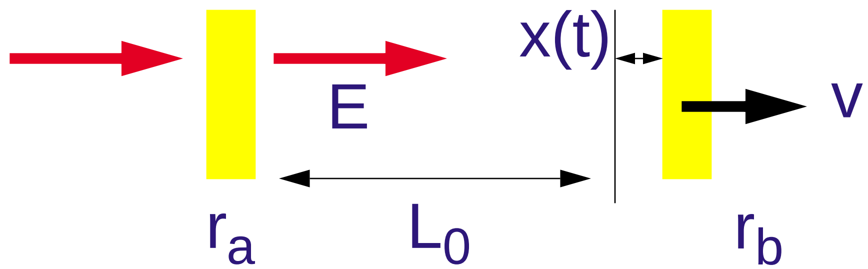
Fringe structure of LHO 2k FP

a modeler's view

- Presented by Hiro Yamamoto of CIT/LIGO Lab at the lunch time meeting at LHO on December 9, 1999
- The analysis method was developed in 1997 when the 40m one arm FP measurement was done and analyzed by Matt Evans, Malik Rakhmanov and Hiro Yamamoto.
- The derivation of the analytic expression of the fringe structure can be found in the thesis of Malik.
- The fringe structure analysis tool for this 2k FP data has been developed using matlab, and will be installed at LHO.

Fabry-Perot cavity field

simple expression



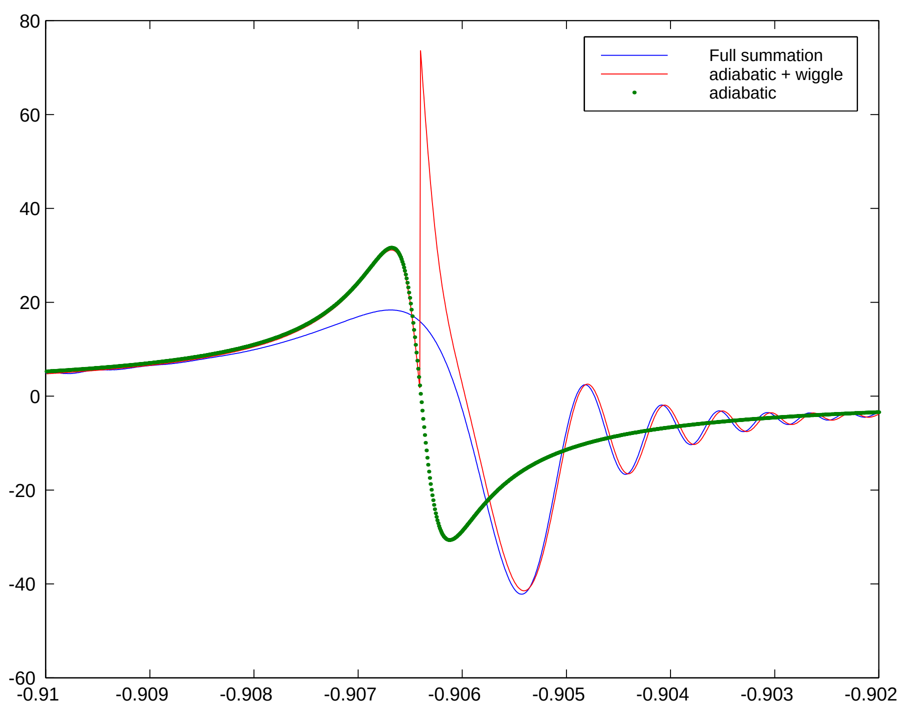
$\tau_0 = L_0/c$

$k = 2\pi/\lambda$

- $$E(t) = \sum_{m=0}^{N=\frac{t}{2\tau_0}} \exp(-(B + iC)\cdot m + i(A\cdot m^2))$$

$$A = 2k\tau_0 v \qquad B = \ln\left(\frac{1}{r_a r_b}\right) \qquad C = 2kx(t)$$

- $$E(t) \approx \frac{1}{1 - r_a r_b \exp(-2kx)} + \sqrt{\frac{i\cdot\pi}{A}}\exp\left(-\frac{\ln\left(\frac{1}{r_a r_b}\right)}{2\tau_0}(t - t_0)\right)\exp\left(i\left(\frac{B^2}{4A} - \frac{kv}{2\tau_0}(t - t_0)^2\right)\right)$$



