2$$

Karl Schwarzschild

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

- Escape Velocity

$$R_s = \frac{2GM}{c^2}$$

- Schwarzschild Radius

Object	Schwarzschild Radius
You	1 thousand, million, million, millionth the thickness of a human hair
Earth	1 cm (size of marble)
Sun	3 km (2 miles)
Galaxy	~ trillion miles

Gravitational Time Dilation

$$\Delta T = \sqrt{1 - \frac{2GM}{rc^2}}\,\Delta t$$

Gravity slows time down!

Photo:http://en.wikipedia.org/wiki/Leaning_Tower_of_Pisa

Clock_Photos:http://en.wikipedia.org/viki/Cuckoo_clock

Gravity Slows Time

- Due to the orbital speed, clocks on the satellite lose 7 microseconds per day
- Due to the weaker gravitational field, clocks on the satellite gain 45 microseconds per day
- Satellite clocks gain a net of 38 microsecond per day
- Distance error = c*38 microseconds; c = 186,000 miles per second.
- Without calibrating clocks to account for Relativity, GPS distance would be off by 7 miles after one day!

See Scientific American, Sept. 1994

Illustration: NASA

Clock_Photos:http://en.wikipedia.org/wiki/Cuckoo_clock

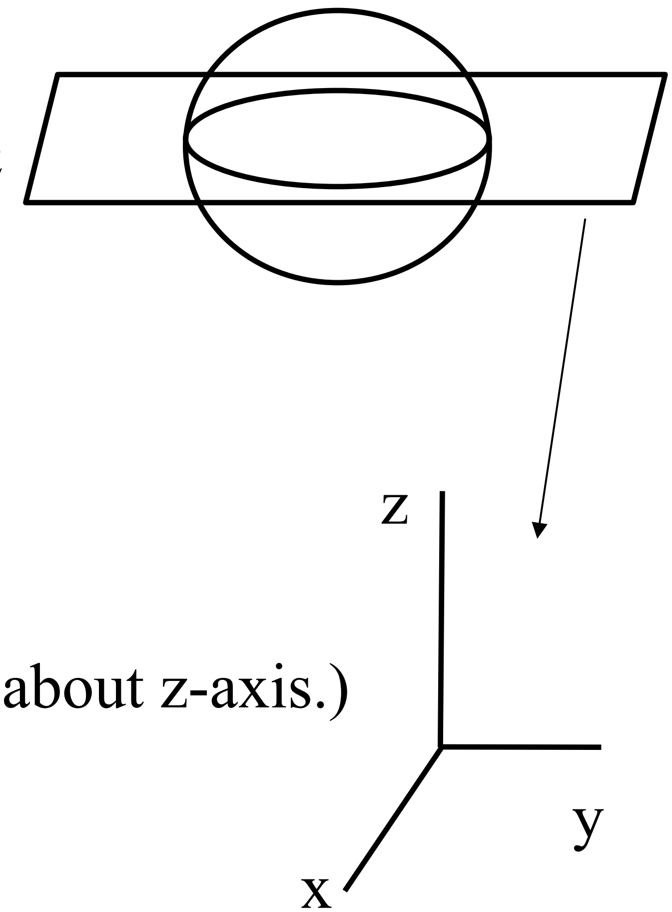
Embedding Diagrams

- If we slice our spacetime with a 2D plane, the geometry on that plane corresponds to some curved surface. If we drop that slice into 3D flat space we can try to visualize this surface.
- In 1935 Einstein and Rosen did this for the Schwarzschild solution as shown on the next two slides.
- The first step is to consider the plane with t and theta constant. This leaves the arc length formula for this plane as a function of r and phi. We then map this to some surface in 3D flat space. Since this surface is symmetric with respect to phi, this must be some surface of revolution about the z-axis, and in general on such a surface z = f(r). Starting with the 3D flat space arc length formula in cylindrical coordinates, the math steps are shown to get the arc length formula on the surface z = f(r). Just matching this with what we get for our slice of Schwarzschild results in a differential equations for f(r).
- The result is the Einstein Rosen Bridge. (IF YOU WANT TO WORK OUT THE SURFACE ON YOUR OWN DO THAT NOW, BEFORE LOOKING AHEAD TWO SLIDES.)

Embedding Diagram

Schwarzschild for $t = 0$, $\theta = \pi/2$:

$$ds^2 = \frac{1}{\left(1 - \frac{2GM}{rc^2}\right)} dr^2 + r^2 d\phi^2$$



Flat space cylindrical coordinates:

