Application of the Maximum Entropy Method to Data from the LIGO Gravitational Wave Detectors

Andrews University

C. W. Greenley, L. S. Finn, T. Z. Summerscales

LSC

Abstract

We apply the Maximum Entropy method, a coherent data analysis method for retrieving a common signal measured by a network of detectors, to data taken by the LIGO detectors during Science Run 5. The method is applied to data from hardware injection times when the detector mirrors were physically moved to simulate a gravitational wave detection. Comparison between the injection signal waveform and that recovered by Maximum Entropy provides a test of the method.

Background Information

General Relativity

The theory of general relativity, proposed by Albert Einstein in 1916, describes falling objects as simply following the most direct path through space. We can think of space as a flat elastic sheet which stretches when a massive object is placed on it, forming an indent or "well". If another object is traveling along in a straight line and encounters this "gravity well" caused by the first object, the path will curve and the two objects will begin to fall towards their center of mass. (See figure 1.)

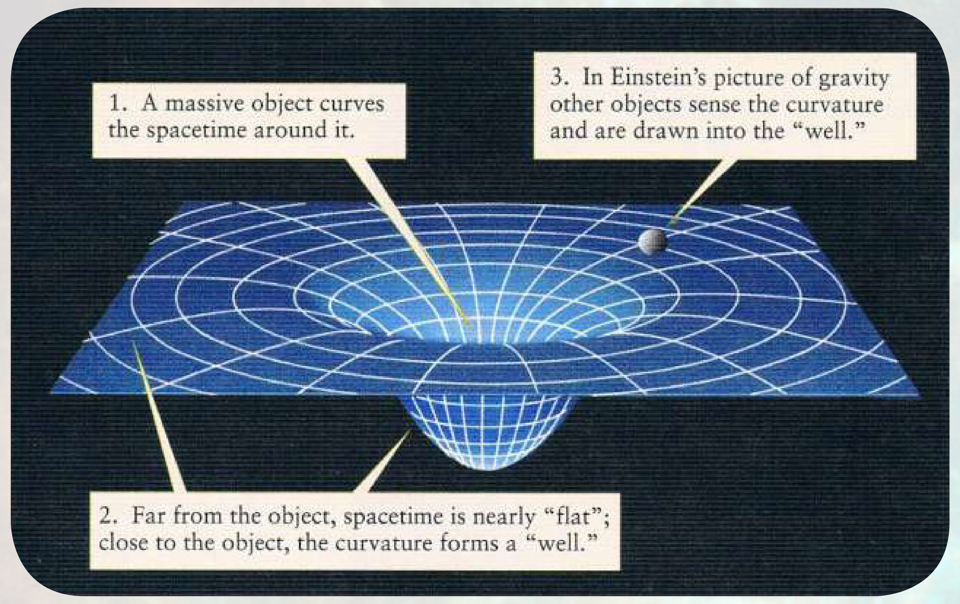

Fig. 1 An artists rendetion of a gravity well

Gravitational Waves

Events such as Type II supernovae and orbiting neutron stars cause ripples in the fabric of space. (See figure 2.) Referring to our previous model, we can imagine the elastic sheet with two heavy balls rolling around each other or, similarly, two speed boats circling a central point, causing ripples to propagate outward. These ripples represent gravitational waves, which travel through space at the speed of light. There is implicit evidence for the existence of gravitational waves, but LIGO hopes to be able to provide explicit evidence by direct detection.

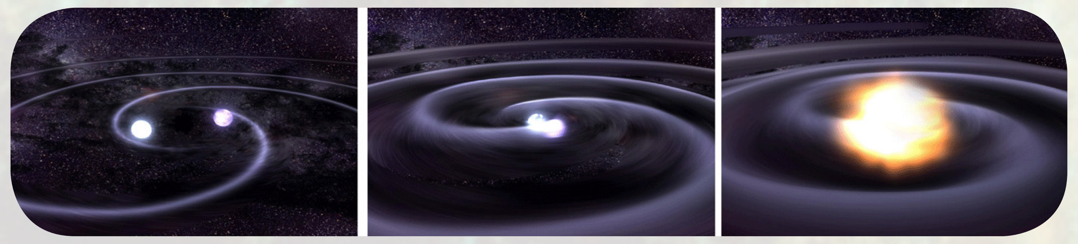

Fig. 2 An artists impression of two orbiting stars generating gravitational waves.

Fig. 3 The LIGO observatories at Livingston, LA and Hanford, WA.

