Actuator Sizing of a Quad Pendulum for Gravitational Wave Detectors

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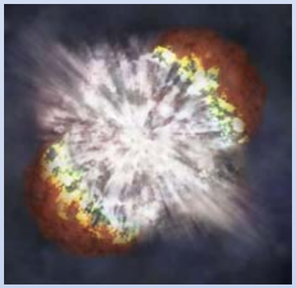
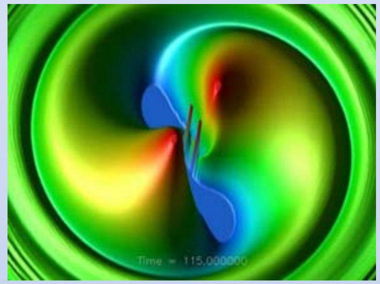
ACC 2011 – San Francisco
June 29

Massachusetts Institute of Technology

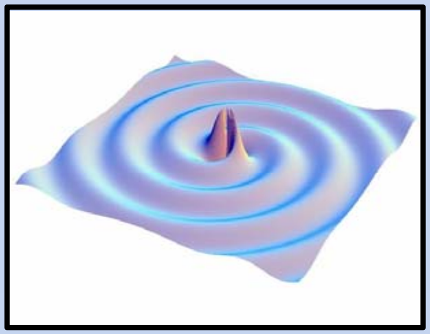
Gravitational Waves

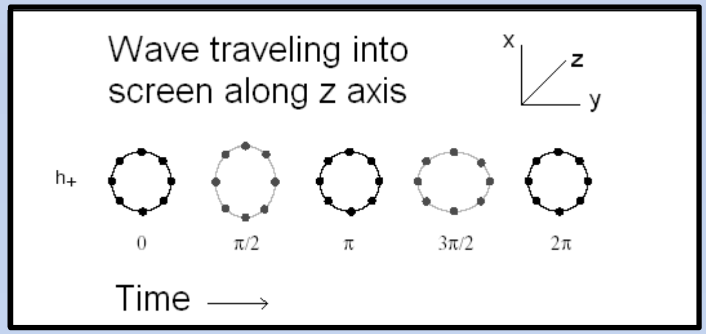

Supernova


Merging Black Holes


Pulsar

Wave of strain amplitude h

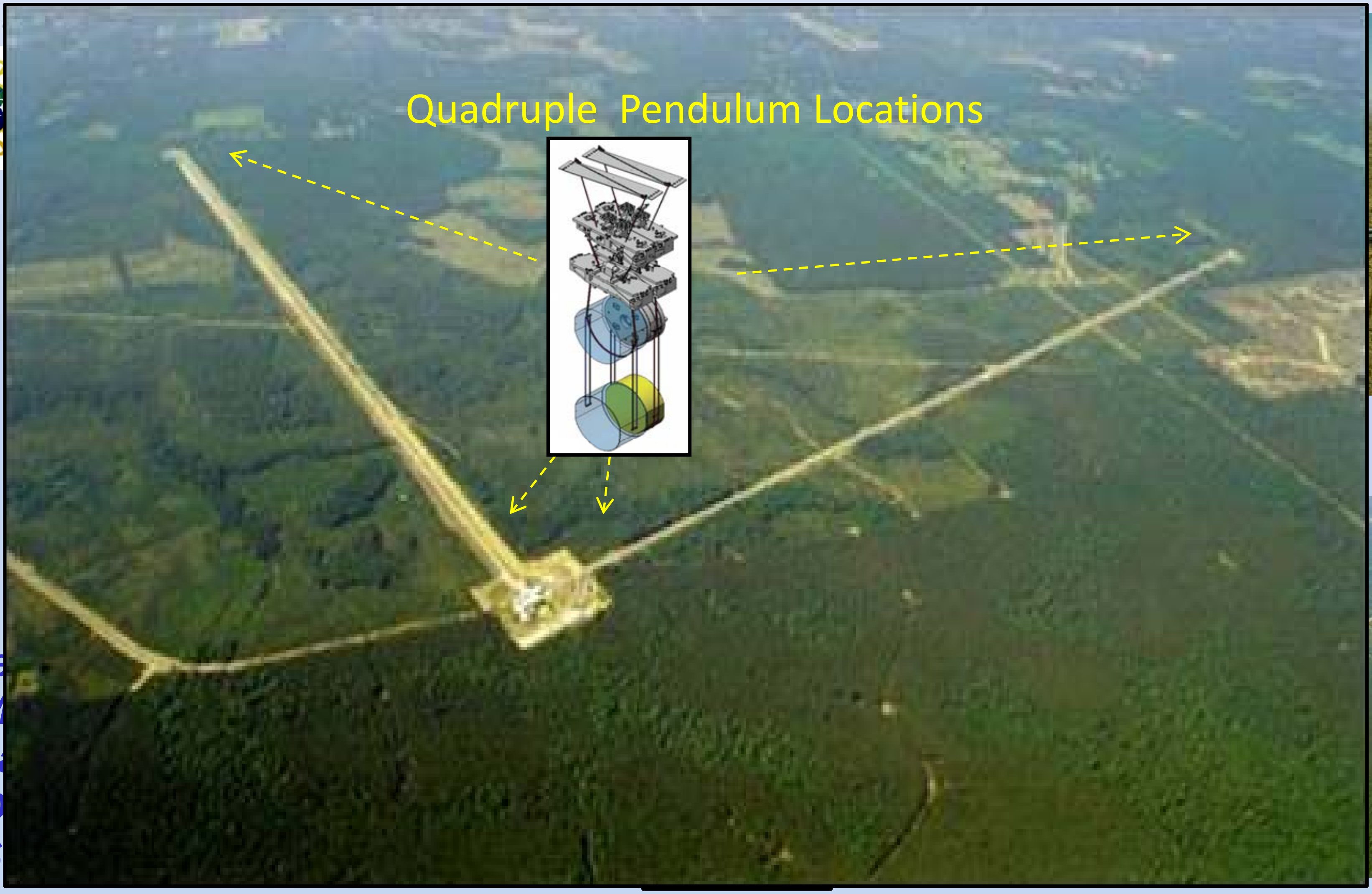


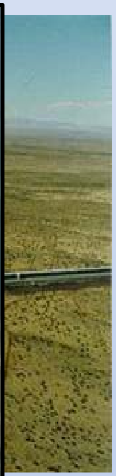


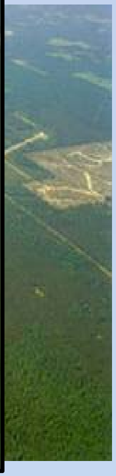
- Supernovae
  - Asymmetry required
- Coalescing Binaries
  - Black Holes or Neutron Stars Mergers
- Pulsars
  - Asymmetry required
- Stochastic Background (Big bang, etc.)

The Laser Interferometer Gravitational-wave Observatory (LIGO)

Funde







- *4*
the
- *M*
F
- *O*
- *S*

Livingston, LA

Quadruple Pendulum

Prototype quad pendulum

≈2 m

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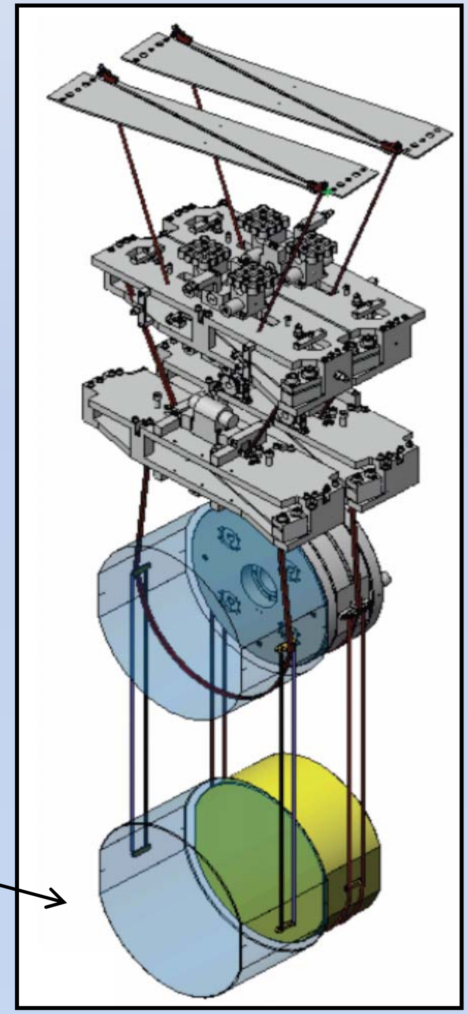
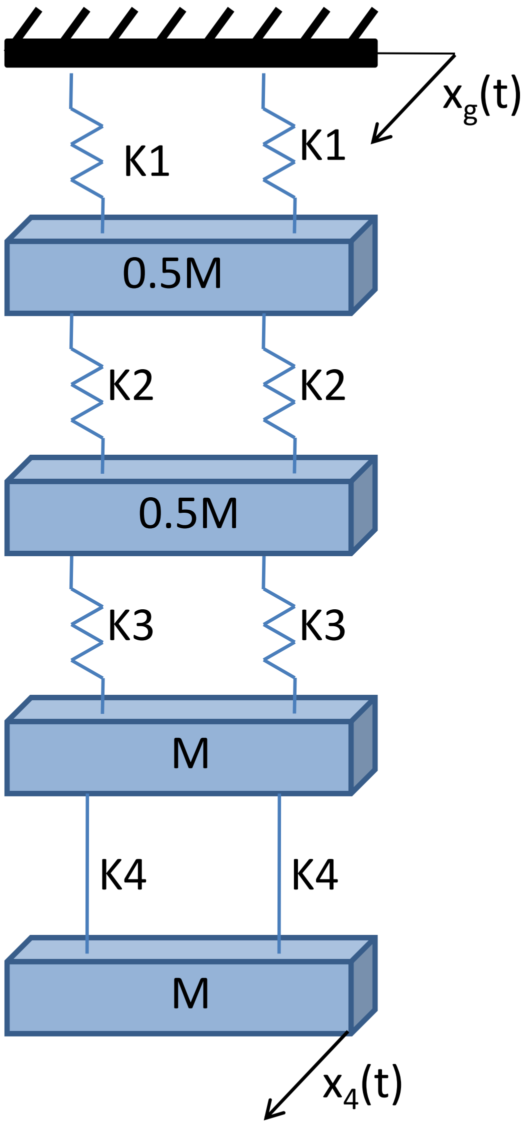
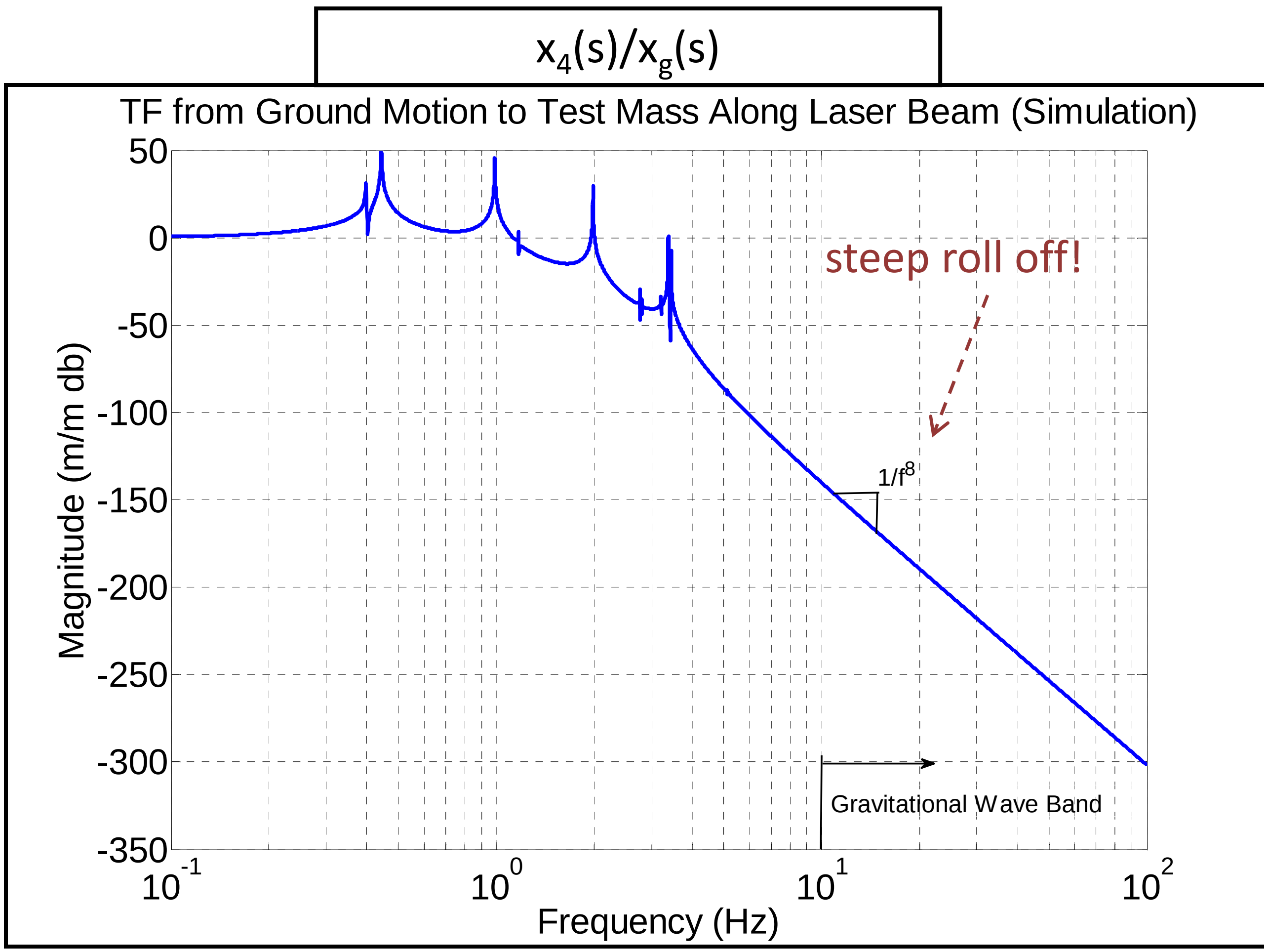
Test Mass
40 Kg