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2$$

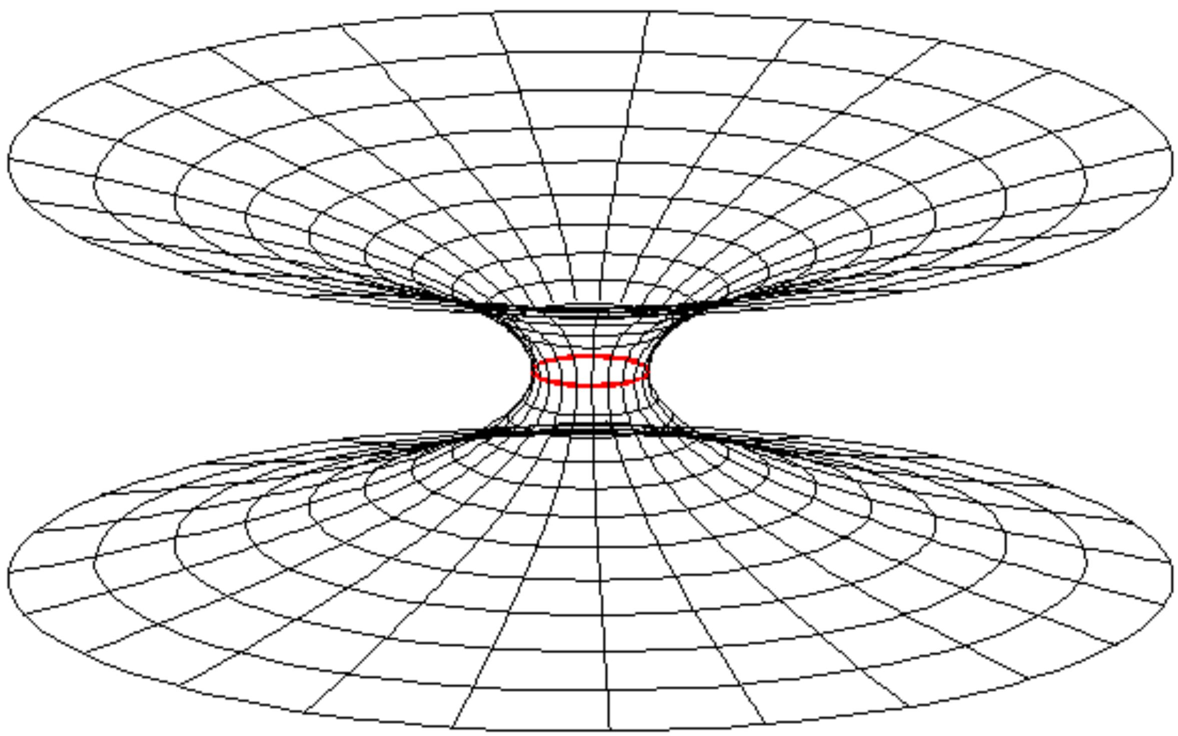
$z = f(r)$ (Surface of revolution about z-axis.)

