LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
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Notes on atom interferometry for LIGO enthusiasts

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1 Definitions

By *atom interferometer* we mean a Mach-Zehnder interferometer where free-falling atoms initially in the ground state first evolve into a superposition of two states (associated with the two hyperfine levels of the ground state of alkali atoms) after interaction with two counter-propagating $\pi/2$ laser pulses. The two lasers are shifted in frequency to compensate the hyperfine splitting and the Doppler effect due to the motion of the atoms. The pulse duration $\pi/2$ refers to the Rabi oscillation period and means that the resulting superposition is balanced with respect to the two states. Since momentum is transferred during this interaction depending on the internal state of the atom, the superposition leads to a spatial splitting of the atom path so that the first interaction with the laser essentially acts as a 50/50 splitter of the atom beam. After a time T another laser-atom interaction occurs. This time it is a π pulse that interacts with both atom paths. It inverts the momenta of the atom such that the two paths converge again to one point and therefore one can think of this interaction as reflection from a mirror. After time T at the point where the two paths converge, a final $\pi/2$ pulse occurs that lets the two atom paths interfere with each other. Atoms are then counted in the two outgoing paths.

By *atom GW detector* we mean two atom interferometers separated by a large distance L that both interact with the same lasers. In this case, the system is characterized by three time scales, the light propagation time L/c, the interaction time τ, which is either a quarter or half of the Rabi oscillation period, and the fall time T of atoms between interactions with the lasers.

2 The response of atom interferometers to GWs

Each atom interferometer by itself has already some sensitivity to GWs, but we can neglect this since the size of the Mach-Zehnder formed by the atom paths is small. To understand how the atom GW detector responds to GWs, we need to know that the lasers imprint their phase on the atom beams at each laser-atom interaction. Since a GW changes the distance between the two interferometers, the relative laser phase imprinted on the atoms at the two interferometers will be modulated at the GW frequency. This means that the two atom interferometers take over the part of the test masses inside a LIGO arm and the GW response in the long-wavelength limit with respect to the relative laser phase is simply

$$\Delta\phi(\Omega) = \frac{\omega_0 L}{2c} h(\Omega) \tag{1}$$

where ω_0 is the laser frequency, and Ω the GW frequency. A complete calculation of the atom interferometer response to GWs and other relativistic effects can be found in [1]. Even though the signal is ultimately given by atom numbers counted in each atom interferometer, it is convenient to express the GW response with respect to the laser phase. This will become clear when we calculate some contributions to the instrumental noise.

The analogy between atom interferometers and LIGO test masses is not perfect since two atom interferometers measure the light phase at two points whereas in LIGO light is only detected at the dark port. This is also one reason why atom GW detectors can potentially have suppressed seismic noise as we will see in the next section.

3 Atom GW detector noise coming from the lasers

We do not want to give a full account of instrumental noise sources. A fairly complete list (more complete than I could have given) can be found in [2]. In this article we only want to focus on noise contributions that help us gaining some intuitive understanding of atom GW detectors. The first noise contribution that we consider is the laser phase noise $\phi_L(\Omega)$. Since both atom interferometers interact with the same lasers, the noise in the relative phase between the two interferometers is given by

$$\Delta\phi(\Omega) = \phi_L(\Omega)\frac{\Omega L}{c} \tag{2}$$

where the fraction accounts for a partial correlation of laser-phase noise at the two atom interferometers. We can write the respective GW sensitivity

$$h(\Omega) = \phi_L(\Omega)\frac{\Omega L}{c}\frac{2c}{\omega_0 L} = 2\phi_L(\Omega)\frac{\Omega}{\omega_0} \equiv 2\frac{\Delta\omega}{\omega_0} \tag{3}$$

The last expression is simply twice the relative frequency noise of the laser. Here we can already see why it is convenient to refer noise and GW response to the laser phase. It does not matter how atoms are counted, whether the atom interferometer is operated as a Mach-Zehnder or in another configuration, or how many laser photons interact with each atom per pulse. The laser-phase-noise related sensitivity of a two-atom-interferometer GW detector with no sophisticated laser interferometry will always be governed by this equation. This also tells us that this type of atom GW detector would not work since relative laser frequency noise cannot be made many orders of magnitude smaller than a currently typical value of $10^{-15}/\sqrt{\mathrm{Hz}}$. Note that it is really the frequency stability of the oscillator used for the AOM to produce a second laser with shifted frequency that matters here, not the frequency noise of the laser itself. This is because of how the two counter-propagating lasers at slightly different frequencies imprint an effective phase on the atoms.

