LIGO Scientific Collaboration

VIRGO

Material Loss Angles from Direct Measurements of Broadband Thermal Noise: Statistical Analysis and Preliminary Comparison with Results from Mixture Theory

Akira E. Villar, Eric. D. Black, Kenneth G. Libbrecht
California Institute of Technology, LIGO Lab

Maria Principe, Innocenzo M. Pinto, Vincenzo Pierro, Riccardo De Salvo
WavesGroup, University of Sannio at Benevento and INFN

Cristophe Michel, Nazario Morgado, Laurent Pinard
Laboratoire Materiaux Avances, CNRS, Lyon FR

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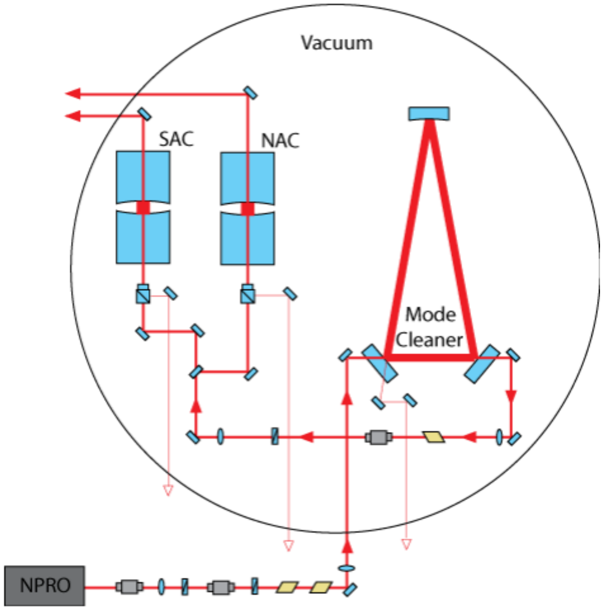
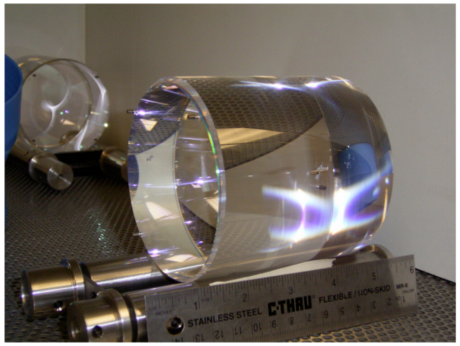
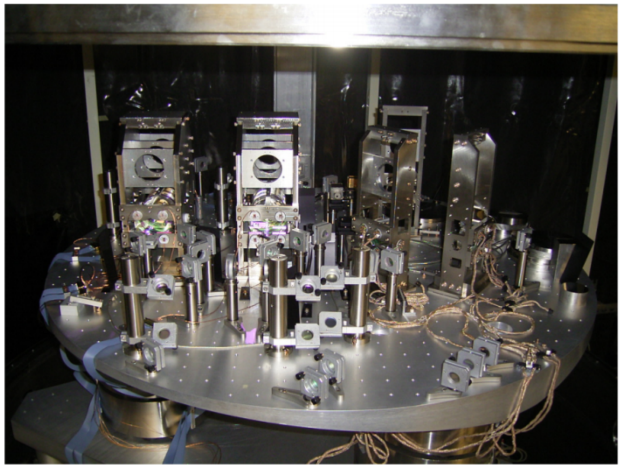
TNI Measurements

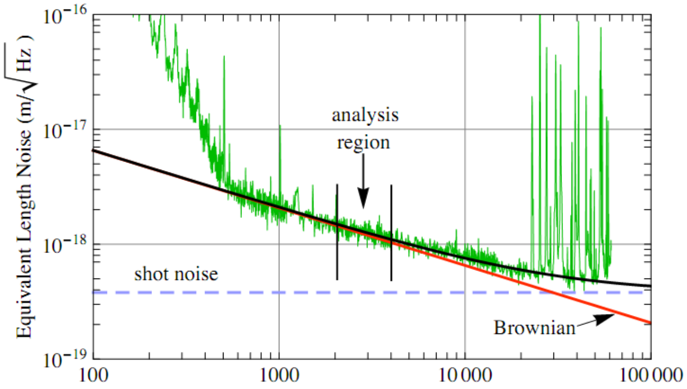






Spectrum Analyzer

calibration



fit data to model

[G. Harry et al., CQG 19 (2002) 897]

$$S_B(f) = \frac{2k_BT}{\pi^{3/2} f}\frac{(1-\sigma^2)}{wY}\phi_c$$

ϕ_c

Individual loss angles

Measurements on different coatings give 2 equations and 2 unknowns, relating Quarter-Wave (QWL) and Optimized (OPT) coating loss to the losses of Silica and Tantala layers

$$\phi_C^{(QWL)} = A\phi_{SiO_2} + B\phi_{Ta_2O_5}$$

$$\phi_C^{(OPT)} = C\phi_{SiO_2} + D\phi_{Ta_2O_5}$$

$$A = b_{SiO_2} d_{SiO_2}^{(QWL)} \qquad B = b_{Ta_2O_5} d_{Ta_2O_5}^{(QWL)}$$

$$C = b_{SiO_2} d_{SiO_2}^{(OPT)} \qquad D = b_{Ta_2O_5} d_{Ta_2O_5}^{(OPT)}$$

$$b_X = \frac{1}{\sqrt{\pi}}\left(\frac{Y_X}{Y_S} + \frac{Y_S}{Y_X}\right)$$

Loss Angle Statistics, I

Fastest way to do this is with matrix algebra:

$$\mathbf{M} \cdot \boxed{\boldsymbol{\phi}} = \mathbf{\Phi_c}$$

, with

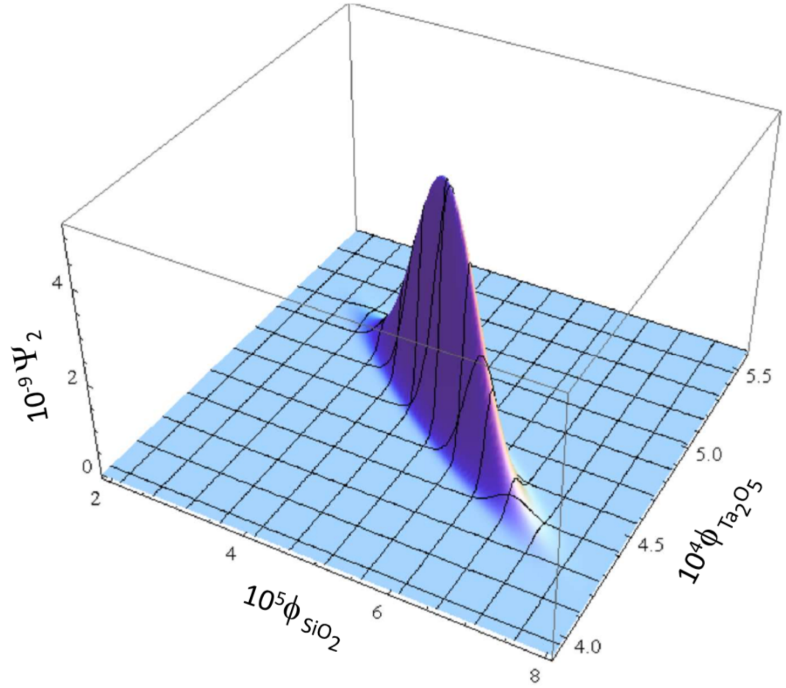
$$\left\{ \begin{array}{l} \mathbf{\Phi_c} = \left\{ \Phi_c^{(QWL)}, \Phi_c^{(OPT)} \right\} \\[2em] \boldsymbol{\phi} \;= \left\{ \phi_{SiO_2}, \phi_{Ta_2O_5} \right\} \\[2em] \mathbf{M} = \begin{bmatrix} b_{SiO_2} d_{SiO_2}^{(QWL)} & b_{Ta_2O_5} d_{Ta_2O_5}^{(QWL)} \\ b_{SiO_2} d_{SiO_2}^{(OPT)} & b_{Ta_2O_5} d_{Ta_2O_5}^{(OPT)} \end{bmatrix} \end{array} \right.$$

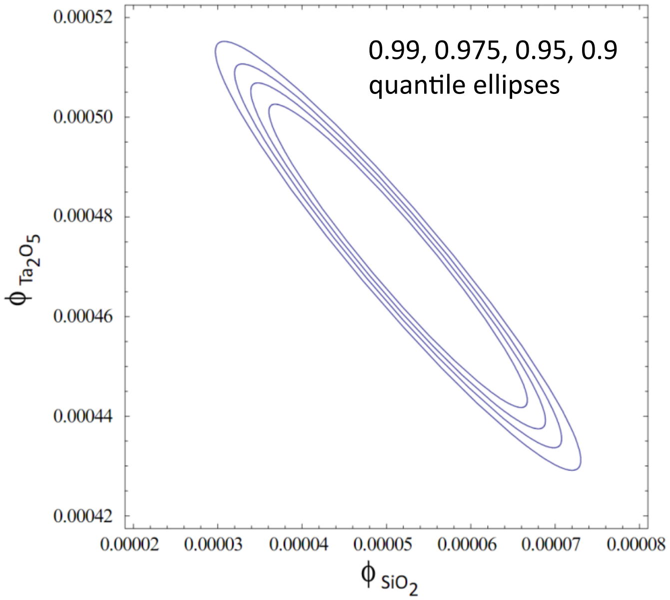
Error bars (assumed independent and Gaussian) are handled consistently, resulting into correlated, Gaussian-distributed material loss angles …

Loss Angle Statistics, II

$$E(\boldsymbol{\phi}) = \mathbf{M}^{-1} E(\boldsymbol{\Phi}_{\mathbf{c}}), \quad cov(\boldsymbol{\phi}) = \mathbf{M}^{-1} \cdot \begin{bmatrix} \sigma^2_{\Phi_c^{(QWL)}} & 0 \\ 0 & \sigma^2_{\Phi_c^{(OPT)}} \end{bmatrix} \cdot \left[\mathbf{M}^{-1}\right]^T$$

Hence, the TNI measurements of QWL and OPT coatings based on plain Tantala give:



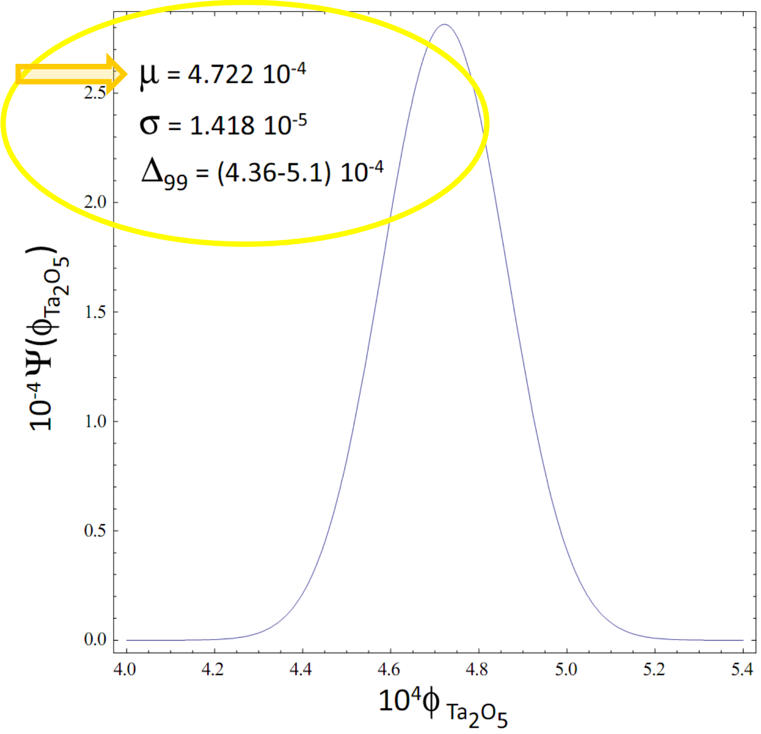


0.99, 0.975, 0.95, 0.9
quantile ellipses

Marginal Distributions

The *marginal* distributions (which take into account uncertainties in *both* numbers) of Silica and plain Tantala loss angles are easily computed