$$dz = f'(r)dr$$

$$ds^2 = [f'(r)^2 + 1]dr^2 + r^2 d\phi^2$$

Einstein-Rosen Bridge

Our Universe

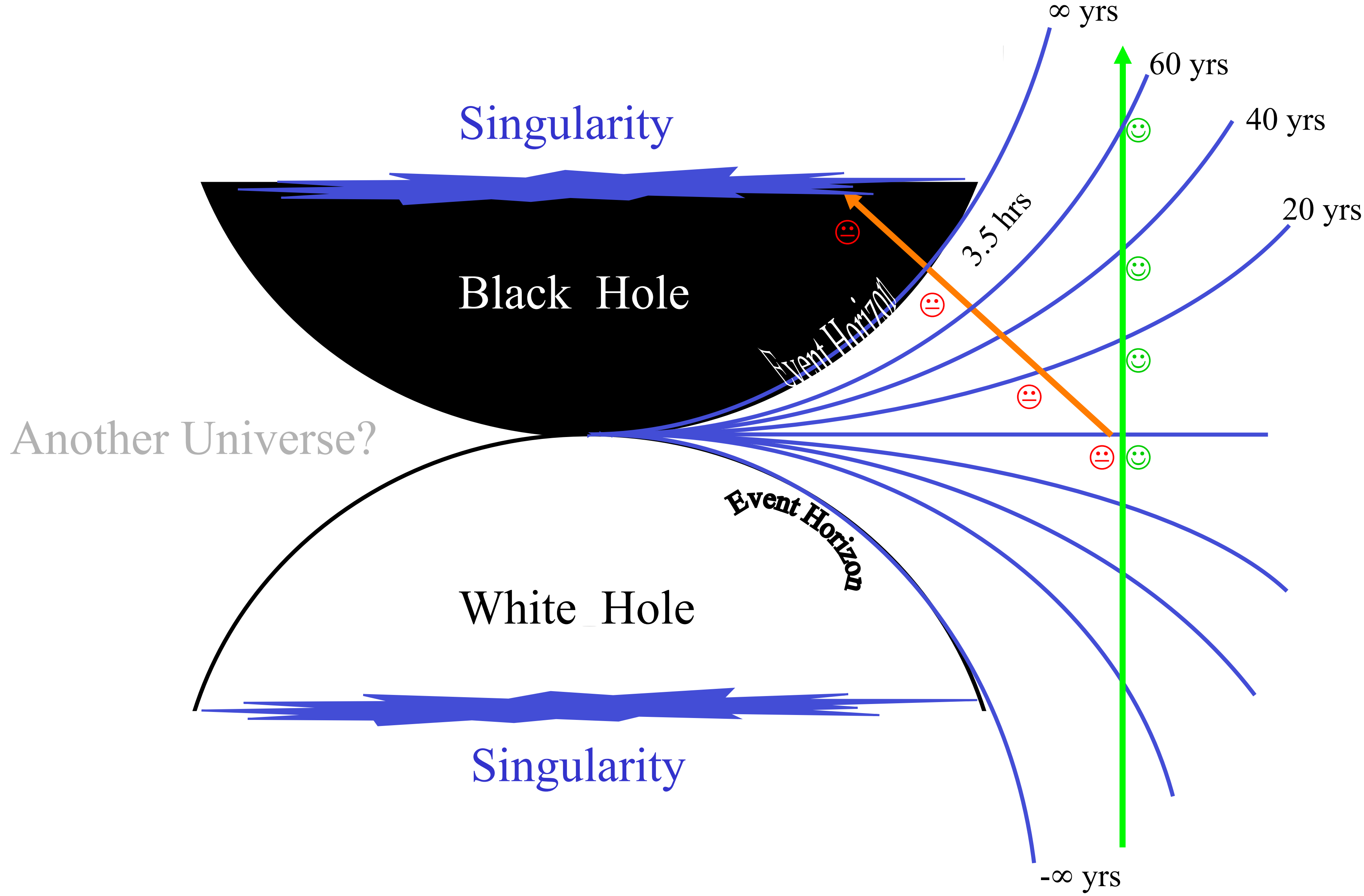


Another Universe?

Einstein-Rosen Bridge and how to extend Schwarzschild inside the Black Hole

- If you work out the embedding surface as described on the previous slides you will get a parabola of revolution about the z-axis. The circle in the z=0 plane is the horizon of the black hole, and the surface above it maps to the 2D slice outside the black hole in our universe. However, it joins onto a mirror image of this, which is the surface outside the black hole somewhere else. This could be another universe, or somewhere else in our universe.
- Why doesn't the slice go inside the horizon? It took until the late 50's and early 60's for this to be completely understood.
- Coordinate introduced by Eddington in 1924 and rediscovered by Finkelstein in 1958 showed the singularity in the Schwarzschild metric at the horizon was just a coordinate singularity that could be removed by a coordinate transformation.
- In 1960 Kruskal and Szekeres found coordinates that fully extend the Schwarzschild solution.
- A fanciful illustration of a Kruskal Szekeres Penrose type spacetime diagram for the Schwarzschild solution is shown on the next page. Note that the Schwarzschild t coordinate extends from minus infinity to plus infinity outside the BH in our universe. Thus the Schwarzschild time outside the BH does not extend inside the BH. One needs another set of Schwarzschild coordinates for the inside of the BH, and for the WH, and for the other universe!
- Also on the next slide is Betty and Bob again. Bob gets the clever idea to fall towards the black hole, while leaving Betty in orbit. He falls near the horizon of a supermassive black hole in just 3.5 hrs, while for Betty 60 years go by. Bob is now younger than Betty! However, he does not have long to enjoy her passing him in age, as he crosses the horizon and find himself in a singularity in just 10's of seconds. (Note the singularity is a place in time, not a place in space. Everything that enters the black hole becomes part of the sigularity a finite amount of time.)

Falling Into A Black Hole



Embedding diagram for the inside of the Schwarzschild Black Hole

- The Schwarzschild solution is also a solution inside the horizon of the black hole.
- However, because the sign changes on the factors in front of dt^2 and dr^2 in the Schwarzschild solution inside the horizon, the meaning of r and t changes. The t coordinate now describes places in space, and the r coordinate now describe places in time. Since the Schwarzschild solution is a function of r, it is no longer static inside the black hole!
- Thus, inside the horizon we need to look at a slice with constant r and theta, and see what it corresponds to in flat 3D space. This is done on the next slide.

Embedding Diagram Inside The Black Hole

Schwarzschild for $r = R$, $\theta = \pi/2$:

$$ds^2 = c^2[2GM/(Rc^2)-1]dt^2 + R^2\phi^2.$$

Flat space cylindrical coordinates:

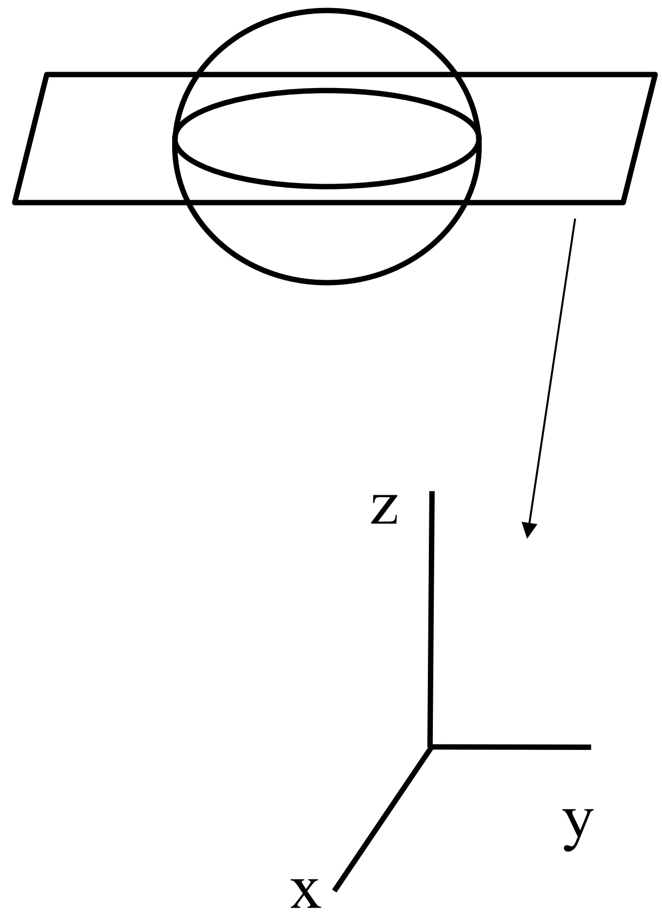
$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2.$$