About LIGO

LIGO utilizes a laser interferometer to measure the distortion in space caused by gravitational waves. (See figures 4 & 5.) A laser beam is split equally into two perpendicular beams which travel down 4 km tunnels, before reflecting back to recombine causing an interference pattern. If there are no disturbances the beams will be offset so as to perfectly cancel. However, a gravitational wave would cause the relative length of the tunnels to change. This would result in the beams failing to cancel and the photodiode detecting light.

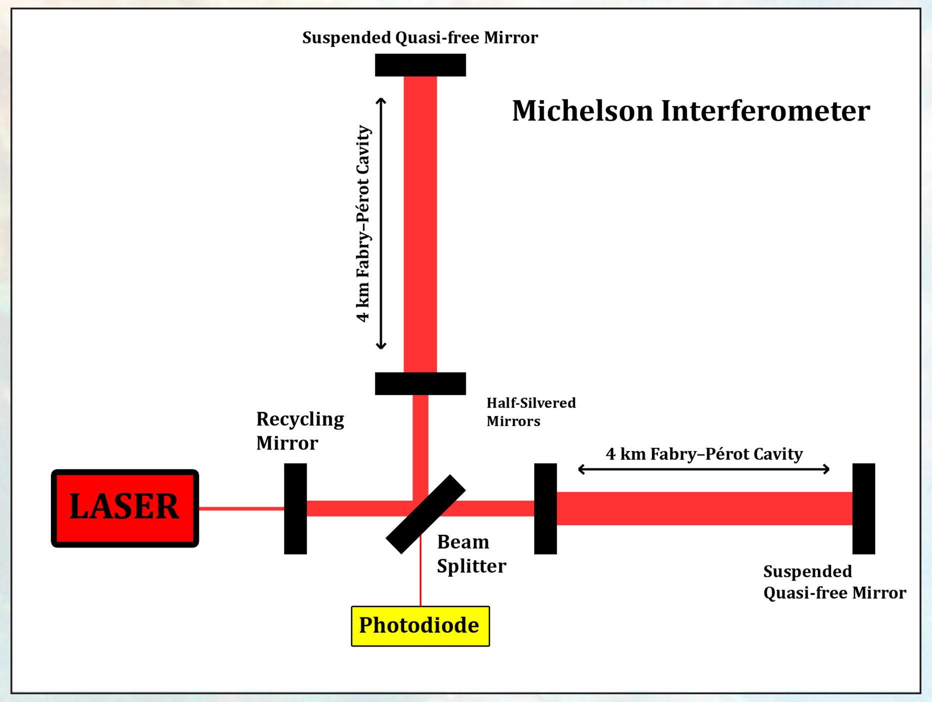

Fig. 4 The LIGO laser interferometer setup.

Hardware Injections

The best way to test the whole system is to generate fake gravitational wave signals, and then attempt to extract them from the data. One of the methods used is called a "hardware injection," and is done by physically vibrating the mirrors in the interferometer to simulate the effects of a gravitational wave. The parameters of the signals that are fed in are known, and can be compared to the parameters of the signals that are extracted. I apply the Maximum Entropy (MaxEnt) method to extract these signals and then compare the injected and extracted signals as a test of the method.

References

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Maisinger, K., Hobson, M. P. and Lasenby, A. N. (2004), Maximum-entropy image reconstruction using wavelets. Monthly Notices of the Royal Astronomical Society, 347: 339–354. doi: 10.1111/j.1365-2966.2004.07216.x

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[Fig. 1] http://www.astro-photography.net/Dark-Matter.html

[Fig. 2] http://www.ligo.org/science/GW-Inspiral.php

[Fig. 3] http://www.mpa-garching.mpg.de/mpa/research/current_research/hl2012-2/ and http://www.astro.caltech.edu/research/ligo/

[Background] http://enlightenmentjunkie.wordpress.com/

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Methodology

Maximum Entropy

The data gathered by the LIGO detectors is a measurement of the strain on the detector caused by a gravitational wave as a function of time, **h(t)**. The resultant data (**d**) is a combination of the signal (**h**) multiplied by the detector response (**R**) and added to the noise (**n**).

$$\mathbf{d} = \mathbf{R}\mathbf{h} + \mathbf{n}$$

The probability of obtaining any (**h**) given (**d**) and additional information (*I*) must be maximized. This is done using Bayes' Theorem.

$$P(\mathbf{h}|\mathbf{d}, I) \propto P(\mathbf{d}|\mathbf{h}, I)P(\mathbf{h}|I)$$

The probability P(**d** | **h**, *I*) is known as the "likelihood function" for (**d**) given (**h**), and P(**h** | *I*) takes into account the *a priori* expectations about the signal. Assuming the noise is Gaussian, then the likelihood can be defined as

$$P(\mathbf{h}|\mathbf{d}, I) \propto e^{-\frac{1}{2}(\mathbf{Rh}-\mathbf{d})^T\mathbf{N}^{-1}(\mathbf{Rh}-\mathbf{d})} = e^{-\frac{1}{2}\chi^2(\mathbf{R},\mathbf{h},\mathbf{d},\mathbf{N})}$$

