A Modelled Cross-Correlation Search for Gravitational Waves from Scorpius X-1

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Outline

1. Searches for Gravitational Waves
   - Crash Course in Gravitational Wave Physics
   - Gravitational-Wave Observations & Detectors
   - Gravitational Waves from Low-Mass X-Ray Binaries
2. Cross-Correlation Method
   - Application to Stochastic Background
   - Application to Quasiperiodic Gravitational-Wave Signals
   - Tuning Search by Choice of Data Segments to Correlate
3. Application to LMXB Searches
   - Parameter Space Metric
   - Sensitivity Estimates
   - Summary

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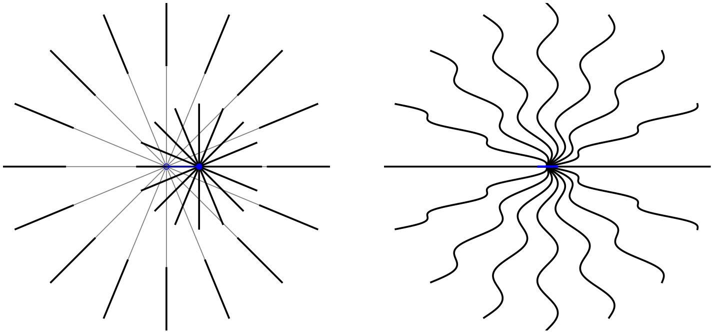
Motivation



- In Newtonian gravity, force dep on distance btwn objects
- If massive object suddenly moved, grav field at a distance would change instantaneously
- In relativity, no signal can travel faster than light $\longrightarrow$ time-dep grav fields must propagate like light waves

Gravity as Geometry

- Minkowski Spacetime:

$$ds^2 = -c^2(dt)^2 + (dx)^2 + (dy)^2 + (dz)^2$$
$$= \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix}^{\text{tr}} \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} dt \\ dx \\ dy \\ dz \end{pmatrix} = \eta_{\mu\nu} dx^\mu dx^\nu$$

- General Spacetime:

$$ds^2 = \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix}^{\text{tr}} \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \begin{pmatrix} dx^0 \\ dx^1 \\ dx^2 \\ dx^3 \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu$$

Gravitational Wave as Metric Perturbation

- For GW propagation & detection, work to 1st order in $h_{\mu\nu} \equiv$ difference btwn actual metric $g_{\mu\nu}$ & flat metric $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

  ($h_{\mu\nu}$ "small" in weak-field regime, e.g. for GW detection)
- Convenient choice of gauge is transverse-traceless:

$$h_{0\mu} = h_{\mu 0} = 0 \qquad \eta^{\nu\lambda} \frac{\partial h_{\mu\nu}}{\partial x^{\lambda}} = 0 \qquad \eta^{\mu\nu} h_{\mu\nu} = \delta^{ij} h_{ij} = 0$$

  In this gauge:
  - Test particles w/constant coörds are freely falling
  - Vacuum Einstein eqns $\Longrightarrow$ wave equation for $\{h_{ij}\}$:

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right) h_{ij} = 0$$

Gravitational Wave Generation

- Generated by moving/oscillating mass distribution
- Lowest multipole is quadrupole

$$h_{\mu\nu} = \frac{2G}{c^4 d} P^{\mathrm{TT}\hat{k}\lambda\sigma}_{\mu\nu} \ddot{I\!\!\!\!-}_{\lambda\sigma}(t - d/c)$$

- Rotating neutron star w/non-axisymmetric perturbation gives sinusoidally-varying quadrupole moment
- Other sources: compact binary inspiral, bursts (supernova etc), stochastic backgrounds. . .

Gravitational Wave Polarization States

- Far from source, GW looks like plane wave prop along $\hat{k}$
  TT conditions mean, in convenient basis,

