LIGO-G990079-31-M

Simultaneous dynamical tracking and removal of multiple violin modes

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Background & Motivation

- Violin modes are narrowband noise in the output of a detector that arise due to excitation of mirror suspension wires.
- Gravitational waves incident on an interferometer change the distance between the mirrors without disturbing the mirror suspensions.
- Study of dynamics of the violin modes can thus be used as a diagnostic to separate the mechanical disturbances from gravitational waves.
- Instrumental artifacts can be removed from the data to leave a cleaner residual for subsequent search for signals of astrophysical interest.
- Strip chart operator display helps track each mode for correlations and sudden nonstationarities.
- Instantaneous mode temperature estimation can be done.

Dynamical model

A viscously damped simple harmonic oscillator driven by white noise.

$$\ddot{x}_0 + (\omega_0 / Q)\dot{x}_0 + \omega_0^2 x_0 = F$$

Measurement equation :

$$y(t) = H\, x(t) + V(t)$$

Problem :
Given y(t), estimate x(t) .

Method

A Kalman filter estimates the state of a dynamical system driven by a random force.

State equation :

$$X_{k+1} = A_k X_k + W_k$$

Measurement equation :

$$Y_k = H_k X_k + V_k$$

How does a Kalman filter work ?

How ~~does~~ a Kalman filter work ?

$$\hat{X}_{K|K-1} \longrightarrow \hat{Y}_K$$

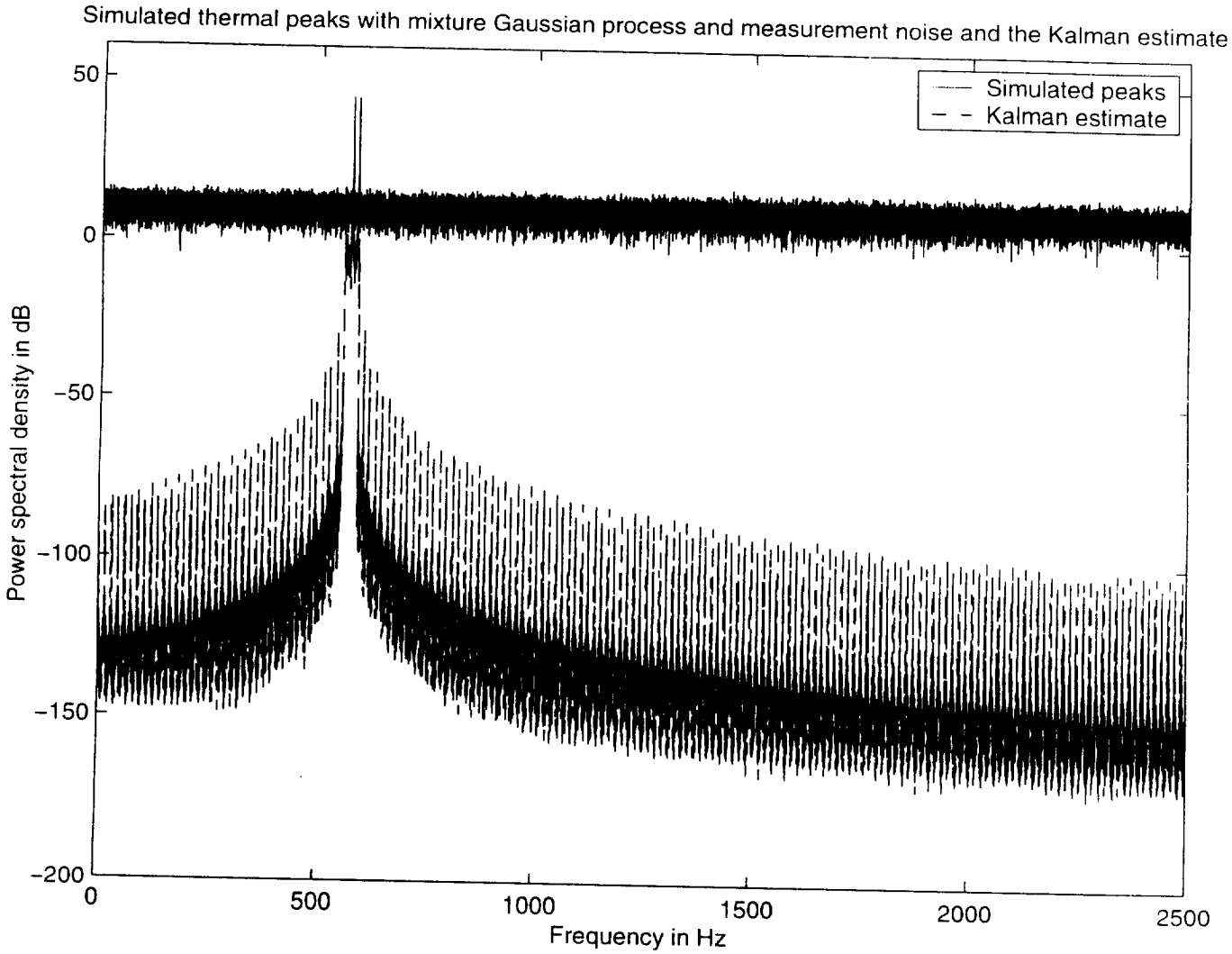
$$Y_K - \hat{Y}_K = \varepsilon_K$$

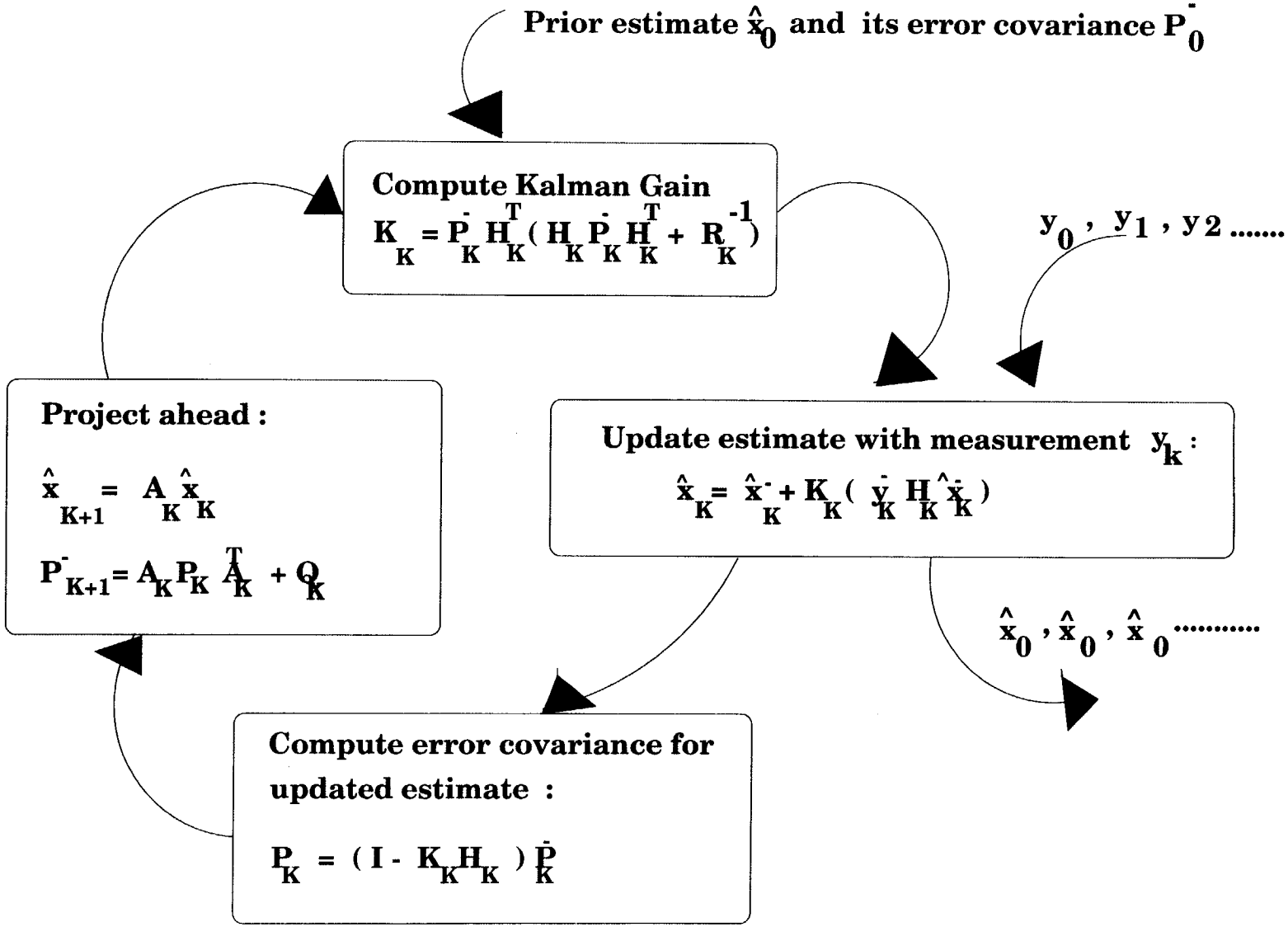
$$G_K \varepsilon_K + \hat{X}_{K|K-1} \longrightarrow \hat{X}_K$$