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Data vs calculation

preview of results

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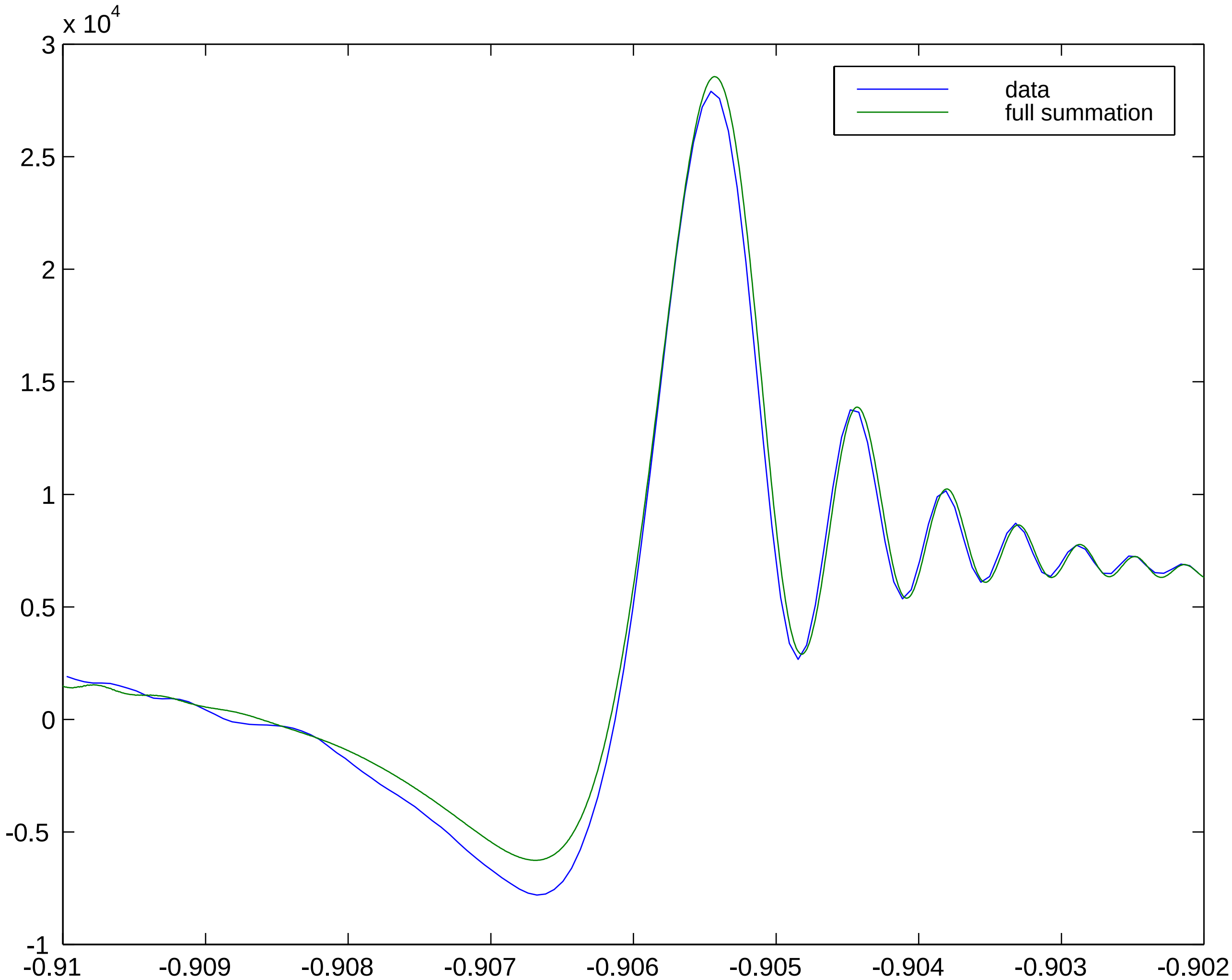

- Calculation is done using the parameters determined by the procedures described blow, except for the DC offset and mixing angle (assumbed to be ideal).