If we then choose as a prior

$$P(\mathbf{h}|, I) = e^{\alpha S(\mathbf{h})}$$

we can say that

$$P(\mathbf{h}|\mathbf{d}, I) \propto e^{-\frac{1}{2}\chi^2(\mathbf{R},\mathbf{h},\mathbf{d},\mathbf{N})}e^{\alpha S(\mathbf{h})} = e^{-[\frac{1}{2}\chi^2(\mathbf{R},\mathbf{h},\mathbf{d},\mathbf{N})-\alpha S(\mathbf{h})]}$$

where alpha is a Lagrange coefficient used to balance accurately representing the data while avoiding fitting the noise, and S(h) is related to the Shannon information entropy. Maximizing the right hand side of the equation is the same as minimizing the function (F) below

$$F(\mathbf{h}|\mathbf{d}, \mathbf{R}, \mathbf{N}) = \chi^2(\mathbf{R}, \mathbf{h}, \mathbf{d}, \mathbf{N}) - \alpha S(\mathbf{h})$$

To test the method, the recovered signal (**h**) is compared to a clean signal generated using the known injection parameters.

Testing the Method

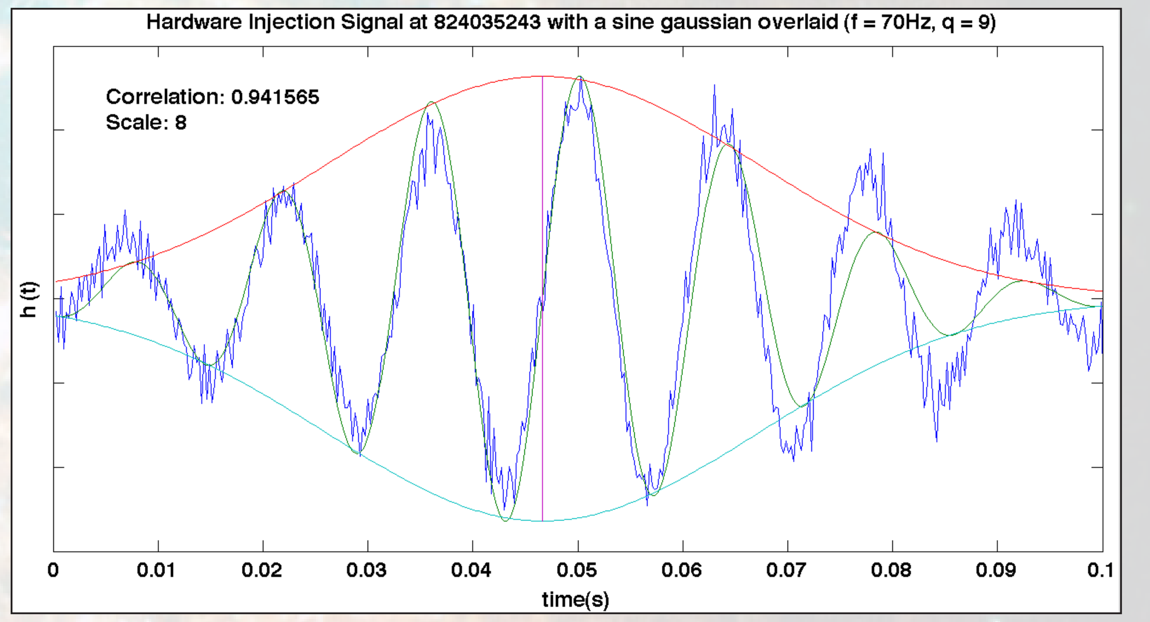
Specific parameters are used to generate the wave function for each hardware injection. The events I have been analyzing are sine waves modulated by a Gaussian envelope with parameters Q = 9 and f_0 = 70 Hz where (t) is time and (T_0) is the center of the Gaussian envelope.

