An overview of LSC-OWG-related research activities at the University of Sannio

Vincenzo Galdi

Waves Group, University of Sannio, Benevento, Italy
vgaldi@unisannio.it

In collaboration with:
G. Castaldi, V. Pierro, I.M. Pinto (University of Sannio)
J. Agresti, E. D'Ambrosio, R. DeSalvo (CalTech)

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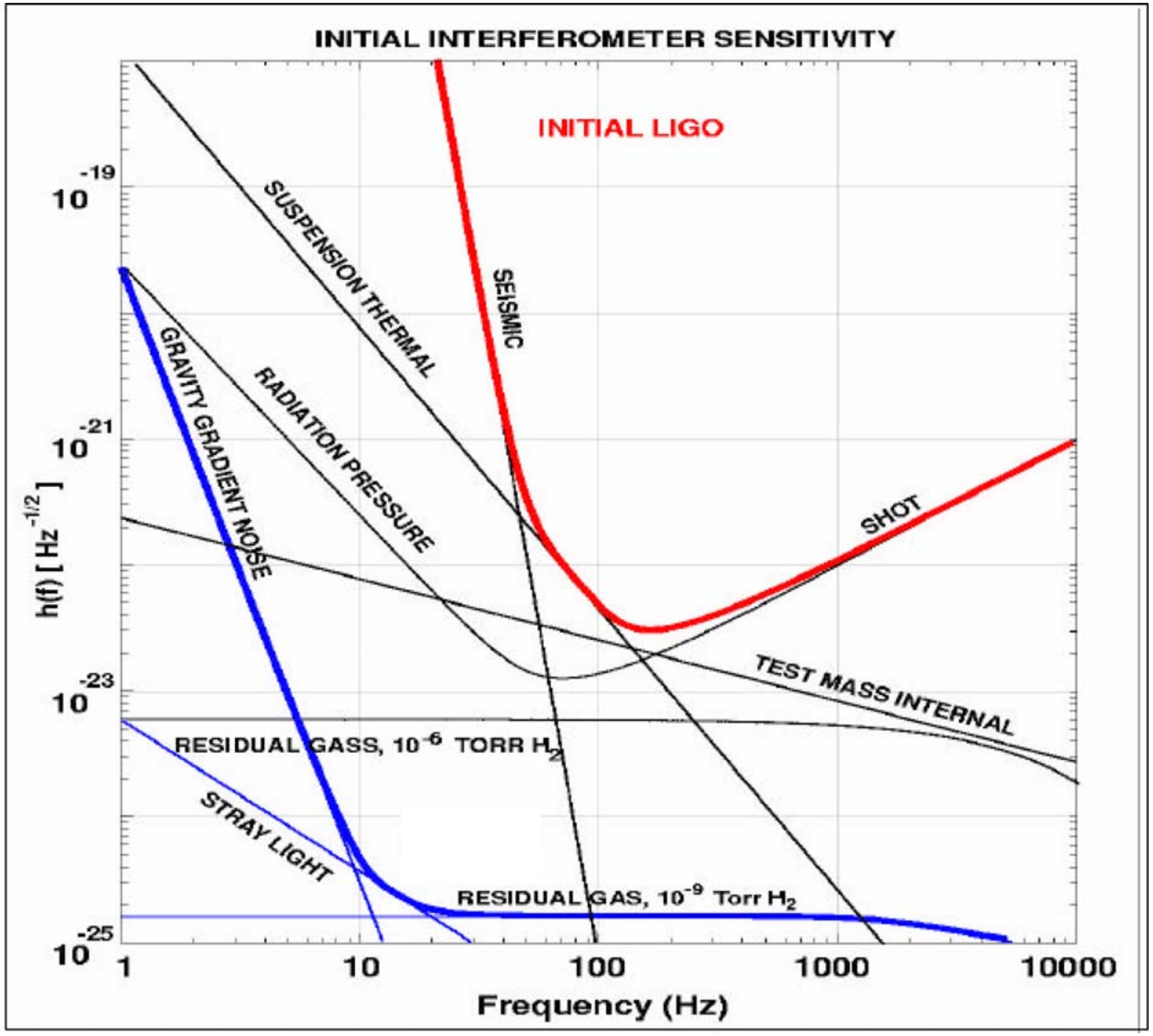
Agenda

- Optimized coatings
  - Background
  - Genetically-optimized coatings
  - Stacked-doublet design
  - Results

- Analytic structure of “hyperboloidal” beams
  - Background: From *nearly-flat* (FM) and *nearly-concentric* mesa (CM) beams to Bondarescu-Thorne (BT) *hyperboloidal* beams
  - Rapidly-converging Gauss-Laguerre (GL) expansion
  - Some results: Beam shapes and mirror corrections
  - Generalized duality relations (lowest-order mode)
    - *Complex-order* Fourier transform

Fundamental IFO noises



Thermal noise budget

Suspension thermal noise

mirror th. noise

Substrate thermal noise
Coating thermal noise - structural damping losses

Substrate thermoelastic noise
Coating thermoelastic noise

Thermorefractive noise - Refractive index fluctuations

Photon-driven (photoelastic) noise - Laser-driven therm. fluct.

see., e.g., Shanti Rao's thesis (Caltech, 2003) for a nice review

Thermal noise budget

will be dominant
for Silica substrates

Suspension thermal noise

mirror th. noise

Substrate thermal noise
Coating thermal noise

- structural damping losses

Substrate thermoelastic noise
Coating thermoelastic noise

Thermorefractive noise - Refracti

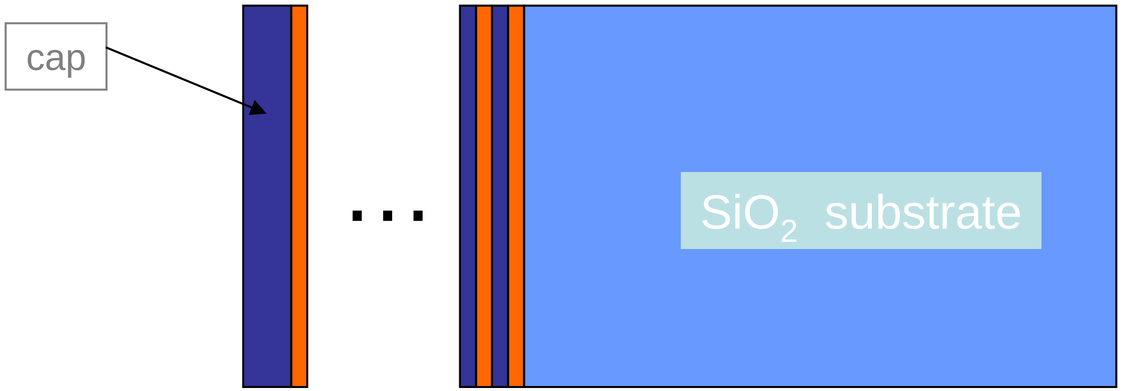
will be dominant
for Sapphire substrates

Photon-driven (photoelastic) noi ct.

see., e.g., Shanti Rao's thesis (Caltech, 2003) for a nice review

Coating geometry (as of today)

Alternating high/low - index layers grown on top of substrate
First/last layer: high-index
Further ($\lambda/2$) low-index protective layer on top (“cap”)



Current design: Quarter wavelength layers (QWL)
High-index (red): Ta_2O_5
Low-index (blue): SiO_2

Coating thermal noise PSD

[G. Harry, LIGO-T040029-00-R, 2004]

$$S_x(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{(1-\sigma^2)}{wY} \phi_{eff}$$

- Position noise PSD ($S_x(f)$)
- Boltzmann constant (k_B)
- Absolute temperature (T)
- Poisson ratio of substrate (σ)
- Beam half-width (w)
- Young modulus of substrate (Y)
- Effective loss - angle of mirror (complicated function of layers' thicknesses, loss-angles, Young moduli & Poisson coeffs.) (ϕ_{eff})

Reducing coating thermal noise

$$S_x(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{(1-\sigma^2)}{wY} \phi_{eff}$$

Position noise PSD

Boltzmann constant

Absolute tempe

Poisson ratio of substrate

Reduce coating loss-angle (materials & geometry)

Increase spotsize (mesa beams)

alf-width

Young modulus of substrate

Effective loss-angle of mirror (complicated function of layers' thicknesses, loss-angles, Young moduli & Poisson coeffs.)

Coating loss-angle

[G. Harry, LIGO-T040029-00-R, 2004]

$$\phi_{\rm eff}^{\rm coat} = \frac{d_{\rm coat}}{\sqrt{\pi}w}\frac{1}{Y_\perp}\left\{\left[\frac{Y}{1-\sigma^2} - \frac{2\sigma_\perp^2 Y Y_\parallel}{Y_\perp(1-\sigma^2)(1-\sigma_\parallel)}\right]\phi_\perp + \right.$$

$$+ \frac{Y_\parallel \sigma_\perp (1-2\sigma)}{(1-\sigma_\parallel)(1-\sigma)}(\phi_\parallel - \phi_\perp) +$$

$$\left. + \frac{Y_\parallel Y_\perp (1+\sigma)(1-2\sigma)^2}{Y(1-\sigma_\parallel^2)(1-\sigma)}\phi_\parallel\right\}, \quad d_{\rm coat} = d_1 + d_2$$

d_1 , d_2 = high/low index layer thickness

$\sigma, \sigma_\perp, \sigma_\parallel$ = Poisson ratios
$Y, Y_\perp, Y_\parallel$ = Young moduli
$\phi, \phi_\perp, \phi_\parallel$ = Loss angles

(substrate, coating - $\perp$, coating - $\parallel$)

Coating loss-angle (cont'd)

[G. Harry, LIGO-T040029-00-R, 2004]

$$Y_\perp = \frac{1+d_2/d_1}{Y_1^{-1}+(d_2/d_1)Y_2^{-1}}, \quad Y_\parallel = \frac{Y_1+(d_2/d_1)Y_2}{1+d_2/d_1},$$

$$\phi_\perp = Y_\perp \frac{\phi_1 Y_1^{-1}+(d_2/d_1)\phi_2 Y_2^{-1}}{1+d_2/d1}, \quad \phi_\parallel = Y_\parallel^{-1}\frac{Y_1\phi_1+(d_2/d_1)Y_2\phi_2}{1+d_2/d_1}$$

$$\sigma_\perp = \frac{\sigma_1 Y_1+(d_2/d_1)\sigma_2 Y_2}{Y_1+(d_2/d_1)Y_2},$$

defines implicitly $\sigma_\parallel$

$$\frac{\sigma_1 Y_1}{(1+\sigma_1)(1-2\sigma_1)}+\frac{(d_2/d_1)\sigma_2 Y_2}{(1+\sigma_2)(1-2\sigma_2)} = -\frac{Y_\parallel(\sigma_\perp^2 Y_\parallel+\sigma_\parallel Y_\perp)(1+d_2/d_1)}{(\sigma_\parallel+1)[2\sigma_\perp^2 Y_\parallel-(1-\sigma_\parallel)Y_\perp]}$$

Basic ingredients: $\phi_{1,2}, Y_{1,2}, \sigma_{1,2}, d_{1,2}$

Reducing coating thermal noise (cont'd)

$$Y_\perp = \frac{1+d_2/d_1}{Y_1^{-1}+(d_2/d_1)Y_2^{-1}}, \quad Y_\parallel = \frac{Y_1+(d_2/d_1)Y_2}{1+d_2/d_1},$$

$$\phi_\perp = Y_\perp \frac{\phi_1 Y_1^{-1}+(d_2/d_1)\phi_2 Y_2^{-1}}{1+d_2/d1}, \quad \phi_\parallel = Y_\parallel^{-1}\frac{Y_1\phi_1+(d_2/d_1)Y_2\phi_2}{1+d_2/d_1}$$

$$\sigma_\perp = \frac{\sigma_1 Y_1+(d_2/d_1)\sigma_2 Y_2}{Y_1+(d_2/d_1)Y_2},$$

defines implicitly $\sigma_\parallel$

$$\frac{\ldots\sigma_2 Y_2}{(1\ldots-2\sigma_2)} = -\frac{Y_\parallel(\sigma_\perp^2 Y_\parallel+\sigma_\parallel Y\ldots}{(\sigma_\parallel+1)[2\sigma_\perp^2 Y_\parallel\ldots}$$

"Better" materials (lower losses)

"Better" geometry (?)

