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Requirement For Adjustment Procedure Of Advanced LIGO Transmission Monitor Telescope

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Abstract

Requirement for TMS telescope is discussed in the context of focusing and alignment equipments and procedures.

1 Requirements

The requirement for the telescope is described in LIGO-T0900385, but here is a reducted explanation.

TMS telescope comprises two off-axis parabolic mirrors ("primary" and "secondary") and two folding mirrors attached on two parallel plates. The telescope receives the 1064 μm IFO beam coming out of the arm cavity through ETM on the primary and reduces the beam size by about a factor of 20 on the secondary side. There are two requirements for the telescope, i.e. the focus and the astigmatism.

1.1 Focus Requirement

Due to the lens effect of the ETM substrate, the IFO beam through ETM looks as if the waist is located 1520 m into the arm cavity with the waist radius of 8.2 mm, and at the primary position the beam radius is about 62 mm and the radius of curavature is about 1550 m.

With nominal parameters (2m primary focal length, -10cm secondary focal length, 1902.6mm spacing) the telescope reduces the beam size by about a factor of 20 with the new waist at the secondary.[1]

There are two QPDs on an optical bread board on top of the telescope, and the lens system for these QPDs (colloquially called Gouy telescope, which is not to be confused with TMS telescope) is designed in such a way that the Gouy shift of the beam on two QPDs differ by 90 degrees. This Gouy shift difference is directly related to the orthogonality of the alignment signals obtained by QPDs, and the requirement is that this difference is 90$\pm$10 degrees.

[1] Though nominal telescope spacing is written as 1902.6 mm, the actual number used for numerical calculations in this document is 1902.58636 mm. There's no practical difference between the two, but the plots look somewhat different, especially if you plot the waist position with a good accuracy.

When the waist coming out of the telescope is at the different location than the secondary, the Gouy shift at two QPDs starts to deviate from 90 degrees separation.

And the spacing between the primary and the secondary acts mainly on the position of the waist, not the size. Thus this the Gouy shift difference requirement puts a tolerance on the spacing between the primary and the secondary, which is

$$z_{12}^{\mathrm{reqired}} = 1902.6 + 0.2 - 0.5 \text{ mm}$$

(see e.g. Eq. 9 in T0900385-V07).

Note however, that this number only comes from the requirement for the QPDs, which is the main functionality of the TMS. The deviation from nominal length makes the mode matching of the green beam for ALS worse than it should be, and though the mode matching of the green beam is not as important as that of the IR beam, it is desirable (though not required) to have

$$z_{12}^{\mathrm{desirable}} = 1902.6 \pm 0.1 \text{ mm}.$$

1.2 Astigmatism Requirement

The telescope shall satisfy the focus requirement described above for all directions lateral to the beam axis.

2 ABCD Matrix of an Ideal Telescope Test Setup

The test setup is such that a beam is injected from the secondary, goes through the telescope, is reflected by a mirror placed on the primary side, and comes back to the secondary.

The ABCD matrix of such a double-path system can always be written by an equivalent single mirror system where a mirror is sitting some distance away.[2]

Table 1 shows the ABCD matrices as well as their equivalent mirror ROC and the distance for four possible test configurations, i.e. afocal with a flat mirror, afocal with ETM (ROC=2245m, fused silica), correctly detuned with a flat mirror, correctly detuned with ETM. In all four cases, the retro reflecting mirror (be it a flat one or ETM) was placed 60cm

[2] This is really easy to show, and is left for the readers to prove.

	afocal	detuned
flat	$\begin{pmatrix} 1 & 0.193 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} R = \infty \\ d = 0.0965 \text{ m} \end{pmatrix}$	$\begin{pmatrix} .9500 & .1882 \\ -.5177 & .9500 \end{pmatrix}, \begin{pmatrix} R = 3.8629 \text{ m} \\ d = 0.0965 \text{ m} \end{pmatrix}$
ETM	$\begin{pmatrix} 1.0498 & 0.1978 \\ 0.5166 & 1.0498 \end{pmatrix}, \begin{pmatrix} R = -3.872 \text{ m} \\ d = 0.0965 \text{ m} \end{pmatrix}$	$\begin{pmatrix} 1.0000 & .1930 \\ -2.33 \times 10^{-4} & 1.0000 \end{pmatrix}, \begin{pmatrix} R = 8596.3 \text{ m} \\ d = 0.0965 \text{ m} \end{pmatrix}$