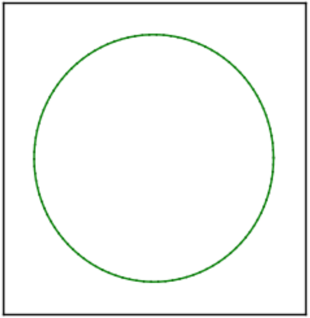
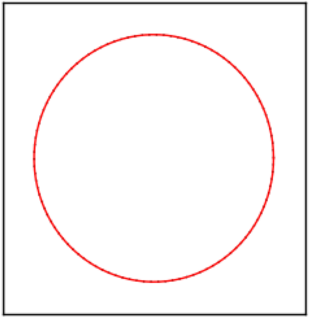
$$\{k_i\} \equiv \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \{h_{ij}\} \equiv \mathbf{h} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

  where $h_+\left(t - \frac{x^3}{c}\right)$ and $h_\times\left(t - \frac{x^3}{c}\right)$ are components in "plus" and "cross" polarization states
- More generally

$$\overleftrightarrow{h} = \left[h_+\left(t - \frac{\hat{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_+ + h_\times\left(t - \frac{\hat{k}\cdot\vec{r}}{c}\right)\overleftrightarrow{e}_\times \right]$$

Effects of Gravitational Wave

Fluctuating geom changes distances btwn particles in free-fall:

Plus (+) Polarization	Cross (×) Polarization
	

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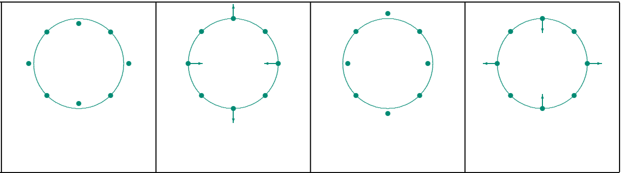
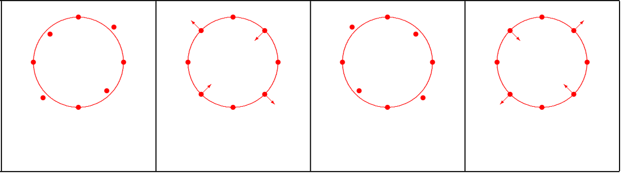
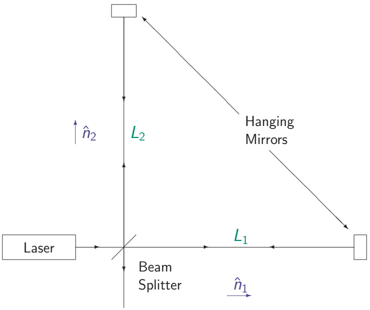
Plus (+) Polarization	
Cross (×) Polarization	

Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



- Measure small change in

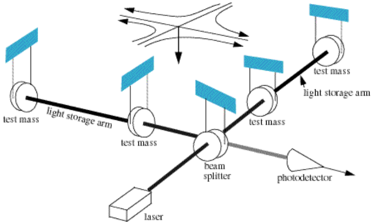
$$L_1 - L_2 = \sqrt{g_{11} L_0^2} - \sqrt{g_{22} L_0^2}$$
$$= \sqrt{(1 + h_{11}) L_0^2} - \sqrt{(1 + h_{22}) L_0^2}$$
$$\approx L_0 \frac{h_{11} - h_{22}}{2} \sim L_0 h_+$$

- More gen,
$(L_1 - L_2)/L_0 = \overleftrightarrow{h} : \overleftrightarrow{d}$
with "response tensor"
$$\overleftrightarrow{d} = \frac{\hat{n}_1 \otimes \hat{n}_1 - \hat{n}_2 \otimes \hat{n}_2}{2}$$

(also when $\hat{n}_1$ & $\hat{n}_2$ not $\perp$)

Measuring GWs w/Laser Interferometry

Interferometry: Measure GW-induced distance changes



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Rogues' Gallery of Ground-Based Interferometers

LIGO Hanford (Wash.)

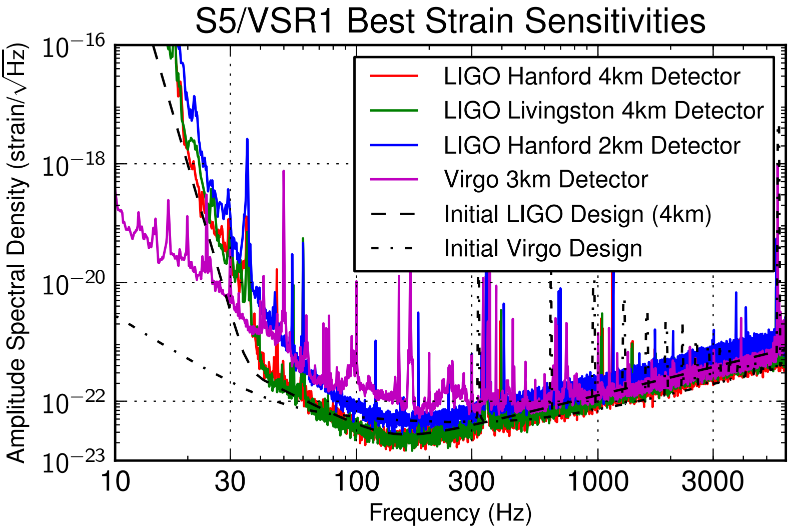
LIGO Livingston (La.)