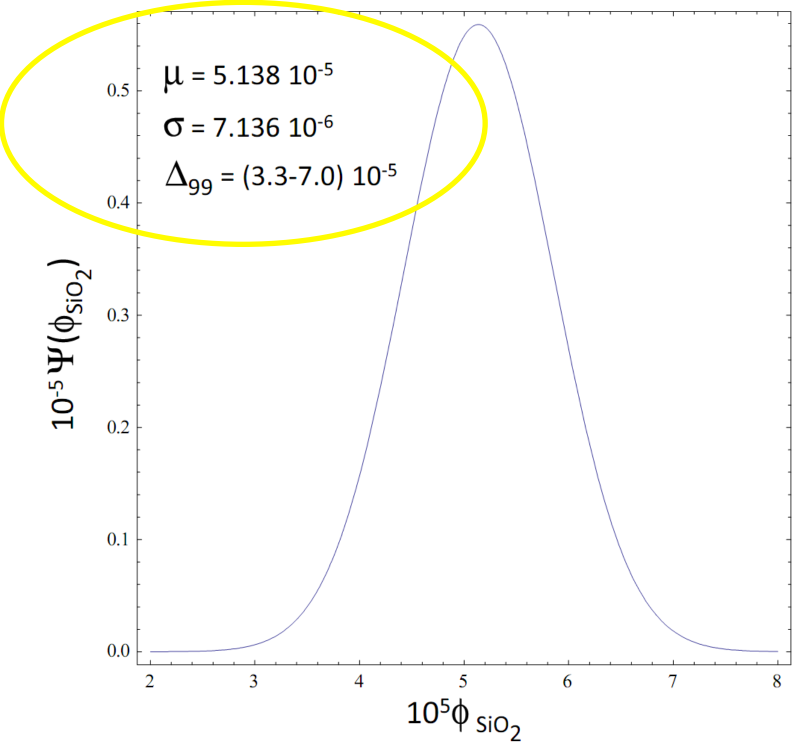
$$\Psi(\phi_{SiO_2}) = \int_{-\infty}^{\infty} d\phi_{Ta_2O_5}\Psi_2(\phi_{SiO_2},\phi_{Ta_2O_5}), \quad \Psi(\phi_{Ta_2O_5}) = \int_{-\infty}^{\infty} d\phi_{SiO_2}\Psi_2(\phi_{SiO_2},\phi_{Ta_2O_5})$$



μ = 4.722 10^{-4}

σ = 1.418 10^{-5}

Δ_{99} = (4.36-5.1) 10^{-4}



μ = 5.138 10^{-5}

σ = 7.136 10^{-6}

Δ_{99} = (3.3-7.0) 10^{-5}

Doped Tantala

Doped Tantala is harder because the system is *ill-conditioned*, or

$$\det M \approx 0$$

However, the low-index layers here are made from fused silica, i.e., the same material used in the undoped coatings. This means we can use the previous value for the fused-silica loss,

$$\phi_{SiO_2} = (5.138 \pm 0.714) \times 10^{-5}$$

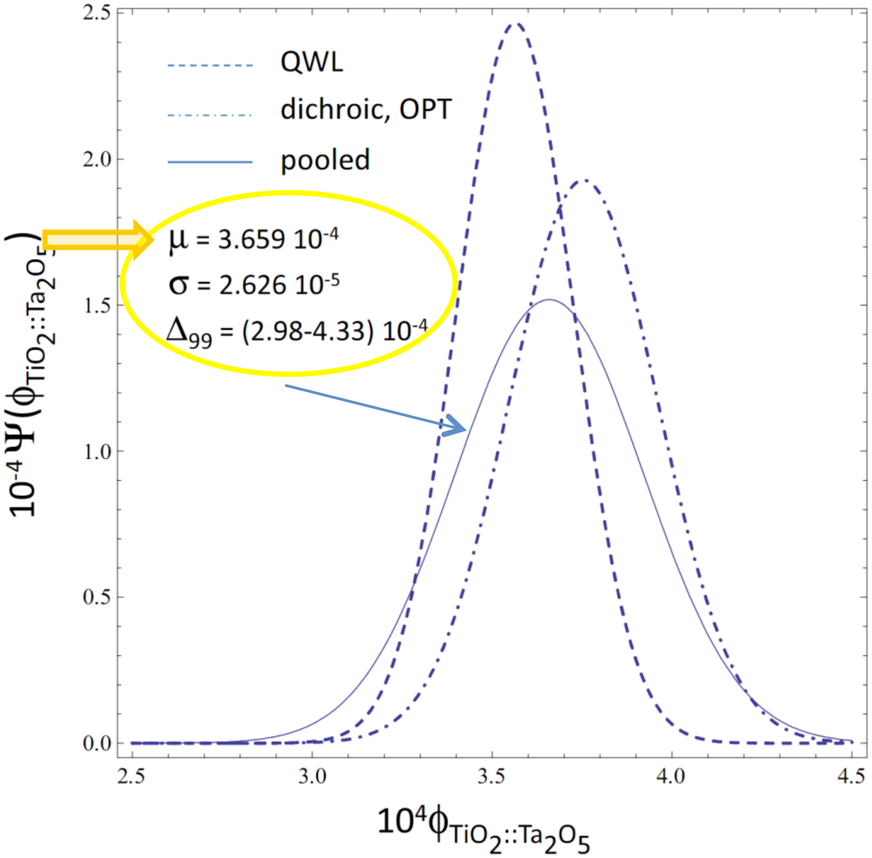
leaving us with only one unknown. We still have two equations, and we can use this previous value for the silica loss to get two (Gaussian) distributions for the doped-tantala loss. We expect them to agree, and they do, yielding

$$\phi_{Ta_2O_5:TiO_2} = (3.56 \pm 0.16) \times 10^{-4}$$

$$\phi_{Ta_2O_5:TiO_2} = (3.75 \pm 0.21) \times 10^{-4}$$

Doped Tantala

We may pool (combine) the two distributions of the doped-Tantala loss. Average and uncertainties propagate in the usual way.