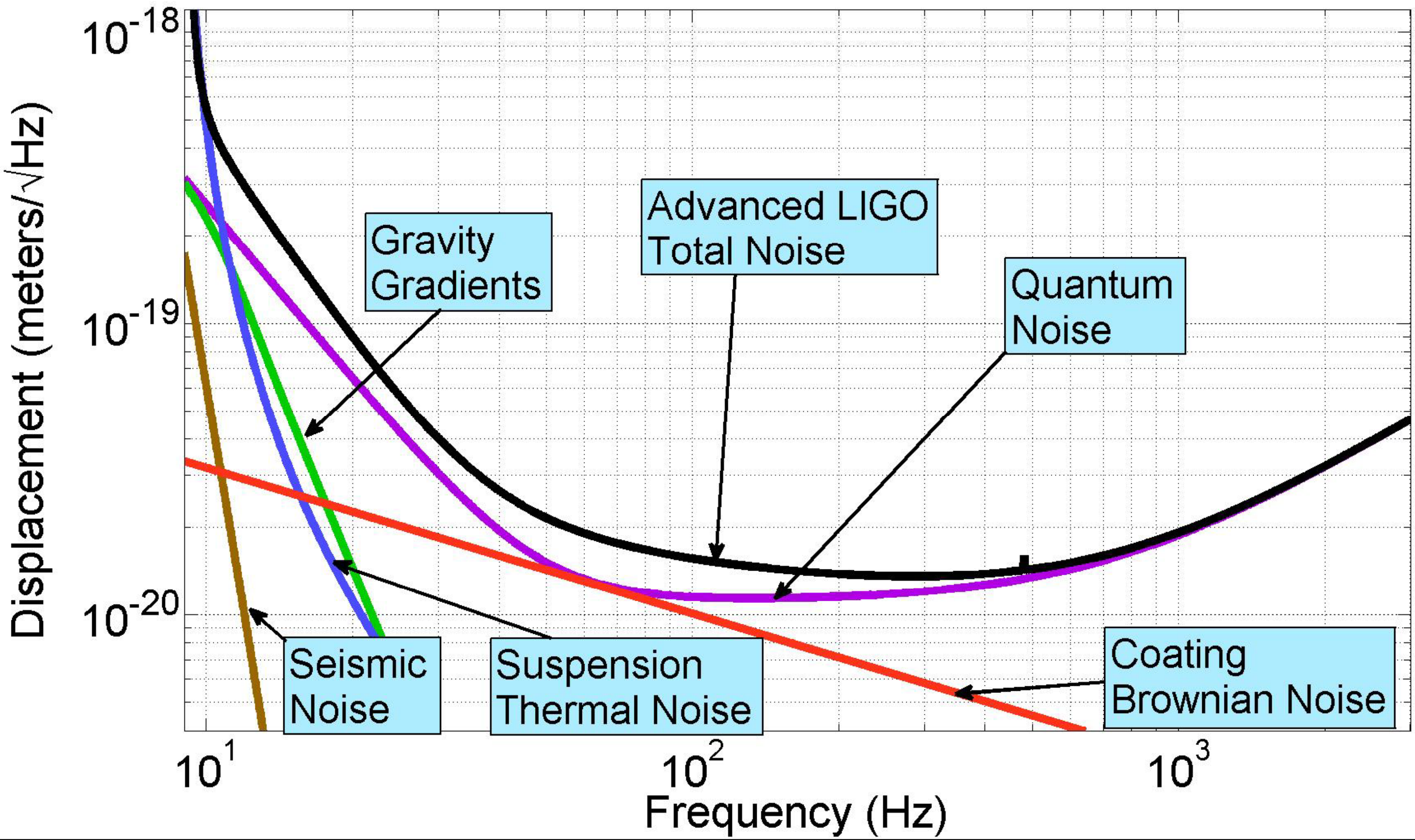


Quadruple Pendulum (Quad) Isolation





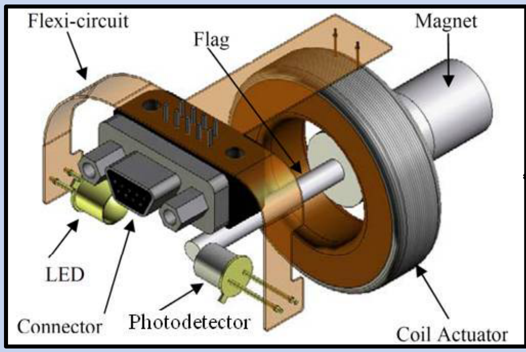
Advanced LIGO Sensitivity



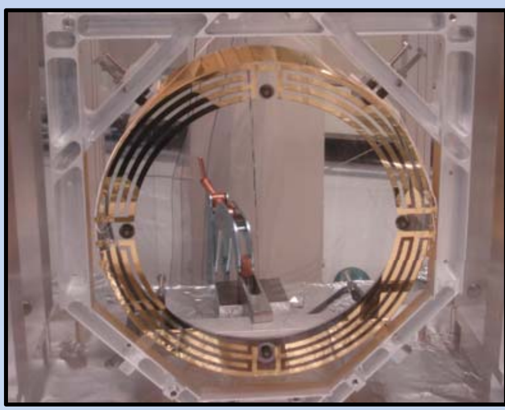
Quad Pendulum Actuators

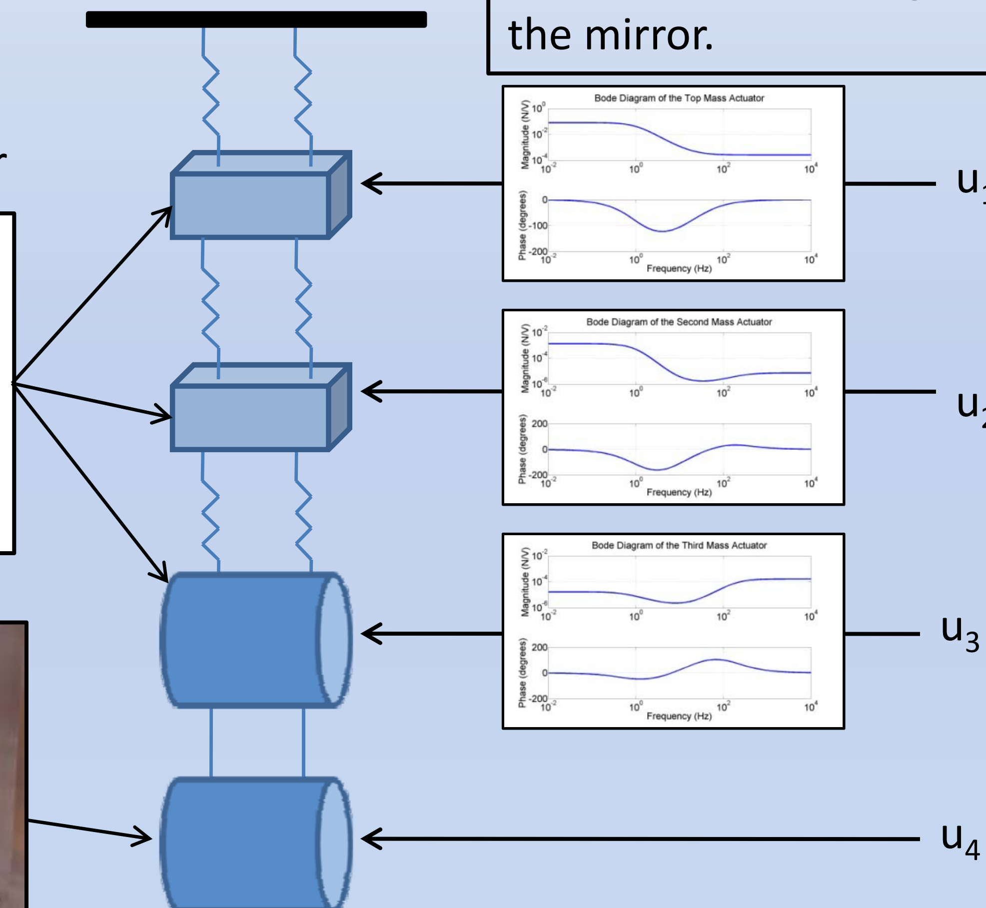
Goal is to determine whether the actuators have enough drive to position the mirror.