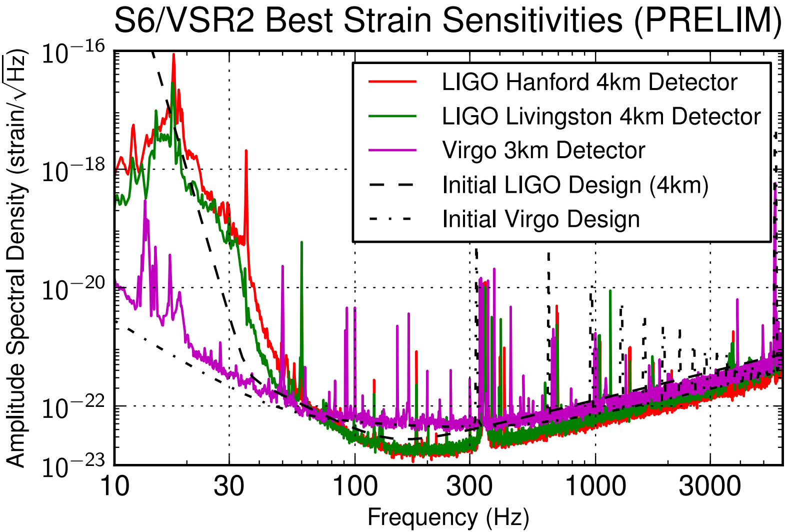
GEO-600 (Germany)

Virgo (Italy)

GW Observatory Network

- LSC detectors conducting science runs since 2002
  - LIGO Hanford (4km H1 & 2km H2)
  - LIGO Livingston (4km L1)
  - GEO-600 (600m G1)
- Virgo (3km V1) started science runs in 2007
- Recent long runs:
  - LIGO/GEO S5: Nov 2005-Sep 2007: LIGO @ design sens
  - Virgo VSR1: May-Sep 2007: Begin joint LSC-Virgo analysis
  - LIGO (H1 & L1) S6: Jul 2009-Oct 2010
  - Virgo VSR2 Jul 2009-Jan 2010 & VSR3 Aug-Oct 2010
  - Virgo VSR4 Jun-Sep 2011: joint run w/GEO-600
- LIGO & Virgo going offline 2010 & 2011
  to begin upgrade to Advanced Detectors
  expect $\sim 10\times$ sensitivity


S5/VSR1 Best Strain Sensitivities
Amplitude Spectral Density (strain/√Hz)
10^−16
10^−18
10^−20
10^−22
10^−23
10
30
100
300
1000
3000
Frequency (Hz)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
LIGO Hanford 2km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design


S6/VSR2 Best Strain Sensitivities (PRELIM)
LIGO Hanford 4km Detector
LIGO Livingston 4km Detector
Virgo 3km Detector
Initial LIGO Design (4km)
Initial Virgo Design
Amplitude Spectral Density (strain/√Hz)
10^{-16}
10^{-18}
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10^{-22}
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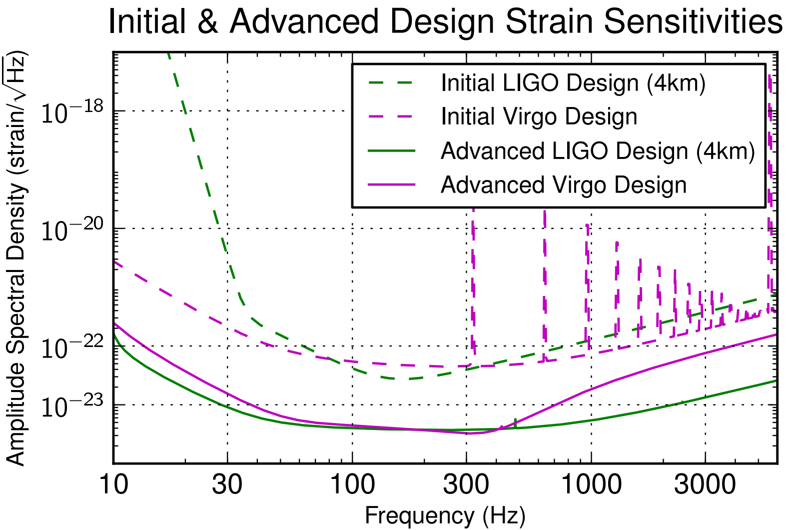
Frequency (Hz)