Velocity and resonant point

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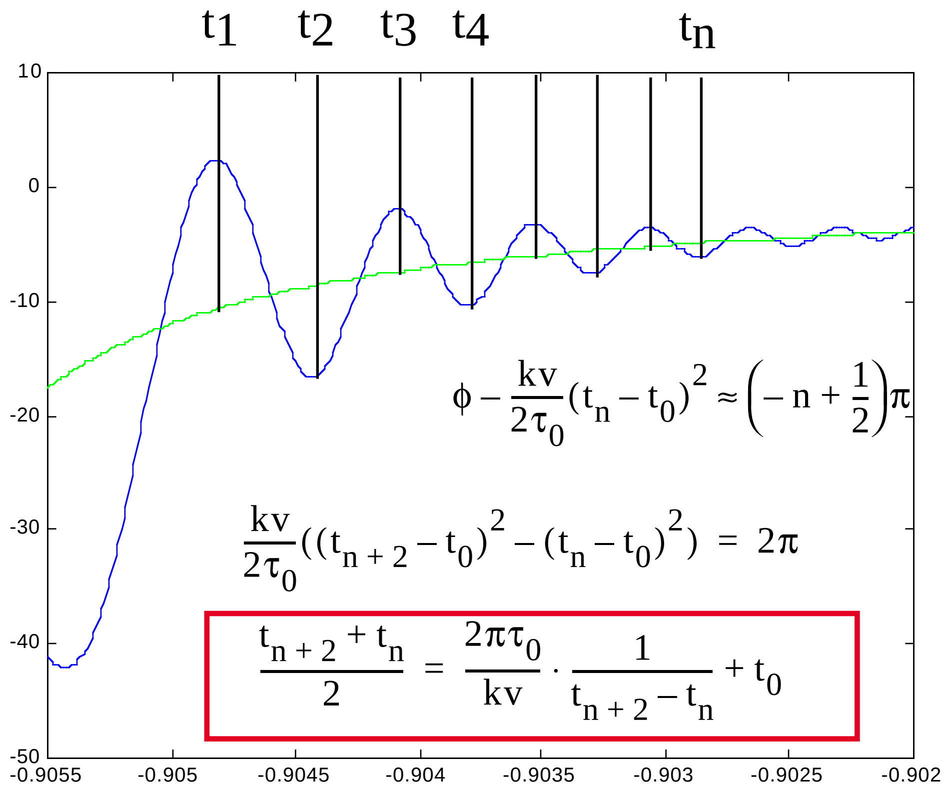

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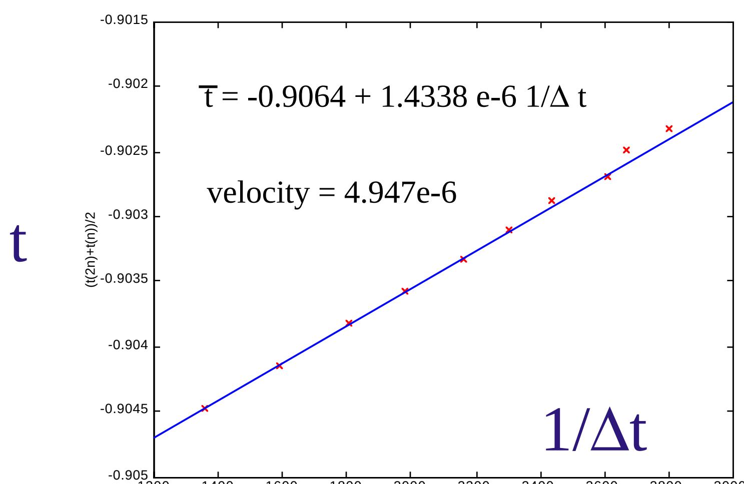

Finesse

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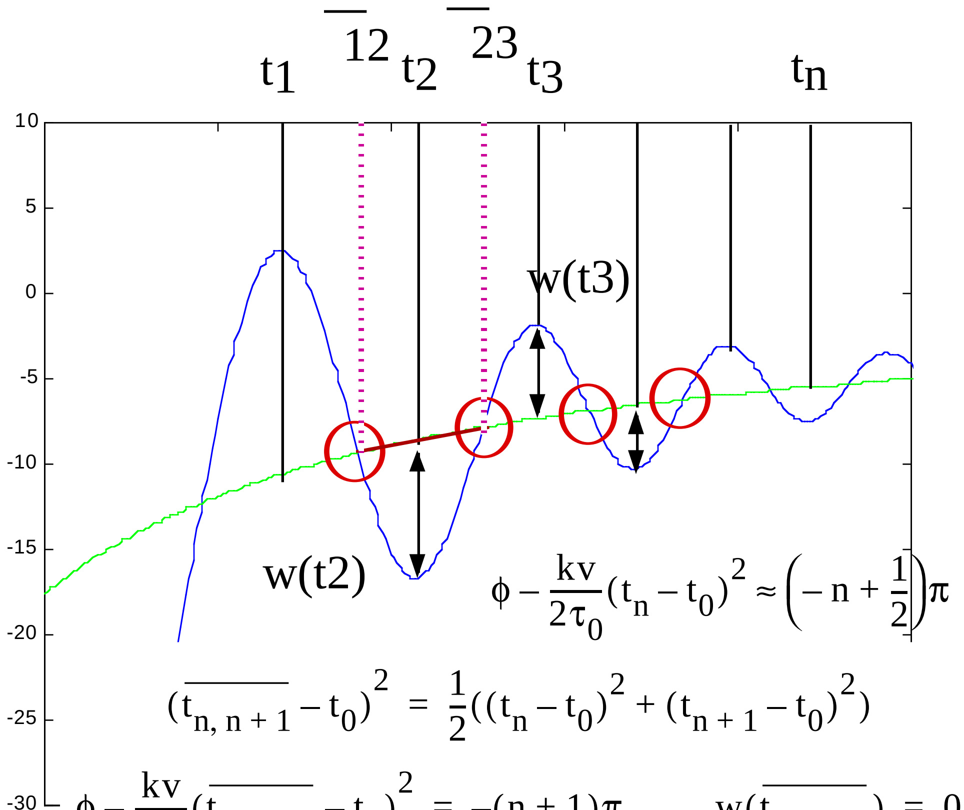