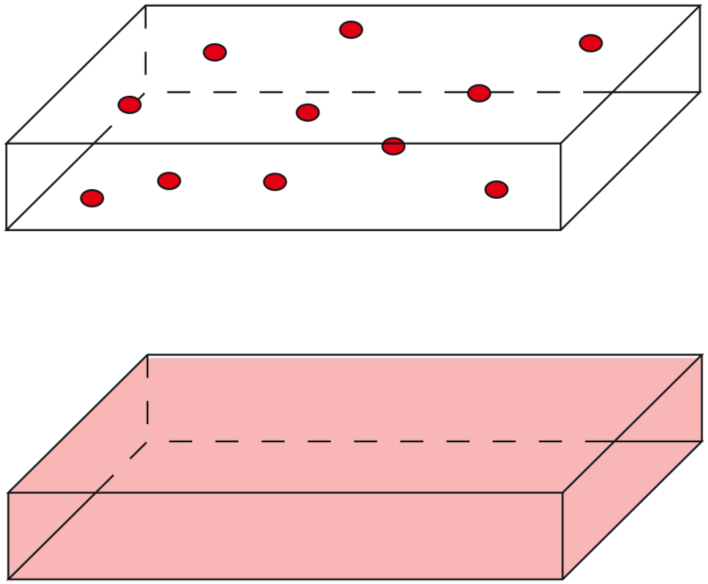


$$\langle \phi_{TiO_2::Ta_2O_5} \rangle = \frac{\langle \phi_{TiO_2::Ta_2O_5} \rangle_1 + \langle \phi_{TiO_2::Ta_2O_5} \rangle_2}{2}$$

$$\sigma_{\phi_{TiO_2::Ta_2O_5}} = \left(\sigma^2_{\phi_{TiO_2::Ta_2O_5},1} + \sigma^2_{\phi_{TiO_2::Ta_2O_5},2} \right)^{1/2}$$

$$\phi_{Ta_2O_5:TiO_2} = (3.66 \pm 0.26) \times 10^{-4}$$

Effective-Medium Theory



Composite materials (mixtures) can be modeled by an appropriately-weighted average of macroscopic properties of both components.

Replace actual, “composite” system with a homogeneous, “effective” medium.

Effective for a wide variety of properties
- dielectric constant
- index of refraction
- elastic modulus
- loss angle
- etc.

Results depend somewhat on inclusion concentration, morphology, orientation.

Titania Fraction

First model the index of refraction and check the Titania volume fraction.
(Use Bruggemann formula for modeling the effective medium.)

$$\eta_2 \frac{\epsilon_2 - \epsilon_{mix}}{\gamma\epsilon_2 + (1-\gamma)\epsilon_{mix}} + (1-\eta_2)\frac{\epsilon_1 - \epsilon_{mix}}{\gamma\epsilon_i + (1-\gamma)\epsilon_{mix}} = 0 \ , \quad \epsilon = n^2$$

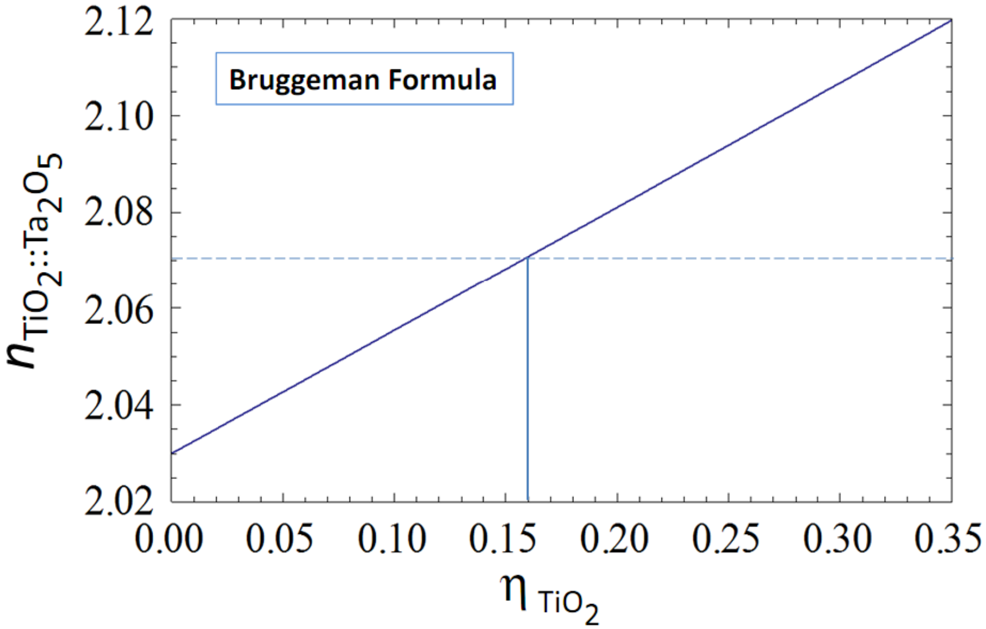
using (LIGO) reference values for pertinent optical indexes

$$n_{Ta_2O_5} = 2.03$$

$$n_{TiO_2} = 2.29$$

$$n_{TiO_2::Ta_2O_5} = 2.07$$

⟹ $\eta = 0.16$



Barta-Bruggeman EMT

Next calculate Young's modulus and Poisson's ratio.