Initial & Advanced Design Strain Sensitivities
Initial LIGO Design (4km)
Initial Virgo Design
Advanced LIGO Design (4km)
Advanced Virgo Design
Amplitude Spectral Density (strain/√Hz)
10^−18
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Frequency (Hz)

Gravitational Waves from Low-Mass X-Ray Binaries

- LMXB: compact object (neutron star or black hole) in binary orbit w/companion star
- If NS, accretion from companion provides "hot spot"; rotating non-axisymmetric NS emits gravitational waves
- Bildsten *ApJL* **501**, L89 (1998) suggested GW spindown may balance accretion spinup; GW strength can be estimated from X-ray flux
- Torque balance would give ≈ constant GW freq
- Signal at solar system modulated by binary orbit

Scorpius X-1

- 2nd brightest X-Ray source in the sky, after the Sun
- Favored model is $1.4 M_{\odot}$ NS + $0.42 M_{\odot}$ companion
  Steeghs & Casares *ApJ* **568**, 273 (2002)

Parameters (see LSC *PRD* **76**, 082001 (2007) for refs)

RA	α	$16^{h}19^{m}55^{s}$
dec	δ	$-15°38'25''$
orb period	P_{orb}	(68023.84 ± 0.08) s
ref time	$\widetilde{T}$	(731163327 ± 299) s
proj orb radius	a_p	(1.44 ± 0.18) s

GW Searches for Sco X-1

- Fully coherent $\mathcal{F}$-statistic search
  Jaranowski, Królak & Schutz *PRD* **58**, 063001 (1998)
  ☞ w/6 hours of LIGO S2 data LSC *PRD* **76**, 082001 (2007)
- Directed stochastic ("radiometer") search
  Ballmer *CQG* **23**, S179 (2006)
  ☞ w/LIGO S4 data LSC *PRD* **76**, 082003 (2007)
- Sideband search Messenger & Woan *CQG* **24**, S469 (2007)
- Modelled cross-correlation search

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

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Cross-Correlation Search for Stochastic Background

- Noisy data from GW Detector:
  $x(t) = n(t) + h(t) = n(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}$
- Correlate data btwn detectors (Fourier domain)

$$E[\tilde{x}_1^*(f)\tilde{x}_2(f')] = E[\tilde{h}_1^*(f)\tilde{h}_2(f')] = \overleftrightarrow{d}_1 : E[\overleftrightarrow{\tilde{h}}{}_1^*(f) \otimes \overleftrightarrow{\tilde{h}}_2(f')] : \overleftrightarrow{d}_2$$

- For stochastic backgrounds

$$E[\tilde{h}_1^*(f)\tilde{h}_2(f')] = \delta(f - f')\gamma_{12}(f)\frac{S_{\mathrm{gw}}(f)}{2}$$

$S_{\mathrm{gw}}(f)$ encodes spectrum; $\gamma_{12}(f)$ encodes geometry

Detection Statistic

- Optimally filtered cross-correlation statistic

$$Y = \int df\, \tilde{x}_1^*(f)\, Q(f)\, \tilde{x}_2(f)$$

- Filter encodes expected spectrum & spatial distribution (isotropic, pointlike, spherical harmonics ...)

$$Q(f) \propto \frac{\gamma_{12}^*(f) S_{gw}^{exp}(f)}{S_{n1}(f) S_{n2}(f)}$$

- “Radiometer” search for ptlike srcs incl targeting Sco X-1: known sky location, unknown frequency
Ballmer, *CQG* **23**, S179 (2006); LSC, *PRD* **76**, 082003 (2007)