Table 1: ABCD matrix and its equivalent single mirror configuration (a mirror at distance d from the secondary).

away from the primary. This distance is not critical to the test as the Rayleigh range of the beam on the primary side is about 200m for the IR beam. Lens effect of the ETM substrate was taken into account.

Interestingly, the only difference between four configurations is the ROC of the equivalent mirror. The distance from the secondary to the equivalent mirror, which has a weak dependence on the distance from the primary to the retro-reflection mirror, is the same for all four configurations.

Figure 1 shows the curvature, ROC and position of the equivalent one-mirror system for two cases of interest, i.e. detuned+ETM and detuned+flat, as a function of offset in detuning.

3 Telescope Alignment Requirement

It's important to understand that the requirement for the telescope itself is different from the requirement for a specific alignment procedure. This section studies the latter, which includes the requirement for the measurement apparatus.

From Table 1 one can see that "aligning TMS telescope" ultimately means to make the telescope look like a mirror with a specific ROC (8.6km in the case of the telescope with ETM surrogate mirror or 3.86m with flat mirror) at 9.05 cm away from the secondary. ROC and the position of the equivalent mirror should be the same for vertial and horizontal directions.

Therefore, our adjustment procedure is conceptually summarized as "inject a beam, and adjust the telescope until the wave front of the returning beam becomes almost unchanged (with ETM surrogate mirror) or with the curvature of the wavefront increased by $2/3.86\text{ m}^{-1}$ than injected (with a flat mirror) over the entire cross section".

No matter what the details of the procedure is, there is no doubt that we can set things