Barta's microscopic derivation of Bruggemann-like mixture formulas for viscoelastic parameters of a glassy-oxide composite yields

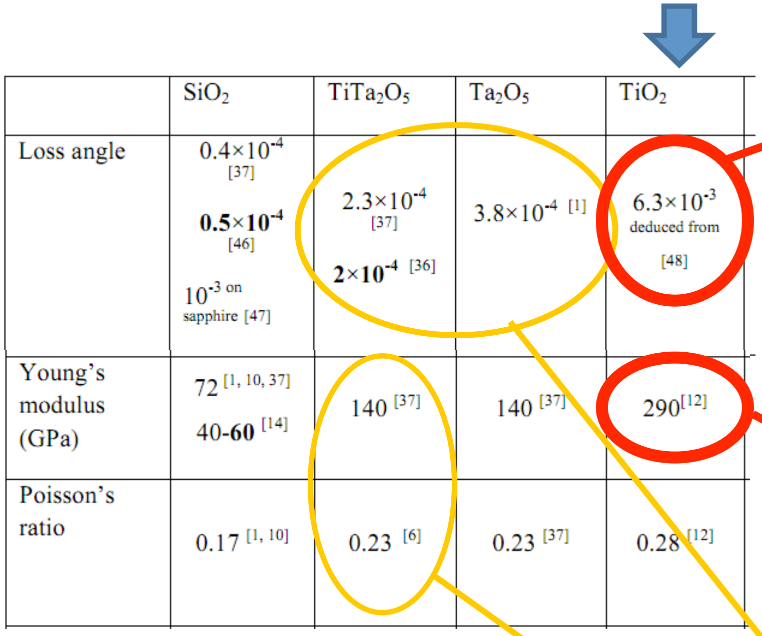
$$\begin{cases} (1-\eta_2)\dfrac{X-X_1}{2X+(X_1/y_1)(\sigma_1+1)}+\eta_2\dfrac{X-X_2}{2X+(X_2/y_2)(\sigma_2+1)}=0 \\ (1-\eta_2)\dfrac{(X/y)-(X_1/y_1)}{2X+(X_1/y_1)(\sigma_1+1)}+\eta_2\dfrac{(X/y)-(X_2/y_2)}{2X+(X_2/y_2)(\sigma_2+1)}=0 \end{cases},$$

$$X=\frac{\sigma Y}{\sigma+1},\quad y=\sigma-2$$

System can be solved in closed form . [S. Barta, «Effective Young modulus and Poisson's ratio for the particulate composite," J. Appl. Phys. 75 (1994) 3258].

Titania Parameters

Frequently quoted values must be handled w. care, especially for Titania …



	SiO_2	$TiTa_2O_5$	Ta_2O_5	TiO_2
Loss angle	0.4×10^{-4} [37] **0.5×10^{-4}** [46] 10^{-3} on sapphire [47]	2.3×10^{-4} [37] **2×10^{-4}** [36]	3.8×10^{-4} [1]	6.3×10^{-3} deduced from [48]
Young's modulus (GPa)	72 [1, 10, 37] **40-60** [14]	140 [37]	140 [37]	290[12]
Poisson's ratio	0.17 [1, 10]	0.23 [6]	0.23 [37]	0.28 [12]

J. Franc et al., ET-021-09 (2009).

Values from a *single* experiment: 25-doublets QWL Silica/Titania coating [P. Amico et al., J. Phys. Conf. Ser. 32 (2006) 413]. **Thickness of Titania layers was 116nm. Well above limit-thickness for preventing crystallization upon annealing** [S. Chao et al., J. Opt. Soc. Am. A16 (1999) 1477]. **Reported loss angle most likely due to crystallization.**

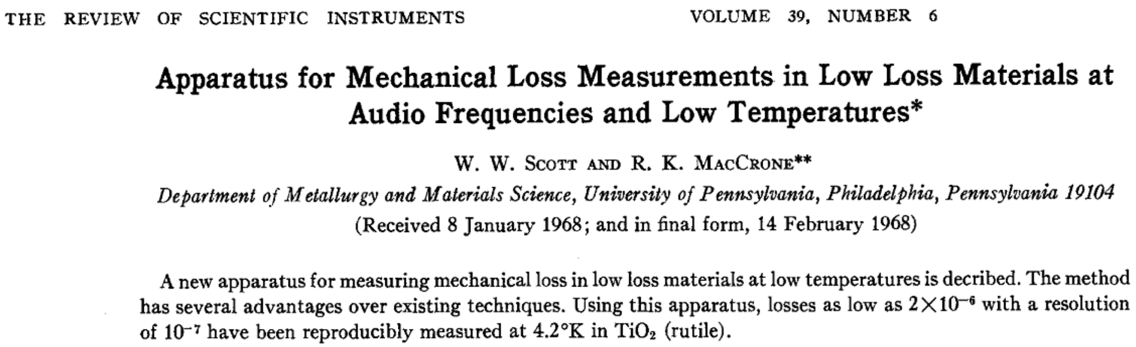
In the amorphous phase Y_{TiO_2} is 160 - 170 Gpa. [T. Modes et al., Surf. Coat. Technol. 200 (2005) 306]

Quoted Y = 290 Gpa is OK for the *crystalline* (Rutile) phase [O. Zywitzki, et al., Surf. Coat. Technol. 180 (2004) 538].

Large spreads among values obtained from different measurement techniques…

Conjectured values - *No* direct measurements of Y or σ on doped Tantala reported yet.