$$\phi - \frac{kv}{2\tau_0}(t_n - t_0)^2 \approx \left(-n + \frac{1}{2}\right)\pi$$

$$(\overline{t_{n,\,n+1}} - t_0)^2 = \frac{1}{2}((t_n - t_0)^2 + (t_{n+1} - t_0)^2)$$

$$\phi - \frac{kv}{2\tau_0}(\overline{t_{n,\,n+1}} - t_0)^2 = -(n+1)\pi \qquad w(\overline{t_{n,\,n+1}}) = 0$$

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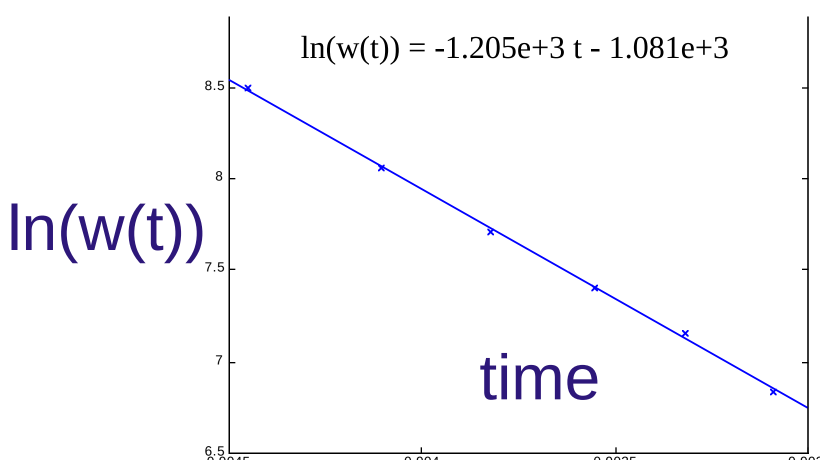