Basic ingredients: $\phi_{1,2}, Y_{1,2}, \sigma_{1,2}, d_{1,2}$

“Better” materials

- Select among alternative low/high index mates for best tradeoff in terms of dielectric contrast, acoustic losses, optical absorption. Votes for tantala/silica. Niobia/silica, tantala/alumina, alumina/silica also tested

- Ongoing research at LMA (and Glasgow) on Ti-doped tantala. Reduced acoustic losses observed; Young’s modulus and optical absorption almost unchanged. Mechanism yet unclear

- Alternative materials/dopants (e.g., silica-doped titania, rare-earth dopants, etc.) ?

“Better” geometry?

- Current coating design: Stacked doublets of *quarter-wavelength* (QWL) SiO_2 - Ta_2O_5 layers
- Yields *largest reflectivity* among all stacked-doublet designs for any *fixed* no. of layers (or equivalently, *smallest* no. of layers at any *fixed reflectivity*)
- Does *not* yield the mimimum noise for a prescribed reflectivity, hence *not optimal*

“Better” geometries: Possible directions

General (non-periodic) Explored via Genetic optimization

Highest design flexibility; no
a-priori assumption on structure

Stacked non-QWL doublets

Truncated-periodic

Most obvious generalization of
stacked QWL doublets; also
suggested by GA optimization

Regular-non periodic Fractal & substitutional

“perfect” (polarization/incidence
angle insensitive) dielectric mirrors…

Non-periodic coatings: Genetic optimization

- **Key Features**
  - Multiple, *heterogeneous* mixed continuous/discrete constraints
  - Multi-objective and/or best tradeoff optimization
  - Robust. Slow.

Available options include:
-structural-related constraints
-multiple-wavelength operation
-several (> 2) materials, etc.…

Educated ignorance attitude (almost *no a-priori assumption* on structure of sought solution - will shed light on it !);

Effective & well established (e.g. microwave antenna and filter design)…

Genetic Algorithms in Engineering Electromagnetics

J. Michael Johnson and Yahya Rahmat-Samii
Department of Electrical Engineering
University of California, Los Angeles
405 Hilgard Ave.
Los Angeles, CA 90095-1594
Tel: (310) 206-4801 (JMJ); (310) 206-3847 (YRS)
Fax: (310) 206-8495
E-mail: johnson@ee.ucla.edu ; rahmat@ee.ucla.edu
http://www.antlab.ee.ucla.edu

Keywords: Genetic algorithms, absorbing media, antenn
array synthesis, microstrip antennas, natural modes, rada
identification

1. Abstract

This paper presents a tutorial and overview of ge
rithms for electromagnetic optimization. Genetic-algor
optimizers are robust, stochastic search methods mode
concepts of natural selection and evolution. The r
between traditional optimization techniques and the
cussed. Step-by-step implementation aspects of th
detailed, through an example with the objective of provi
guidelines for the potential user. Extensive use is made
and graphical presentation to facilitate understanding. T
is followed by a discussion of several electromagnetic a
in which the GA has proven useful. The applications
include the design of lightweight, broadband microwave
the reduction of array sidelobes in thinned arrays, the
shaped-beam antenna arrays, the extraction of natural
modes of radar targets from backscattered response da
design of broadband patch antennas. Genetic-algorithm
tion is shown to be suitable for optimizing a broad cla
lems of interest to the electromagnetic community. A c
sive list of key references, organized by application c
also provided.

2. Introduction

The application of modern electromagnetic theory t
and scattering problems often either requires, or at
fits from, the use of optimization. Among the typica
requiring optimization are shaped-reflector antenna desi
get image reconstruction [2], and layered material, anti
coating design for low radar cross section (RCS) [3]. C
lems, such as antenna-array beam-pattern shaping [4,
solvable without optimization, are often more readil
using optimization. This is particularly true when one is
realization constraints imposed by manufacturing consid
environmental factors.

Electromagnetic optimization problems generally
large number of parameters. The parameters can be eith
ous, discrete, or both, and often include constraints in
values. The goal of the optimization is to find a solution
sents a *global* maximum or minimum. In addition, th
domain of electromagnetic optimization problems ofte

Application of a Microgenetic Algorithm (MGA) to the Design of Broad-Band Microwave Absorbers Using Multiple Frequency Selective Surface Screens Buried in Dielectrics

Sourav Chakravarty, Raj Mittra, *Life Fellow, IEEE*, and Neil Rhodes Williams

***Abstract*—Over the years, frequency selective surfaces (FSSs) have found frequent use as radomes and spatial filters in both commercial and military applications. In the literature, the problem of synthesizing broadband microwave absorbers using multilayered dielectrics through the application of genetic algorithms (GAs) have been dealt with successfully. Recently, spatial filters employing multiple, freestanding, FSS screens have been successfully designed by utilizing a domain-decomposed GA. In this paper, we present a procedure for synthesizing broadband microwave absorbers by using multiple FSS screens buried in a dielectric composite. A binary coded microgenetic algorithm (MGA) is applied to optimize various parameters, viz., the thickness and relative permittivity of each dielectric layer; the FSS screen designs and materials; their x- and y-periodicities; and their placement within the dielectric composite. The result is a multilayer composite that provides maximum absorption of both transverse electric (TE) and transverse magnetic (TM) waves simultaneously for a prescribed range of frequencies and incident angles. This technique automatically places an upper bound on the total thickness of the composite. While a single FSS screen is analyzed using the electric field integral equation (EFIE), multiple FSS screens are analyzed using the scattering matrix technique.**

***Index Terms*—Frequency selective surfaces (FSSs), genetic algorithm, microwave absorbers, multilayered media, scattering matrices.**

I. Introduction

GENETIC ALGORITHMS (GAs) are robust stochastic search methods modeled on the principles of natural selection. The powerful heuristic of the GA, as an optimizer, is useful for solving complex combinatorial problems. It is particularly effective in searching for near-global maxima in domains that are both multidimensional and multimodal. The GA simultaneously processes a population of points in the optimization space, and uses stochastic operators to transition from one generation of points to the next, resulting in a decreased probability of their being trapped in local extrema.

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S. Chakravarty was with The Pennsylvania State University, University Park, PA 16802-2707 USA. He is now with Intel Corporation, Hillsboro, OR 97124 USA.
R. Mittra is with The Pennsylvania State University, University Park, PA 16802-2707 USA (e-mail: mittra@engr.psu.edu).
N. R. Williams was with The Pennsylvania State University, University Park, PA 16802-2707 USA. He is now with W. L. Gore & Associates, Advanced Electromagnetic Products, Newark, DE 19714–9236 USA.
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Design of Lightweight, Broad-Band Microwave Absorbers Using Genetic Algorithms

Eric Michielssen, Jean-Michel Sajer, S. Ranjithan, and Raj Mittra, *Fellow, IEEE*

***Abstract*— In this paper, a novel procedure for synthesizing multilayered radar absorbing coatings is presented. Given a predefined set of N_m available materials with frequency-dependent permittivities $\epsilon_i(f)$ and permeabilities $\mu_i(f)$ $(i = 1, \cdots, N_m)$, the proposed technique simultaneously determines the optimal material choice for each layer and its thickness. This optimal choice results in a screen which maximally absorbs TM and TE incident plane waves for a prescribed range of frequencies $\{f_1, f_2, \cdots, f_{Nf}\}$ and incident angles $\{\theta_1, \theta_2, \cdots, \theta_{N\theta}\}$. The synthesis technique presented herein is based on a genetic algorithm. The present technique automatically places an upper bound on the total thickness of the coating, as well as the number of layers contained in the coating, which greatly simplifies manufacturing. In addition, the thickness or surface mass of the coating can be minimized simultaneously with the reflection coefficient. The algorithm was successfully applied to the synthesis of wide-band absorbing coatings in the frequency ranges of 0.2–2 GHz and 2–8 GHz.**

I. Introduction

THIS paper focuses on the design of wide-band, multilayered radar absorbing (RAM) coatings. In view of the application of these coatings in the area of low observability, the coatings not only need to exhibit a low reflection coefficient over a wide frequency range, but also need to be lightweight and thin. The primary goal of the research reported in this paper is the development of a simple technique for designing such coatings, and for investigating the tradeoff between a coating thickness (or weight) and reflectivity.

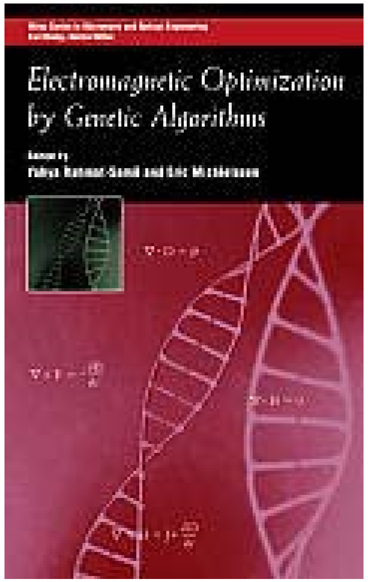
In the past, several techniques (e.g., Salisbury, graded index, Jaumann, and Dallenbach screens [1]–[3]) were proposed for designing absorbing coatings. In these techniques, the screens are usually designed using approximate closed-form expressions, or relatively simple optimization schemes [4]–[5]. Recently, Pesque *et al.* [6] proposed a technique for designing absorbing coatings which is based on an optimal control approach. In this method, thin absorbing coatings are designed

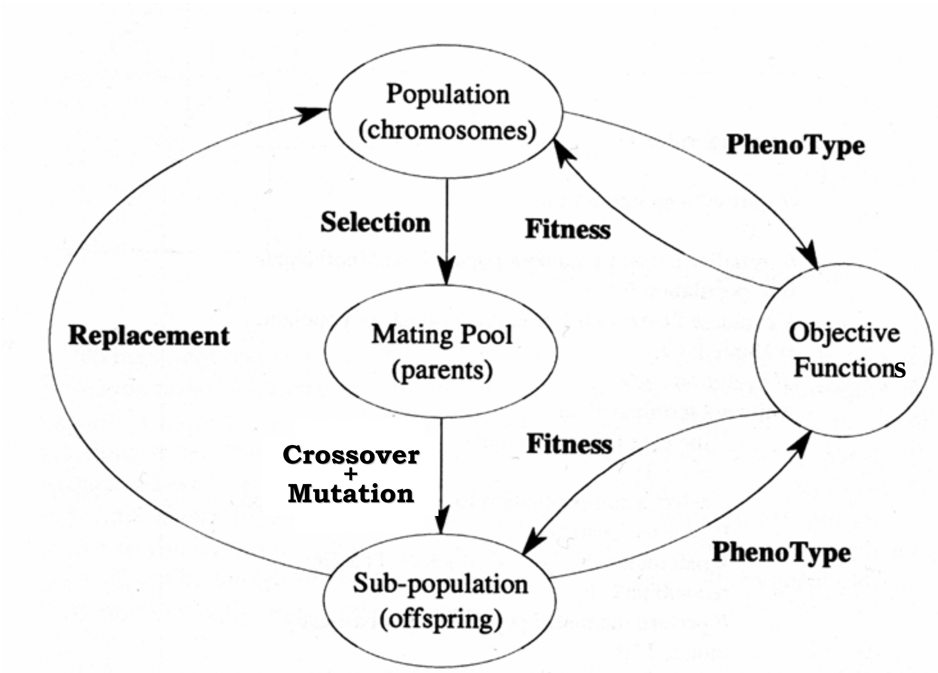
major drawback of convergence to only a local minimum of the cost function, which was defined as the maximum reflection coefficient over the frequency band of interest. The possibility of existence of better absorbing, or equally absorbing but thinner coatings, can therefore not be excluded. To overcome this drawback, the authors also present a design technique which is based on the combinatorial optimization technique of simulated annealing (SA) [7]. In this technique, the coating is subdivided into a large number of thin layers with fixed thicknesses, each of which is assigned a material chosen from a predefined set of available materials. The optimal solution is found through iterative random perturbations of the material choices for each layer, and evaluations based on the well-known Metropolis criterion [8]. Although convergence to a global minimum can never be guaranteed, this technique usually leads to less reflective and thinner coatings when compared to the optimal control approach. However, the coatings synthesized using this particular implementation of the simulated annealing technique typically contain far more and consequently thinner layers than those obtained using the optimal control method. This leads to manufacturing problems due to the fragile nature of the typically available materials.