$$\hat{X}_{K+1|K} = A_K \hat{X}_K \quad \ldots\ldots\ldots\ldots$$

Minimum mean square estimation (MMSE).

Kalman, R. E. 1960 Trans. ASME, J Basic Eng. 82D pp 35-45.

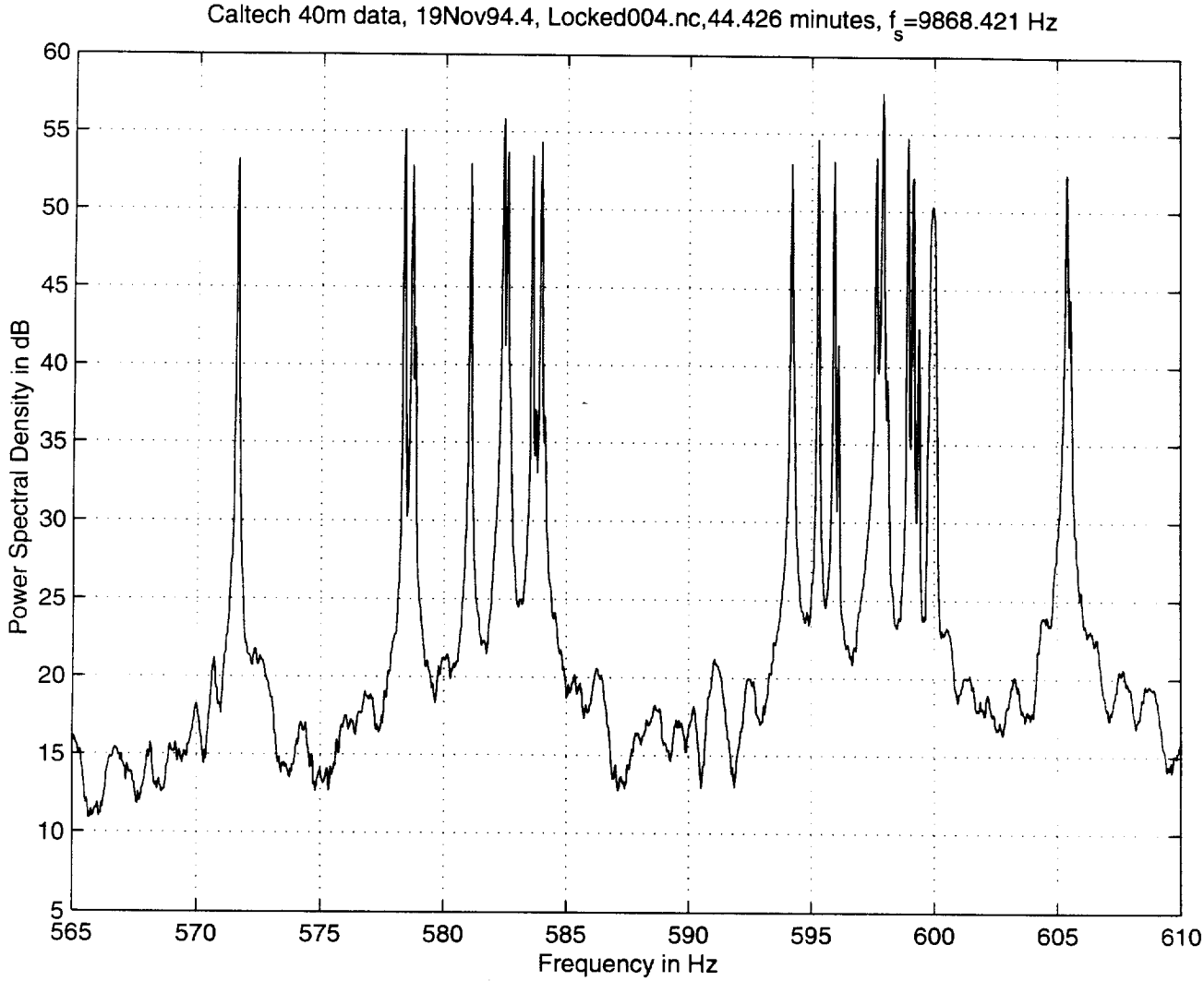

Simulated thermal peaks with mixture Gaussian process and measurement noise and the Kalman estimate
Simulated peaks
Kalman estimate
Power spectral density in dB
50
0
-50
-100
-150
-200
0
500
1000
1500
2000
2500
Frequency in Hz


Prior estimate $\hat{x}_0^-$ and its error covariance P_0^-
Compute Kalman Gain
$K_K = P_K^- H_K^T (H_K P_K^- H_K^T + R_K)^{-1}$
y_0 , y_1 , y_2 .....
Update estimate with measurement y_k :
$\hat{x}_K = \hat{x}_K^- + K_K (y_K - H_K \hat{x}_K^-)$
$\hat{x}_0$, $\hat{x}_0$, $\hat{x}_0$ .....
Compute error covariance for updated estimate :
$P_K = (I - K_K H_K) P_K^-$
Project ahead :
$\hat{x}_{K+1} = A_K \hat{x}_K$
$P_{K+1}^- = A_K P_K A_K^T + Q_K$