Gravitational Waves from Quasiperiodic Sources

- Sco X-1 is Low-Mass X-Ray Binary: accreting neutron star in orbit w/companion
- Rotating NS w/deformation emits nearly sinusoidal signal

$$\overleftrightarrow{h}(t) = h_0 \left[\mathcal{A}_+ \cos\Phi(\tau(t))\overleftrightarrow{e}_+ + \mathcal{A}_\times \sin\Phi(\tau(t))\overleftrightarrow{e}_\times\right]$$

  - $\mathcal{A}_+ = \frac{1+\cos^2\iota}{2}$; $\mathcal{A}_\times = \cos\iota$
  - $\Phi(\tau)$: phase evolution in rest frame;
  - $\tau(t)$: Doppler mod from detector motion (& binary orbit)
- Features of signal model missing from stoch search:
  - Doppler shift @ each detector: correlations peaked @ different freqs
  - Long-term coherence: can correlate data @ different times

Doppler Modulation in Cross-Correlation Searches

- Max Doppler shift from Earth's rotation: $\frac{|\vec{v}_{\oplus\text{rot}}|}{c} \lesssim 1.5 \times 10^{-6}$
  Doppler shift at 2000 Hz is $\lesssim 0.003$ Hz.
- Max Doppler shift from Earth's orbit: $\frac{|\vec{v}_{\oplus\text{orb}}|}{c} \lesssim 1.0 \times 10^{-4}$
  Doppler shift at 2000 Hz is $\lesssim 0.2$ Hz.
- Stochastic searches use FTs of e.g., 120 s duration, so

  $$\delta f \approx 0.0083\,\text{Hz}$$

  Cross-correlation between detectors uses same freq bin
- Stochastic search combines fine bins into coarse bins of

  $$\Delta f = 0.25\,\text{Hz}$$

  Cross-corr power collected in single bin for most freqs
- Correlating detectors at different times, or with longer FTs means including Doppler effects

Basics of Cross-Correlation Method

Dhurandhar, Krishnan, Mukhopadhyay & JTW *PRD* **77**, 082001 (2008)

- [BTW, other targets include SN1987A supernova remnant; see Chung, Melatos, Krishnan & JTW *MNRAS* **414**, 2650 (2011)]
- Divide data into segments of length T_{sft} & take "short Fourier transform" (SFT) $\tilde{x}_I(f)$
- Label segments w/indices I, J, etc
  - ☞ I & J can be same or different times or detectors
- Use CW signal model ($\mathcal{A}_+ = \frac{1+\cos^2\iota}{2}$; $\mathcal{A}_\times = \cos\iota$)

$$h(t) = h_0 \left[\mathcal{A}_+ \cos\Phi(\tau(t)) F_+ + \mathcal{A}_\times \sin\Phi(\tau(t)) F_\times\right]$$

to determine expected cross-correlation btwn SFTs I & J

$$E\left[\tilde{x}_I^*(f_{k_I})\,\tilde{x}_J(f_{k_J})\right] = \tilde{h}_I^*(f_{k_I})\,\tilde{h}_J(f_{k_J})$$
$$= h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{T_{\text{sft}}}(f_{k_I} - f_I)\,\delta_{T_{\text{sft}}}(f_{k_J} - f_J)$$

Expected Cross-Correlation & Optimal Statistic

- Cross-correlation of signal w/intrinsic frequency f_0:

$$E\left[\tilde{x}_I^*(f_{k_I})\,\tilde{x}(f_{k_J})\right] = \tilde{h}_I^*(f_{k_I})\,\tilde{h}(f_{k_J}) = h_0^2\,\tilde{\mathcal{G}}_{IJ}\,\delta_{T_{\text{sft}}}(f_{k_I}-f_I)\,\delta_{T_{\text{sft}}}(f_{k_J}-f_J)$$