In this paper, a combinatorial optimization technique is presented for designing absorbing coatings which is based on a genetic algorithm. This algorithm offers several advantages over the existing optimal control and SA techniques. First, in contrast to the optimal control method [6], the present technique provides a mechanism for global search. In contrast to the SA-based technique presented in [6], the current technique succeeds in designing high-performance coatings consisting of only few layers, and therefore almost always leads to physically realizable structures. Second, the execution of the algorithm typically results in a number of "high-performance" designs rather than a single solution as offered by other techniques. A specific design can be selected

Genetic optimization in a nutshell

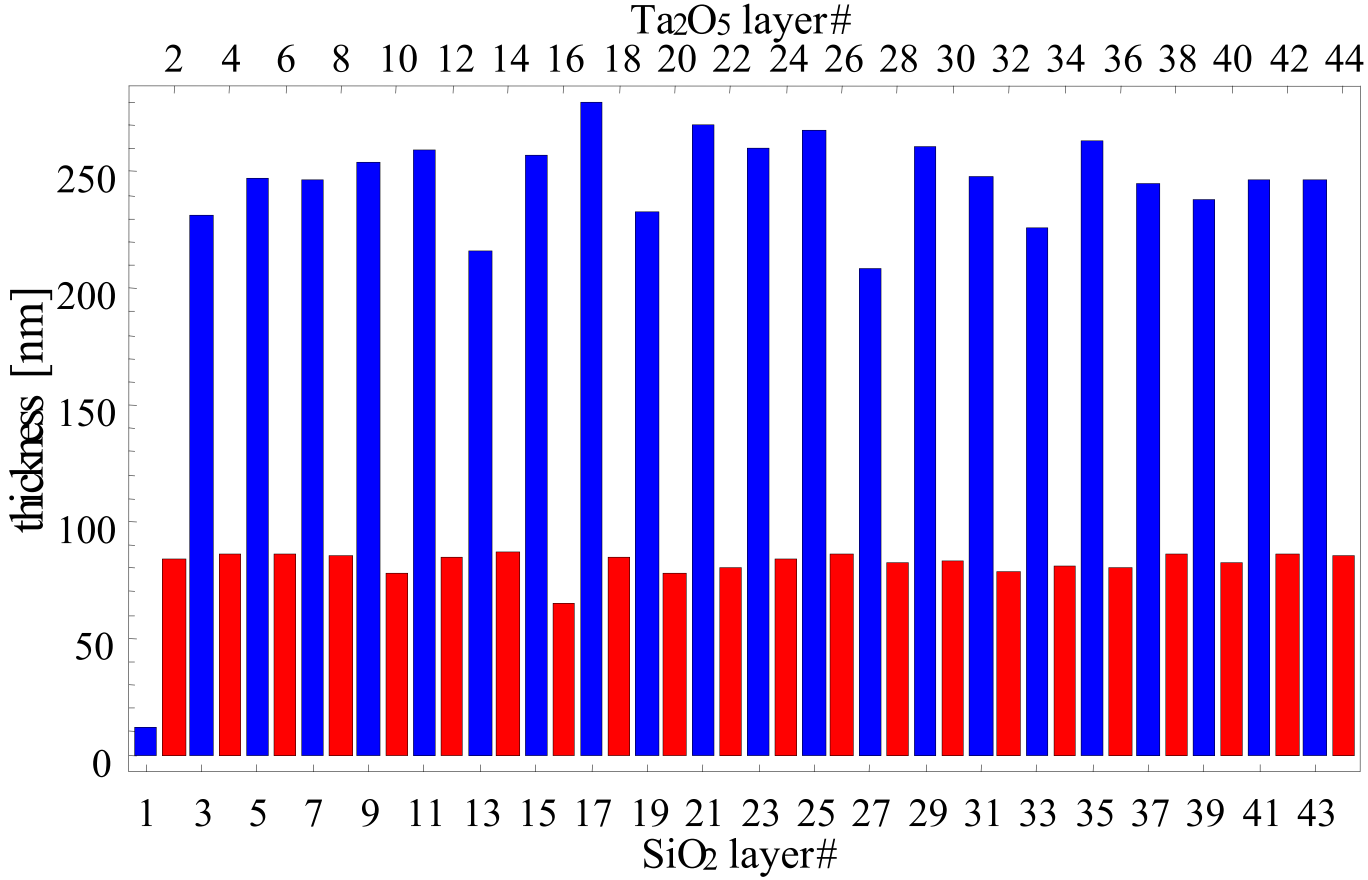
-Problem unknowns: *genes*
-Point in search space: *chromosome*
-Set of points in search space: *population*
-Evolve *random* initial population
according to *evolutionary schedule*





GA-engineered prototype

Goal: $1-|\Gamma|^2 < 15$ ppm. $L[Ta_2O_5] < 2000$ nm

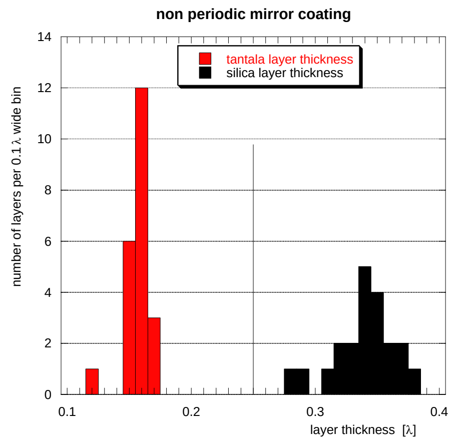


GA-engineered vs. nearest-neighbor QWL

	QWL-1	**Genetic**	QWL-2
N (cap included)	36	**44**	28
$1-\|\Gamma\|^2$ ppm	16.20	**14.91**	**235.46**
$L(Ta_2O_5)$ nm	**2359.43**	**1815.61**	1835.11
$L(SiO_2)$ nm	3479.98	**5217.4**	2747.35
L_{tot} nm	5839.41	**7033.01**	4582.46

Lessons learned from genetic optimization

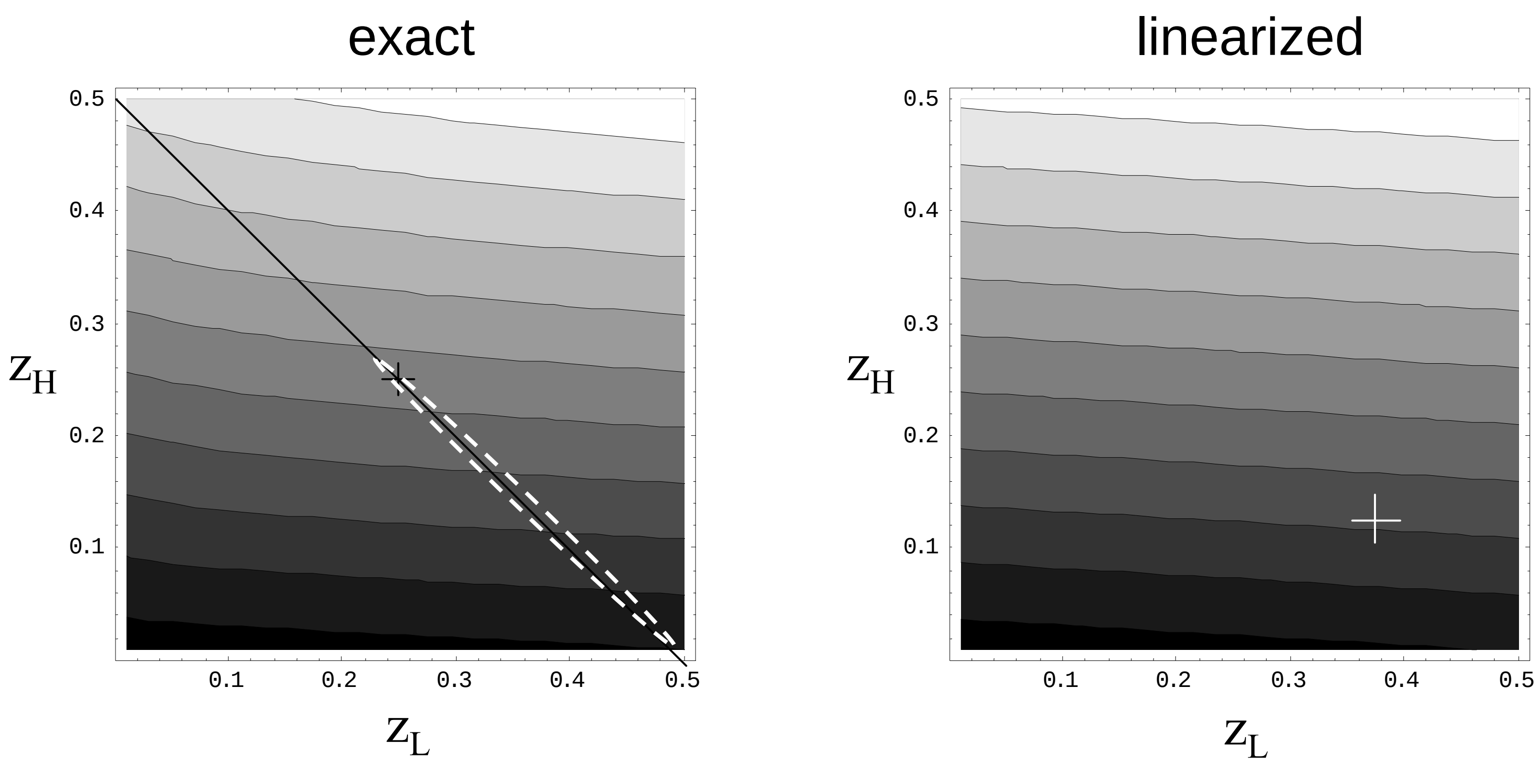
2 – dielectric media based GA-engineered coatings for minimum noise at prescribed show trend toward non-QWL stacked-doublet configurations, except for the terminal layers



Suggests the following practical design criterion:

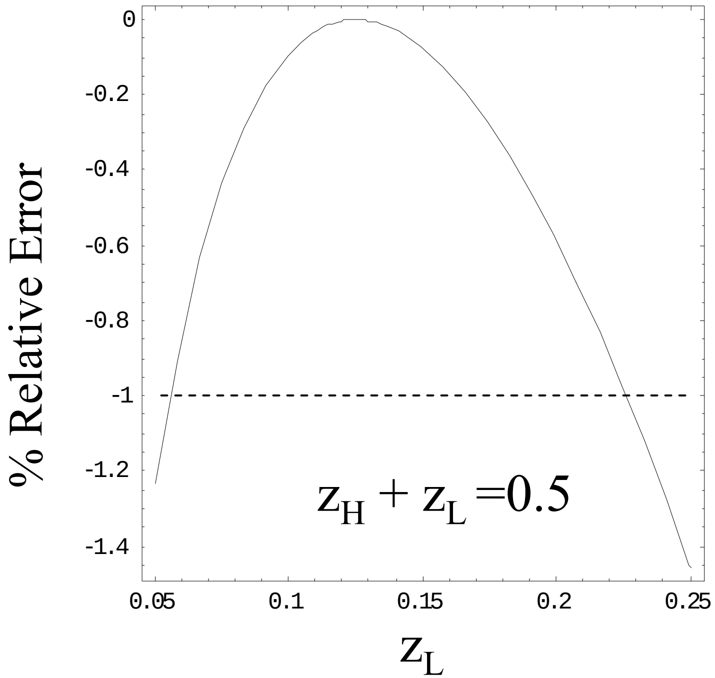
a) Design minimum noise configuration
b) Tweak terminal layer thicknesses, e.g., using GA

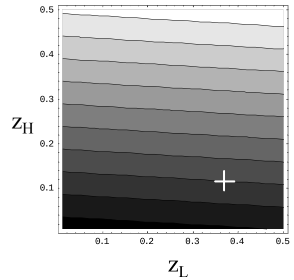
Coating thermal noise PSD: Linearization



Single doublet noise contour plots vs. $z_H = n_{Ta_2O_5}\Delta_{Ta_2O_5}/\lambda_0$ **and** $z_L = n_{SiO_2}\Delta_{SiO_2}/\lambda_0$.
Left panel: exact. Region of interest highlighted.
Right panel: first-order truncated Taylor-McLaurin expansion with initial point $z_H = 1/8$, $z_L = 3/8$ **(white-cross marker)**.