Electromagnetic actuator



Electrostatic actuator





205 mN DC

3.4 mN DC

0.043 mN DC

0.095 mN

Outline

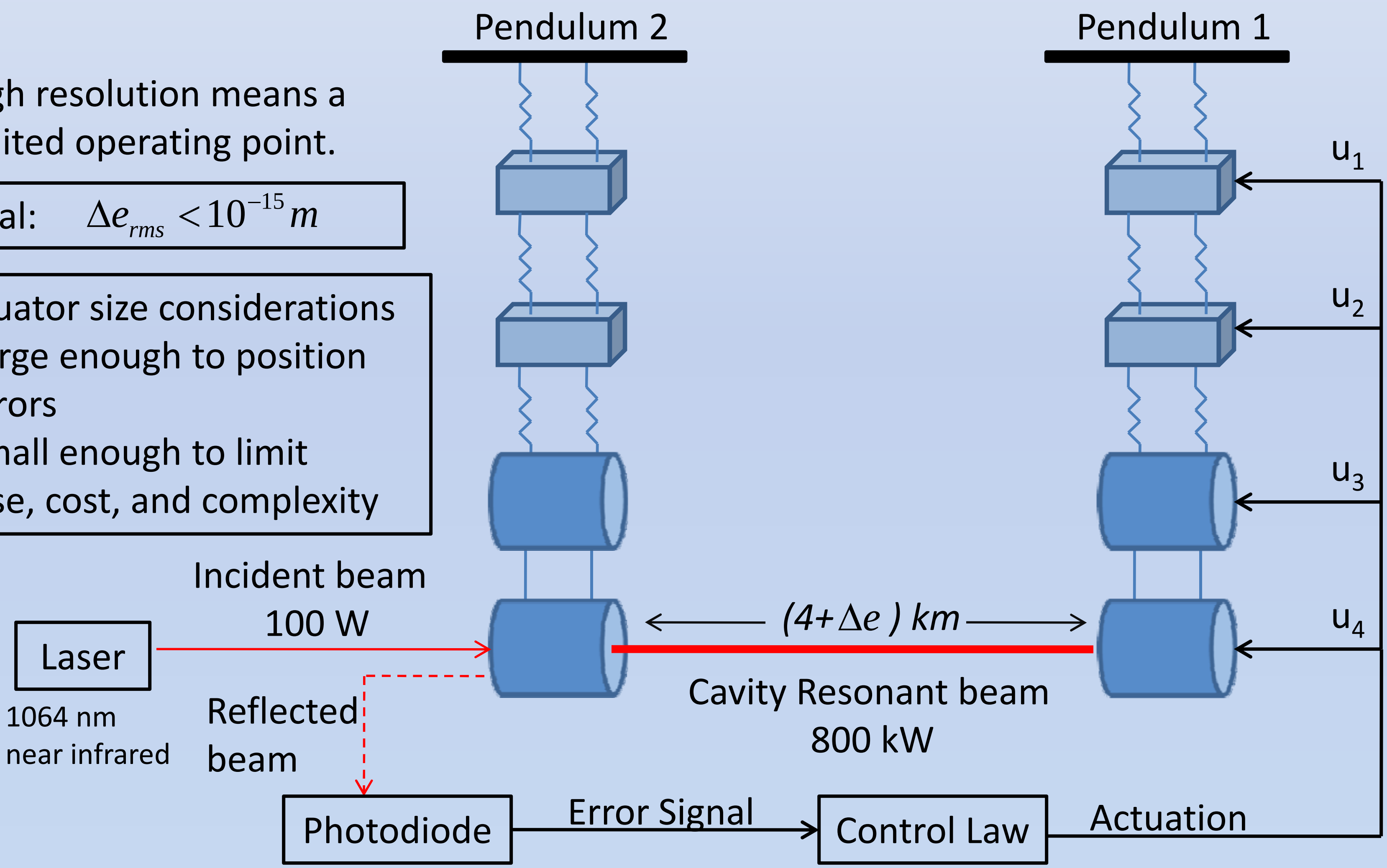
- Gravitational Waves and LIGO
- Quadruple Pendulum
- Control of the Quad Pendulum
- Actuator Sizing
- Conclusion

Optical Arm Cavity Control

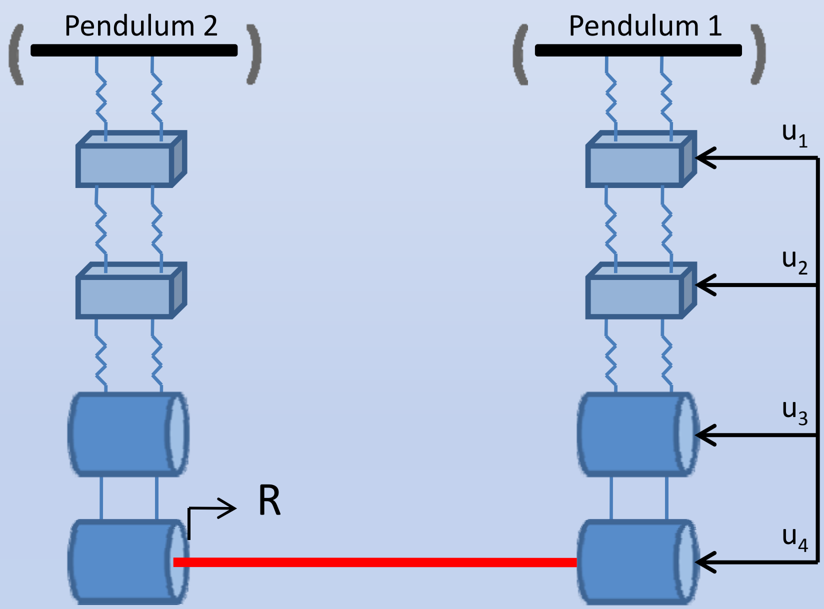
High resolution means a limited operating point.

Goal: $\Delta e_{rms} < 10^{-15}\,m$

Actuator size considerations
- Large enough to position mirrors
- Small enough to limit noise, cost, and complexity

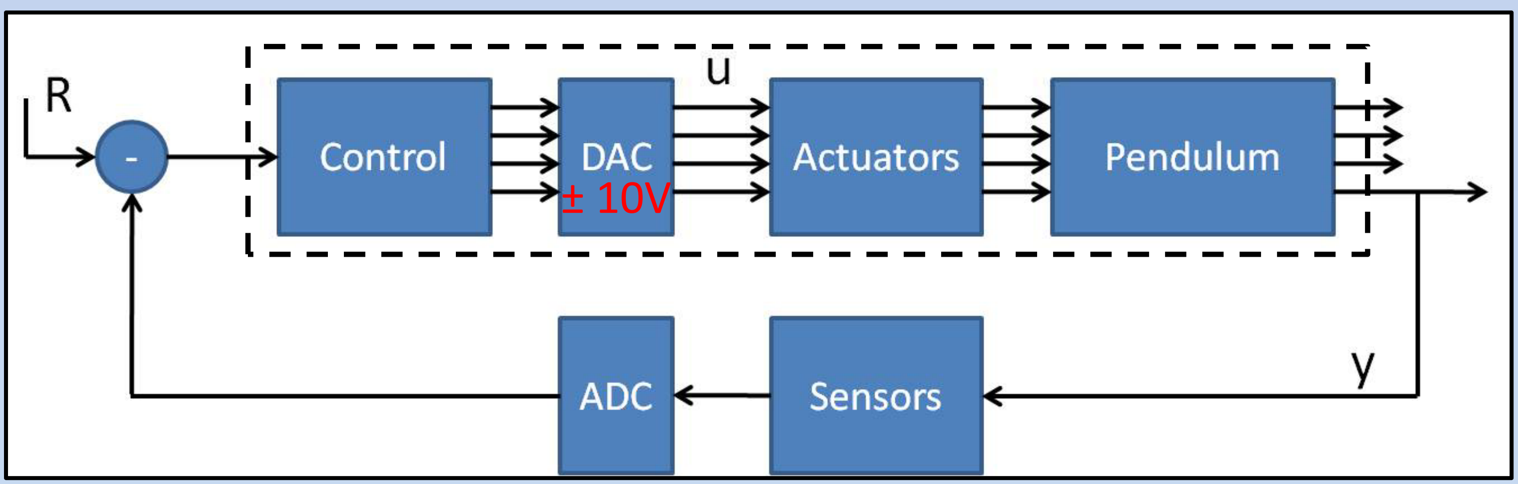


Optical Arm Cavity Control

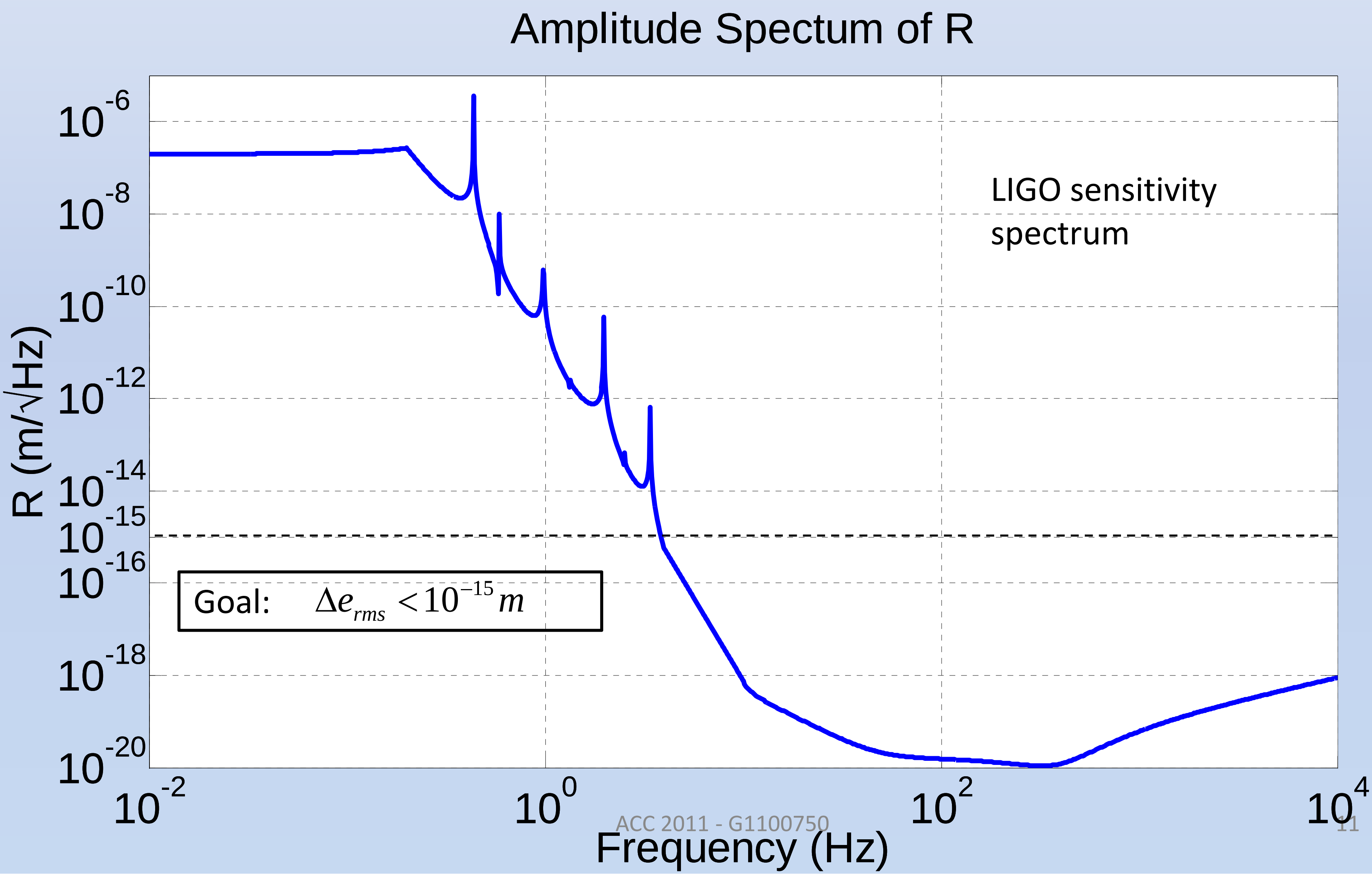


- DAC saturates at ± 10V
- What is the minimum size for each actuator, without considering an infinite set of possible feedback designs?