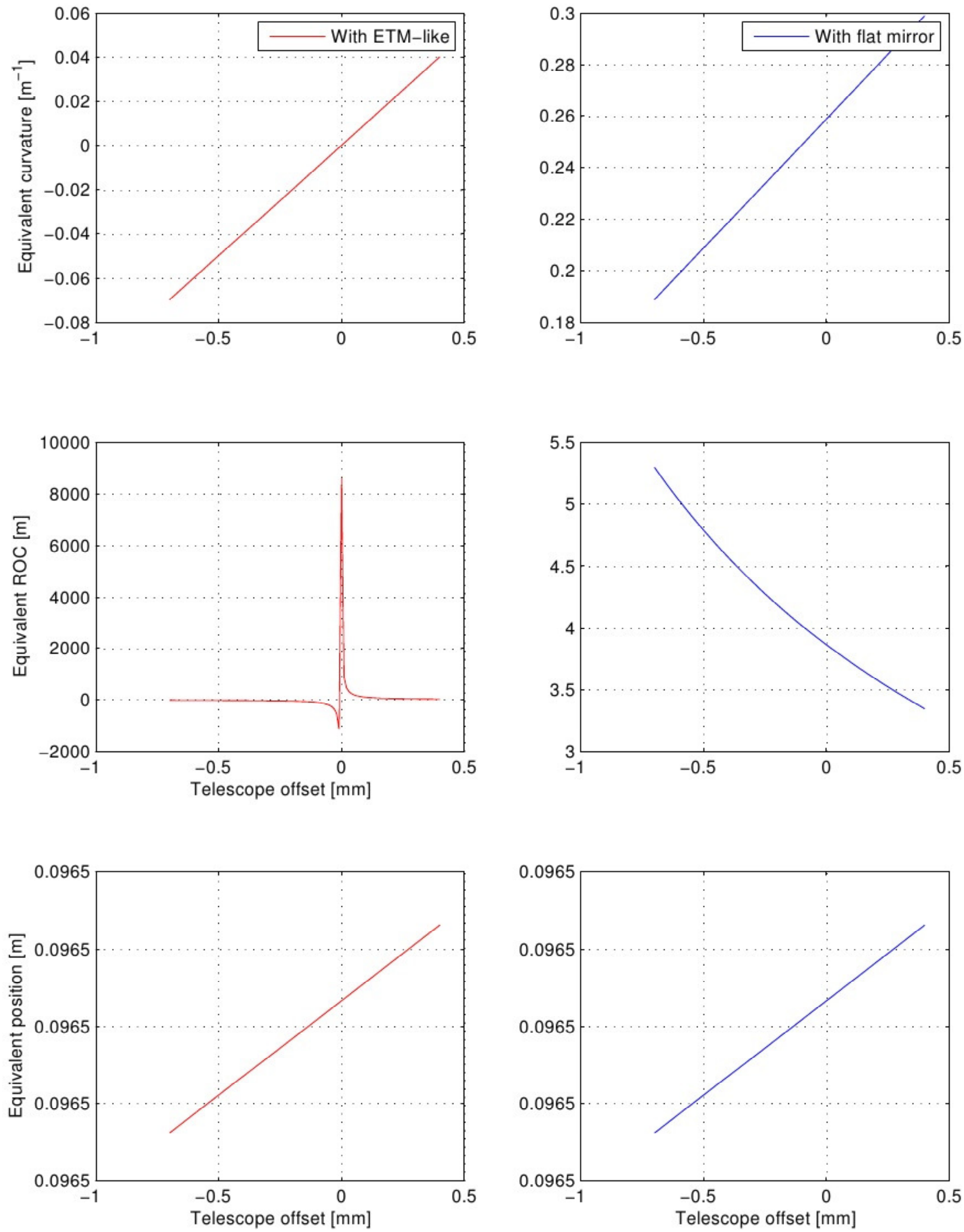

Figure 1: Offset in telescope length VS the curvature, ROC and the position of its equivalent one mirror system for two configurations, i.e. detuned+ETM and detuned+flat. $offset = 0$ means that the telescope is perfectly detuned.

up properly if our measurement apparatus is perfect. That is already demodstated e.g. in T1100258.

However, the real question is the quality of the beam we inject, the tolerance of the auxiliary optics we use, and the tolerance of the equipments. Without an asessment of these, the usefulness of the alignment procedure is quite limited.

3.1 Tolerance of the retro-reflecting mirror

If we use the ETM-surrogate mirror, the ROC should be 2245 ± 100 m and we need a large enough wedge.

If we use a flat mirror, the ROC should be larger than 40 km, which means that 6 inch diameter, 633 nm/10 flat optics suffices.

3.1.1 With ETM surrogate mirror

One way to adjust the telescope is to use ETM surrogate mirror with ROC of 2245 m placed in front of the primary. This mirror retro reflects the light sent from the secondary side back to the secondary. To mimic the real ETM, the light enters to the back surface end is reflected by the coating of the front surface.

Figure 2 shows the equivalent curvature of the telescope as a function of the detuning offset for the detuned telescope with ETM. Also shown are the traces for ETM-like mirrors that have +200 or -200 m different ROC than the ETM. In the actual testing process, the adjustment effort is ultimately equivalent of getting the "right" equivalent ROC. For the nominal ETM, this target is shown by a small circle at $offset = 0$ for a green trace. In the focusing procedure, one tries to set the curvature to be almost zero.

If the ROC of ETM surrogate mirror is smaller than the real ETM by 200m (red), and if we do not know that the ROC is off by this amount, one would set the detuning to $offset = 0.25$ mm to obtain a "correct" curvature, which is out of the tolerance. It's safer to specify ±100 m ROC error.

Note, however, that this doesn't mean we cannot use a mirror with, say, -300m ROC error, if the real ROC is known with ±100 m accuracy. In that case, we simply change the adjustment target by our knowledge about the true ROC.