$$\text{slope} = \frac{\ln\frac{1}{ra \cdot rb}}{2 \cdot \tau_0}$$

$$RaRb = 0.9684$$

$$T_a J_0 J_1 Pwr = 575$$

Summary of velocity and finess
preliminary

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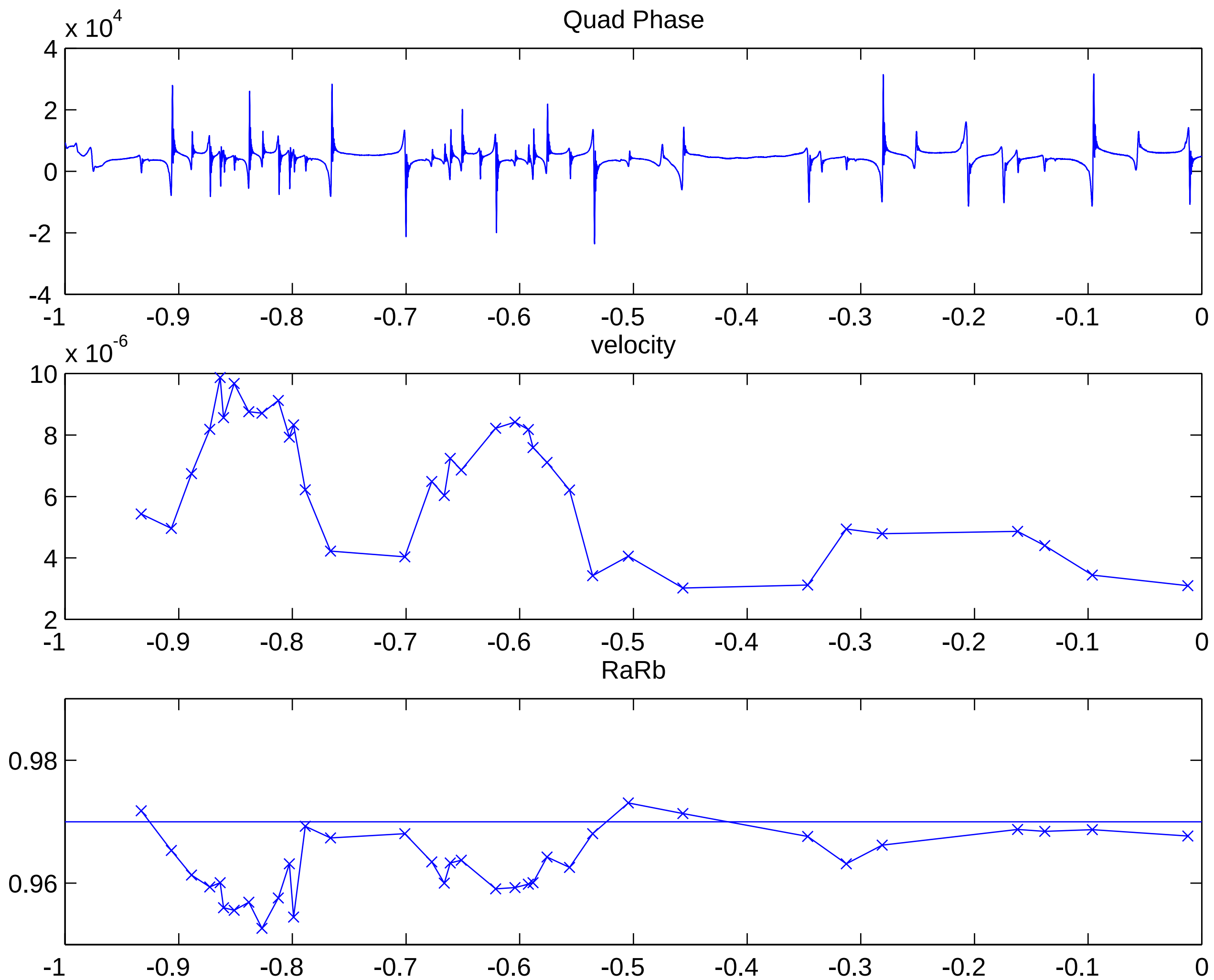

Signal normalization

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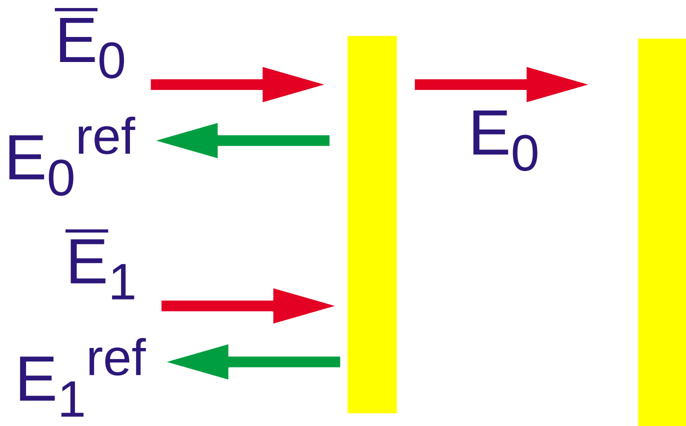

$$E_1^{ref} = r_a \overline{E_1}$$

$$E_0^{ref} = r_a \overline{E_0} + t_a E_0$$

$$E_0 = \frac{1}{1 - r_a r_b e^{-i2kl}} + \text{wiggle}$$

$$\text{demod} \sim \overline{E_1} \cdot E_0$$

- Error signal and slope - static limit

$$\text{error} = 2 \cdot \text{Pwr} \cdot J_0(\Gamma) \cdot J_1(\Gamma) \cdot T_a \cdot r_a \cdot \text{Imag}\left(\frac{1}{1 - r_a r_b \exp(-2ikl)}\right)$$

$$= 575 \cdot \text{Imag}\left(\frac{1}{1 - r_a r_b \exp(-2ikl)}\right)$$

$$\text{slope} = \frac{d}{dl}\text{error} = 575 \cdot \frac{4\pi r_a r_b \frac{1}{\lambda}}{(1 - r_a r_b)^2} = 575 \times 5.1\text{e}10 = 2.7\text{e}13$$