Coating thermal noise PSD: Linearization (cont'd)



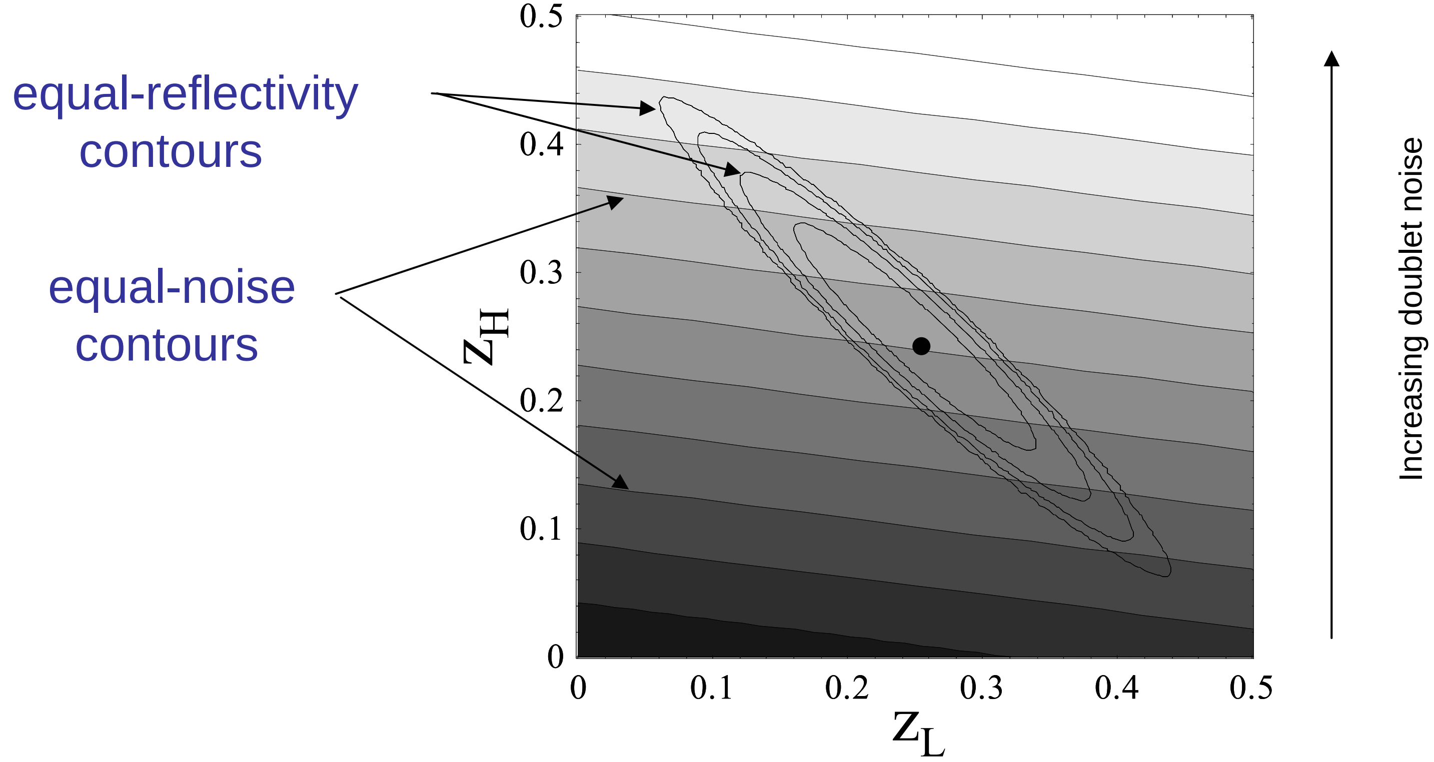


Single doublet noise. Percent relative error between exact and first-order truncated Taylor (linearized) expansion w. initial point $z_H = 1/8$, $z_L = 3/8$ as a function of z_L in the range $0 < z_L < 1/4$, $z_H = 1/2 - z_L$

Coating noise well approximated by: $PSD = C(z_H + \gamma^{-1} z_L)$

$$\gamma \sim \frac{Y_H \phi_H}{Y_L \phi_L}$$

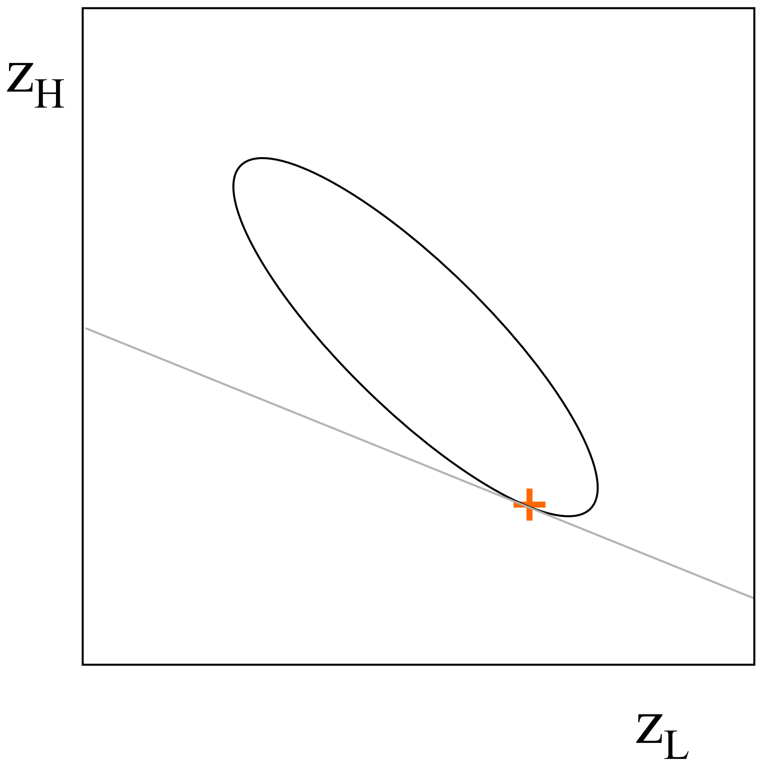
Nd stacked-doublets: Reflectivity & noise



z_L = thickness of low-index (SiO_2) layer
z_H = thickness of high-index (Ta_2O_5) layer

In units of local wavelength $\lambda_0/n_{H,L}$

Minimizing stacked doublet noise for prescribed reflectivity



Constructing stacked-doublet minimum-noise vs. reflectivity tradeoff curves

Assign number N_d of doublets;

Compute $\tau_{QWL}(N_d)$;

For $\tau_{QWL}(N_d) \le \tau^* \le \tau_{max}$,

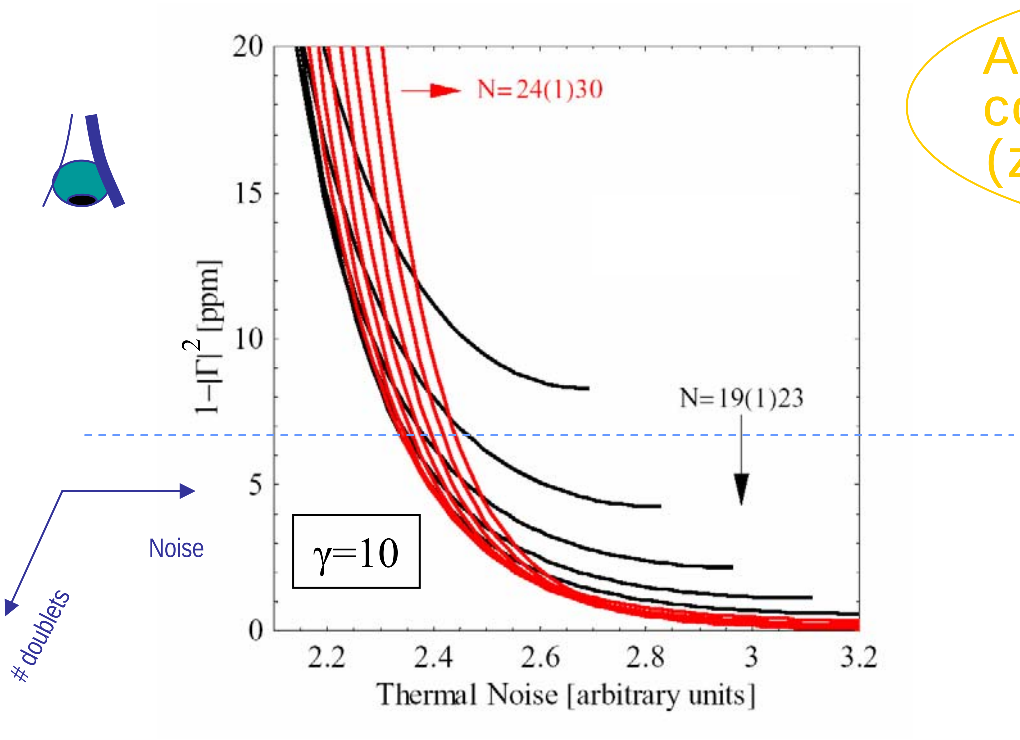
identify point (z_L^*, z_H^*) on $\tau(z_L, z_H, N_d) = \tau^*$ contour

such that $PSD(z_L^*, z_H^*, N_d) = PSD^*$ is a minimum;

construct curve through $(PSD^*, \tau^*)[z_L^*, z_H^*]$ points.

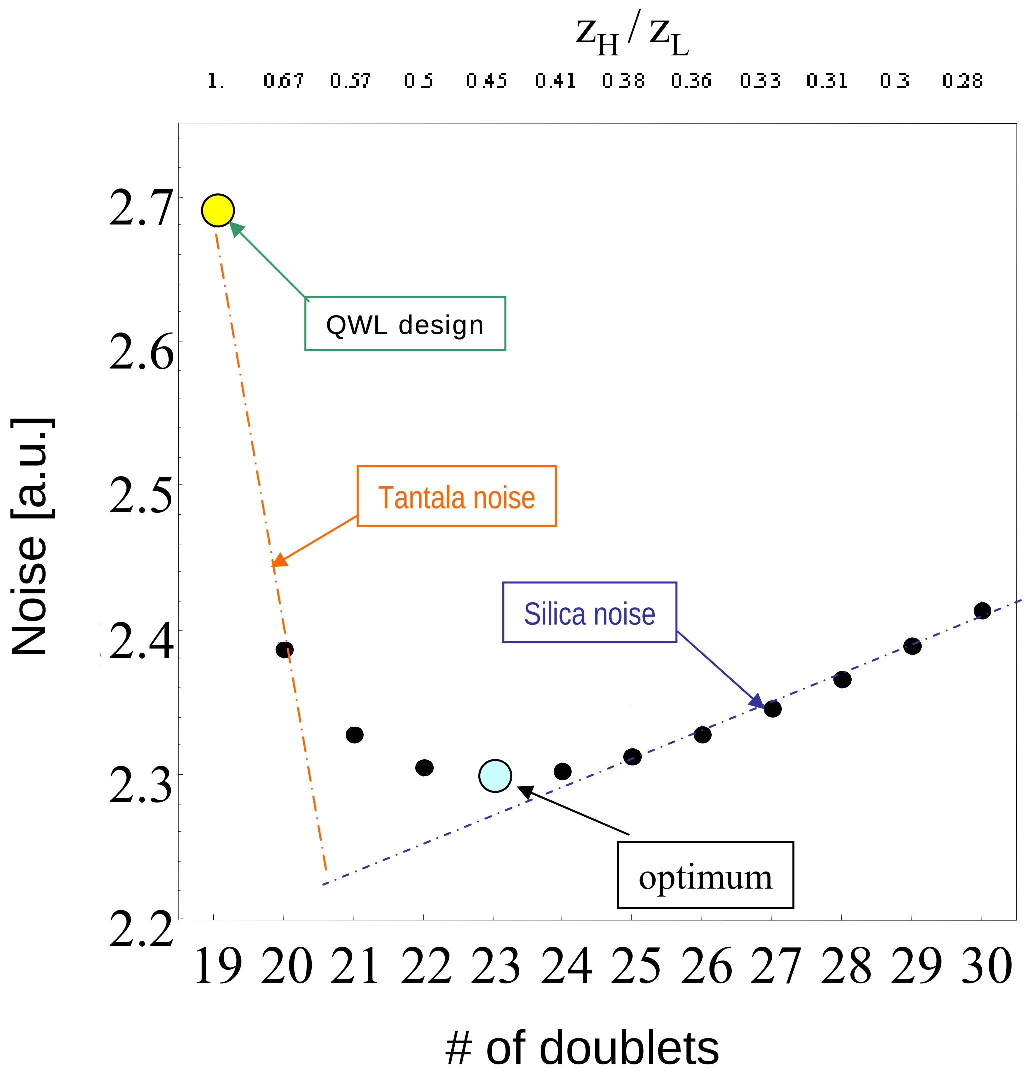
Loop.