A Direct Measurement of TiO2 Loss Angle



THE REVIEW OF SCIENTIFIC INSTRUMENTS VOLUME 39, NUMBER 6 JUNE 1968

Apparatus for Mechanical Loss Measurements in Low Loss Materials at Audio Frequencies and Low Temperatures*

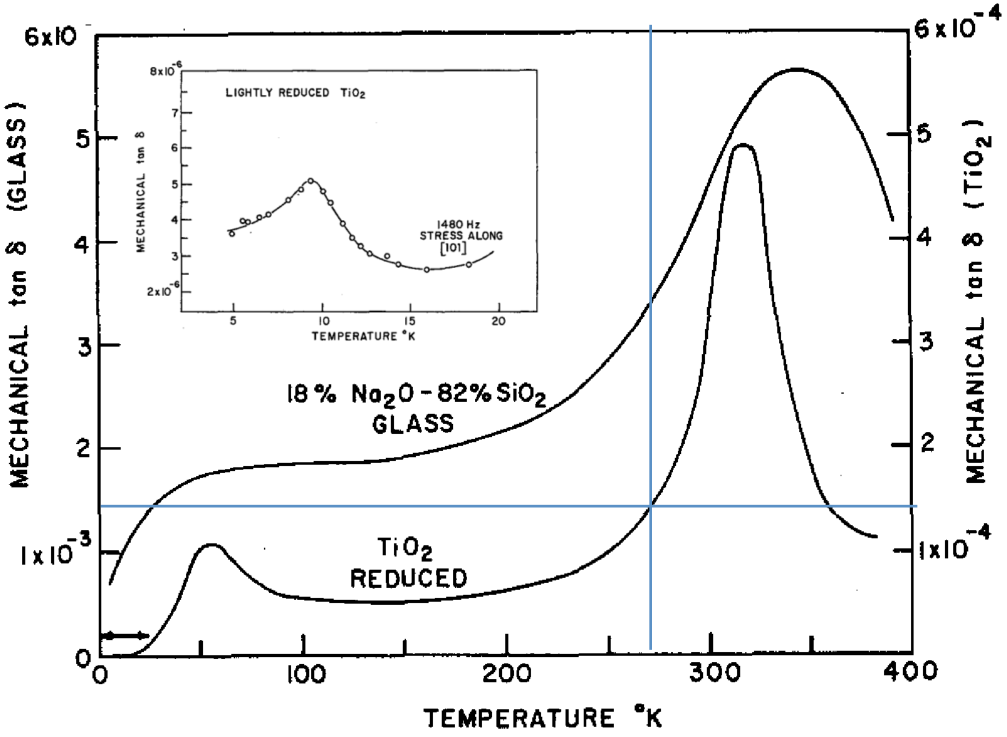
W. W. SCOTT AND R. K. MACCRONE**

Department of Metallurgy and Materials Science, University of Pennsylvania, Philadelphia, Pennsylvania 19104

(Received 8 January 1968; and in final form, 14 February 1968)

A new apparatus for measuring mechanical loss in low loss materials at low temperatures is decribed. The method has several advantages over existing techniques. Using this apparatus, losses as low as 2×10^{-6} with a resolution of 10^{-7} have been reproducibly measured at 4.2°K in TiO_2 (rutile).

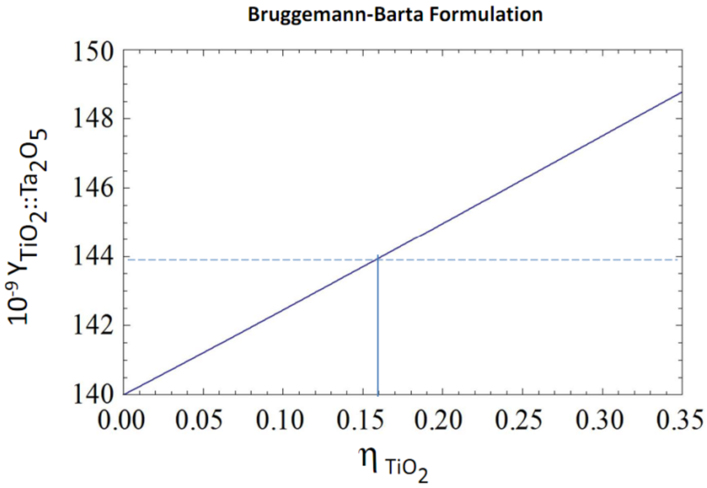
- Introduces an apparatus *conceptually similar* to the familiar «cantilever».
- Reported results over a wide range of temperatures from 5K to 400K…
- Little details about tested material.