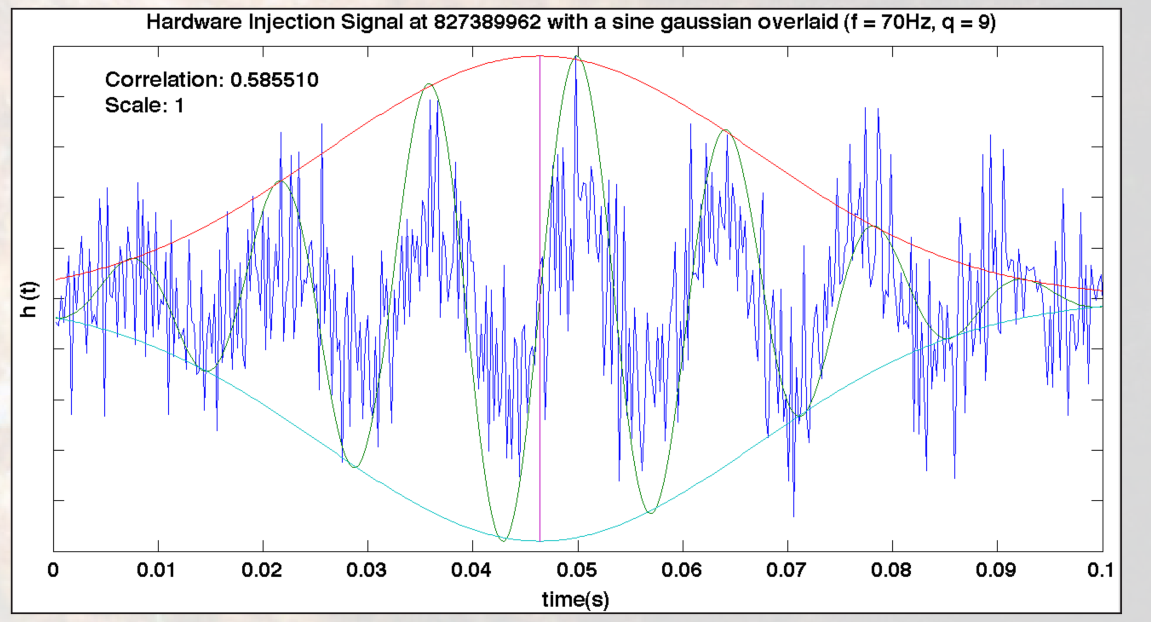
$$h(t) = sin[2\pi(t - T_0)f_0]e^{-\frac{(t-T_0)^2}{\tau^2}} \qquad \tau = \frac{Q}{f_0\sqrt{2}\pi}$$

These injections occurred during the first year of science run 5, during which time events were simulated as occurring directly overhead of each detector. A constant offset was introduced to account for the time it would take a gravitational wave to travel from one detector to the other. The waveform was scaled to a predefined h_{rss} (root-sum-squared strain) and then on injection varied by scale factors of 1, 2, 4, and 8. As a consequence the method sensitivity can be tested by applying it to different events with various signal-to-noise ratios.

Results

I have applied MaxEnt to data from thirteen hardware injections from science run 5 (between 821921000 and 834742000). The extracted and the pure signal are compared graphically and by calculating the cross-correlation between the waveforms. This provides both a visual and mathematical basis for comparison. The following graphs are representative of the correlations obtained. The average correlations for the scale factors 1, 2, 4, and 8 are 0.53, 0.78, 0.83, and 0.94 respectively.



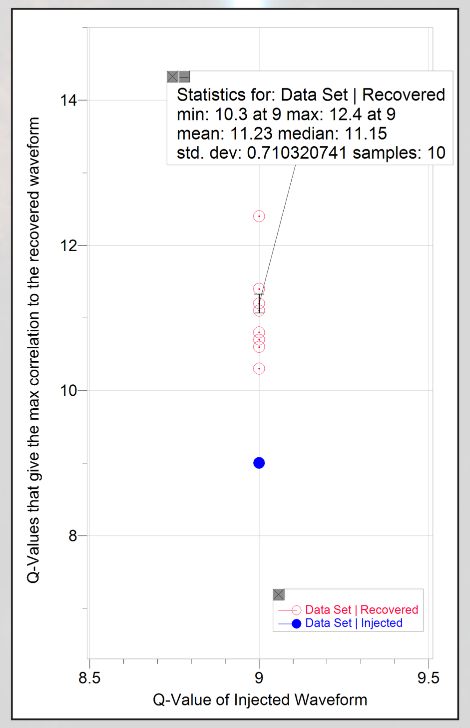


Conclusions

The MaxEnt method can extract injected waveforms with good sensitivity and the successful extraction of hardware injection waveforms shows that the overall data collection and analysis process is reliable.

Further Research

While the MaxEnt method is able to extract the waveform quite accurately, there is one oddity in the data. The Gaussian envelope constraining the extracted sine wave seems to go to zero more slowly than it should. This property is controlled by the (Q) value. I iterated over (Q) values to find best possible match for each extracted waveform, and a pattern emerges. (See graph on right.) The next step is to look into why the extracted waveforms all appear to have (Q) values around 11 rather than 9.



Background: The Crab Nebula - remnants of a supernova.