Constructing stacked-doublet minimum-noise vs. reflectivity tradeoff curves (cont'd)



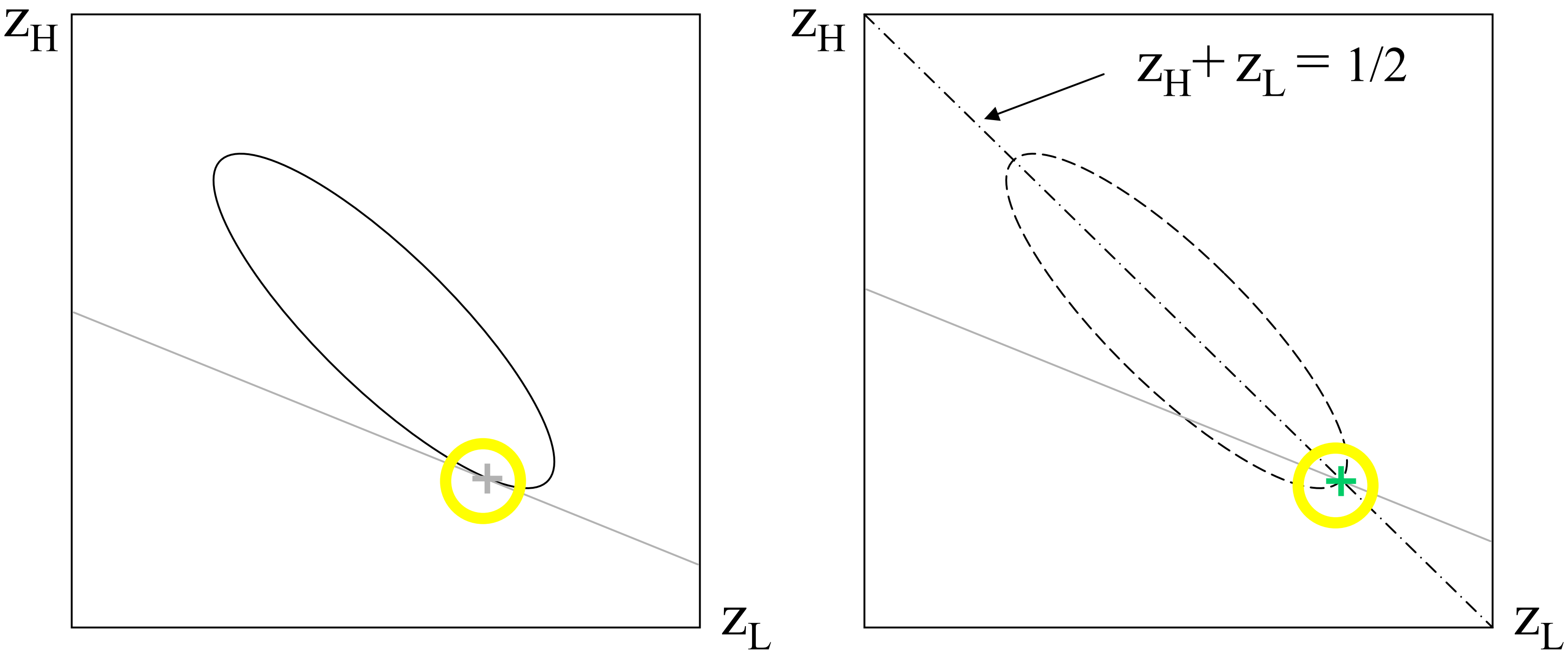
Any point on any curve corresponds to some (z_L, z_H) value.

$\tau = const.$



z_H / z_L
1. 0.67 0.57 0.5 0.45 0.41 0.38 0.36 0.33 0.31 0.3 0.28
Noise [a.u.]
2.7
2.6
2.5
2.4
2.3
2.2
QWL design
Tantala noise
Silica noise
optimum
19 20 21 22 23 24 25 26 27 28 29 30
# of doublets

Minimum-noise solution: Exact vs. approximate



Reflectivity contour lines *very thin*: negligible difference

(~10% uncertainties in material parameters may blur the differences)

Minimum-noise stacked-doublet design

Prescribe transmissivity : $\tau_P = 1 - \left|\Gamma_p\right|^2$

Find smallest N_d : $\tau_{QWL}(N_d) = \tau^* \le \tau_P$

Do $N_d = N_d + 1$,

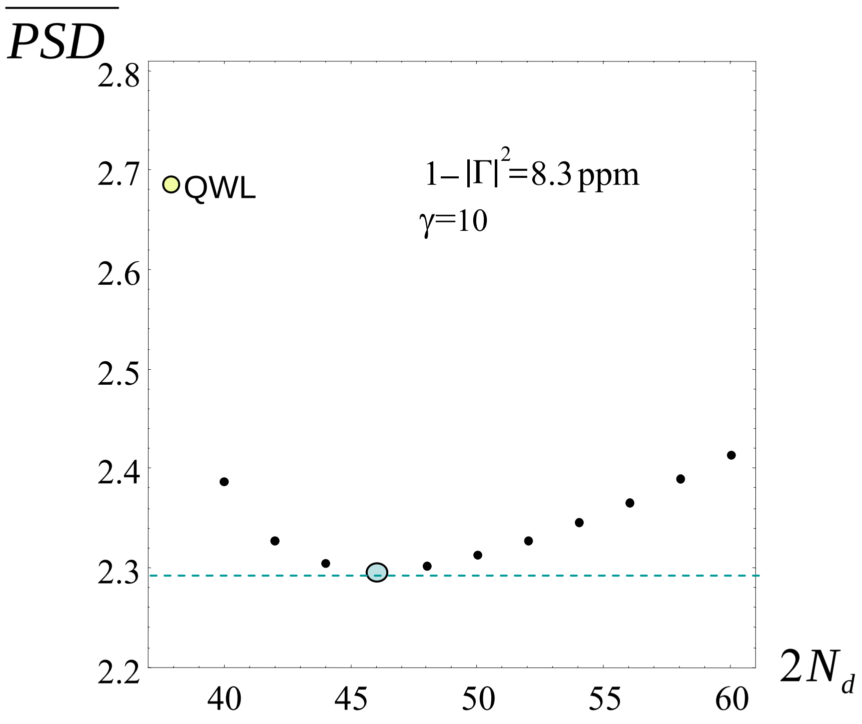
find (z_L, z_H) :

$$\begin{cases} z_L + z_H = 1/2 \text{ (\sim minimum noise)} \\ \tau(z_L, z_H, N_d) = \tau^* \end{cases}$$

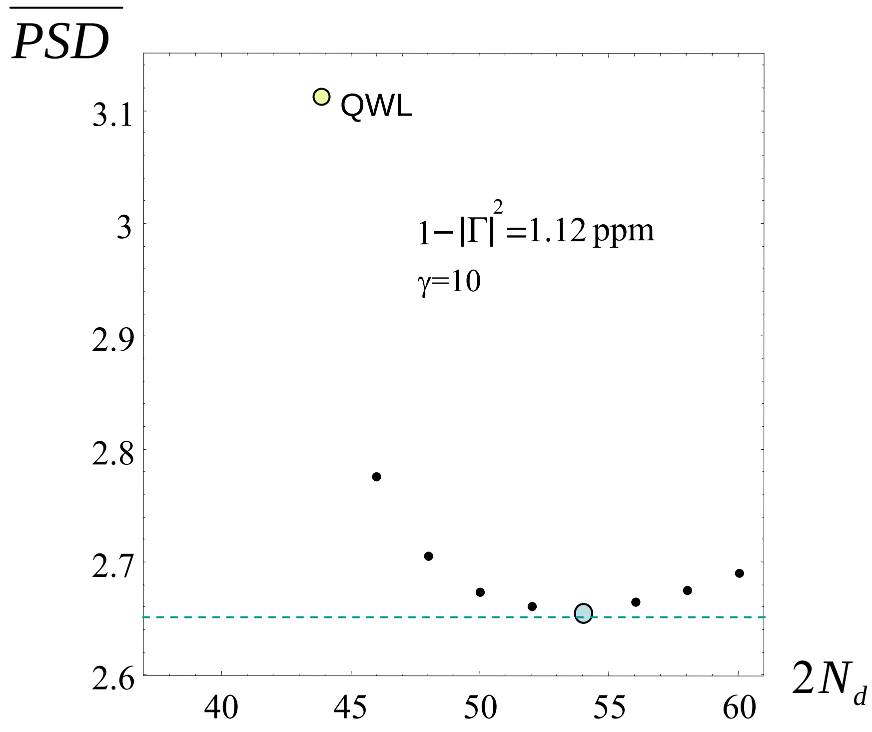
while $PSD(z_L, z_H, N_d) \le PSD(z_L, z_H, N_d - 1)$

Minimum-noise stacked-doublet design (cont'd)

[J. Agresti et al., LIGO G-050363-00-R, 2005]