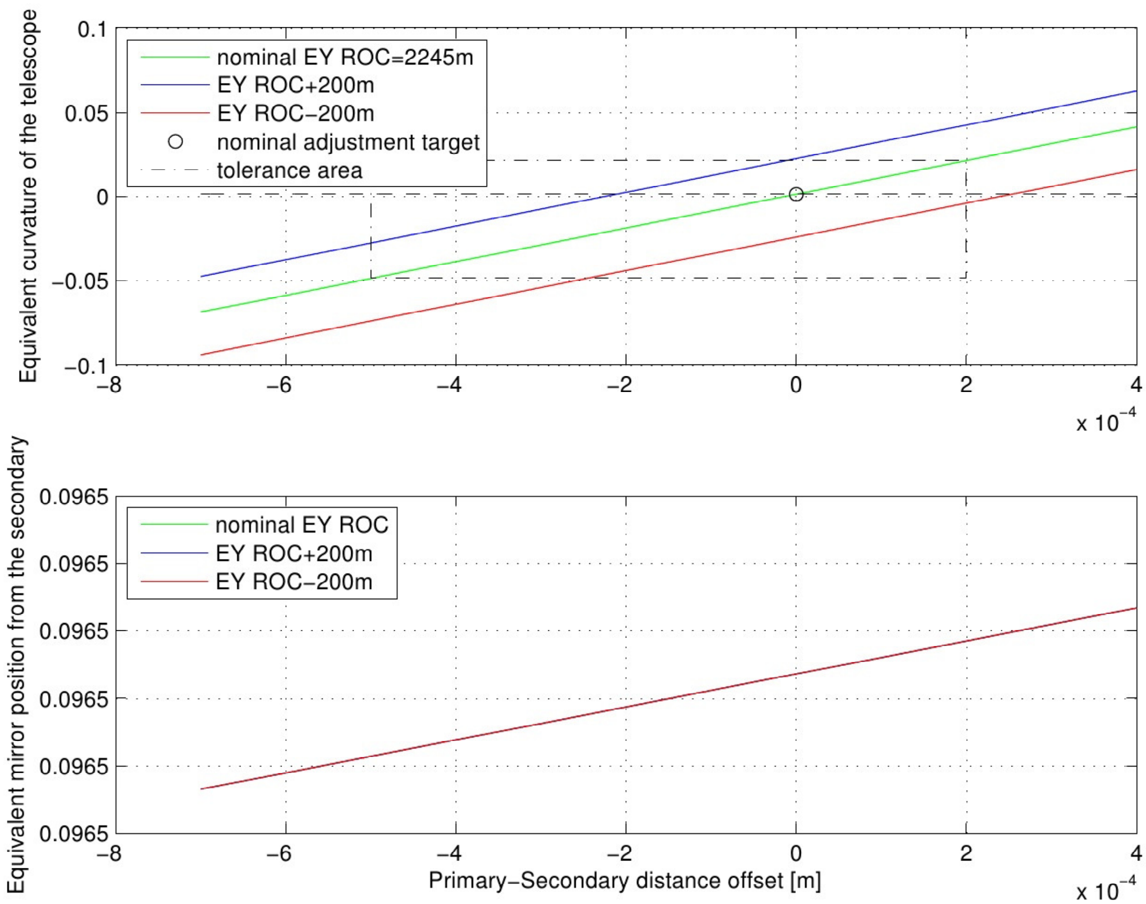

Figure 2: Tuning offset of the telescope VS the curvature and the position of the equivalent one-mirror system for telescope and a nominally ETM-like mirror. Nominally detuned position is at $offset = 0$. If the ROC of ETM-like mirror is 200m smaller than the actual ETM, it is already out of adjustment tolerance. We need something like +-100m ROC tolerance.