Comparing, it looks like in the flat space $r = R =$ constant, so

$$ds^2 = dz^2 + R^2 d\phi^2.$$

We need to match up:

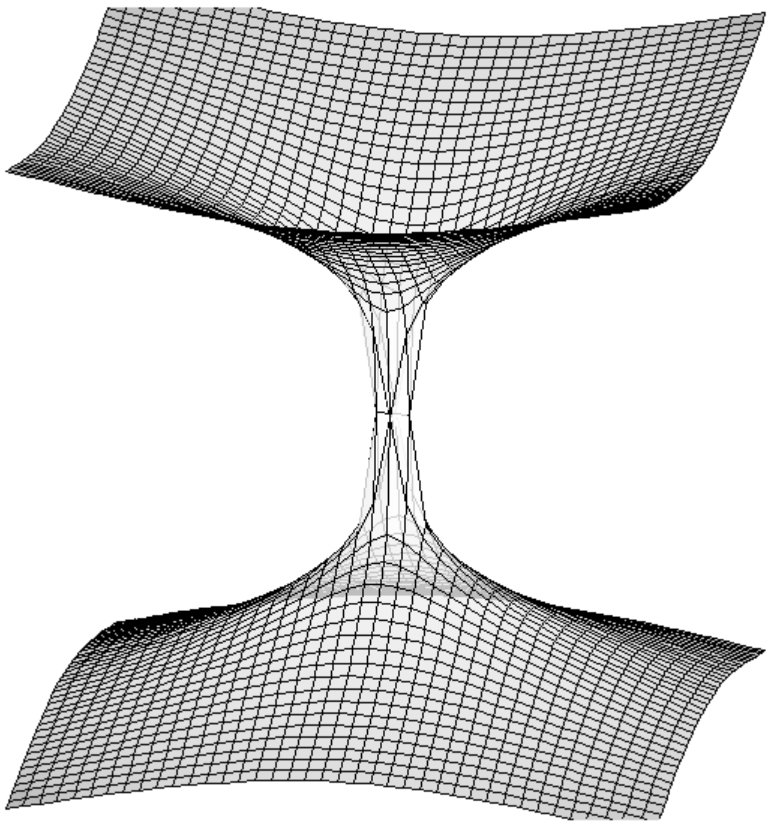
$$dz^2 = c^2[2GM/(Rc^2)-1]dt^2.$$



Embedding diagram for the inside of the Schwarzschild Black Hole, Part 2.

- On the previous slide we see r = R in the embedding space, so we have a cylinder. However, inside the BH, r is a time coordinate, and r goes from 2GM/c2 to 0. Thus, as r decreases (and time goes forward) the radius of the cylinder in the embedding space decreases!
- Note the relation between t and z. Inside the BH t is a place in space, and different t values map onto different z values on the cylinder. Note as the time r goes to zero (i.e, as R, the radius of the cylinder, goes to zero) that the distance between two points in space for two different values of t becomes an infinite distance along z.
- Thus, inside the black hole, as the cylinder's radius shrinks to zero the length of the cylinder grows infinite. This is the singularity! It is an infinite stretching in one direction and an infinite squeezing in the other. And it happens within a finite time for all particles that enter the black hole. The singularity is a place in time, and it happens everywhere in the space inside the black hole.
- In fact, this interior solution is unstable, and spacetime actually will oscillate wildly inside the black hole.
- In any case, spacetime is breaking down inside the black hole.
- On the next slide is a "qualitative" illustration of the parabola of revolution joined onto the cylinder inside the BH, giving a wormhole going between two universes. It probably represents some mapping to the embedding space, but it is not an analytical mapping.
- Note that you cannot get through the wormhole. That would require going faster than light. Otherwise the wormhole squeezes shut before you can traverse it. (Some sort of exotic matter could maybe hold a wormhole open, e.g, look for the work by Morris and Thorne, but that is beyond the scope of this presentation.)
- To get an analytical embedding diagram that joins the exterior to the interior of the Schwarzschild solution, I will use Eddington-Finklestein coordinates, introduced on the slide after next.