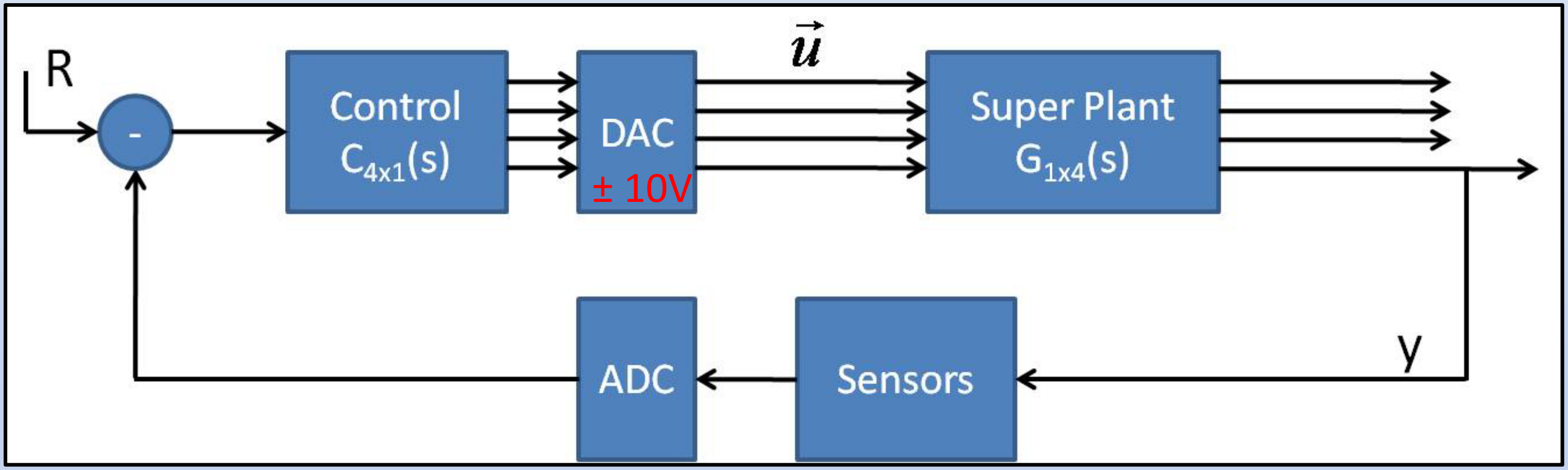
Standard tracking diagram



Disturbance Spectrum



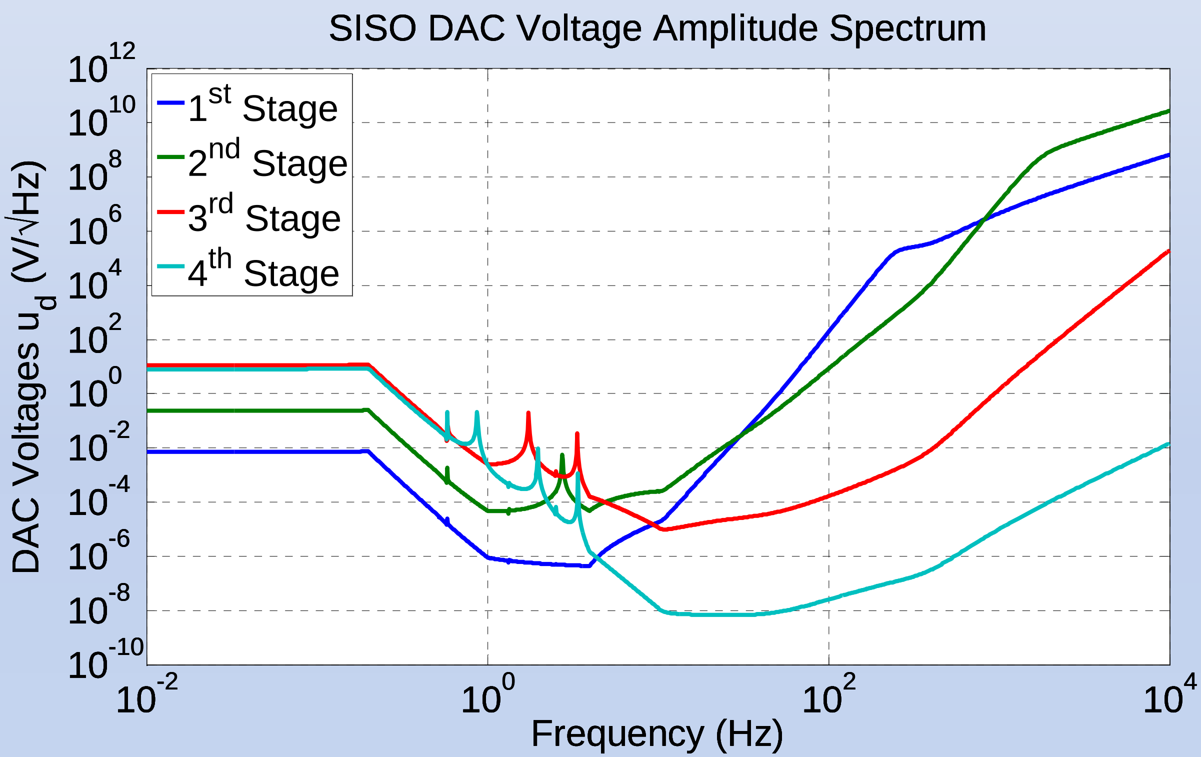
Arm Control Block Diagram



- $y = G\vec{u} = R \Rightarrow \boxed{\vec{u}_{\min} = G^{\dagger} R}$
- $y = \dfrac{GC}{1+GC} R \approx R \qquad GC >> 1$
- $\vec{u} = \dfrac{C}{1+GC} R \qquad GC >> 1 \Rightarrow \vec{u} \approx G^{\dagger} R = \vec{u}_{\min}$

† -> Moore-Penrose pseudoinverse

Minimum SISO Actuation



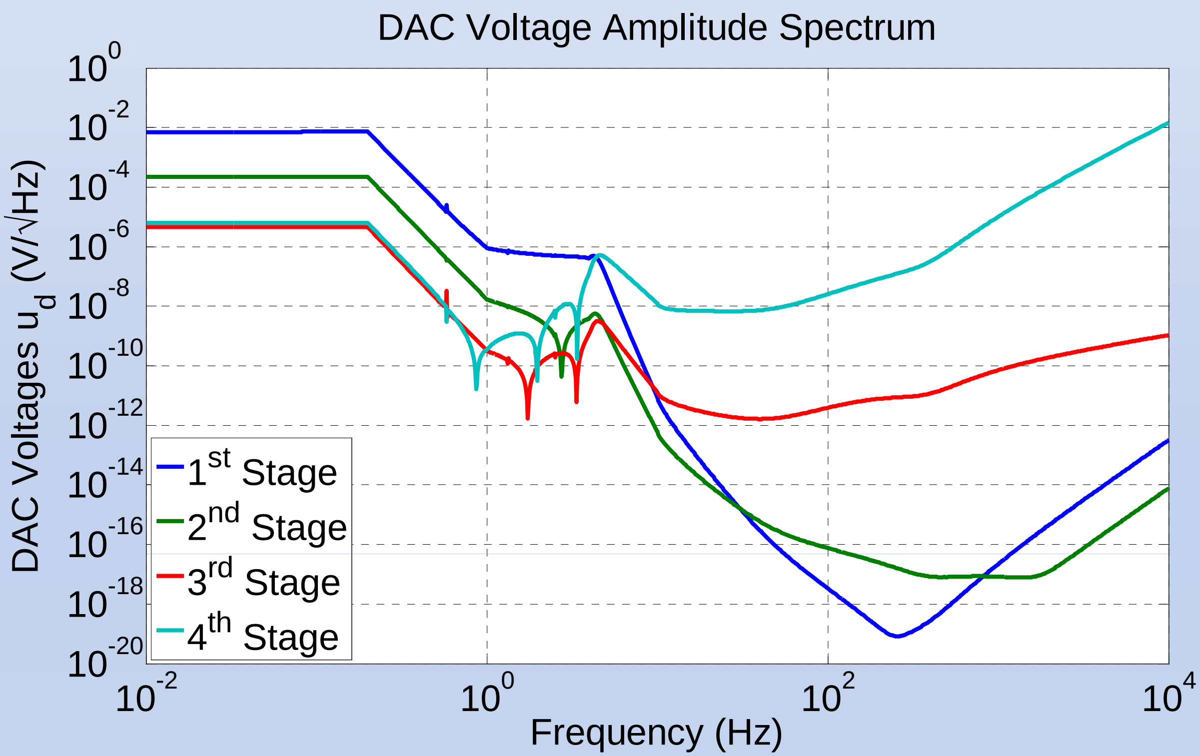
$$u_i(f) = G_i^{-1}(f) R(f)$$

$$u_{i,RMS} = \sqrt{\int u^2(f)\, df}$$

$$p_i = 1 - erf\left(\frac{10}{\sqrt{2}u_{i,RMS}}\right)$$

Actuator	RMS (V)	Saturation Probability (p)
1st	3.05×10^9	1
2nd	1.27×10^{12}	1
3rd	6.29×10^6	1
4th	3.82	0.0087766

Minimum Least Squares Actuation



$$\vec{u}_{\min} = G^{\dagger} R$$

- The relative magnitudes between curves quantifies the relative 'effectiveness' of an actuator.
- Pseudoinverse guides feedback design.

Actuator	RMS (V, 10 V max)	Saturation Probability (p)
1st	3.29×10^{-3}	0
2nd	1.00×10^{-4}	0
3rd	2.09×10^{-6}	0
4th	0.569	3.3×10^{-69}

Conclusions

- The pseudoinverse of the combined plant TF shows the actuators have enough range with reasonable margin.
- Also shows which actuator is most effective at each frequency.
- No feedback design is required.
- The pseudoinverse can guide feedback design.

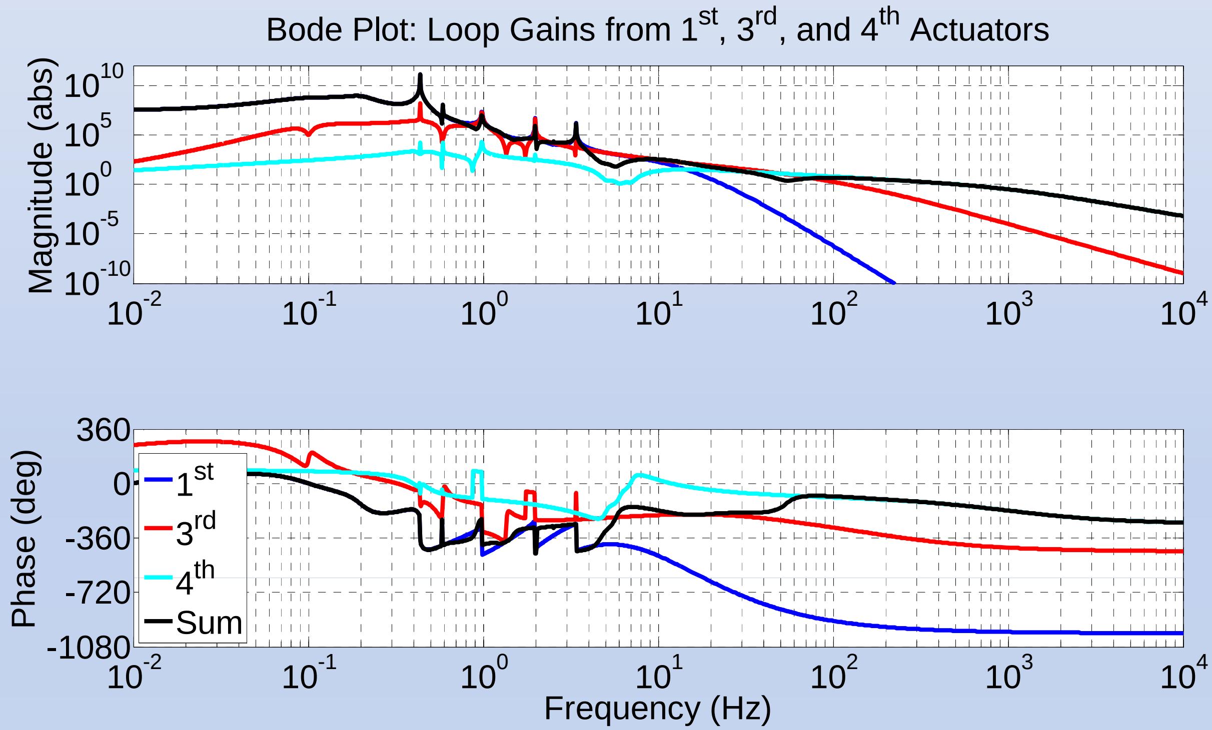
LIGO Scientific Collaboration

- Australian Consortium for Interferometric Gravitational Astronomy
- The Univ. of Adelaide
- Andrews University
- The Australian National Univ.
- The University of Birmingham
- California Inst. of Technology
- Cardiff University
- Carleton College
- Charles Sturt Univ.
- Columbia University
- CSU Fullerton
- Embry Riddle Aeronautical Univ.
- Eötvös Loránd University
- University of Florida
- German/British Collaboration for the Detection of Gravitational Waves
- University of Glasgow
- Goddard Space Flight Center
- Leibniz Universität Hannover
- Hobart & William Smith Colleges
- Inst. of Applied Physics of the Russian Academy of Sciences
- Polish Academy of Sciences
- India Inter-University Centre for Astronomy and Astrophysics
- Louisiana State University
- Louisiana Tech University
- Loyola University New Orleans
- University of Maryland
- Max Planck Institute for Gravitational Physics