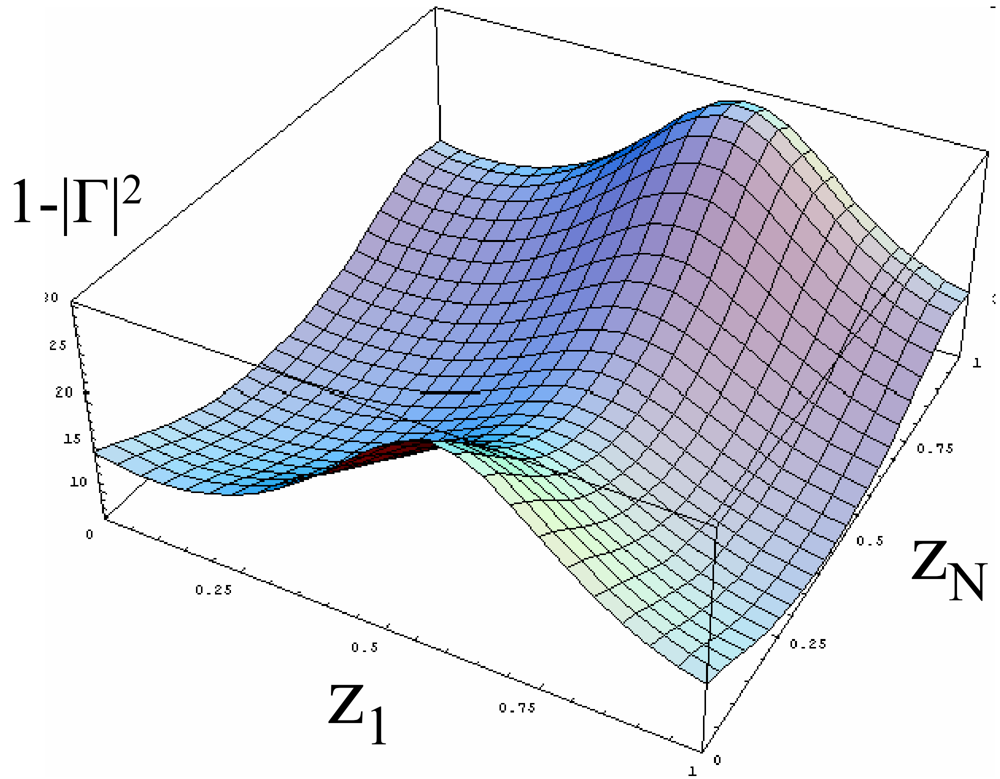


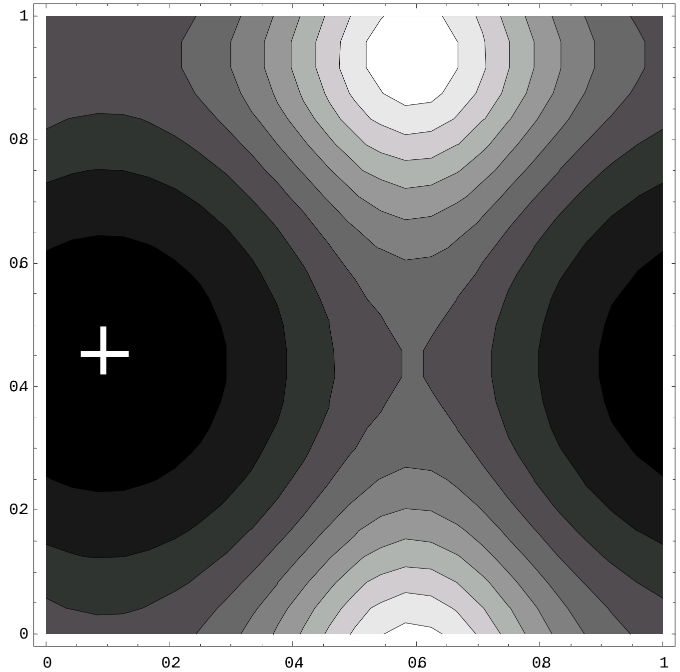
~14% noise reduction



~14% noise reduction

Tweaking the end layers





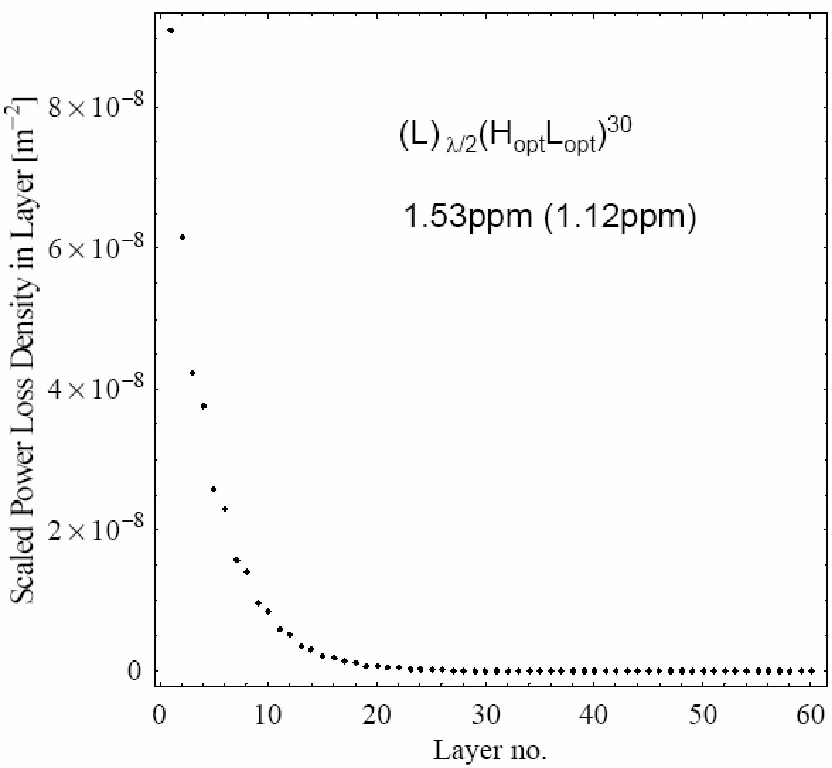
($z_{L(1st)}$ = 0.0943, $z_{H(last)}$ =0.437, in units of local wavelength)

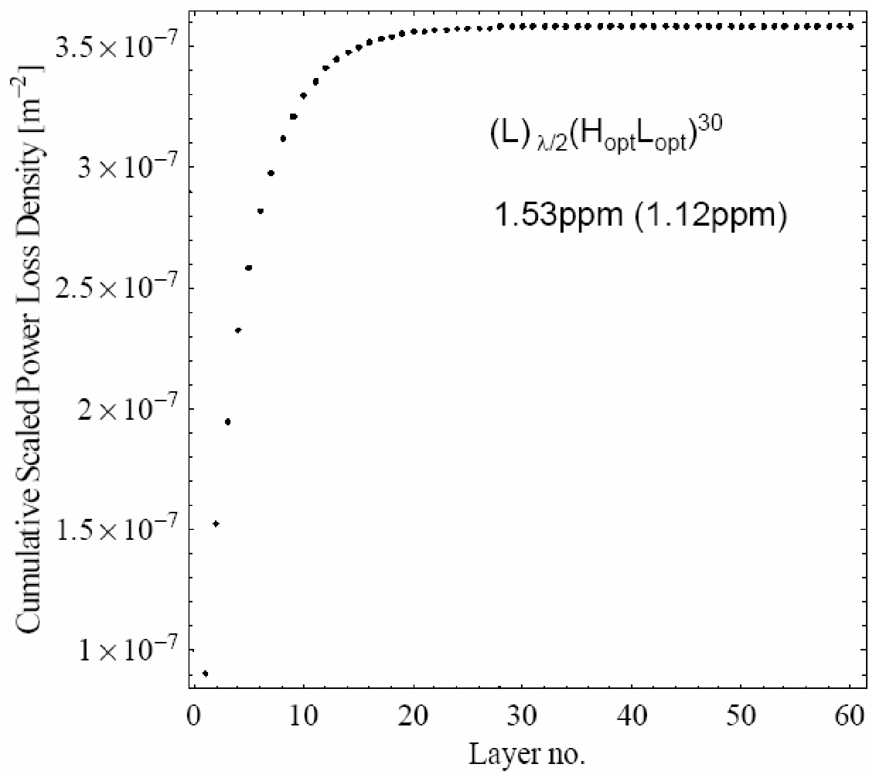
reflectivity + 10%, noise almost unchanged

Optical absorption

$$\text{Refr. index} = \begin{cases} 1.45231 & [SiO_2] \\ 2.0293 & [Ta_2O_5] \end{cases} \quad \phi = 4.5 \cdot 10^{-4}$$

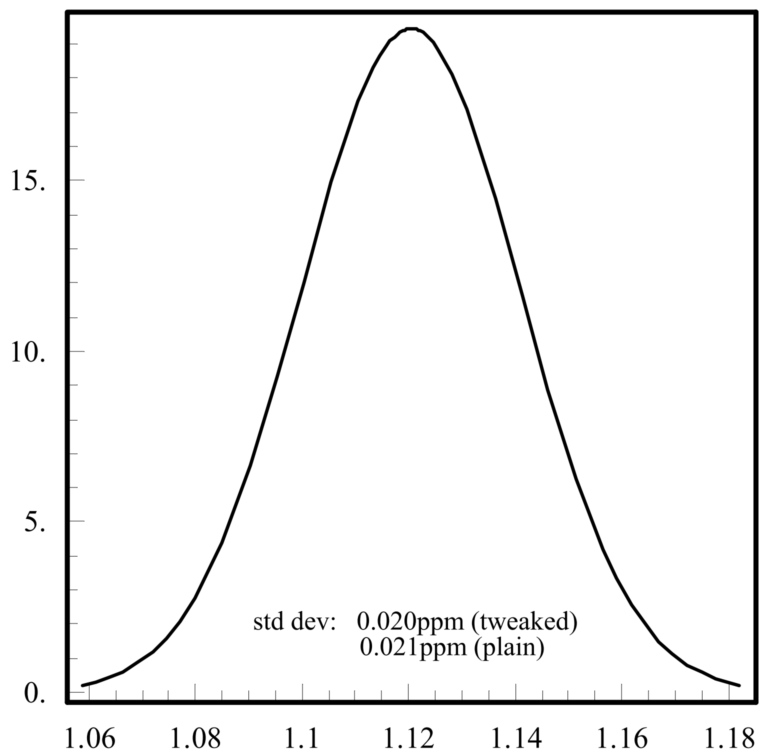
$$\text{Extinction coeffs.} \begin{cases} (4 \pm 0.8) \cdot 10^{-8} & [SiO_2] \\ (1.8 \pm 0.4) \cdot 10^{-7} & [Ta_2O_5] \end{cases}$$



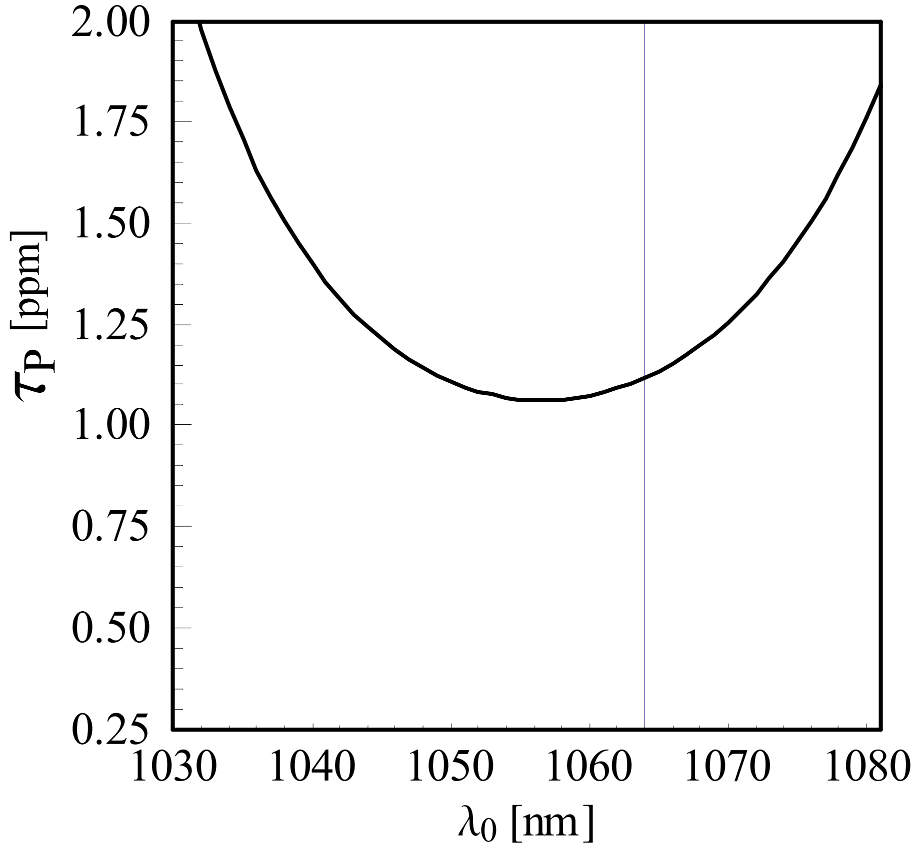


Frequency response and robustness

Transmissivity [ppm]distribution over 10^4 realizations featuring random uniform layer thickness thickness errors within ±1nm .