Schwarzschild Worm Hole



Eddington Finkelstein Coordinates

If we introduce the following form of the Eddington Finkelstein time coordinate, t',

$$ct = ct' - (2GM/c^2)\ln|rc^2/(2GM) - 1|$$

outside the horizon, and

$$ct = ct' - (2GM/c^2)\ln|1 - rc^2/(2GM)|$$

inside the horizon, then inside or outside, we get

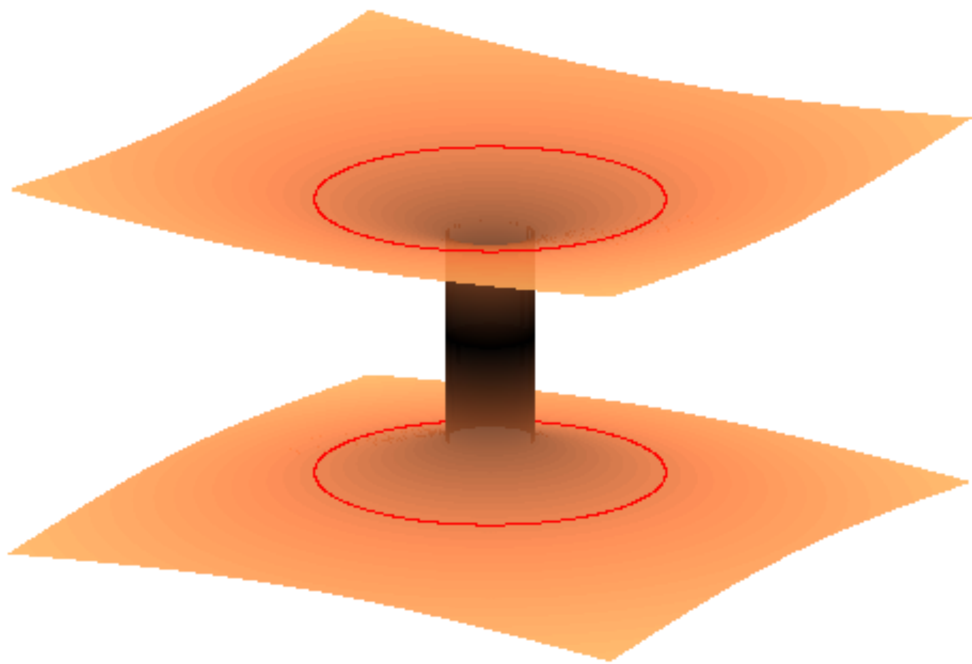
$$ds^2 = -c^2[1-2GM/(rc^2)]dt'^2 + 4GM/(rc^2)dt'dr + [1+2GM/(rc^2)]dr^2 + r^2d\theta^2 + r^2\sin\theta^2 d\phi^2.$$

Note that there is no coordinate singularity at the horizon.

Schwarzschild Wormhole Embedding Diagrams.

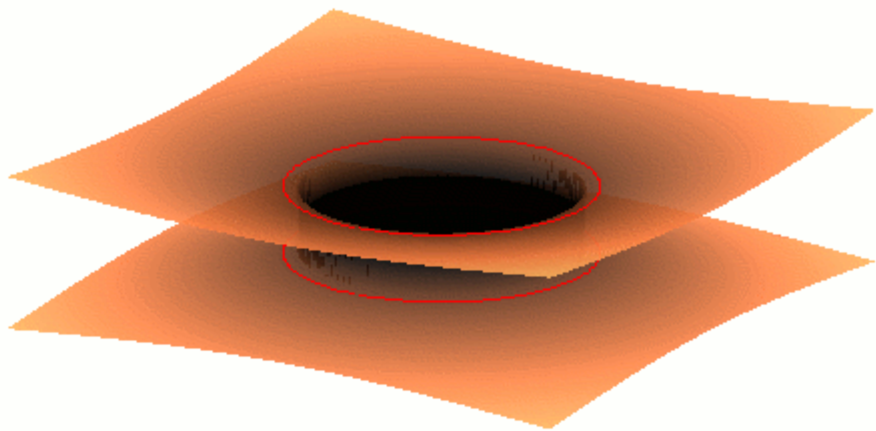
- Note that contant t' surfaces in Eddington Finkelstein coordinate also map to parabolas of revolution about the z-axis in the embedding space, but these parabolas extend inside the horizon.
- Thus, one can join these parabolas onto the cylinders inside the horizon. I do this to get the embedding diagrams on the next 3 slides. The latter 2 are animated gifs that show the spacetime evolution inside the horizons (there is a horizon in each universe).
- To do this, I match the parabolas to the cylinders along the U = 1 line, where U is the Kruskal-Szekeres coordinate, and then do this for various values of the time coordinate r inside the horizon. (Remember r represents time inside the horizon.)
- I also solve for the radial geodesics and map the position of a triangle shaped spaceship that free falls into the black hole. This is seen in the animation on the 3rd of the next 3 slides.
- Note the spaceship experiences tidal stretching and squeezing. Some of this is also frame dependent, since different amounts of "length contraction" will be seen in different frames, and the "simultaneous" position of the nose and back of the spaceship is a frame dependent quantity.
- In any case, note the spaceship cannot traverse the wormhole, but ends up in the sigularity. It does not "hit" the singularity, but becomes part of the singularity, with everything else inside the black hole.
- Also note that the M in the equations has the units of mass, but these are vacuum solutions. Except for the spaceship, there is NO matter in these solutions. Its just space and time, just spacetime! Einstein's theory says spacetime can do what these embedding diagrams illustrate, all by itself!
- The next 3 slides are exact analytical solutions, mapped into the embedding space. The red circles are the horizons.