The second noise contribution that we want to discuss is the seismic noise. We imagine that the lasers themselves are shaking with amplitude $\xi(\Omega)$ along the beam direction, or maybe that the lasers are at some point reflected from a mirror that shakes with this amplitude. In either case, it is easy to see that the resulting phase noise of the laser is

$$\Delta\phi(\Omega) = \frac{\omega_0\xi(\Omega)}{c}\frac{\Omega L}{c} \tag{4}$$

(or twice as high in the case of a shaking mirror). Again, the equation can look different when the atom GW detector has more than two atom interferometers or a more complex optical system. The first fraction is the LIGO type response to seismic noise, and we find that the only type of seismic-noise suppression comes from the partial correlation of seismic noise at the two interferometers. Note that this correlation is produced by the light field and not the seismic field. The free fall of the atoms is one condition why this high degree of correlation can form. It is much harder to establish a high displacement correlation between suspended mirrors. The sensitivity to GW is given by

$$h(\Omega) = \frac{\omega_0\xi(\Omega)}{c}\frac{\Omega L}{c}\frac{2c}{\omega_0 L} = 2\frac{\Omega\xi(\Omega)}{c} \tag{5}$$

In terms of GW sensitivity, both noise contributions, seismic and laser-phase noise, were found to be independent of the distance L between the two atom interferometers. We will see in the next section that this is not the case for noise coming from the atoms.

4 Atom GW detector noise coming from the atoms

As an example, we will calculate the sensitivity curve with respect to atom shot noise. The shot noise of the atom phase is simply

$$\delta\Phi = 1/\sqrt{\eta} \tag{6}$$

where η is the mean number of atoms counted per second. The remaining problem is to express this equation as equivalent laser phase noise, which requires the transfer function from laser phase to atom phase. The full calculation for the Mach-Zehnder interferometer is given in [3]. Here we will simplify the problem by assuming that the interaction between atoms and laser is instantaneous (i.e. that the Rabi frequency is infinite). The result can be applied to ground-based detectors that are much smaller than the length of the GWs. As explained in the introduction, the laser phase ϕ is imprinted onto the atoms at three different times, t, $t+T$ and $t+2T$. The laser phase is amplified by the number N of photons that interact with each atom at each pulse. One can also show that the laser phase obtains a minus sign at the π pulses and a plus sign at the $\pi/2$ pulses. Therefore, assuming instantaneous interaction we obtain

$$\Phi(t) = N\Big(\phi(t-2T) - 2\phi(t-T) + \phi(t)\Big) \tag{7}$$

Modelling more realistic light-atom interactions, the phase terms need to be substituted by some integral over the interaction time that will also depend on the shape of the laser pulse since the Rabi frequency is not a constant (Rabi frequency depends on laser amplitude, which changes over the duration of the pulse).

Transforming the last equation into frequency domain, we obtain the following absolute value of the transfer function from laser to atom phase

$$T(\Omega) = 4N\sin^2(\Omega T/2) \tag{8}$$

and the laser-phase equivalent atom shot noise is given by

$$\phi(\Omega) = \frac{1}{4N\sqrt{\eta}}\frac{1}{\sin^2(\Omega T/2)} \tag{9}$$

Finally we can write the GW sensitivity as

$$h(\Omega) = \frac{c}{2N\omega_0 L\sqrt{\eta}}\frac{1}{\sin^2(\Omega T/2)} \tag{10}$$

Just to emphasize this once again, all sensitivity curves calculated in this article are only valid in the long-wavelength limit. The full expressions valid for detectors of arbitrary size can be found in the cited papers.

References

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