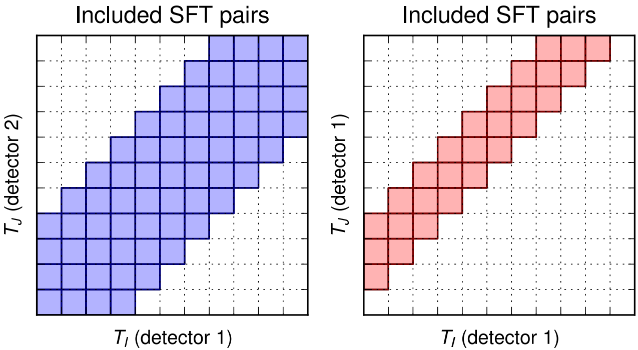
  - $\delta_{T_{\text{sft}}}(f-f') = \int_{-T_{\text{sft}}/2}^{T_{\text{sft}}/2} e^{i2\pi(f-f')t}\,dt$ so $\delta_{T_{\text{sft}}}(0) = T_{\text{sft}}$
  - f_I is signal freq @ time T_I Doppler shifted for detector I
- Label SFTs by I, J, … and pairs by α, β, …
- Construct $\mathcal{Y}_{IJ} = \frac{\tilde{x}_I^*(f_{\tilde{k}_I})\tilde{x}_J(f_{\tilde{k}_J})}{(T_{\text{sft}})^2}$ (where $f_{\tilde{k}_I} \approx f_I$) so that

$$E[\mathcal{Y}_\alpha] \approx h_0^2\tilde{\mathcal{G}}_\alpha \qquad \text{Var}[\mathcal{Y}_{IJ}] \approx \sigma_{IJ}^2 = S_I(f_0)S_J(f_0)/4(T_{\text{sft}})^2$$

- Optimally combine into $\rho = \sum_\alpha(u_\alpha\mathcal{Y}_\alpha + u_\alpha^*\mathcal{Y}_\alpha^*)$ w/$u_\alpha \propto \tilde{\mathcal{G}}_\alpha^*/\sigma_\alpha^2$ so $E[\rho] = h_0^2\sqrt{2\sum_\alpha|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}$ & $\text{Var}[\rho] = 1$

Tuning the Cross-Correlation Search

- Computational considerations limit coherent integration time
- Can make tunable semi-coherent search by restricting which SFT pairs α are included in $\rho = \sum_\alpha (u_\alpha \mathcal{Y}_\alpha + u_\alpha^* \mathcal{Y}_\alpha^*)$
- E.g., only include pairs where $|T_I - T_J| \equiv |T_\alpha| \leq T_{\text{max}}$



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Metric for Cross-Correlation Search

- Consider dependence of ρ on parameters $\boldsymbol{\lambda} \equiv \{\lambda_i\}$
- Parameter space metric $g_{ij} = -\frac{1}{2} \frac{E[\rho_{,ij}]\big|_{\boldsymbol{\lambda}=\boldsymbol{\lambda}_{\text{true}}}}{E[\rho^{\text{true}}]}$ from

$$\frac{E[\rho] - E[\rho^{\text{true}}]}{E[\rho^{\text{true}}]} = -g_{ij}(\Delta\lambda^i)(\Delta\lambda^j) + \mathcal{O}([\Delta\lambda]^3)$$

- Assume dominant contribution to $E[\rho_{,ij}]$ is from variation of $\Delta\Phi_{IJ} = \Phi_I - \Phi_J$; get phase metric

$$g_{ij} = \frac{1}{2} \frac{\sum_\alpha \Delta\Phi_{\alpha,i} \Delta\Phi_{\alpha,j} |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}{\sum_\beta |\tilde{\mathcal{G}}_\beta|^2/\sigma_\beta^2} \equiv \frac{1}{2} \langle \Delta\Phi_{\alpha,i} \Delta\Phi_{\alpha,j} \rangle_\alpha$$

- Note $\langle\ \rangle_\alpha$ is average over pairs weighted by $|\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2$
- If you ignore that weighting factor you get back usual metric

$$\langle \Phi_{I,i} \Phi_{I,j} \rangle_I - \langle \Phi_{I,i} \rangle_I \langle \Phi_{J,j} \rangle_J$$

Signal Phase for LMXB

- Assuming constant intrinsic freq f_0, phase is

$$\Phi_I = \Phi_0 + 2\pi f_0 \left(T_I - (\vec{r}_{\text{det}} - \vec{r}_{\text{orb}}) \cdot \hat{k}/c \right)$$
$$= \Phi_0 + 2\pi f_0 \left\{ T_I - d_I + a_p \cos \left[2\pi (T_I - \widetilde{T})/P_{\text{orb}} \right] \right\}$$