- University of Michigan
- University of Minnesota
- The University of Mississippi
- Massachusetts Inst. of Technology
- Monash University
- Montana State University
- Moscow State University
- National Astronomical Observatory of Japan
- Northwestern University
- University of Oregon
- Pennsylvania State University
- Rochester Inst. of Technology
- Rutherford Appleton Lab
- University of Rochester
- San Jose State University
- Univ. of Sannio at Benevento, and Univ. of Salerno
- University of Sheffield
- University of Southampton
- Southeastern Louisiana Univ.
- Southern Univ. and A&M College
- Stanford University
- University of Strathclyde
- Syracuse University
- Univ. of Texas at Austin
- Univ. of Texas at Brownsville
- Trinity University
- Tsinghua University
- Universitat de les Illes Balears
- Univ. of Massachusetts Amherst
- University of Western Australia
- Univ. of Wisconsin-Milwaukee
- Washington State University
- University of Washington

Back Ups

Example Feedback Loop Gains



Crossover Frequencies

1st: 5 Hz

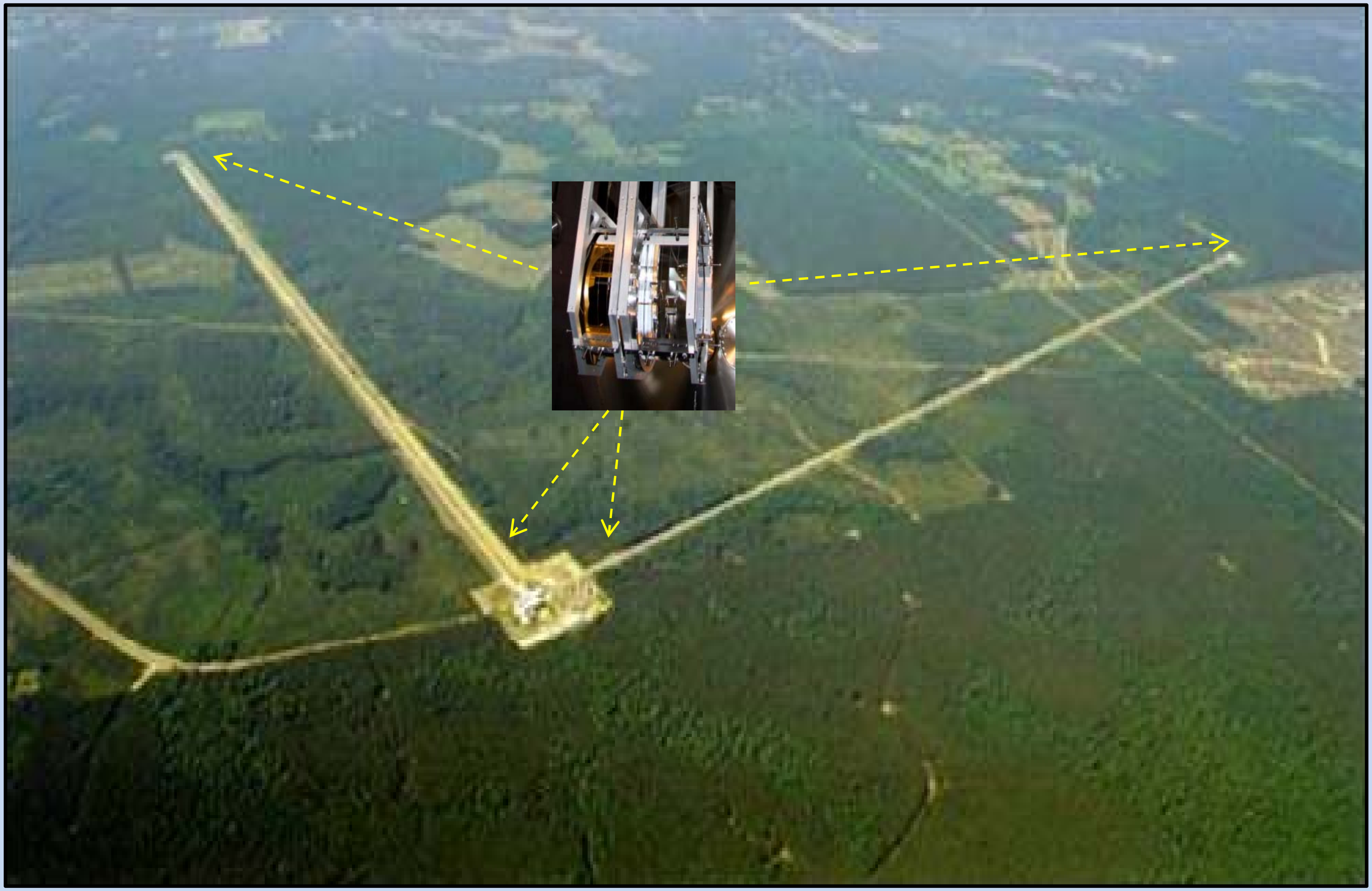
2nd: NA

3rd: 50 Hz

4th: 500 Hz

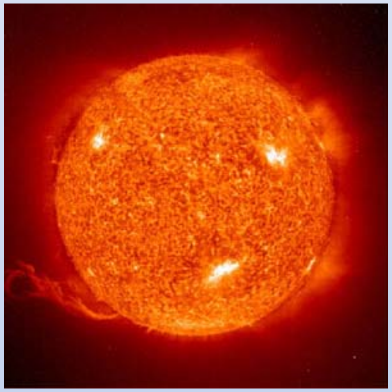
Actuator	Feedforward RMS (V)	Feedback RMS (V)	Saturation Probability (p)
1st	3.29×10^{-3}	3.7 X 10^-3	0
2nd	1.00×10^{-4}	0	0
3rd	2.09×10^{-6}	2 X 10^-2	0
4th	0.569	8 X 10^-5	0

Location of Quadruple Pendulums



Seismic Noise

- Tides - ≈ 10^{-5} Hz



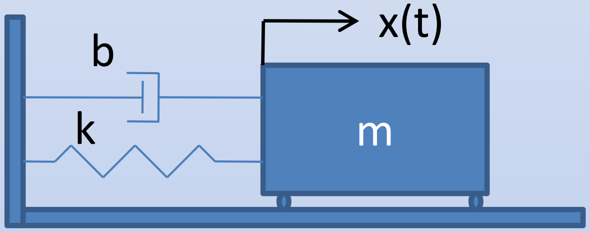
- Microseismic peak - ≈ 0.1 to 0.3 Hz

- Anthropogenic Noise - ≈ 1 to 10 Hz

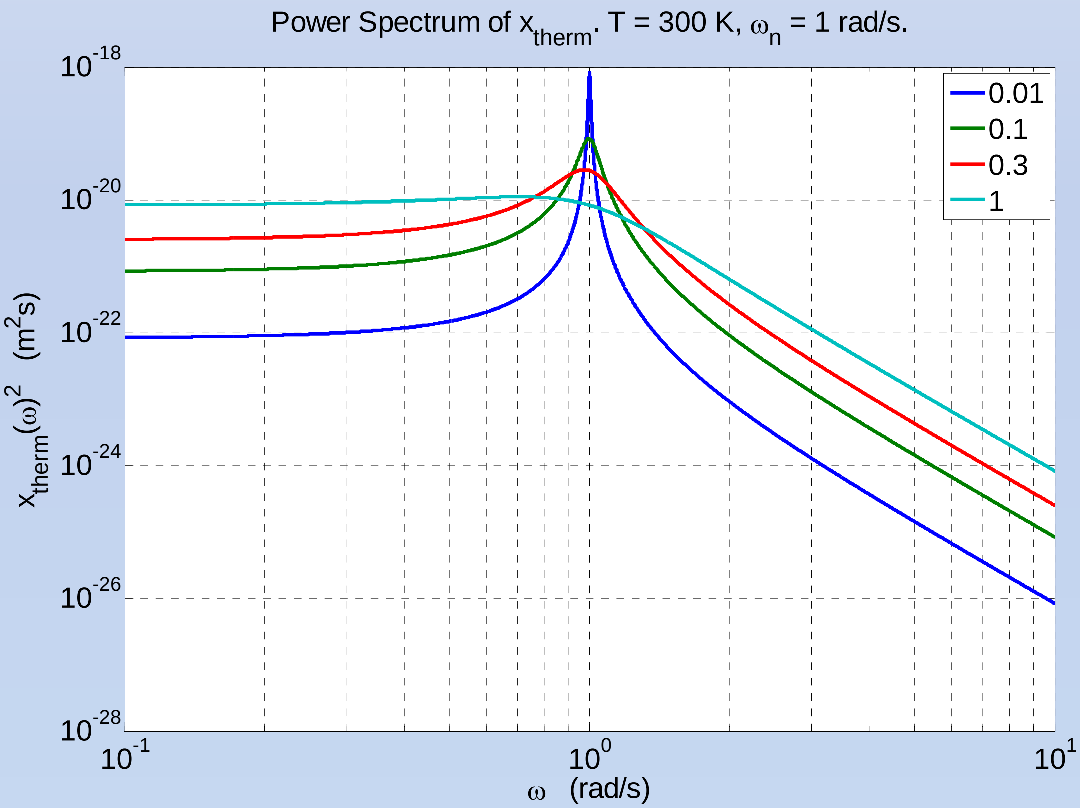
Thermal Noise

$$x_{therm}^2(\omega) = \frac{2k_B T b}{\omega^2[b^2 + m(\omega - \frac{\omega_n^2}{\omega})^2]}$$

System in *thermal equilibrium*



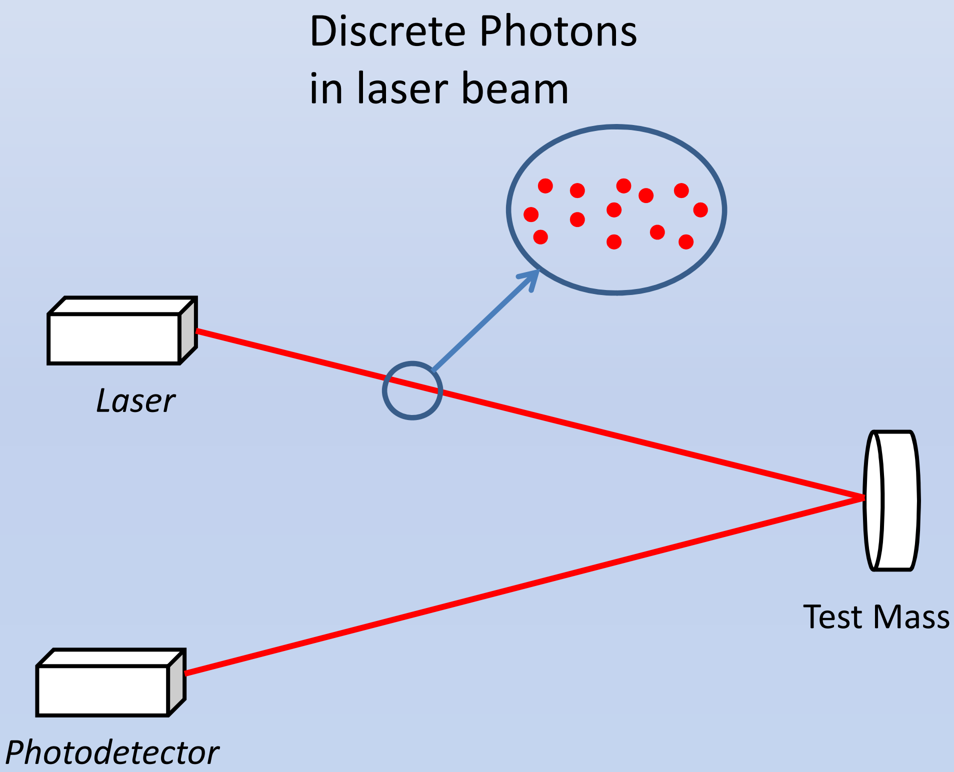
b = dissipation factor (damping)
T = Temperature (Kelvin)
k_b = Boltzmann's Constant