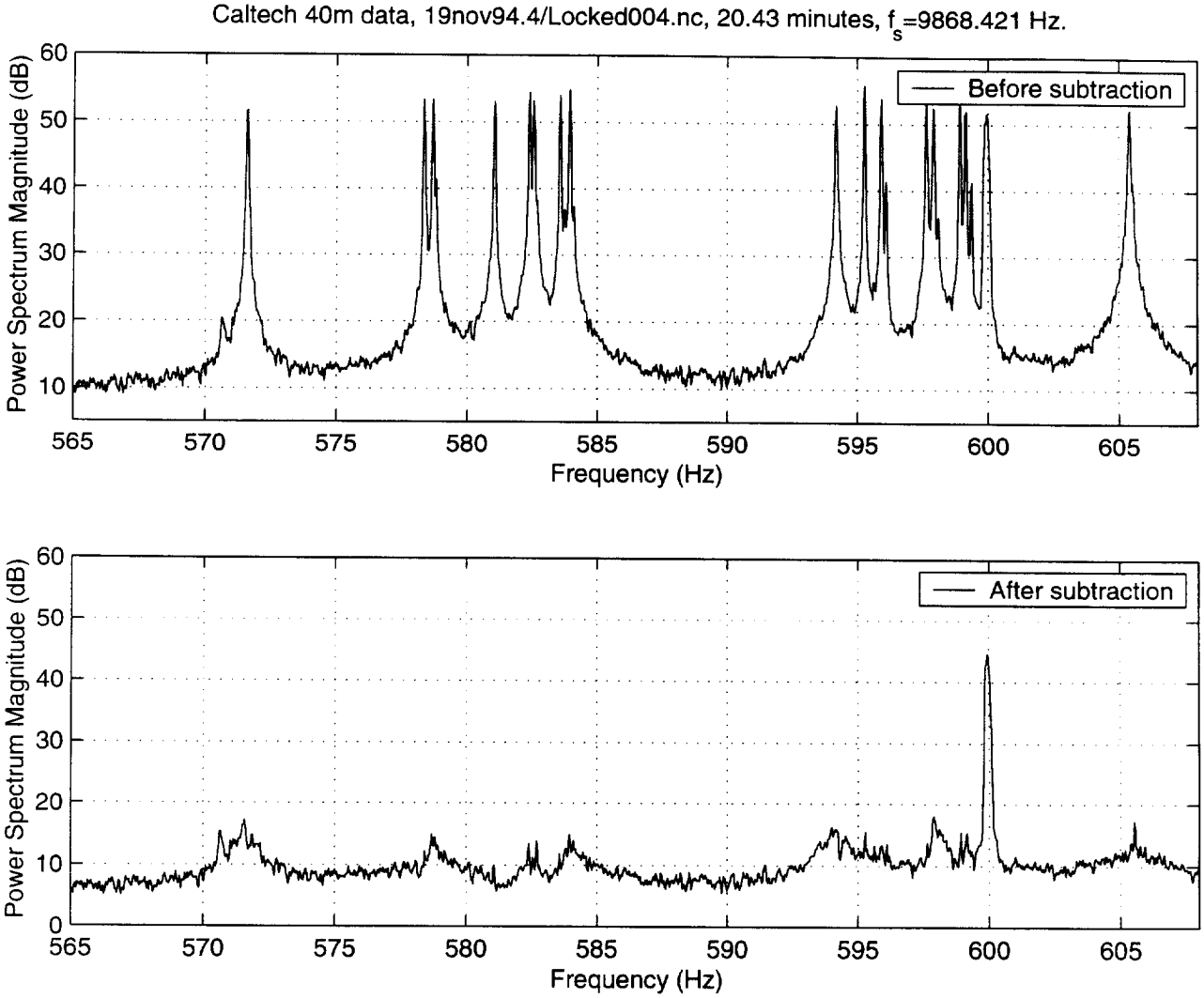
Data

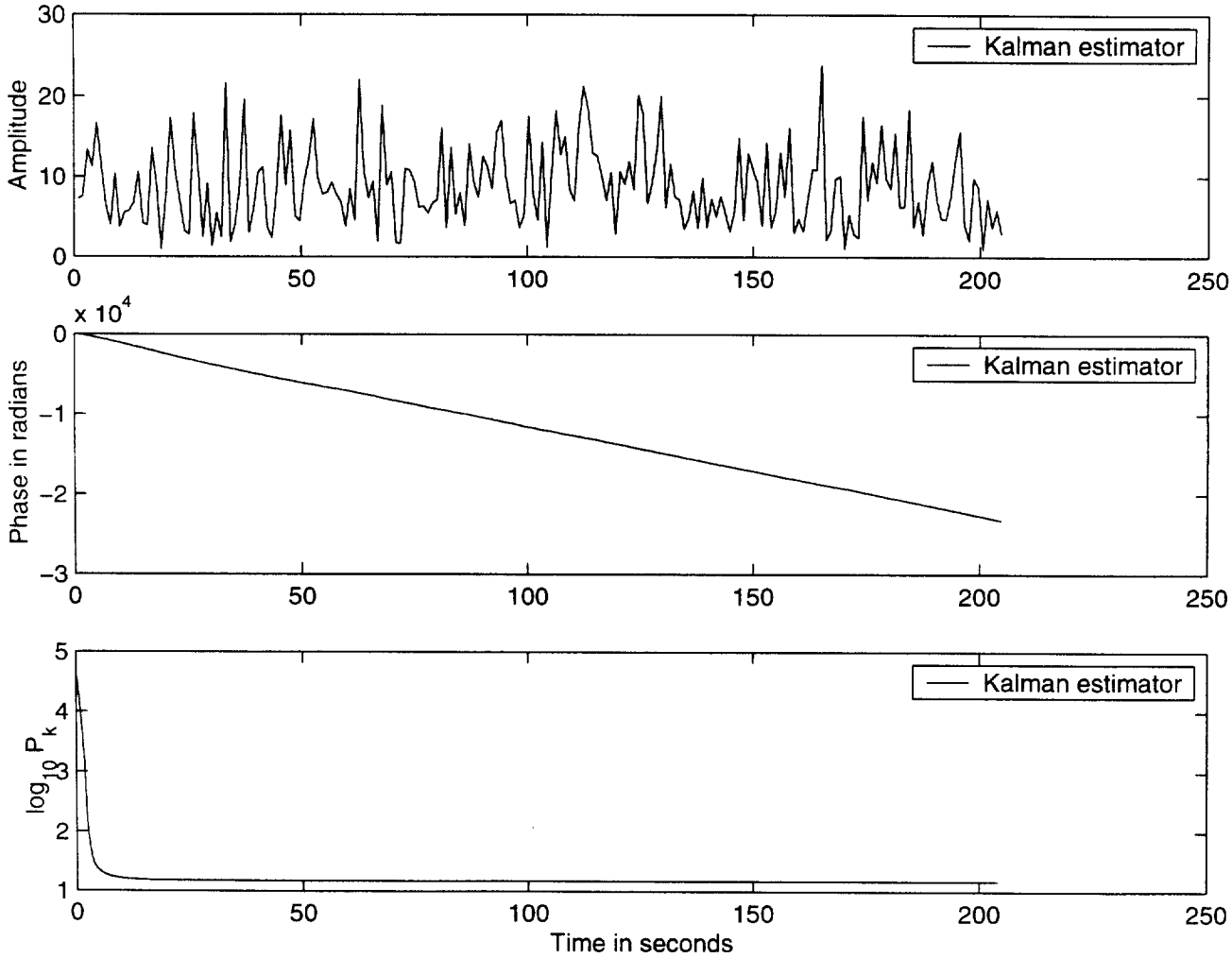
- ✦ Caltech 40 meter data : 19 November , 1994 run.
- ✦ Sampling frequency : 9868.421 Hz
- ✦ IFO output from a 12 bit AD converter : -2047 -- 2048