Schwarzschild Worm Hole



Embedding With Interior Dynamics

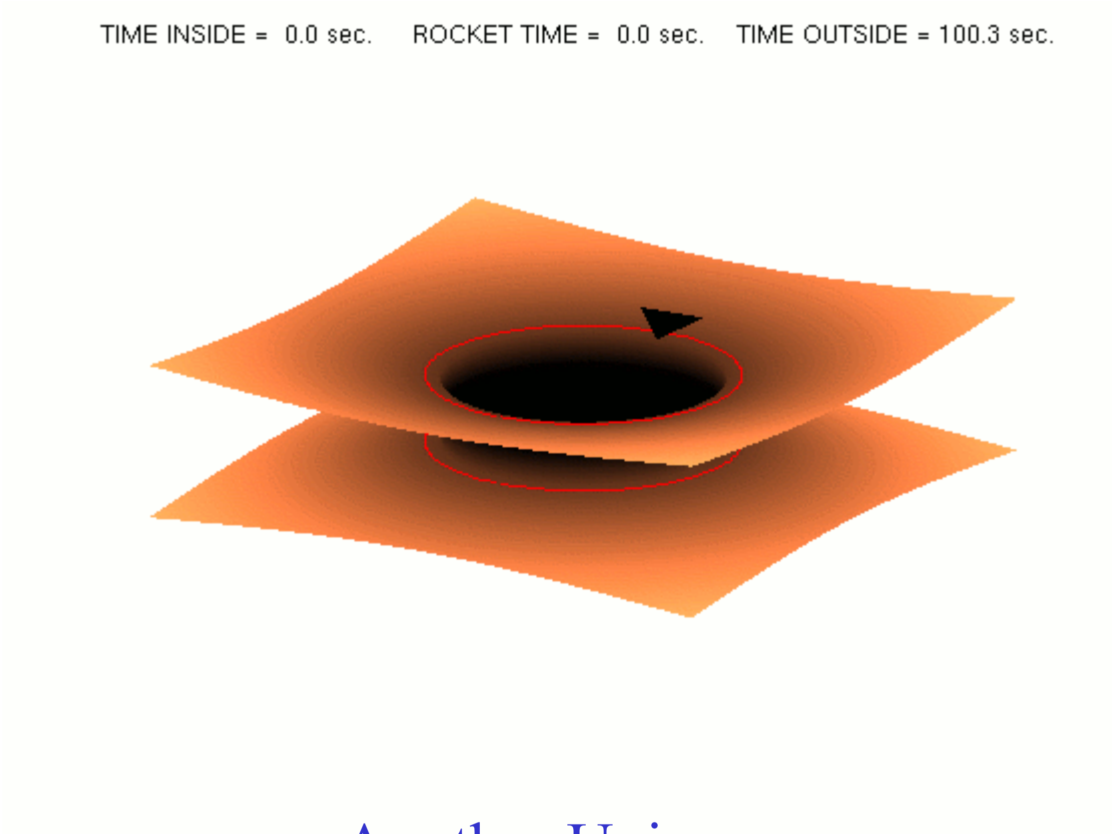
Our Universe



Another Universe

Nontraversable Wormhole

Our Universe

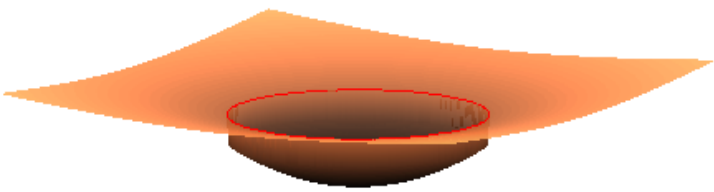


Another Universe

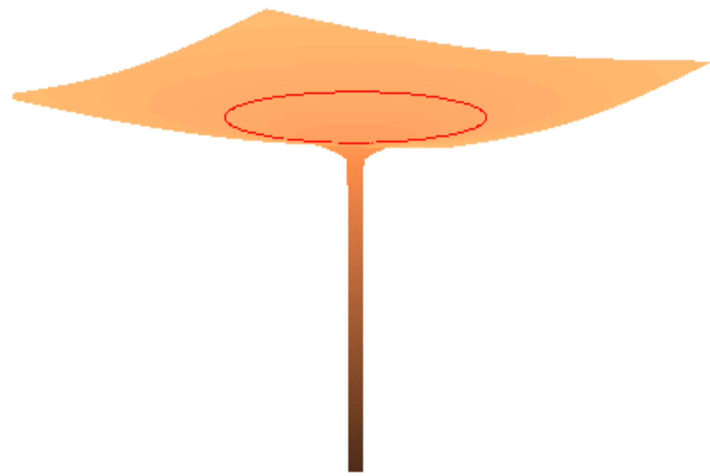
Astrophysical Black Holes

- The previous slide are mathematical solutions of Einstein's equations. They represent things that could exist (in principle) but we do not know how to form such objects.
- Astrophysical black holes form from the collapse of stars. In this case, the geometry outside a static black hole is the same as shown before. And inside the matter collapses down to a singularity, and shown by theorems due to Hawking and Penrose.
- Without using precise mathematics one can infer the mass of a collapsing star forming a black hole will cause space to stretch and