Mean energy: (equipartition function)

$$\overline{x^2} = \frac{k_B T}{k}$$

Quantized Optical Noise

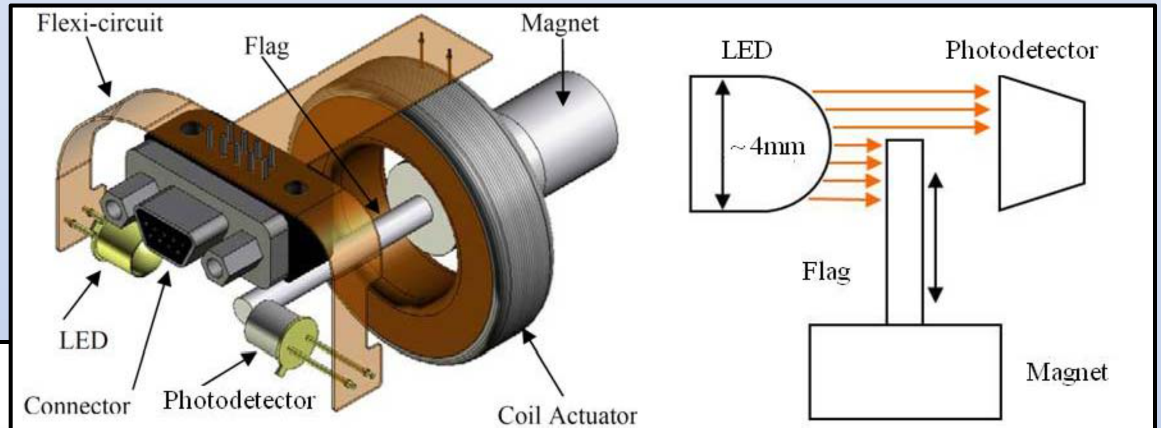


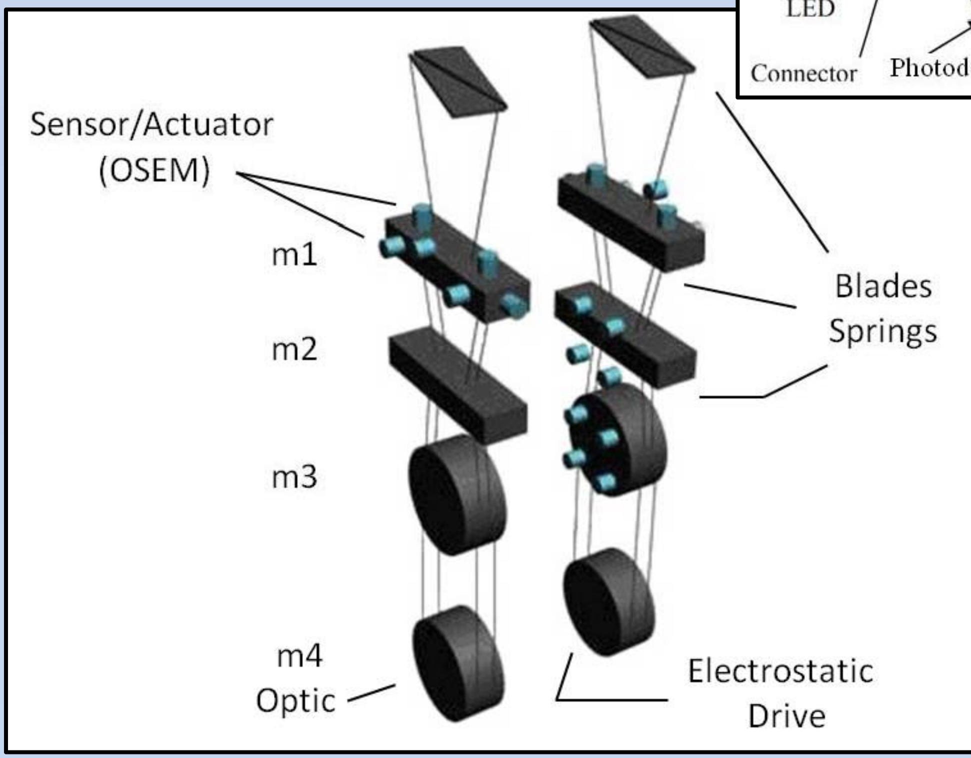
Laser light consists of a finite (and uncertain) numbers of photons.

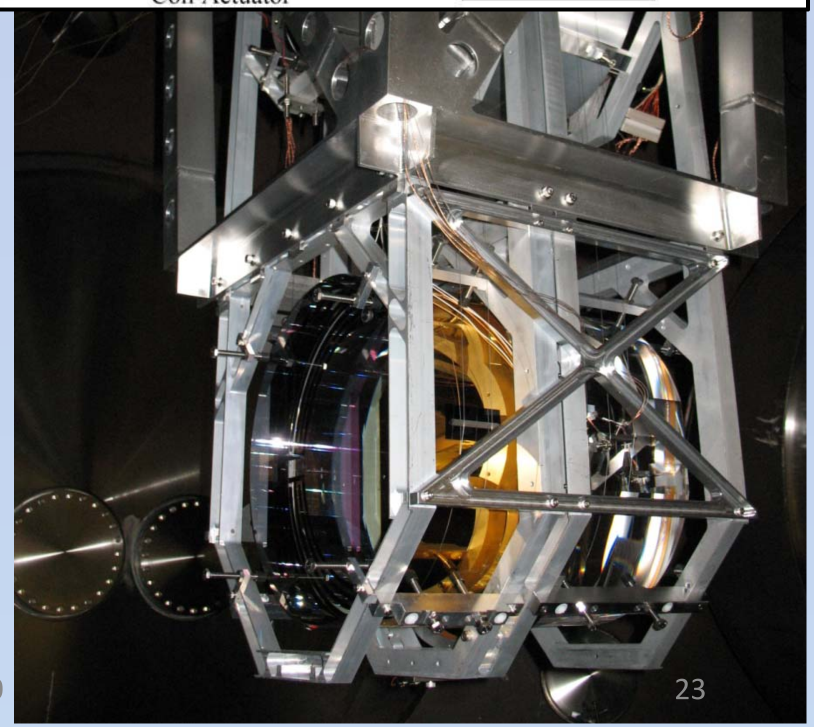
The momentum transfer onto the test mass from a random distribution of photons produces ***radiation pressure noise***.

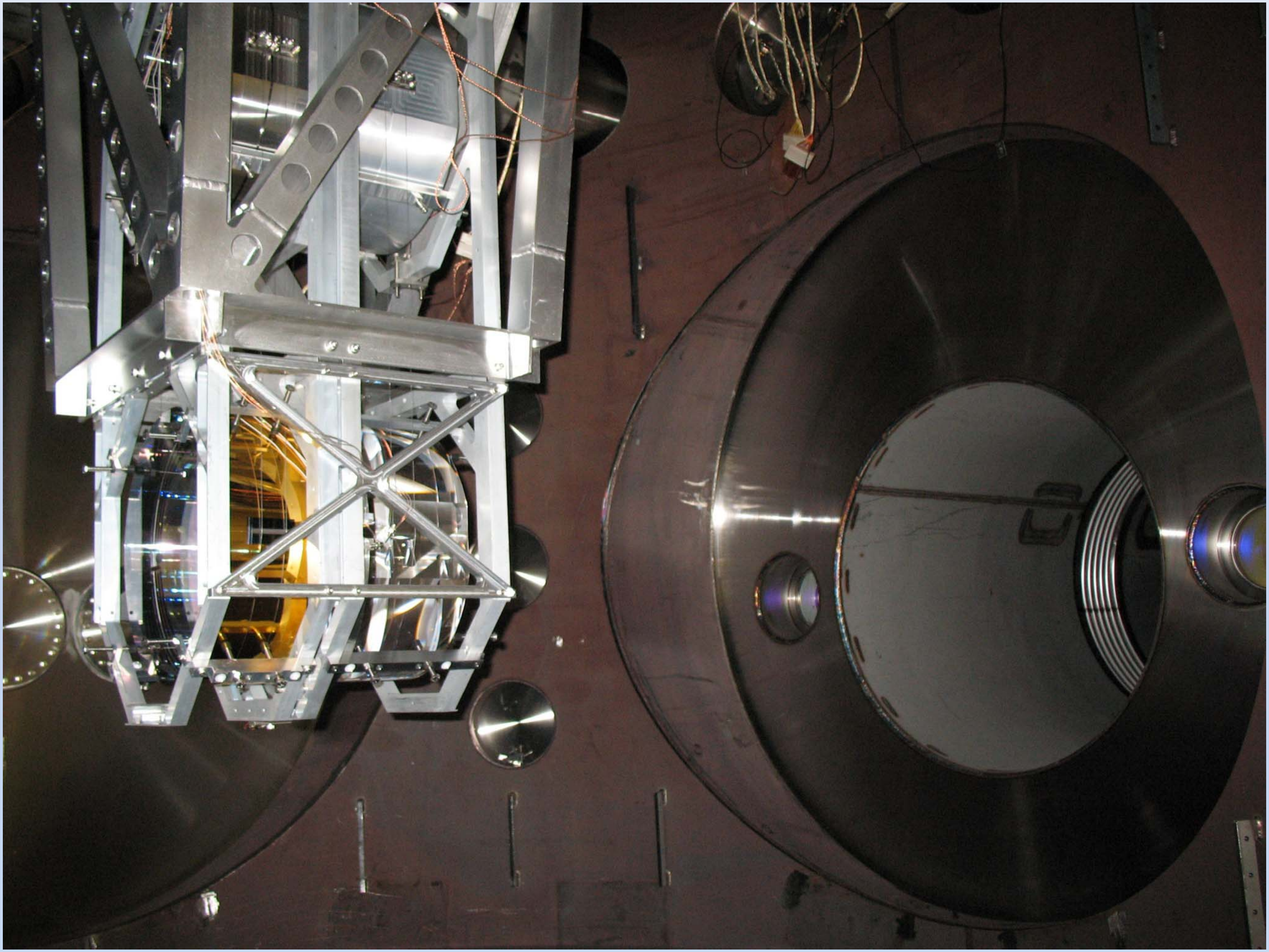
The random distribution of photons arriving at the photodetector produces ***shot noise***.