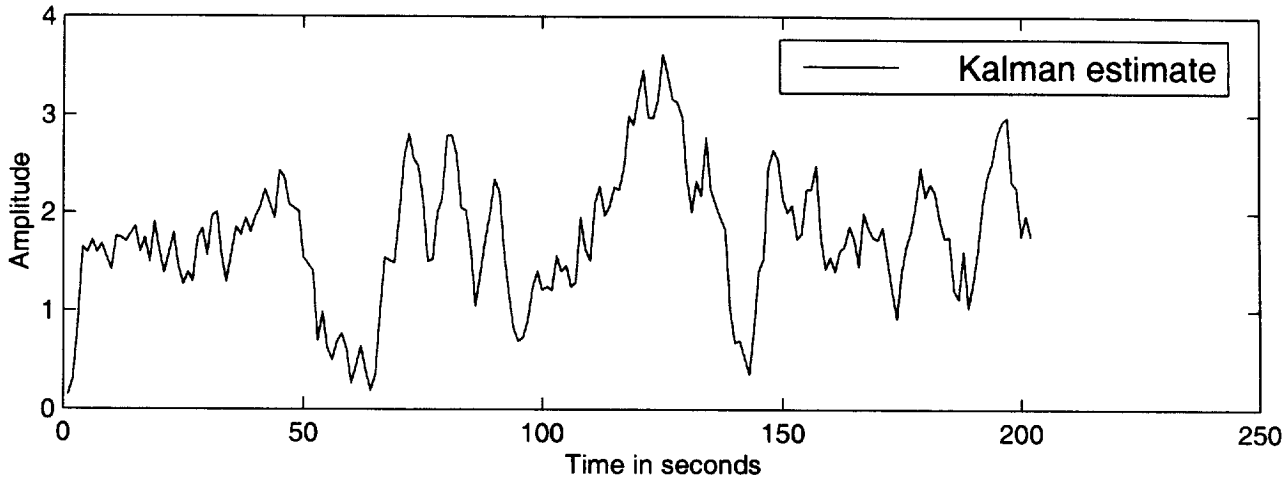
- ✦ Fundamental violin modes lie between 571.6 Hz and 605.425 Hz.
- ✦ More lines than the expected number.
- ✦ Higher frequency counterparts.

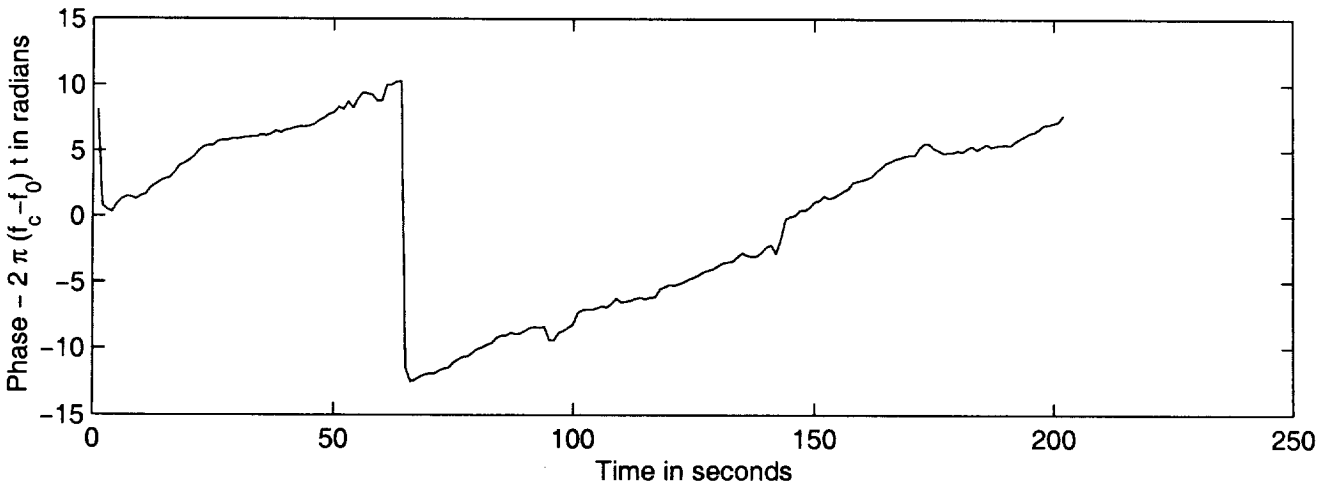

Caltech 40m data, 19Nov94.4, Locked004.nc,44.426 minutes, f_s=9868.421 Hz
Power Spectral Density in dB
Frequency in Hz
60
55
50
45
40
35
30
25
20
15
10
5
565
570
575
580
585
590
595
600
605
610

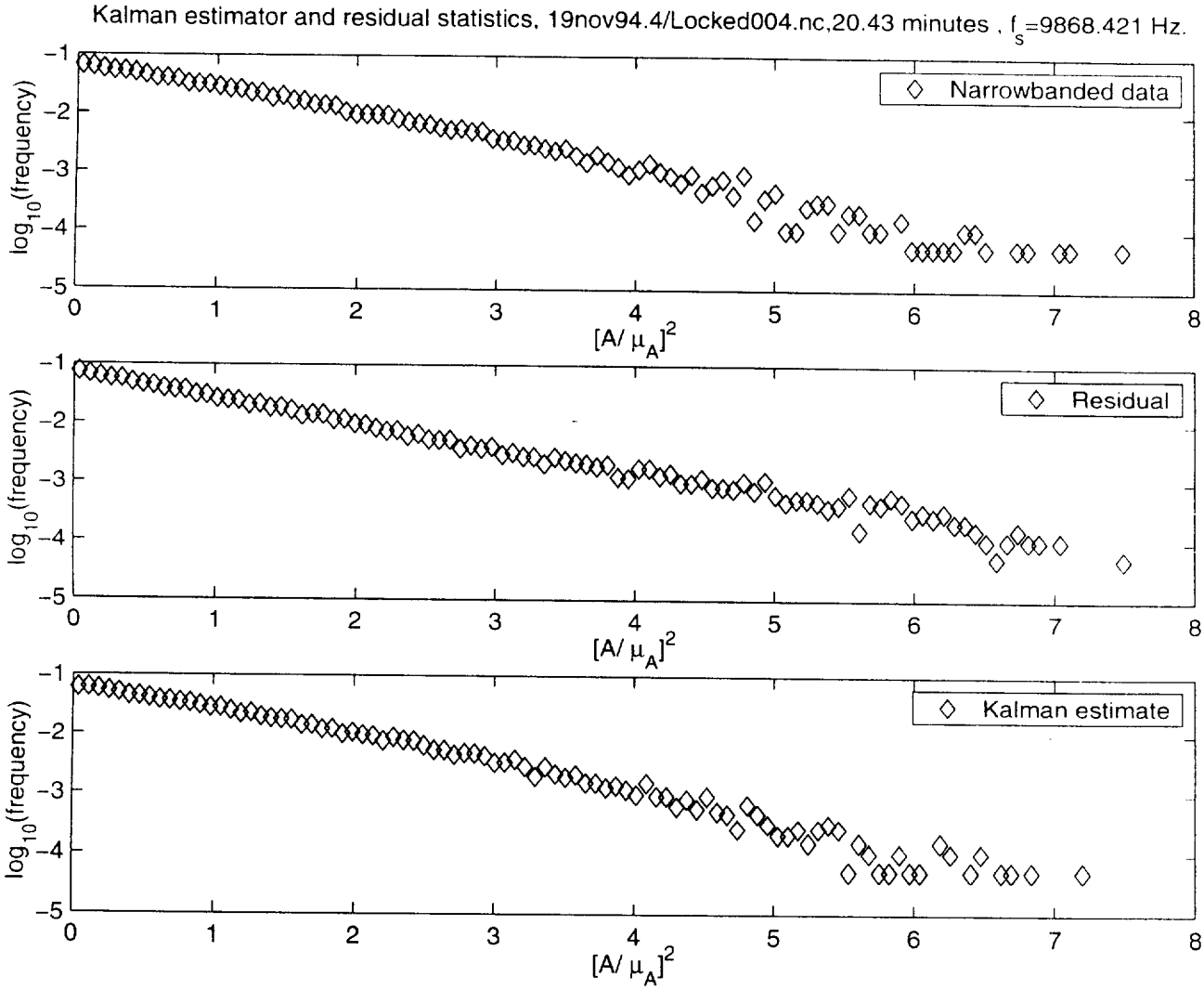
571.5869	0.0126	45508.5111
571.6820	0.0210	27222.9524
578.3400	0.0101	57261.3861
578.4185	0.0140	41315.6107
578.6820	0.0200	28934.1000
578.8050	0.0131	44183.5878
581.0550	0.0200	29052.7500
582.3957	0.0170	34258.5706
582.5586	0.0260	22406.1000
583.5750	0.0155	37650.0000
583.7379	0.0164	35593.7744
583.9429	0.0190	30733.8368
584.1077	0.0220	26550.3482
594.1913	0.0170	34952.4294
594.2902	0.0220	27013.1909
595.2734	0.0130	45790.2615
595.9235	0.0110	54174.8636
596.1029	0.0150	39740.1933
597.6456	0.0180	33202.5333
598.9301	0.0200	29946.5050
599.0271	0.0130	46079.0077
599.1425	0.0140	42795.8929
599.3713	0.0118	50794.1780
599.9756	0.2850	2105.1775
605.3880	0.0450	13453.0667
618.3612	0.0107	57790.7664