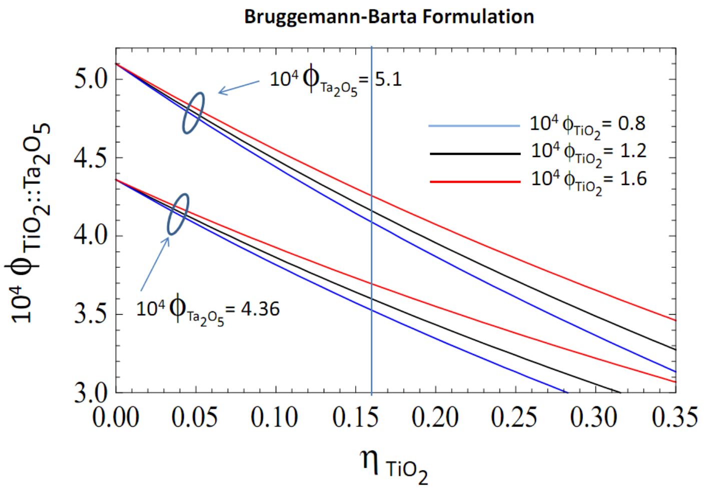


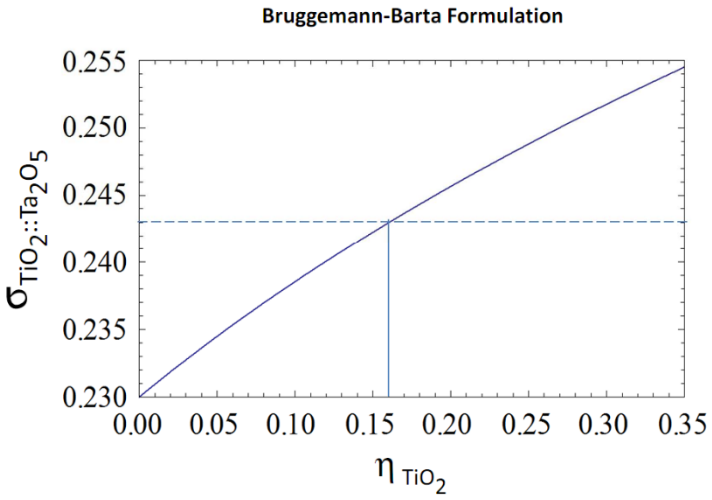
Doped Tantala from EMT

We use Barta-Bruggemann formula to estimate the viscoelastic params of doped Tantala, using reference values for Titania and plain Tantala

$$Y_{TiO_2} = 165GPa$$
$$Y_{Ta_2O_5} = 140GPa$$
$$\sigma_{Ta_2O_5} = 0.23$$
$$\sigma_{TiO_2} = 0.28$$





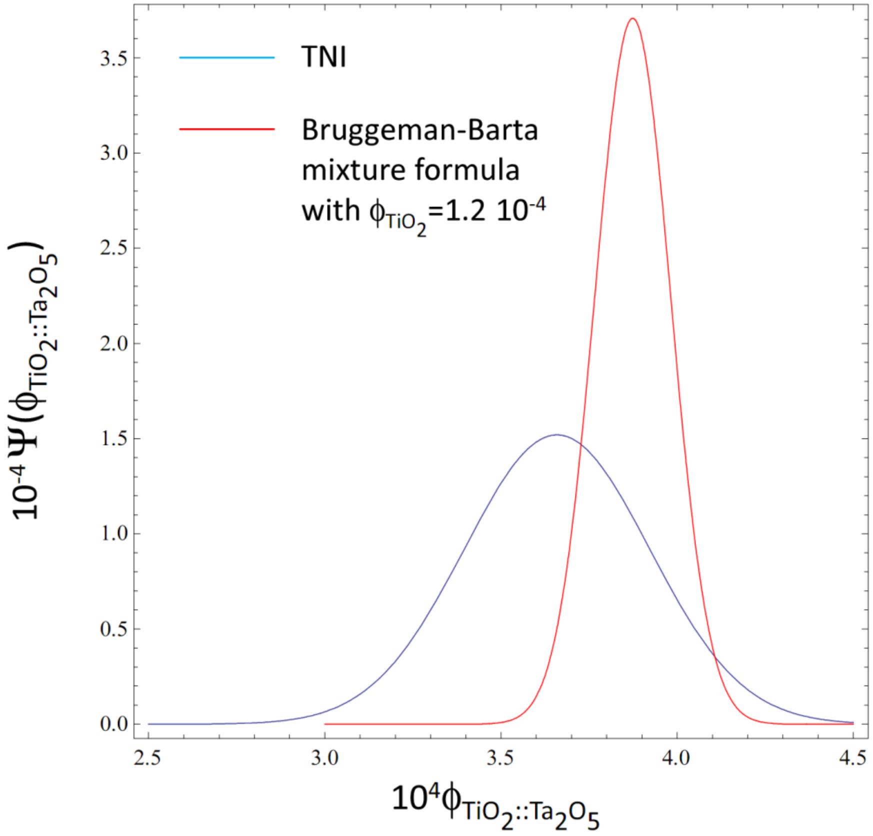


Mixtures vs Measured

We can compare the doped Tantala loss angle distribution obtained from TNI measurements to the prediction of EMT, using Scott-MacCrone loss angle for Titania, and the TNI result for plain Tantala

TNI : distribution deduced from doped coating measurement, using the marginal distribution of Silica loss angle from the undoped coating measurements.

Bruggeman-Barta : distribution deduced using Scott-MacCrone value for Titania loss angle, with plain Tantala loss-angle distribution from un-doped coating measurements.



Conclusions

1. First extraction of individual-material loss angles from direct observations of thermal noise.
2. Simple predictive model for composite material properties including elastic moduli, loss angle.
3. Loss angles derived from direct noise measurements not entirely in agreement with Qs.
4. Loss-angle ratios between doped and undoped are consistent.

Caveats:

- a) Agreement is good, but only one material (doped tantala) tested.
- b) Reliable values for pure materials (components) still needed.

Next: predict and test other materials, and/or other morphologies (subwavelength films)

Back Up Material

Conclusions/Issues

TNI allows the direct measurement of coating noise spectra, and the estimation of coating loss and material loss angles. Experiments were carried out on four different Prototypes, two of which used Titania doped Tantala.

Estimated loss angles are fully consistent with early results in [S.D. Penn et al., CQG 20, 2917 (2003)], but e.g. differ from those by LMA [Ch. Comtet, L-V thermal Noise Workshop, Cascina, 2007] and Glasgow [I. Martin, PhD Thesis, 2009], based on different setups, which yield losses larger for Silica and smaller for (doped) Tantala.
The loss-angle ratio between the doped and undoped material is nonetheless almost the same.

The estimated loss angle distribution for doped Tantala is consistent with that predicted by EMT, using the estimated loss angle distribution of plain Tantala, and a fiducial value for Titania loss angle from Scott-MacCrone.

Loss Angle Numbers

D. Crooks et al., CQG 23 (2006), 4953.
G. Harry et al., CQG 24 (2007) 405.

Material	**Loss Angle (x 10^4)**
SiO_2	1.0 ± 0.2
Ta_2O_5	3.8 ± 0.2 ; 3.0 ± 0.2 (‖)
LMA no. 5	2.0 ± 0.1 (‖)

76.2 mm Ø x 25.4mm substrate + 30 QWL doublets

P.G. Murray, PhD Thesis, Glasgow (2008).