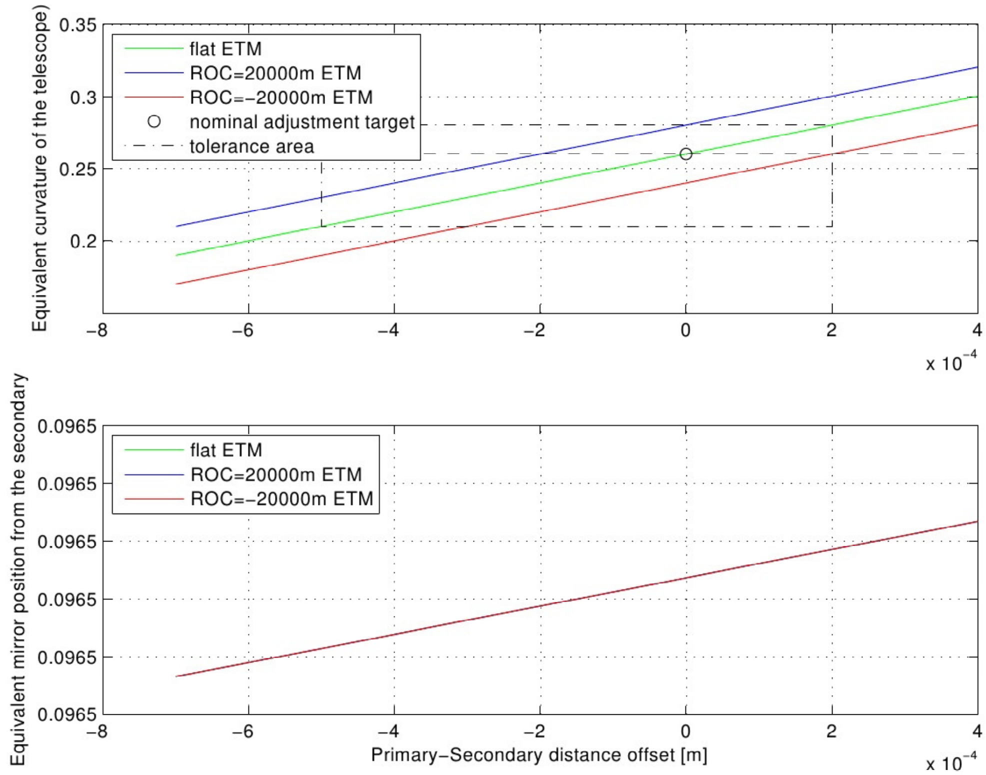

Figure 3: Tuning offset of the telescope VS the curvature and the position of the equivalent one-mirror system for telescope and a nominally flat retro reflecting mirror. ROC of 20km is already too small, and we need something like 40km. For a 6 inch mirror, 633nm/10 flat optics means roughly 45km ROC.

Requirement for the back surface figure TBD.

We also need a wedge for the substrate. Otherwise it is quite likely that the interference between the ghost beam produced by the back and the main beam will make the wave front distorted quite a large amount.

3.1.2 With a flat mirror

Figure 3 shows the equivalent curvature of the telescope as a function of the detuning offset for the detuned telescope with flat mirror, ROC=20km mirror and ROC=-20km mirror. It seems that $|ROC| > 20$ km as the requirement for the flat mirror is in this case equivalent of $ROC = 2245 \pm 200$ m in the case of the ETM-like mirror. If one uses a flat mirror, it's safer to specify $|ROC| > 40$ km. This is roughly equivalent of the $\lambda/10$ (633nm) for 6 inch flat mirror: Sag of 633nm/10 over 3 inch radius equals roughly 45km ROC.

3.2 Requirement for Shack-Hartman sensor

3.2.1 Wavefront sensitivity

The most important metric of a wavefront sensor is its wavefront sensitivity, which is a relative accuracy of the wavefront after subtracting the systematic in the hardware itself. For Thorlab Shack-Hartman sensors, depending on the model, this is either 1/30, 1/50, 1/100 or 1/150 lambda ($\lambda = 633$nm).