squeeze in some analogous way to what was shown before, forming a singularity. It is probably unstable and will oscillate wildly. But our understanding of spacetime is breaking down at this point. The next 2 slide are NOT precise mathematical solutions (like the previous slides) but just illustrations of what the formation of an astrophysical black hole might be like.
- In the astrophysical formation of a black hole there is no worm hole (no tunnels to other universes) and no white hole (the time reverse of the black hole, not discussed here).
- Still the spacetime is about as weird as can be imaged....

Stellar Collapse To Form A Black Hole



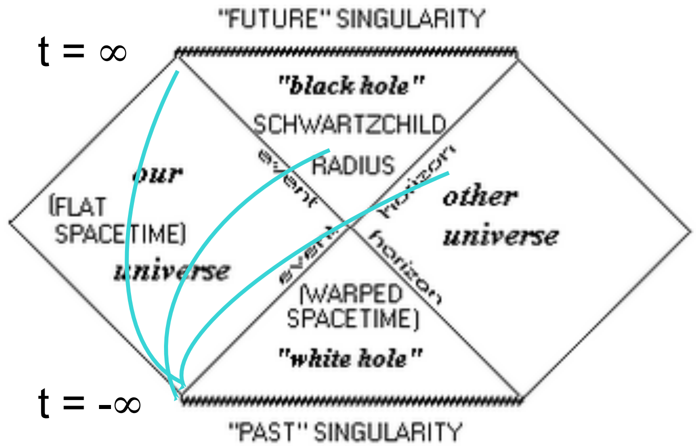
When pressure can no longer support a star's gravity its mass falls through its horizon.



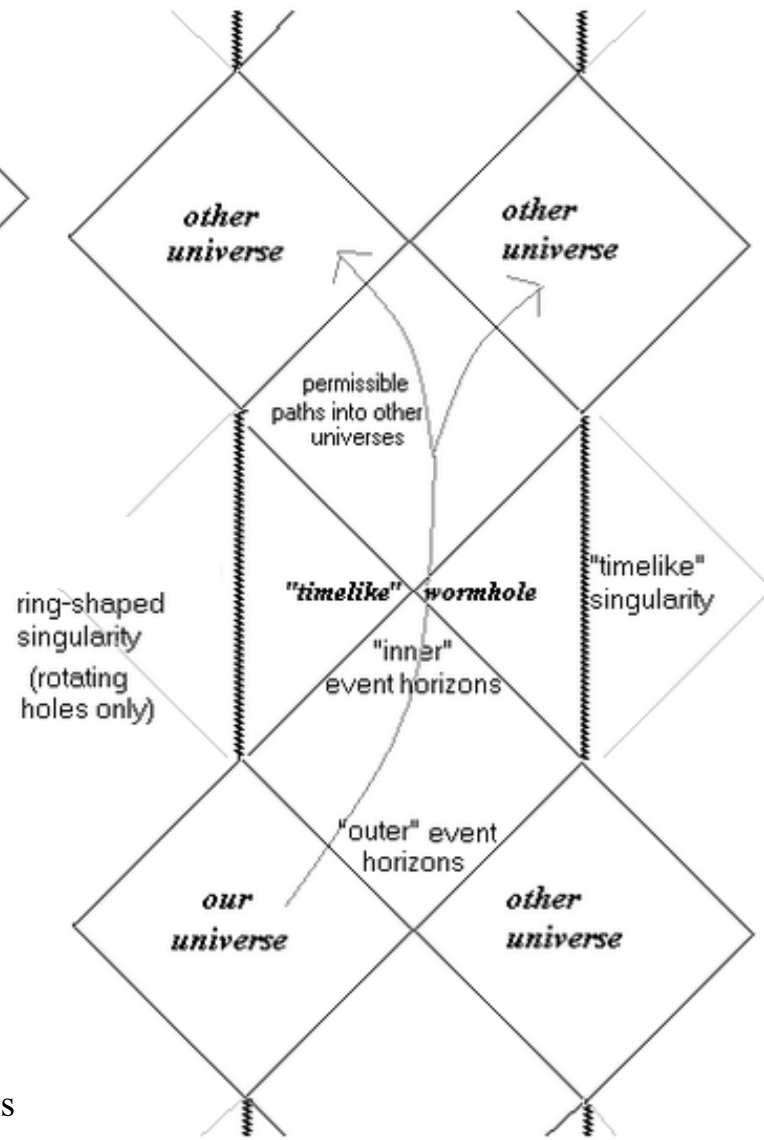
And it collapses to a Singularity.