Caltech 40m data, 19nov94.4/Locked004.nc, 20.43 minutes, f_s=9868.421 Hz.
Power Spectrum Magnitude (dB)
Frequency (Hz)
Before subtraction
After subtraction
60
50
40
30
20
10
0
565
570
575
580
585
590
595
600
605


Amplitude
Kalman estimator
Phase in radians
x 10^4
Kalman estimator
log10 Pk
Kalman estimator
Time in seconds

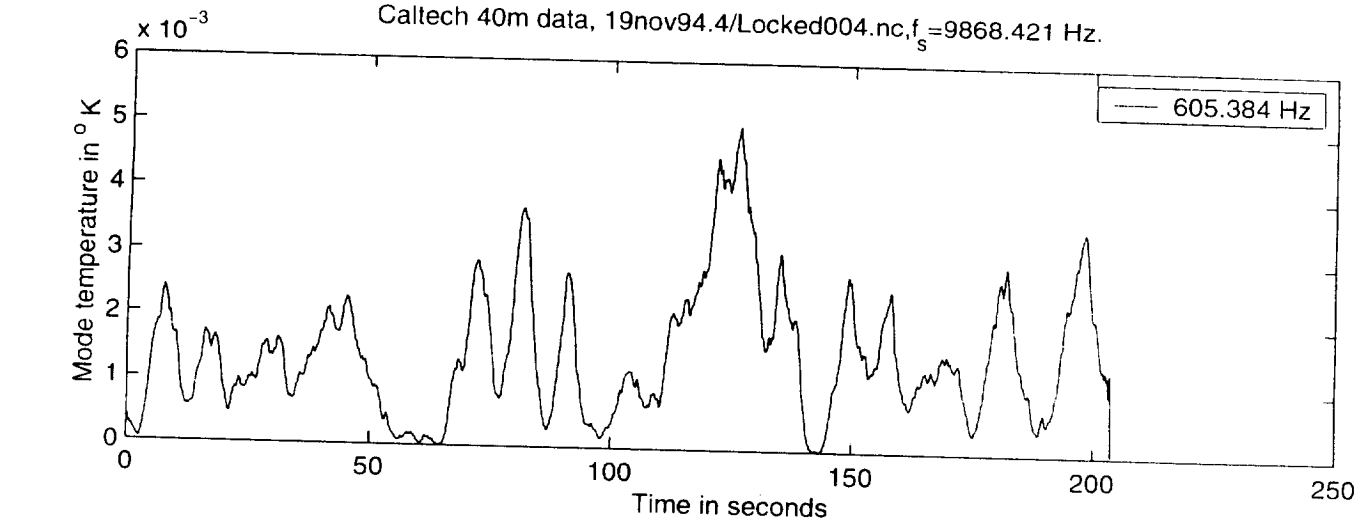

Kalman estimate
Amplitude
Time in seconds
0
1
2
3
4
0
50
100
150
200
250

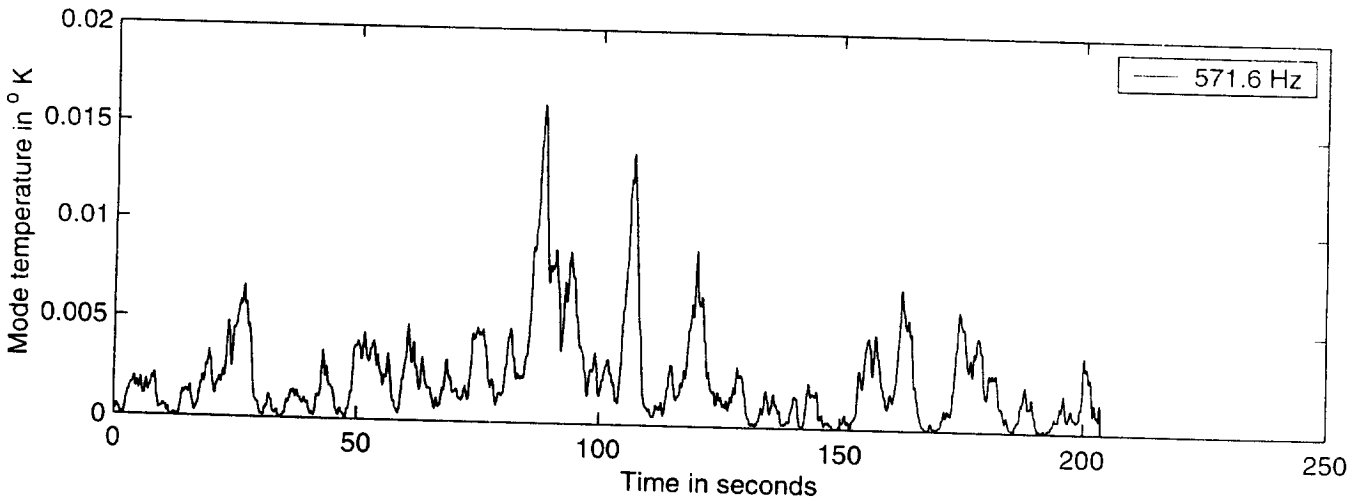

Phase - 2 π (f_c−f_0) t in radians
Time in seconds
15
10
5
0
−5
−10
−15
0
50
100
150
200
250