Shack-Hartman sensor shall have a resolution that assures that the telescope is inside the adjustment range in Figures 2 (with ETM surrogate) or 3 (with a flat mirror).

With ETM surrogate, the adjustment target range in terms of the curvature of the telescope is $-0.0485 < \rho < 0.0215$ [m^{-1}]. Roughly speaking, the target range of the curvature of the returning beam is twice as large as that of the telescope, or $-0.097 < \rho < 0.043$ [m^{-1}]. This means that we need a much better resolution than $0.043 + 0.097 = 0.14$ [m^{-1}]. This corresponds to about 633nm/2 of sag over a 2mm radius pupil. To reliably judge if the telescope with within this range, the sensor shall therefore have a much smaller sensitivity than 633nm/2. All models are adequate.

With a flat mirror, the situation is the same. Adjustment target range in terms of the curvature of the telescope is $0.210 < \rho < 0.280$ [m^{-1}], that of the returning beam is $0.42 < \rho < 0.56$ [m], therefore we need a much better resolution than $0.56 - 0.42 = 0.14$ [m^{-1}].

3.2.2 Sensor area

Thorlabs sensors have about 5×6 mm aperture, though the microlens array is larger. Therefore, practically, one is forced to set the limiting software aperture ("pupil") to be about 2mm or so. If one inject a beam with 3.1 mm diameter, one cannot observe the entire wavefront of the beam. This is not a fatal flaw, but one needs to verify the beam profile across the entire beam cross section using something else.

3.3 Quality of the injected beam

3.3.1 Use of Gaussian

It is very important to measure the quality of the injected beam, not only the wave front but also the intensity distribution, to make sure that we understand the propagation property of the beam. If we do not understand the propagation property of the beam, all adjustment effort is meaningless.

For all practical purposes, we should inject a reasonably Gaussian beam in the alignment procedure. One may be able to simulate the propagation of other beams (a beam with a hard shoulder, a beam coming out of a small iris etc.) in the telescope in principle, but such a simulation was never done, and is unneccesary if we inject a Gaussian.

3.3.2 Wavelength

There is no fundamental requirement for the wavelength of the light used, as the discussion based on the ABCD matrix is independent of the wavelength.

Of course any differences that come from using different wavelength should be taken into account when writing an alignment and tuning test procedure, e.g. if we are to use a surrogate, the focal length of the surrogate substrate should be calculated using a correct inex of refraction.

3.3.3 Beam size

There is no fundamental requirement for the beam size of the light injected except that the beam should sample about the same area as the 1064nm IFO beam coming from the arm cavity. Since the real interferometer beam has a waist radius (not a diameter) of about 3mm on the secondary mirror, the injected beam should reasonably mimic it. Much smaller than 3mm and it samples too small an area to be useful, much larger than 3mm and the beam is clipped by the tightest aperture, which is the large input aperture of the telescope on the ETM end.

Since the telescope is close to afocal (though not exactly), if you inject a beam of about 3mm radius, it samples the correct area of the telescope regardless of the wavelength.

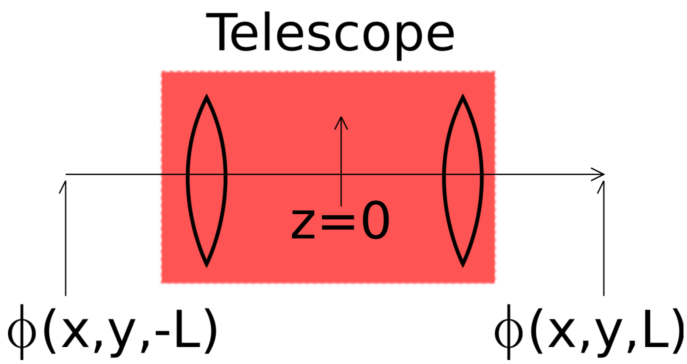

Figure 4: Simplified diagram of the TMS telescope double-path test setup.