Dopant (cation conc.)	**Mixture**	**Loss Angle (x 10^4)**	
		CSIRO	LMA
none	Ta_2O_5	3.8 ± 0.1	4.4 ± 0.2
15% TiO_2	TiO_2::Ta_2O_5	2.4 ± 0.2	2.0 ± 0.2
35% SiO_2	SiO_2::Ta_2O_5	2.5 ± 0.4	-
35% TiO_2	TiO_2::Si_2O_2	1.7 ± 0.4	-

76.2 mm Ø x 25.4mm substrate + 30 QWL mixture/silica doublets

Warning: - uses *naive* mixture formulas for Y and σ;
- assumes $Y_{TiO_2} \cong 290$ Gpa.

C. Comtet, LV –TNW 07, Cascina (2007).

Material	**Loss Angle (x 10^4)**
Ta_2O_5	3.02 ± 0.15
LMA no. 5	2.47 ± 0.05
LMA no. 5**	1.93 ± ?

LMA cantilever testbench

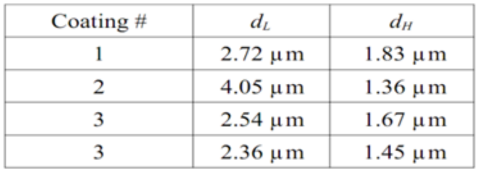
I. Martin, PhD Thesis, Glasgow (2009).

Coating (LMA)	**Cantilever**	**Loss Angle (x 10^4)**
Ta_2O_5	unwelded	4.2 ± 0.3
LMA no. 5*	unwelded	3.0 ± 0.01
LMA no. 5**	welded	1.5 ± 0.1
LMA no. 5***	welded	2.0 ± 0.4
SiO_2	unwelded	0.97 ± 0.02

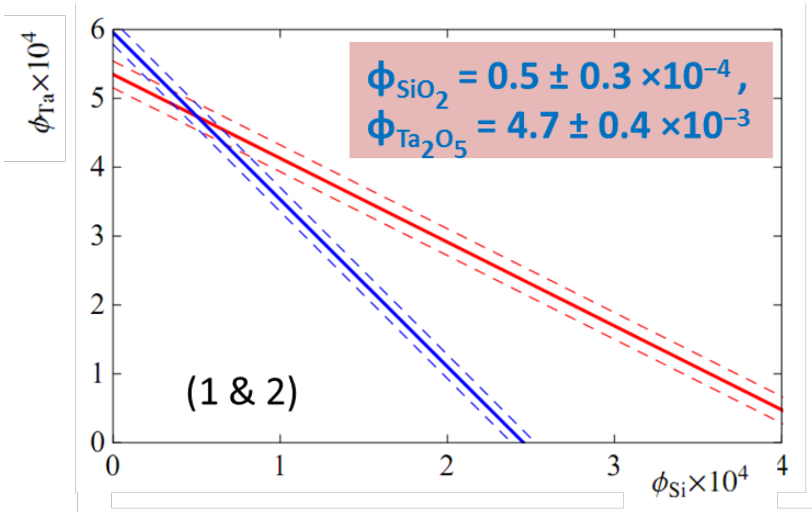
110 μm thick cantilever + 0.5 μm single layer

TNI Coating Prototypes

Coating	Type	Low Index	High Index	Mftr.	$\phi_c \times 10^6$
1	QWL	Silica	Tantala	REO	8.25 ± 0.3
2	Optimized	Silica	Tantala	LMA	6.85 ± 0.2
3	QWL	Silica	Doped Tantala	LMA	6.0 ± 0.25
4	Dichroic	Silica	Doped Tantala	LMA	5.5 ± 0.5

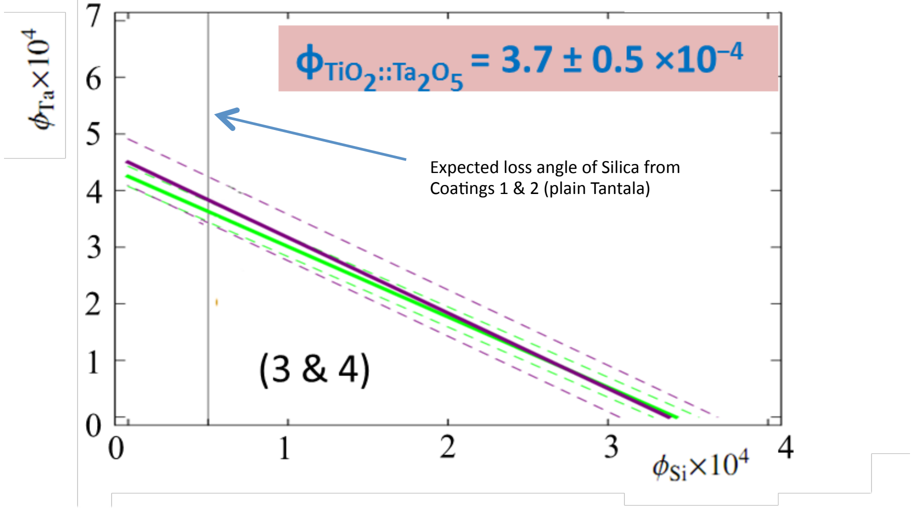


Coating #	d_L	d_H
1	2.72 µm	1.83 µm
2	4.05 µm	1.36 µm
3	2.54 µm	1.67 µm
3	2.36 µm	1.45 µm



$\phi_{SiO_2} = 0.5 \pm 0.3 \times 10^{-4}$,

$\phi_{Ta_2O_5} = 4.7 \pm 0.4 \times 10^{-3}$



$\phi_{TiO_2::Ta_2O_5} = 3.7 \pm 0.5 \times 10^{-4}$

Expected loss angle of Silica from Coatings 1 & 2 (plain Tantala)

A. Villar et al., LIGO-G1000937 (2010)