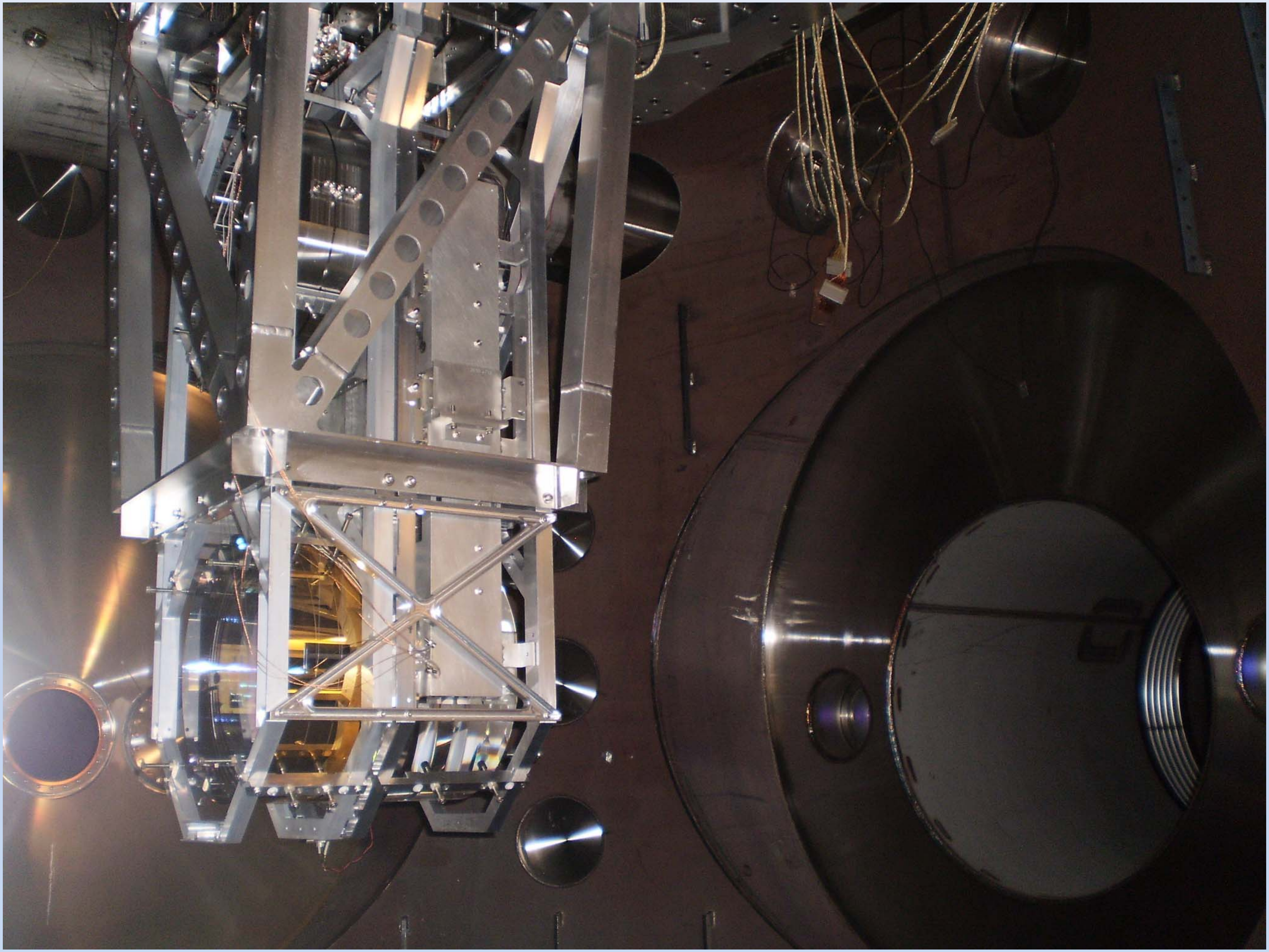
Actuators


Flexi-circuit
Flag
Magnet
LED
Photodetector
~4mm
Flag
Magnet
LED
Connector
Photodetector
Coil Actuator

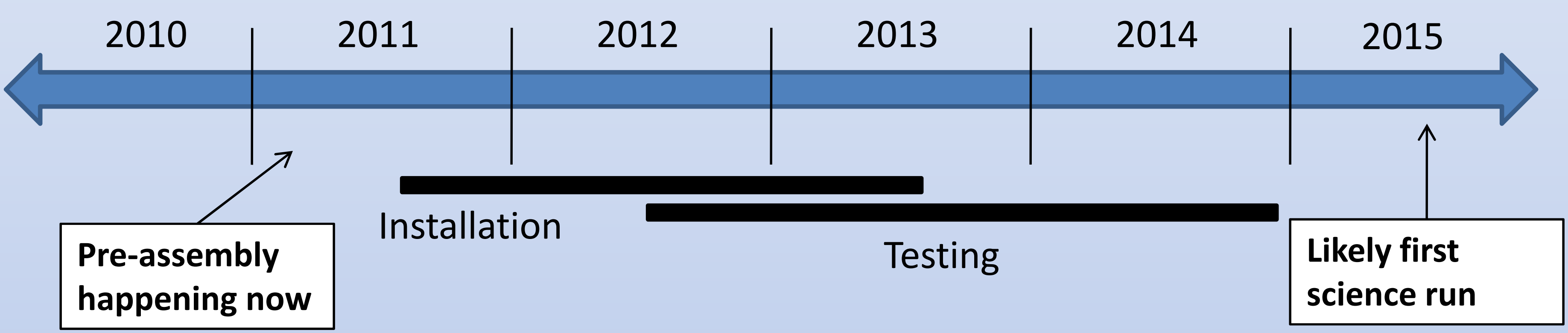

Sensor/Actuator
(OSEM)
m1
m2
m3
m4
Optic
Blades
Springs
Electrostatic
Drive



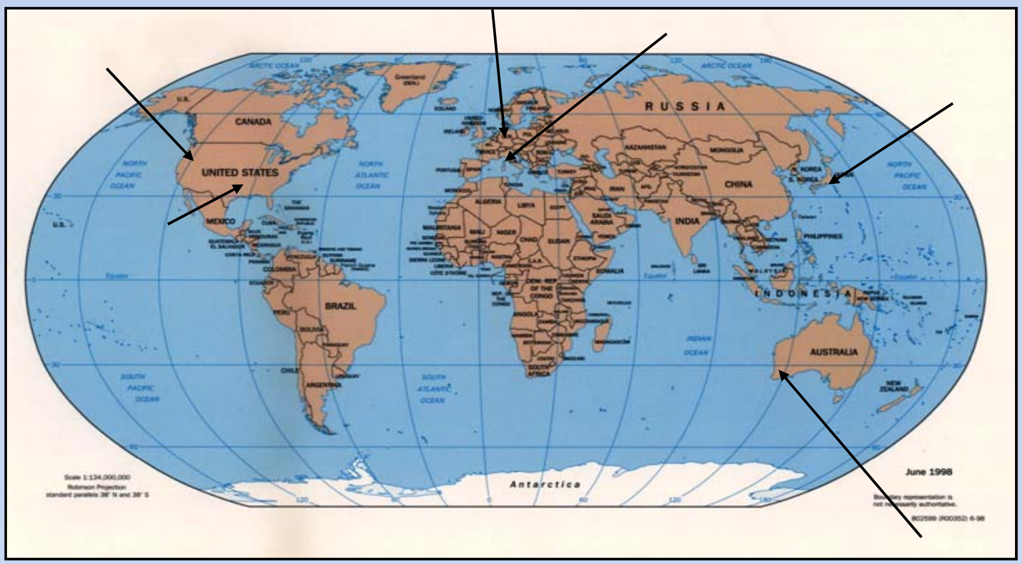




Advanced LIGO Schedule

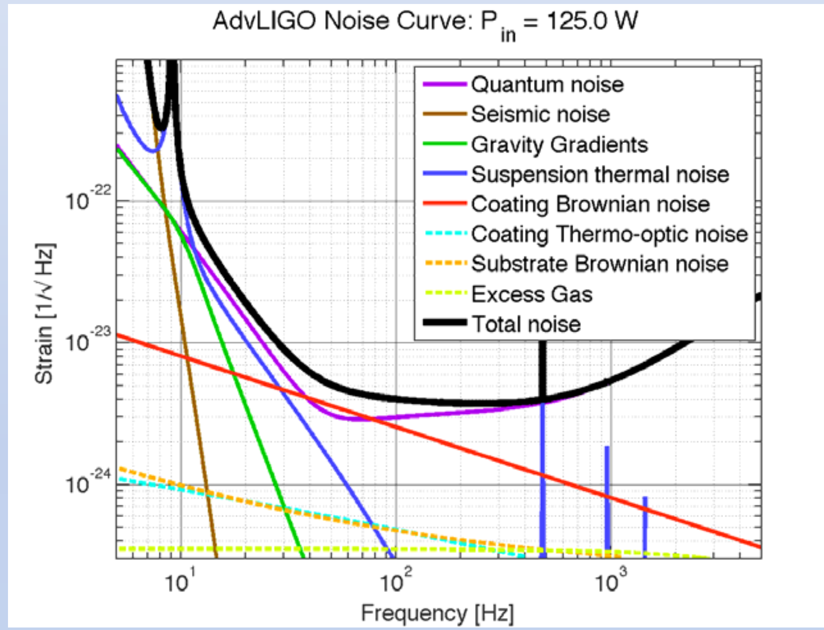


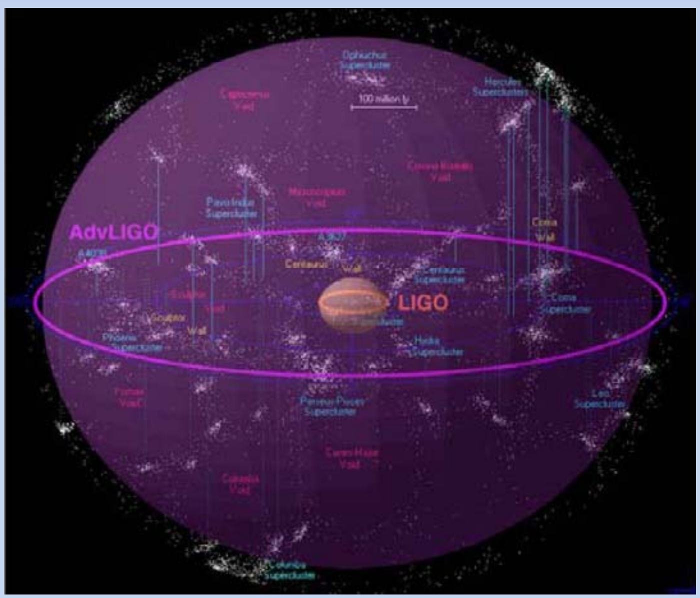
• Other observatories around the world collecting data during this time.



Advanced LIGO

- **Quantum noise limited in much of band**
- **Thermal noise in most sensitive region**
- **About factor of 10 better sensitivity**
- **Expected sensitivity**
  - Neutron star inspirals to about 200 Mpc, ~**40/yr**
  - 10 M_O black hole inspirals to 775 Mpc, ~**30/y**





Advanced LIGO Astronomical Reach

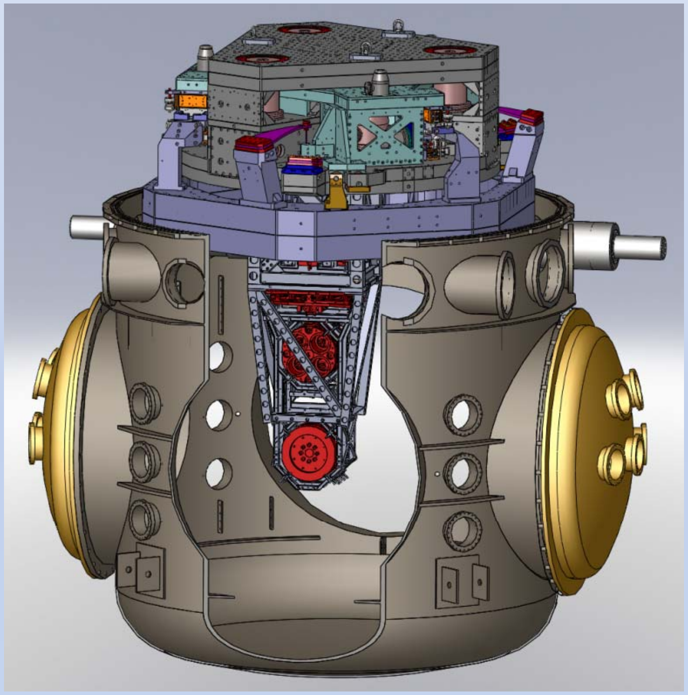
LIGO infrastructure designed for a progression of instruments

- Nominal 30 year lifetime

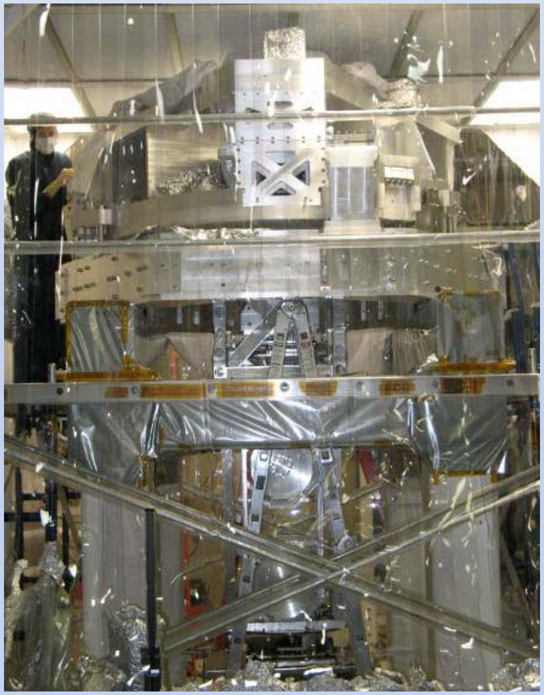
All subsystems to be replaced and upgraded