4 Wave Front Subtraction Using Zernike Coefficient

There is nothing fundamentally wrong with comparing two wavefront shapes using Zernike coefficients. However, the implementation in T1100258 version 1 was flawed. The flaw seems to have been fixed by the version 2 document by the introduction of ETM-surrogate mirror a nice Gaussian light source.

4.1 Basic idea is solid

The idea behind telescope test procedure detailed in T1100258 is as follows: You place some mirror at the primary side so that the beam injected from the secondary side is reflected back to the secondary. Everything is set up in such a way that, when a beam with a specific profile is injected, the beam comes back to itself if the telescope is perfect. So you measure the wavefront shape of the injected beam and the returning beam, and make a comparison. The logic up to here is quite solid.

Suppose that the wavefront of the beam is written by $\phi(x, y, z)$ where z is the coordinate along the unfolded optical path. For convenience, without losing generality we can measure the injected wavefront at $z = -L$, place the retro-reflecting mirror at $z = 0$ and measure the reflected beam at $z = L$ (Figure 4).

The assumption is that, if you have a perfect TMS telescope, you can realize the condition where the wavefront of the retro-reflected beam perfectly matches that of the beam going in to the telescope, i.e.

$$\phi(x, y, -L) = \phi(x, y, L). \tag{1}$$

Therefore, according to this assumption, you can simply subtract the wave front map

$$\delta\phi(x,y) = \phi(x,y,-L) - \phi(x,y,L). \tag{2}$$

By analyzing this quantity, you draw some conclusion about the quality of the telescope alignment and optics themselves in terms of astigmatism as well as the spacing between the primary and the secondary.

4.2 You Also Have To Know How the Wavefront Propagates

In the idea described above, there is an important assumption that one knows how the wavefront propagates. Without this, wavefront comparison becomes meaningless.

The problem arises when one only measures Zernike coefficients, i.e. wavefront shape, of the injected beam of unknown quality. The wavefront shape itself doesn't tell us anything about its propagation. To reliably predict how the wavefront evolves through the telescope, you also need the intensity profile.

If the beam is for example a round Gaussian, this is not the problem as it is quite easy to calculate the wavefront propagation. T1100258 doesn't encourage this. It uses a beam launched from a fiber coupler and an aspheric lens, and the beam indeed has a hard shoulder with an outer ring (which is clearly visible when the beam is collimated). On the other hand, zmax simulation in T1100258 basically only looks at a Gaussian beam, so there might be a disconnect here.

T1100258 procedure only mandates to measure the wavefront shape of the injected beam at two specific locations, and also sets some restriction about the beam size and the waist position, which doesn't assure that the beam is close to Gaussian.

4.3 Simple example

Here is an overly simplified example to illustrate the potential problem in the procedure in T1100258.

In this toy model, injected Gaussian field u_{00} has a waist at $z = -L$ with the waist radius of w_0. Nominally, the beam travels through the telescope and is transformed to U_{00} which has a waist at $z = 0$ and the waist radius of W_0. This is reflected by a flat mirror. After traveling back through the telescope it is transformed again to u_{00} itself.

Now, suppose that we have a perfect telescope but the injected beam is elliptic. The wavefront is still perfectly flat at $z = -L$, the waist radius in x direction w_{x0} is still w_0 but the waist radius in y direction w_y is w_1. Because of this, the beam waist after the single path in y direction is not exactly at $z = 0$, and the waist size is not exactly W_0 either. In other words, the wave front at $z = 0$ is saddle-shaped, not flat. Since our retro-reflection mirror doesn't match the wave front shape of the incoming beam, the return beam is further deformed, and the wave front at $z = +L$ looks more saddle shaped.