Penrose Diagrams & Black Holes

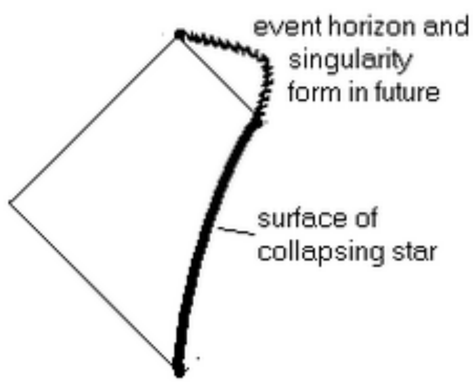
Schwarzschild Black Hole



ELECTRICALLY CHARGED
AND/OR ROTATING WORMHOLE



ACTUAL BLACK HOLE FROM
COLLAPSED STAR



Figures: http://en.wikipedia.org/wiki/Penrose_diagrams

Black Holes After 1960

- Kruskal-Szekeres Coordinates, 1960
- Wormholes, Wheeler and Fuller, 1962
- Black Holes, popularized by Wheeler, 1968
- Penrose Process,1969
- Black Hole Evaporation, Hawking, 1974
- Time Machines, Morris and Thorne, 1988
- BH Information Theory?

LIGO & Gravitational Waves

Gravitational waves carry information about the spacetime around black holes & other sources.

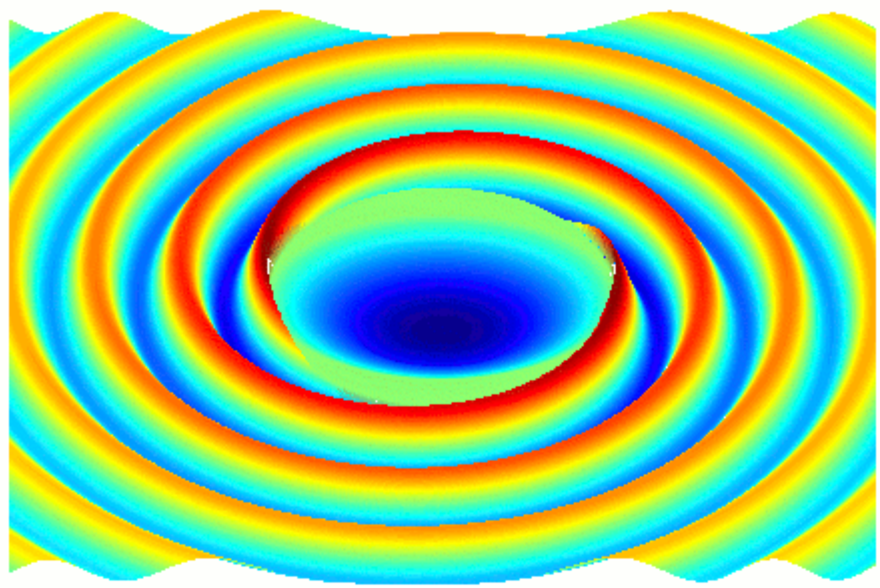
$$dT^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\bar{h}^{\mu\nu} = 0$$

$$h^{TT}_{\hat\theta\hat\theta}(\theta = \pi/2) \propto \frac{1}{r}\cos[2\pi f(t - r/c) + 2\phi]$$

$$h^{TT}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{2\pi i f(t - z/c)}$$



Detector Response

$$g_{\mu\upsilon} dx^{\mu} dx^{\upsilon} = 0$$ (Light Travels On Null Geodesics)

$$c^2 dt^2 - \begin{pmatrix} dx & 0 & 0 \end{pmatrix} \begin{pmatrix} 1+h_{xx} & h_{xy} & h_{xz} \\ h_{yx} & 1+h_{yy} & h_{yz} \\ h_{zx} & h_{zy} & 1+h_{zz} \end{pmatrix} \begin{pmatrix} dx \\ 0 \\ 0 \end{pmatrix} = 0$$

$$c^2 dt^2 = (1+h_{xx}) dx^2$$

$$c\int_0^{\Delta t} dt = \int_0^L \sqrt{1+h_{xx}}\, dx \cong \int_0^L \left(1+\frac{1}{2}h_{xx}\right) dx$$

$$c\Delta t = L_x = L + \frac{L}{2} h_{xx}$$

$$\frac{\Delta L}{L} = \frac{1}{2}(h_{xx} - h_{yy}) = F_{+}(\theta,\phi,\psi) h_{+}(t) + F_{\times}(\theta,\phi,\psi) h_{\times}(t)$$

LIGO

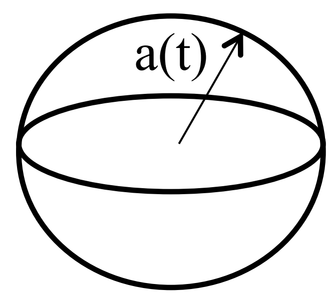
Friedmann-Lemaitre-Roberson-Walker Metric

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{1}{(1-kr^2)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

$$G_{\mu\upsilon} + \Lambda g_{\mu\upsilon} = \frac{8\pi G}{c^4} T_{\mu\upsilon}$$ (Cosmological Constant = Λ)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$ (Curvature constant k = 1,0,-1)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$



$$\rho_c = \frac{3H^2}{8\pi G}$$; $P(\rho)$ (Equation Of State)

LIG

LIGO

The End