- Phase difference between SFTs is

$$\Delta\Phi_\alpha = 2\pi f_0 \left\{ T_\alpha - d_\alpha - a_p \sin[\pi T_\alpha / P_{\text{orb}}] \sin[2\pi (T_\alpha^{\text{av}} - \widetilde{T})/P_{\text{orb}}] \right\}$$

- $d_{IJ} = d_I - d_J$ is proj dist btwn sites, where $d_I = \vec{r}_{\text{det}} \cdot \hat{k}/c$
- a_p, P_{orb} & $\widetilde{T}$ are binary orbital params
- $T_{IJ} = T_I - T_J \equiv T_\alpha$ is time offset btwn SFTs; T_α^{av} is avg time
- For each detector pair, avg over pairs is avg over T_α & T_α^{av}

Approximate Phase Metric for LMXB

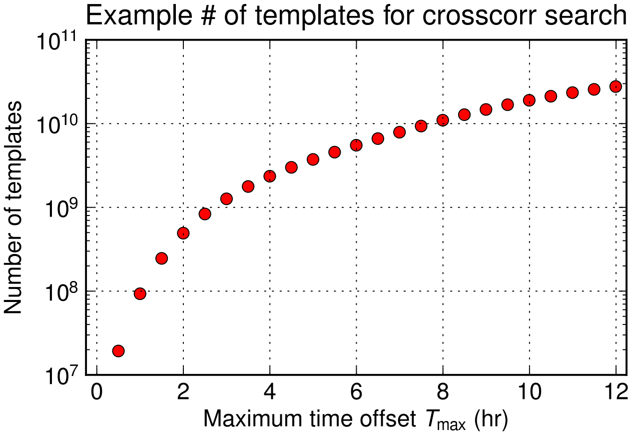
$$\Delta\Phi_\alpha = 2\pi f_0 \left\{ T_\alpha - d_\alpha - a_p \sin[\pi T_\alpha / P_{\text{orb}}] \sin[2\pi (T_\alpha^{\text{av}} - \widetilde{T})/P_{\text{orb}}] \right\}$$

- Assume average over T_α^{av} evenly samples orbital phase
- Metric in $\{f_0, a_p, \widetilde{T}\}$ space is

$$\mathbf{g} = \begin{pmatrix} 2\pi^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \langle (T_\alpha - d_\alpha)^2 \rangle_{T_\alpha} + \begin{pmatrix} \pi^2 a_p^2 & \pi^2 f_0 a_p & 0 \\ \pi^2 f_0 a_p & \pi^2 f_0^2 & 0 \\ 0 & 0 & 4\pi^4 f_0^2 a_p^2 / P_{\text{orb}}^2 \end{pmatrix} \langle \sin^2[\pi T_\alpha / P_{\text{orb}}] \rangle_{T_\alpha}$$

- Since $\langle (T_\alpha - d_\alpha)^2 \rangle_{T_\alpha} \approx \langle T_\alpha^2 \rangle_{T_\alpha} \gg a_p^2 \langle \sin^2[\pi T_\alpha / P_{\text{orb}}] \rangle_{T_\alpha}$ (recall $a_p \approx 1.4\,\text{s}$), metric approximately diagonal

Ballpark Estimate of Template Count

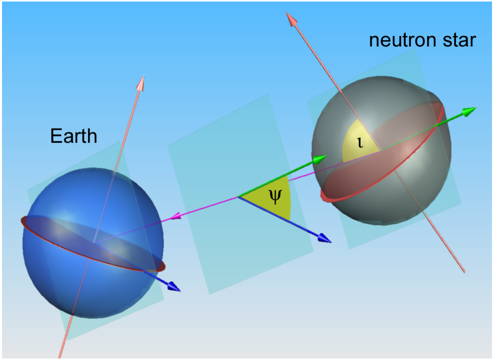


For illustrative purposes, to show dependence on T_{max};

Don't read too much into absolute numbers

Sensitivity Estimates

- Sensitivity of search is $h_0 = \left(\frac{\mathcal{S}^2}{\sum_\alpha |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}\right)^2$
- $\tilde{\mathcal{G}}_\alpha$ depends on (unknown) spin orientation angles ι & ψ; standard approach is to average value of $\tilde{\mathcal{G}}_\alpha$ over $\cos\iota$ & ψ