Frequency response (plain & end-tweaked almost the same)



Conclusions (partial)

- Stacked-doublet coatings with tweaked end-layer thick-nesses are the optimal design for 2-dielectric mirrors; 25% event rate boost obtained for $\gamma \approx 10$

- Genetic optimization may handle more general (multi-objective, multi-dielectric) problems/configurations

- Optimum thicknesses and achievable noise reduction critically dependent on coating loss – angles

Coating loss-angle measurements

Several techniques:

Indirect [loss-angles from mechanical ringdown]

Several geometries:
- thin/thick membranes (Glasgow, MIT)
- clamped cantilever (LMA)

Sources of uncertainty:
- σ, Y same for coating and bulk could be retrieved from redundant measurement but ill-conditioning
- ratio between energies stored in substrate/coating from FEM analysis; analytic result for cantilever geometry

Direct [coating loss angle from noise PSD]

-thermal noise interferometers (Caltech, Tokyo)

Indirect coating loss-angle measurement

Table –I . MECHNICAL LOSS MEASUREMENTS ON SiO2/Ta2O5 COATINGS

	[1] (2003)	[2] (2004)	[3] (2005)	[4] (2005)	[5] (2006)	[6] (2006)
ϕ_{SiO_2}	$(0.5 \pm 0.3) \cdot 10^{-4}$	$(0.4 \pm 0.3) \cdot 10^{-4}$	$(0.2 \pm 0.5) \cdot 10^{-4}$	$(0.4 \pm 0.3) \cdot 10^{-4}$	$(1.2 \pm 0.2) \cdot 10^{-4}$	$(1.0 \pm 0.2) \cdot 10^{-4}$
$\phi_{Ta_2O_5}$	$(4.4 \pm 0.2) \cdot 10^{-4}$	$(4.2 \pm 0.4) \cdot 10^{-4}$	$(5.2 \pm 0.4) \cdot 10^{-4}$	$(4.2 \pm 0.4) \cdot 10^{-4}$	$(3.2 \pm 0.1) \cdot 10^{-4}$	$(3.8 \pm 0.2) \cdot 10^{-4}$

new Y; better FEM code

[1] S.D. Penn et al., "Mechanical Losses in Tantala/Silica Dielectric Mirror Coatings,", Class. Quantum Grav., **20** (2003) 2917;
[2] D.R.M. Crooks et al., "Experimental Measurement of Coating Mechanical Quality Factors," Class. Quantum Grav., **21** (2004) S1059;
[3] – ILIAS-GRA3 STREGA (M4) 1st Year Report (2005); can be downloaded from: http://www.ego-gw.it/ILIAS-GW/documents/STREGA report2005/Long%20reports/Report_M4_Rowan.doc;
[4] G.M. Harry et al., "Thermal Noise from Optical Coatings in GW Detectors," (2005), can be downloaded from: http://www.ligo.org/pdf_public/armandula.pdf;
[5] G.M. Harry et al., "Titania-doped Tantala/Silica Coatings for Gravitational Wave Detection," (2006) preprint, courtesy E. Black;
[6] D.R.M. Crooks et al., "Experimental Measurement of Mechanical Dissipation associated with Dielectric Coatings using SiO2, Ta2O5 and Al2O3," can be downloaded from http://www.ligo.org/restricted/pdf/crooks.pdf;

TNI measurements [E.Black et al., "Direct Observation of Broadband Coating Thermal Noise in a Suspended Interferometer," Phys. Lett. **A328** (2004), 1; see also K. Numata et al., "Wide-Band Direct Measurement of Thermal Fluctuations in an Interferometer, " Phys. Rev. Lett., **91** (2003) 260602-1] are consistent [4] with values $\phi_{SiO_2} \approx 0.5 \cdot 10^{-4}, \phi_{Ta_2O_5} \approx 5.1 \cdot 10^{-4}$.

Loss-angle confidence interval

Measurement related – Poisson-law distributed ?
(measurement process can only *spoil* quality factors)
⇨ Use *minimum* loss-angles for synthesis ?

Process (technology) related – Gaussian distributed ?
⇨ Use *average* loss-angles for synthesis ?

Few measurements available - Hard to distinguish the two

Least-favorable-case synthesis? Pays little…

Conclusions

- Stacked-doublet coatings with tweaked end-layer thicknesses are the optimal design for 2-dielectric mirrors; 25% event rate boost obtained for $\gamma \approx 10$
- Genetic optimization may handle more general (multi-objective, multi-dielectric) problems/configurations
- Optimum thicknesses and achievable noise reduction critically dependent on coating loss - angles
- More direct/indirect accurate & reliable measurements of coating loss - angles needed; work is in progress (LMA, Glasgow, Urbino, TNI)

For more details …

LIGO-P060027-00-Z

Optimized multilayer dielectric mirror coatings for gravitational wave interferometers

Juri Agresti [a,b], Giuseppe Castaldi [c], Riccardo DeSalvo [a], Vincenzo Galdi [c], Vincenzo Pierro [c], and Innocenzo M. Pinto [c]

[a] LIGO Laboratory, California Institute of Technology, Pasadena, CA 91125, USA;
[b] Department of Physics, University of Pisa, I-56127, Pisa, Italy;
[c] Waves Group, Department of Engineering, University of Sannio, I-82100, Benevento, Italy

ABSTRACT

The limit sensitivity of interferometric gravitational wave antennas is set by the thermal noise in the dielectric mirror coatings. These are currently made of alternating quarter-wavelength high/low index material layers with low mechanical losses. The quarter-wavelength design yields the maximum reflectivity for a fixed number of layers, but *not* the lowest noise for a prescribed reflectivity. This motivated our recent investigation of *optimal* thickness configurations, which guarantee the lowest thermal noise for a targeted reflectivity. This communication provides a compact overview of our results, involving *nonperiodic genetically-engineered* and *truncated periodically-layered* configurations. Possible implications for the advanced Laser Interferometer Gravitational wave Observatory (LIGO) are discussed.

Keywords: Multilayer coatings, dielectric mirrors, gravitational waves, interferometers, thermal noise.

1. INTRODUCTION

Interferometric gravitational wave (GW) detectors like LIGO,[1] VIRGO,[2] GEO,[3] and TAMA[4] are very-long-baseline optical interferometers featuring multilayer dielectric mirrors. These consist of a suitable number of alternating layers of

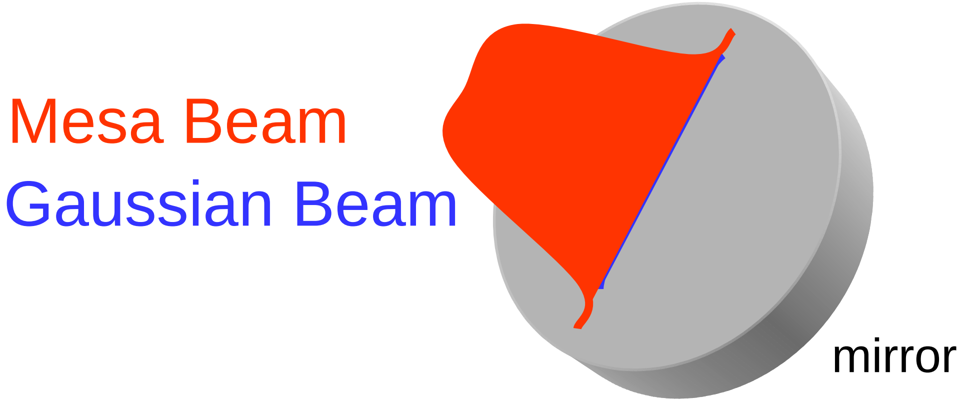
Agenda

- Optimized coatings
  - Background
  - Genetically-optimized coatings
  - Stacked-doublet design
  - Results

- Analytic structure of “hyperboloidal” beams
  - Background: From *nearly-flat* (FM) and *nearly-concentric* mesa (CM) beams to Bondarescu-Thorne (BT) *hyperboloidal* beams
  - Rapidly-converging Gauss-Laguerre (GL) expansion
  - Some results: Beam shapes and mirror corrections
  - Generalized duality relations (lowest-order mode)
    - *Complex-order* Fourier transform

Mesa beams

- Use of *flat-top* ("mesa") beams suggested for mitigating thermal noise effects in Adv-LIGO [D'Ambrosio *et al.*, LIGO-G000223-00-D]



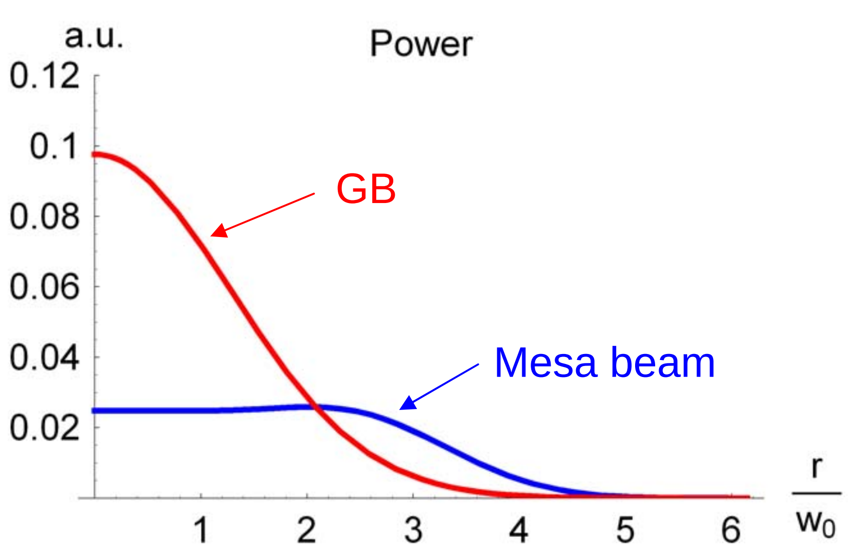
  - Better averaging of thermally-induced mirror surface fluctuations

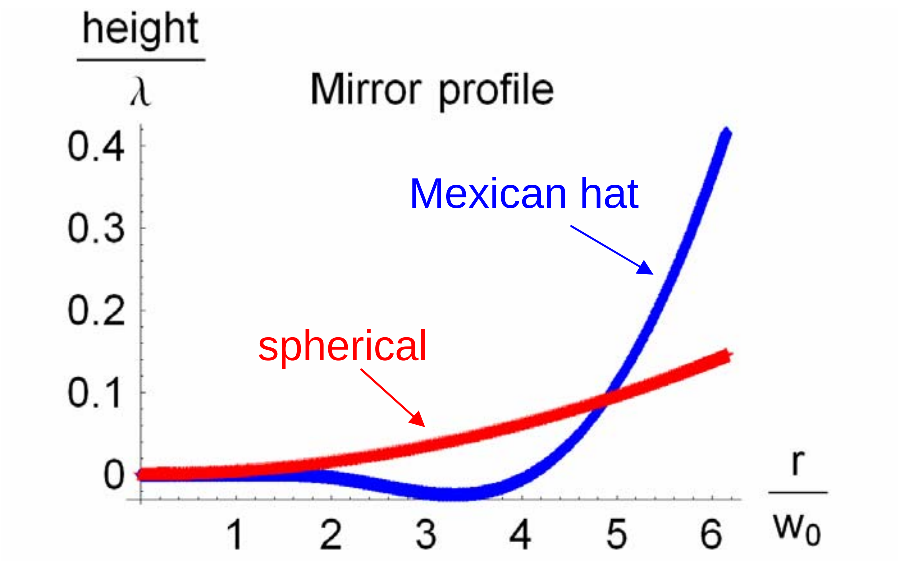
Mesa beams (cont'd)

- *Nearly-flat* mesa (FM) beams
  - Synthesized via coherent superposition of minimum-spreading Gaussian beams (GB) with parallel optical axes

$$u_{mesa}(r) \propto \int_{r' \le R_0} d^2\vec{r}'\, e^{-\frac{|\vec{r}-\vec{r}'|^2 (1-i)}{2 w_0^2}}$$