Figure 5 shows the toy model comprising a single lens, single flat mirror and an elliptic Gaussian beam. Though the parameters are unrealistic, the logic still holds for the real telescope. The beam waist is at $z = -0.7$, the telescope is reduced to a single lens with 1m focal length placed at about $z = -0.33$, and the beam is retro reflected at $z = 0$. For x direction the beam waist radius is chosen such that the beam comes back exactly on itself at $z = 0.7$. For y it's not the case because the injected waist radius is somewhat larger. Note that, since the injected wave front was measured flat at $z = -0.7$, the fact that it is elliptic is not captured by Zernike coefficients corresponding to the astigmatism. Looking at the returning beam, Zernike coefficients capture the saddle-shaped wave front. However, without the knowledge that it was elliptic to start with, one cannot use the measurement of the returning beam to characterize the telescope itself (in this case a single lens). Neither a simple wave front subtraction nor a simple analysis based on Zernike coefficients work.

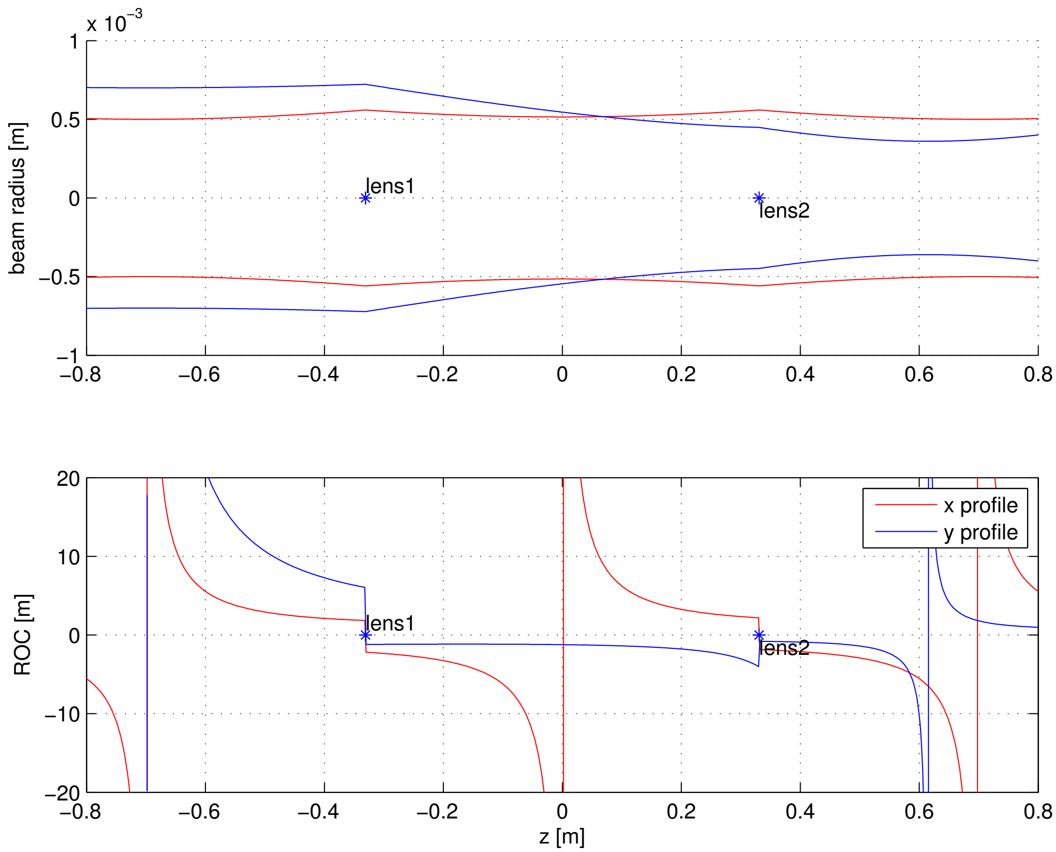

Figure 5: Toy telescope with elliptic input beam. Injected beam waist is at $z = -0.7$, the distance between the waist and the lens is about 0.37 m, and the beam is retro reflected by a flat mirror at $z = 0$. The beam comes back to itself for x direction but not for y, and as a result the wavefront becomes saddle shaped at $z = +0.7$ although the injected beam is flat at $z = -0.7$.