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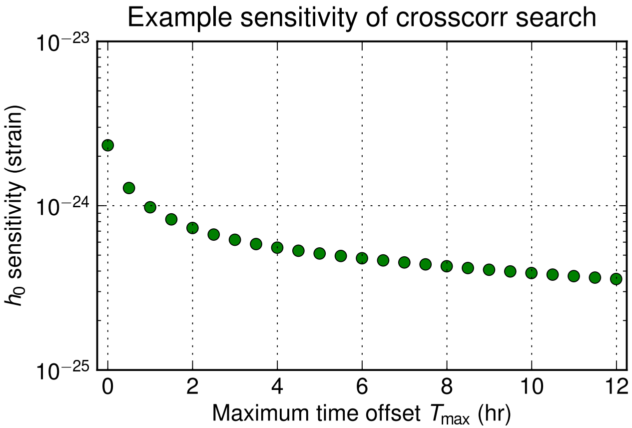
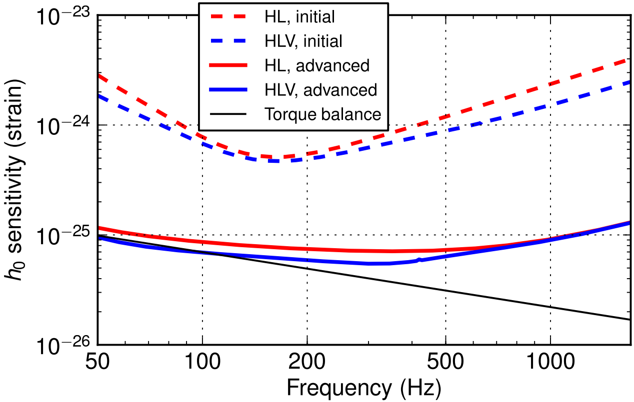
- ψ effect small after average over sidereal time
  ι effect means actually $E[\rho] \approx h_0^2 \frac{\mathcal{A}_+^2 + \mathcal{A}_\times^2}{2} \sqrt{2\sum_\alpha |\tilde{\mathcal{G}}_\alpha|^2/\sigma_\alpha^2}$
  (recall $\mathcal{A}_+ = \frac{1+\cos^2\iota}{2}$ & $\mathcal{A}_\times = \cos\iota$)
- Net effect is to change statistical factor $\mathcal{S}$; for 10% false-alarm & -dismissal, h_0 sensitivity is a factor of 1.4 worse

Dependence of Sensitivity on T_{max}



For illustrative purposes, to show dependence on T_{max};
note $T_{\mathrm{max}} = 0$ measurement $\sim$ stochastic radiometer

Preliminary Sensitivity Estimates



Assumes 10% false-alarm &-dismissal, 1yr @ design, $T_{\mathrm{max}} = 6\,\mathrm{hr}$

Summary

- Cross-correlation method adapted for CW signals
- Inclusion of signal model & Doppler effects
  allows correlation of non-simultaneous data
- Promising target is the low-mass X-ray binary Scorpius X-1
- For Sco X-1, must search over freq & orbital params
- Advanced detector era sensitivity
  should reach torque balance prediction

Tuning the Cross-Correlation Search

- Computational considerations limit coherent time, i.e., possible time lag between correlated segments
- Detectable signal

$$h_0^{\mathrm{th}} \propto \left(\sum_{IJ} |\tilde{\mathcal{G}}_{IJ}|^2\right)^{-1/4} \sqrt{\frac{S_n}{T_{\mathrm{sft}}}} \propto N_{\mathrm{pairs}}^{-1/4} T_{\mathrm{sft}}^{-1/2}$$

(T_{sft} is duration of fourier transformed data segment)

  - If all data used, $N_{\mathrm{pairs}} \sim N_{\mathrm{sft}}^2$, so

$$h_0 \propto (N_{\mathrm{sft}} T_{\mathrm{sft}})^{-1/2}$$

    like coherent search of duration $N_{\mathrm{sft}} T_{\mathrm{sft}}$
  - If only simultaneous SFTs correlated, $N_{\mathrm{pairs}} \sim N_{\mathrm{sft}}$, so

$$h_0 \propto N_{\mathrm{sft}}^{-1/4} T_{\mathrm{sft}}^{-1/2}$$

    like semi-coherent search w/N_{sft} coherent segs of T_{sft} each
  - Can “tune” sensitivity vs comp time by choosing SFT pairs