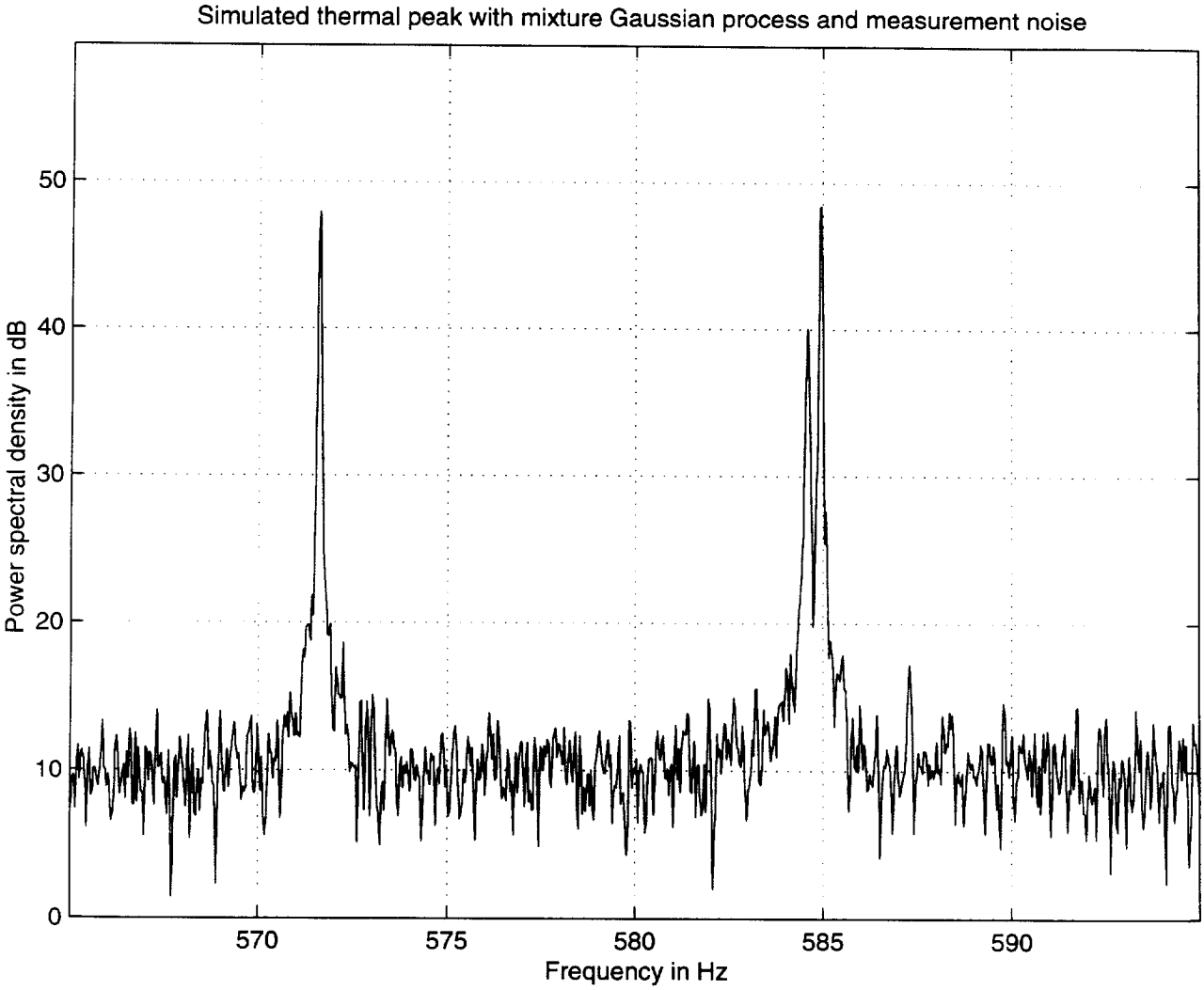

Kalman estimator and residual statistics, 19nov94.4/Locked004.nc,20.43 minutes , f_s=9868.421 Hz.
Narrowbanded data
Residual
Kalman estimate
$\log_{10}$(frequency)
$[A/\mu_A]^2$
−1
−2
−3
−4
−5
0
1
2
3
4
5
6
7
8

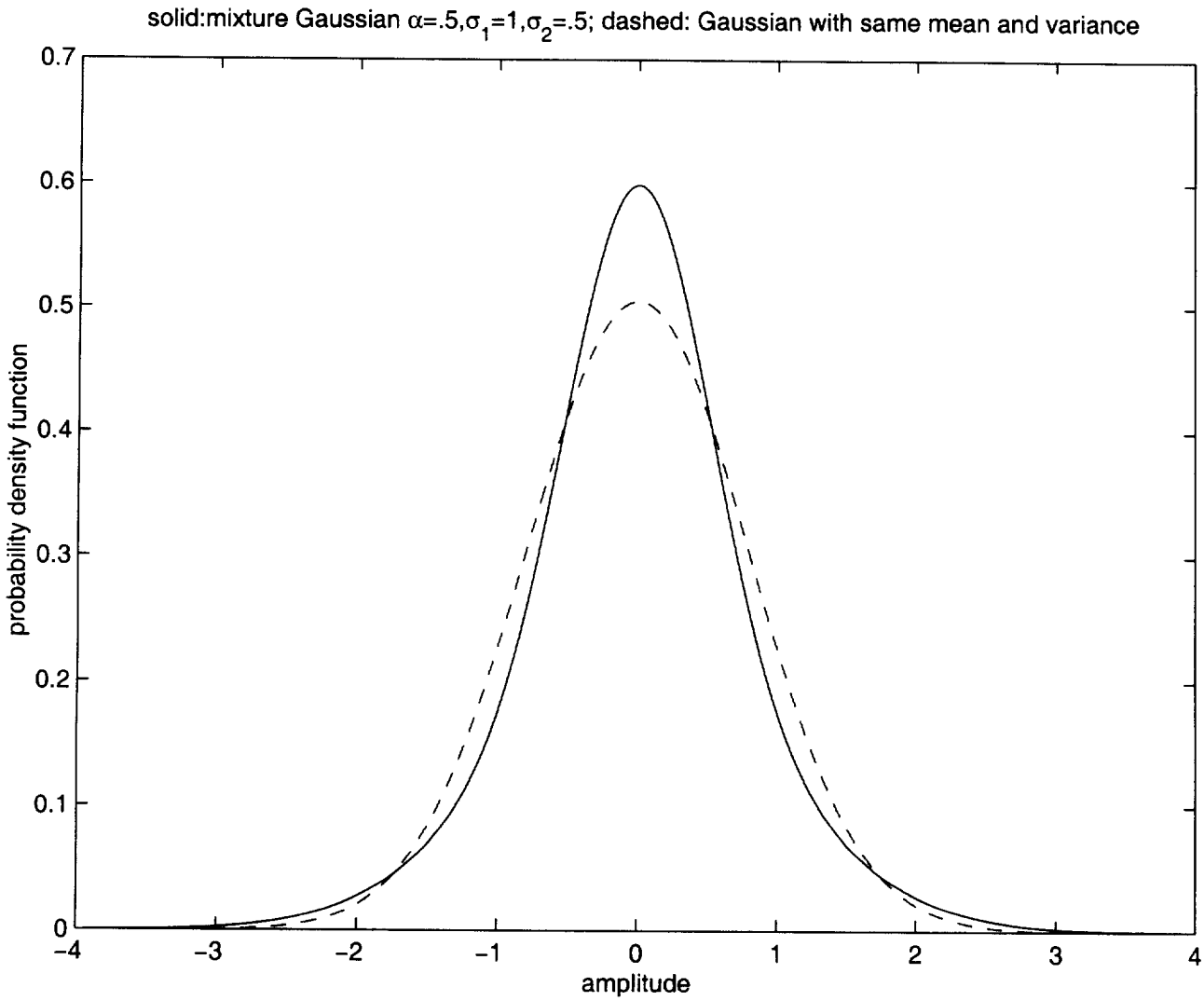
Mode temperature estimation :