- More powerful laser – from 10W to 180 W
- Larger test masses – from 10 kg to 40 kg
- More aggressive seismic isolation
- Lower thermal noise coatings

Seismic Isolation



ISI and Quad in LIGO Vacuum Chamber



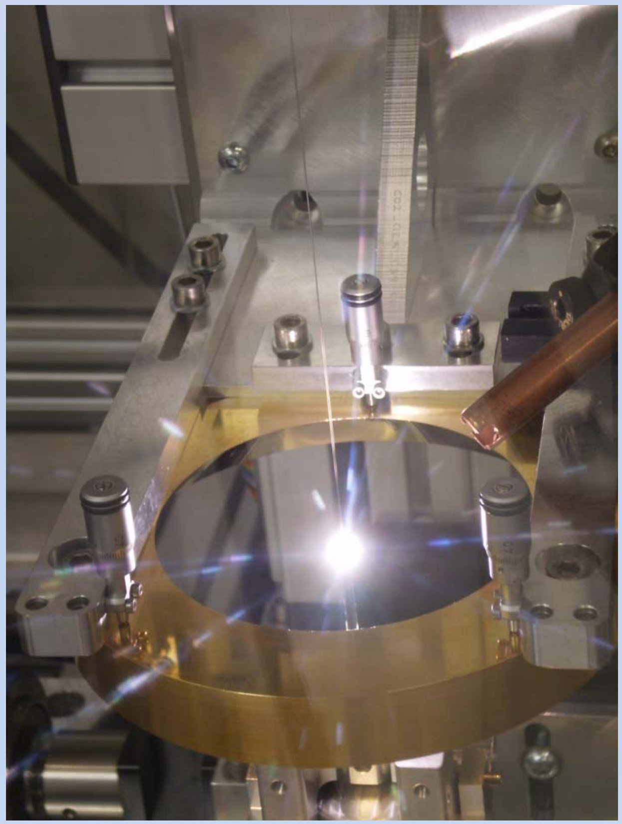
Prototype ISI and Quad at MIT

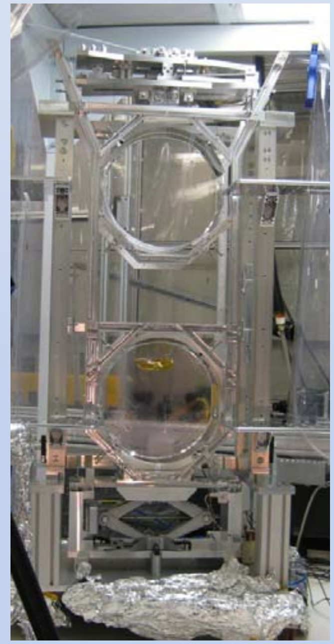
Overall Isolation 9 to 10 orders of magnitude at 10 Hz.

- 7 cascaded stages of seismic isolation
    - External Hydraulic Pre-Isolator (HEPI). Active isolation up to 10 Hz.
    - 2 stage Internal Seismic Isolation (ISI). Active isolation up to 30 Hz, passive above.
    - 4 stage Quadruple Pendulum (Quad). Mirror is the final stage. Passive isolation above 1 Hz.

Mirrors Suspend from Glass Fibers

• Developed by the University of Glasgow
• Suspension thermal noise suppressed by suspending low loss fused silica test masses from fused silica fibers.





• 0.6m long, 400 μm diameter silica fibers pulled from 3 mm diameter stock and laser welded between the two silica lower stages of the quadruple pendulum.

Cavity Error Signal

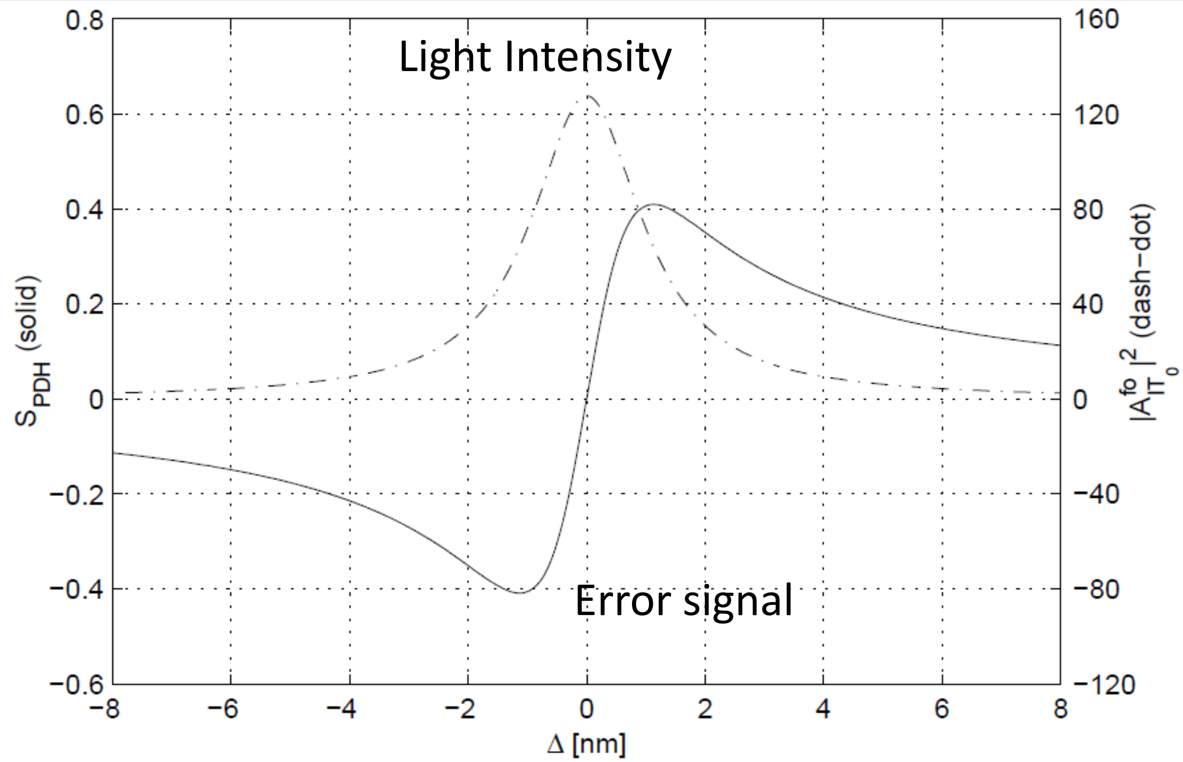


Figure 3.2: The Pound-Drever-Hall error signal for a Fabry-Perot cavity. The cavity parameters are similar to those of the LIGO 1 arm cavities. ($r_{IT} = 0.986$, $r_{ET} = 1$, and $\lambda_0 = 1064\,\text{nm}$.) The "linear region" in S_{PDH} is centered at $\Delta = 0$ and approximately 1 nm wide. In this region a standard linear controller can be used to hold the cavity on resonance. The carrier power in the cavity, $\left|A^{fo}_{IT_0}\right|^2$, is also shown for reference. Note that the key is given along with the axis labels.

Matt Evans. Lock Acquisition in Resonant Optical Interferometers. PhD Thesis. 2002

Cavity Error Signal

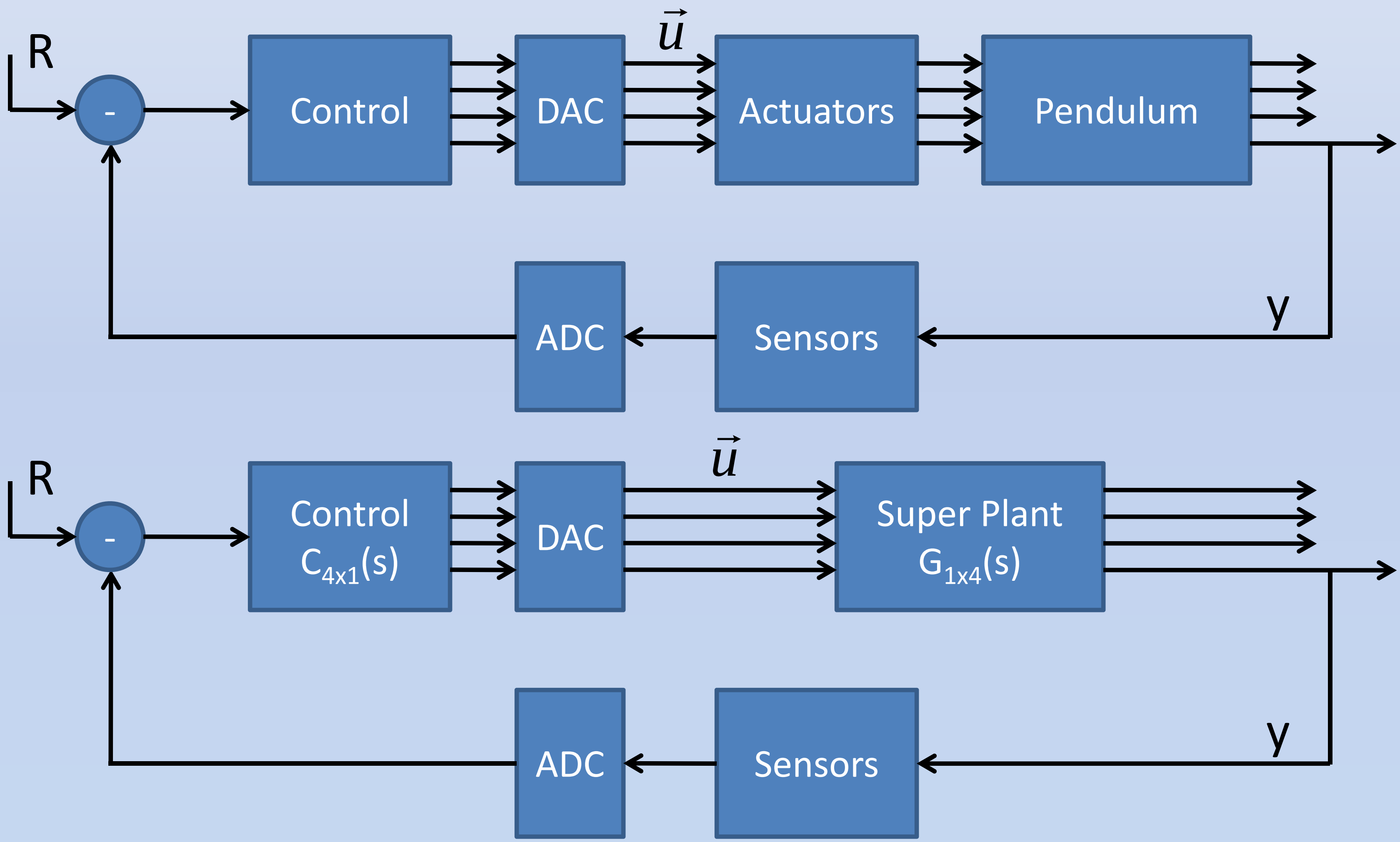
$$S_{PDH} = 2t_{REF}^2 \, r_{ET} \left| A_{IT_0}^{fo} \right|^2 \frac{J_0(\Gamma_{mod})}{J_1(\Gamma_{mod})} \sin(2k_0\Delta)$$

error signal

light power

mirror displacement

Matt Evans. Lock Acquisition in Resonant Optical Interferometers. PhD Thesis. 2002


R
-
Control
DAC
$\vec{u}$
Actuators
Pendulum
y
ADC
Sensors
R
-
Control
$C_{4x1}(s)$
DAC
$\vec{u}$
Super Plant
$G_{1x4}(s)$
y
ADC
Sensors

Notes

- GW calibration
- Find more up to date solid works model
- References

Notes

- Move actuators sooner
- Concise way to explain 10^-15
- Move quad before sensitivity
- Maybe 1 BD
- Write out notes for each slide

- Use lighter background
- Put on flashdisk
- Clarify LIGO figure
- Expand mirror to quad for transition