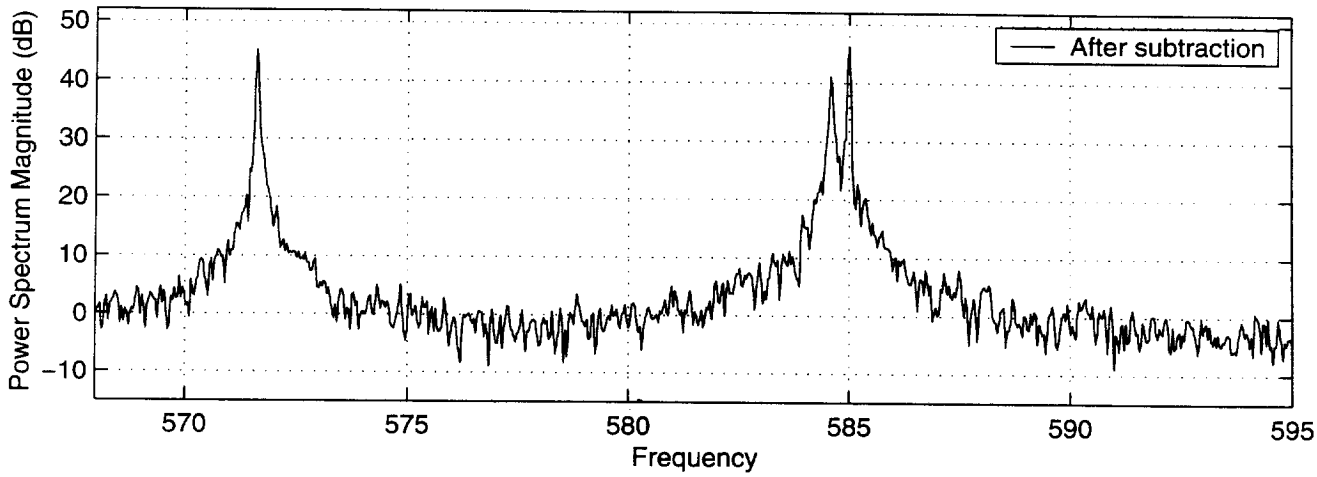
$$\int S_X(f)df = \frac{K_B T f_0^2}{MQ\pi^2} \times \pi \sin\left(\frac{1}{2}\cos^{-1}\left(\frac{1}{2Q^2} - 1\right)\right) \times \text{cosec}\left(\frac{1}{2Q^2} - 1\right) \times \text{cosec}\frac{\pi}{2}$$

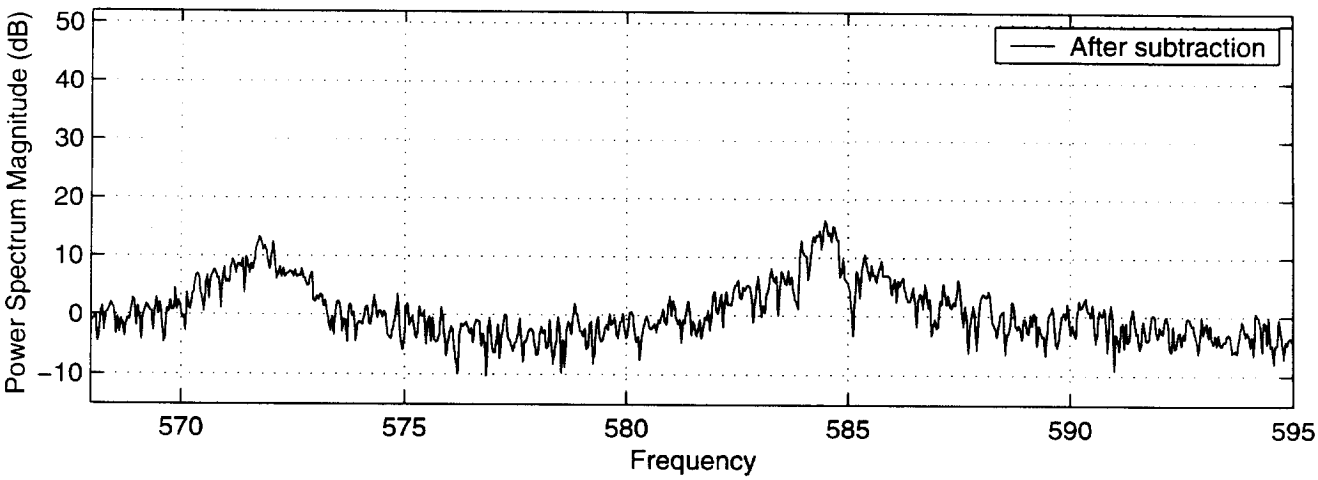

Caltech 40m data, 19nov94.4/Locked004.nc,f_s=9868.421 Hz.
x 10^{-3}
Mode temperature in ^{o}K
605.384 Hz
Time in seconds

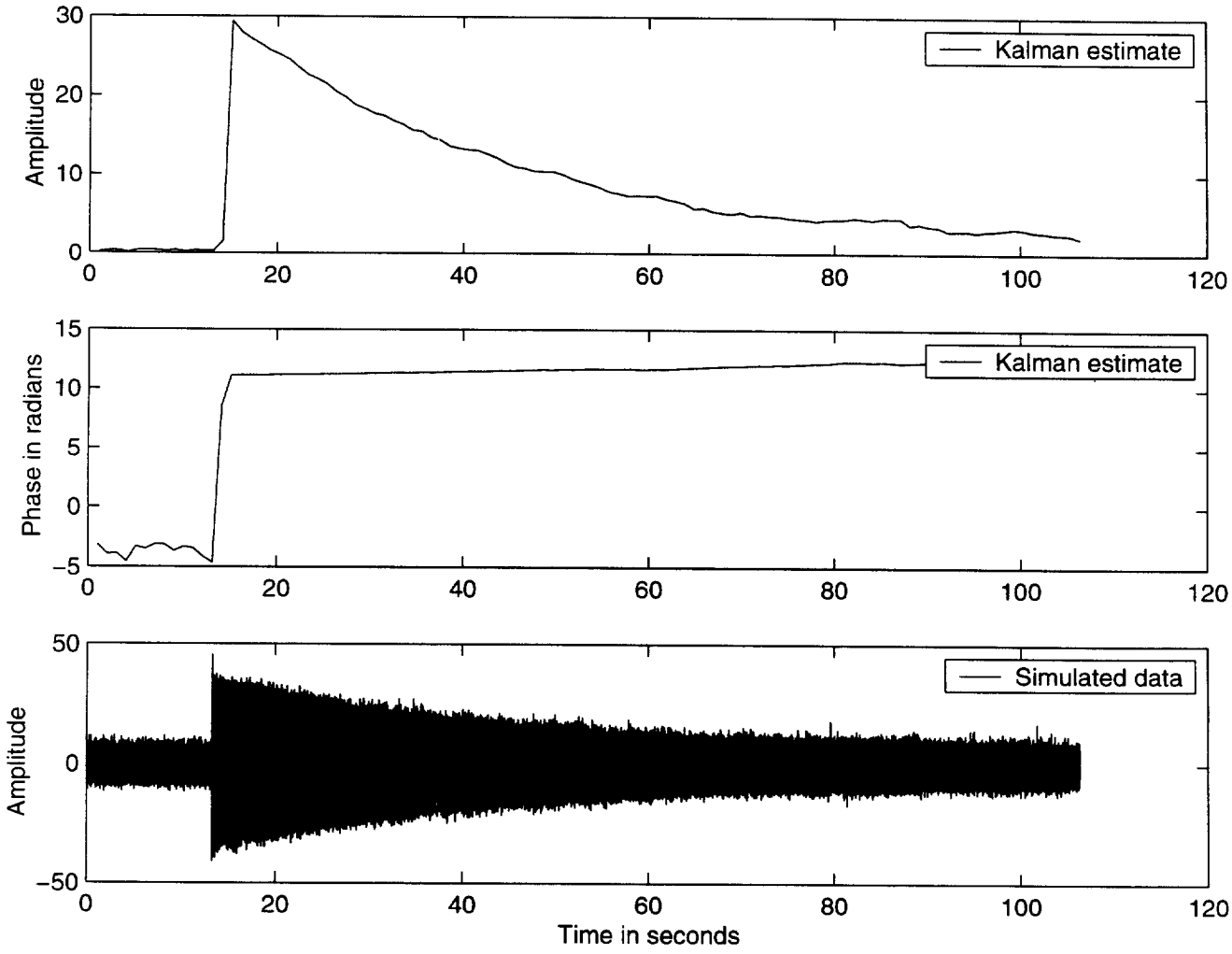

Mode temperature in ^{o}K
571.6 Hz
Time in seconds

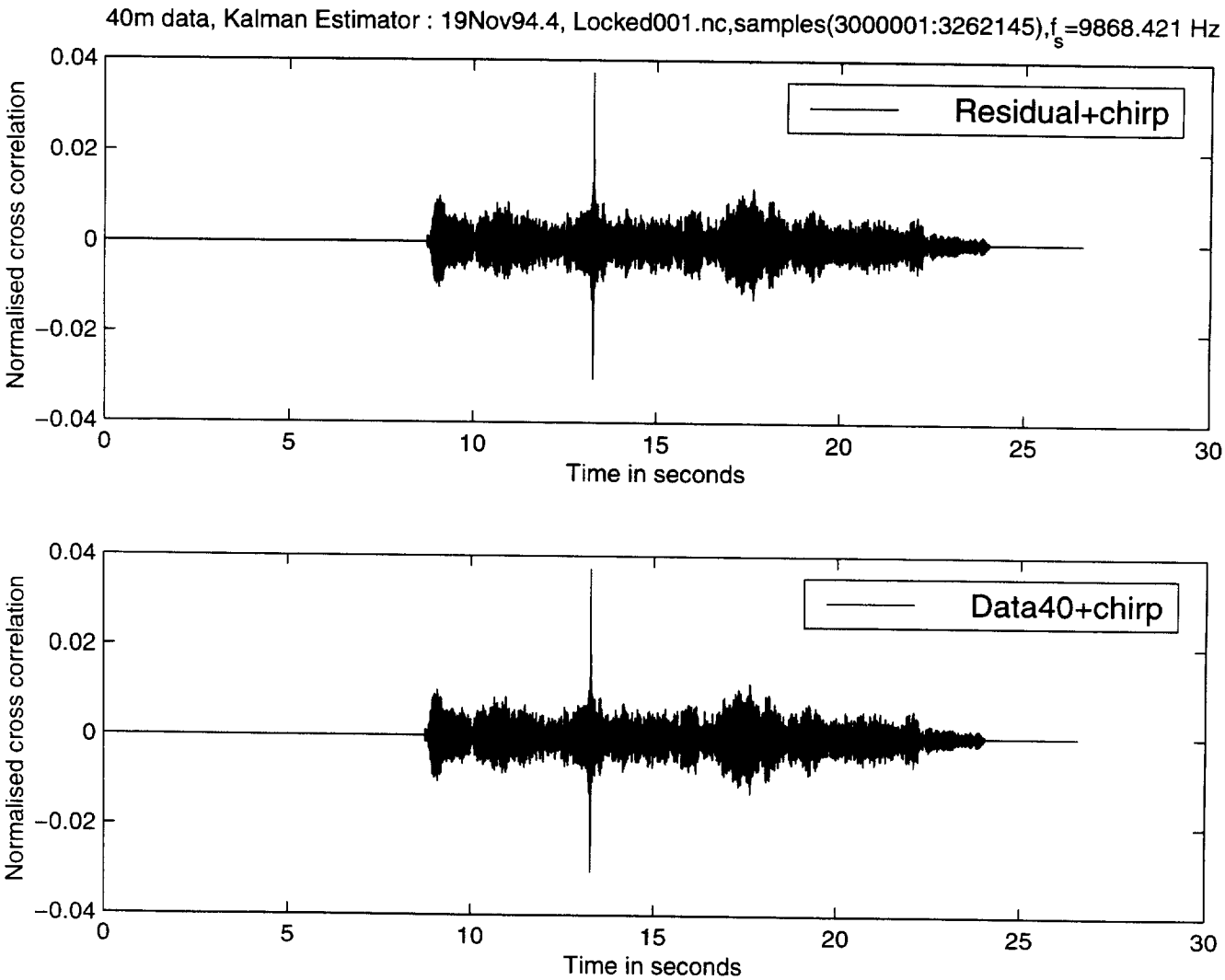

Simulated thermal peak with mixture Gaussian process and measurement noise
Power spectral density in dB
0
10
20
30
40
50
570
575
580
585
590
Frequency in Hz


solid:mixture Gaussian α=.5,σ1=1,σ2=.5; dashed: Gaussian with same mean and variance
probability density function
amplitude
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−4
−3
−2
−1
0
1
2
3
4


Power Spectrum Magnitude (dB)
50
40
30
20
10
0
−10
570
575
580
585
590
595
Frequency
After subtraction


Power Spectrum Magnitude (dB)
50
40
30
20
10
0
−10
570
575
580
585
590
595
Frequency
After subtraction


Kalman estimate
Amplitude
30
20
10
0
0
20
40
60
80
100
120
Kalman estimate
Phase in radians
15
10
5
0
−5
0
20
40
60
80
100
120
Simulated data
Amplitude
50
0
−50
0
20
40
60
80
100
120
Time in seconds


40m data, Kalman Estimator : 19Nov94.4, Locked001.nc,samples(3000001:3262145),f_s=9868.421 Hz
Residual+chirp
Normalised cross correlation
0.04
0.02
0
−0.02
−0.04
0
5
10
15
20
25
30
Time in seconds
Data40+chirp
Normalised cross correlation
0.04
0.02
0
−0.02
−0.04
0
5
10
15
20
25
30
Time in seconds

Note 1, Linda Turner, 08/17/99 08:40:41 PM
LIGO-G990079-31-M