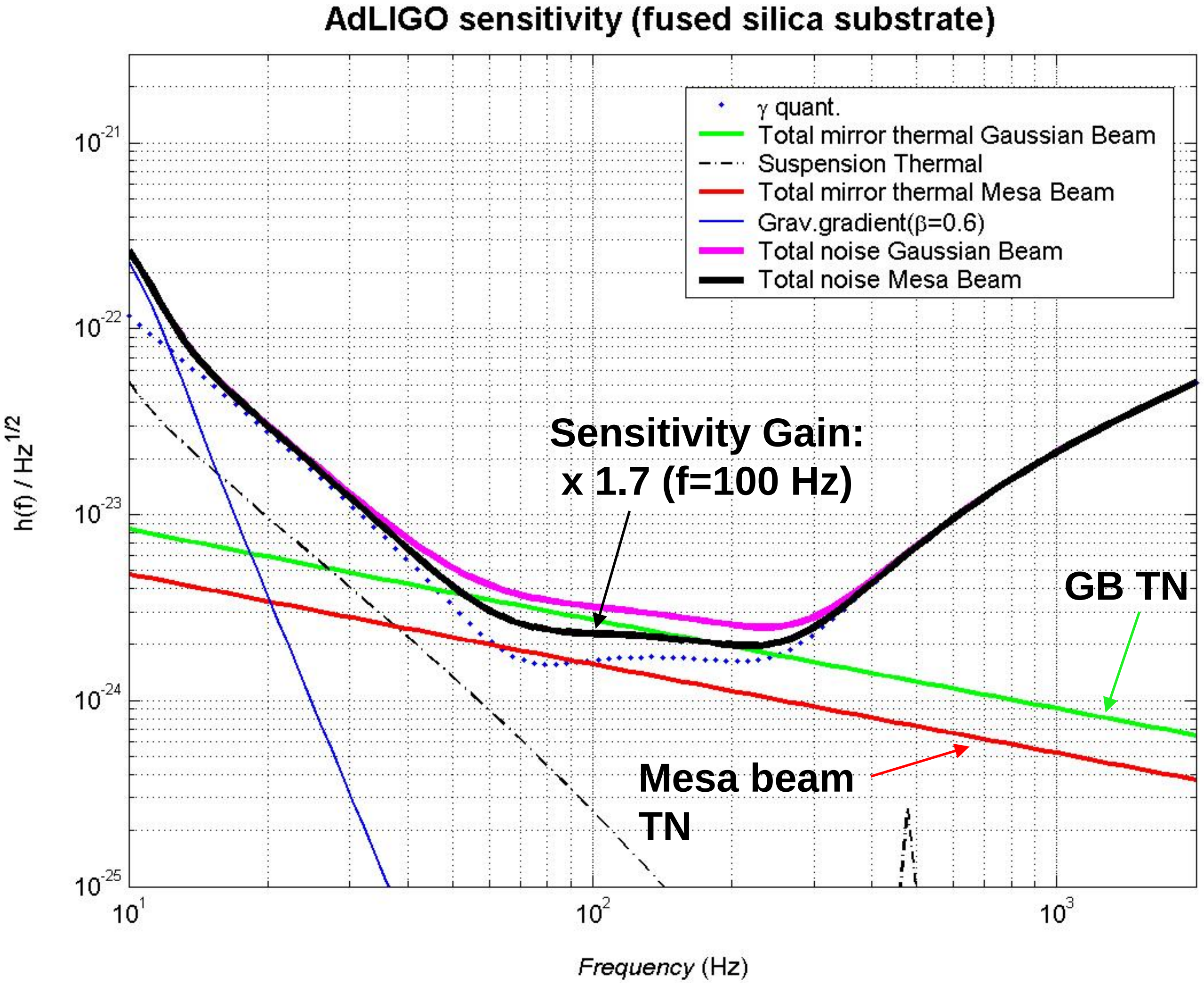
  - Supported by *nearly-flat* mirrors with "Mexican-hat" profile





Mesa beams (cont'd)

[M.G. Tarallo, LIGO G060305-00-Z, 2006]



Mesa beams (cont'd)

- Concerns about *tilt-instability* [Savov & Vyatchanin, gr-qc/0409084]

- *Nearly-concentric* mesa (CM) beams
  - Same intensity distribution at the mirror
  - *Much weaker* tilt-instability [Savov & Vyatchanin, gr-qc/0409084]

- FM and CM configurations connected by *duality* relations
  - One-to-one mapping between eigenstates [Agresti *et al.*, gr-qc/0511062]

- Questions about *optimal* beam-shaping

Coating thermal noise for arbitrary-shaped beams

R. O'Shaughnessy*
Department of Physics and Astronomy, Northwestern University
(Dated: Received ?? Month 2006, printed July 11, 2006)

Advanced LIGO's sensitivity will be limited by coating noise. Though this noise depends on beam shape, and though nongaussian beams are being seriously considered for advanced LIGO, no published analysis exists to compare the quantitative thermal noise improvement alternate beams offer. In this paper, we derive and discuss a simple integral which completely characterizes the dependence of coating thermal noise on shape. The derivation used applies equally well, with minor modifications, to all other forms of thermal noise in the low-frequency limit.

PACS numbers: 04.80.Cc,

I. INTRODUCTION

Though gravitational wave detectors such as LIGO are presently taking data, the best estimates from the astrophysical community for gravitational waves from compact object merger rates [1, 2, 3] (though some disagree [4]), cosmic strings [5], rotating neutron stars [6, 7, 8], and supernovae [9, 10] suggest that discoveries are most likely to begin with next-generation ground based interferometers like advanced LIGO. The present consensus advanced LIGO design has astrophysical reach (e.g., as measured by the distance to which a pair of inspiralling neutron stars could be detected) limited by coating thermal noise [11]. In this context, thermal noise denotes the phase noise in the IFO produced by elastic oscillations of the mirror excited by the thermal bath of the remaining degrees of freedom [12]; coating thermal noise denotes strong contributions to the noise arising when couplings between elastic modes and the thermal bath (i.e., losses) are predominantly located in the thin mirror coating off which the test beam reflects. Thermal noise depends strongly on beam shape: as one can show by applying the fluctuation-dissipation theorem to a low-

where $\tilde{P}(K)$, its two-dimensional fourier transform.

II. SCALING ARGUMENT

According to the fluctuation-dissipation theorem, coating thermal noise is proportional to the power dissipation rate $W_{\rm diss}$ associated with a fluctuating pressure of shape $P(r)$ on the mirror surface. Manifestly (for half-infinite mirrors), $W_{\rm diss}$ must be proportional to a translation-invariant inner product on P, of form

$$W_{\rm diss} \propto \int d^2R \int d^2R' V(R-R')P(R)P(R') \qquad (2)$$

$$\propto \int d^2K G(K)|P(K)|^2 \qquad (3)$$

By definition, the coating thermal noise is the contribution of the coating to the total thermal noise; thus, expanding in powers of coating thickness,

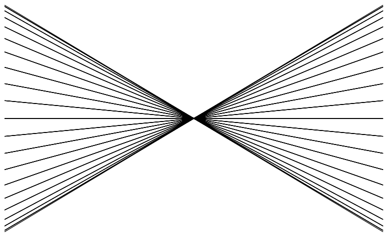
$$G(K,d) \approx G_o(K) + dG_1(K) + \ldots$$

Since no other transverse scale exists in the half-infinite mirror, the kernel G_1 must be scale invariant, and there-

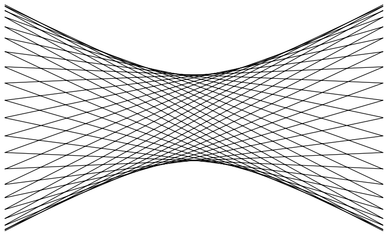
FM/CM vs. BT hyperboloidal beams

- More general ("hyperboloidal") beams [Bondarescu & Thorne, gr-qc/0409083] may be of interest

- FM beams
  - Optical axes are the generators of a *cylinder*



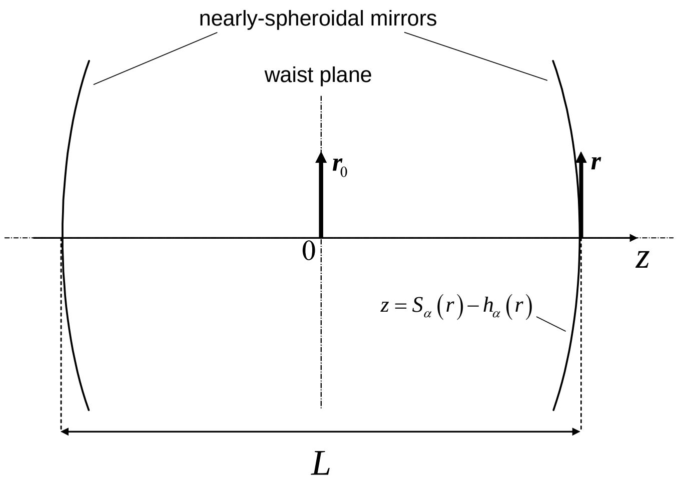
- CM beams
  - Optical axes are the generators of a *cone*



- BT hyperboloidal beams
  - Optical axes are the generators of a *hyperboloid*
  - Parameterized via "twist angle" $0 \le \alpha \le \pi$
  - Contain FM (α= 0) and CM (α=π) as special cases

BT hyperboloidal beams

- Supported by *nearly-spheroidal* mirrors



- Fiducial spheroid: $S_\alpha(r) \approx \frac{L}{2} - \frac{r^2 \sin^2(\alpha/2)}{L}$
- Waist-plane aperture radius: R_0
- GB spot size: $w_0 = \sqrt{\frac{L}{k_0}}$ (minimum spreading)

- Field distribution on fiducial spheroid [Bondarescu & Thorne, gr-qc/0409083]

$$U_\alpha(r, S_\alpha) = \Lambda \int_0^{R_0} dr_0 \int_0^{2\pi} d\theta_0 r_0 \exp\left[i \frac{rr_0}{w_0^2} \sin\theta_0 \sin\alpha - \frac{\left(r^2 + r_0^2 - 2rr_0 \cos\theta_0\right)}{2w_0^2} (1 - i\cos\alpha) \right]$$

  - θ_0-integral generally needs to be computed numerically
  - Closed-form (Gaussian) solution for $\alpha=\pi/2$

BT hyperboloidal beams (cont'd)

- Mirror profile correction

$$\arg\left[U_\alpha\left(r, S_\alpha - h_\alpha\right)\right] = \text{constant} \quad \Rightarrow \quad h_\alpha(r) = \frac{\arg\left[U_\alpha\left(r, S_\alpha\right)\right] - \arg\left[U_\alpha\left(0, S_\alpha\right)\right]}{k_0}$$

- Symmetry/duality relations
  - Field distribution

$$U_{-\alpha} = U_\alpha, \quad \frac{U_{\pi-\alpha}}{\Lambda} = \frac{U_\alpha^*}{\Lambda^*}$$

  - Mirror profile correction

$$h_{\pi-\alpha}(r) = -h_\alpha(r)$$

GL expansions

- FM and CM beams
  - Field distributions at the waist plane related via Fourier transform (FT)
  - Coincide with "flattened" beams in [Sheppard & Saghafi, *Opt. Comm.* **132**, 144, 1996]
    - Gauss-Laguerre (GL) expansions available

$$U_\pi(r,0)=\sum_{m=0}^{\infty}A_m^{(\pi)}\psi_m\left(\frac{\sqrt{2}r}{w_0}\right),\quad U_0(r,0)=\sum_{m=0}^{\infty}A_m^{(0)}\psi_m\left(\frac{\sqrt{2}r}{w_0}\right)$$

$$\psi_m(\xi)=\sqrt{2}\exp\left(-\frac{\xi^2}{2}\right)L_m\left(\xi^2\right)$$ GL (orthonormal) basis functions

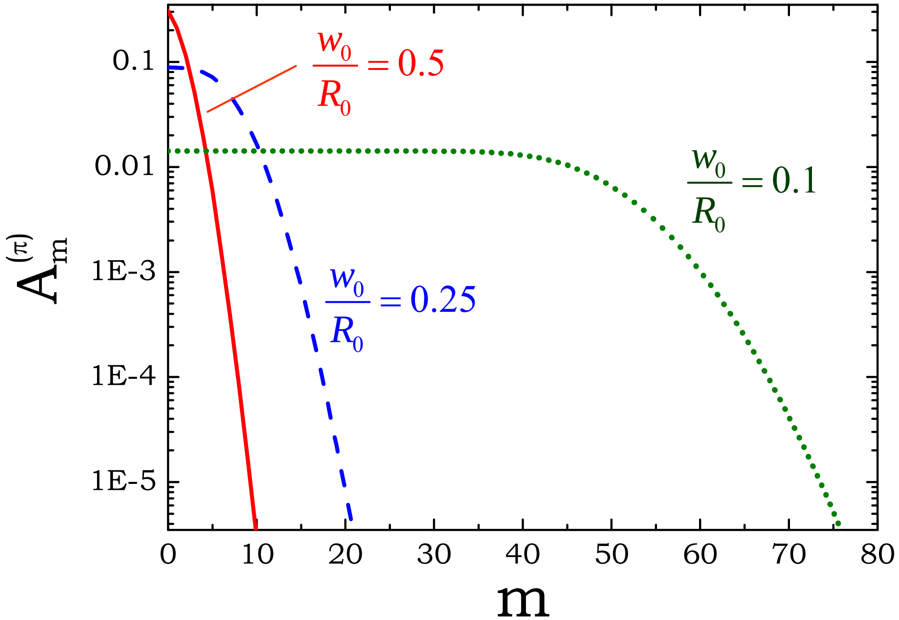
- FT $\Rightarrow$ $\boxed{A_m^{(0)}=(-1)^m A_m^{(\pi)}}$

GL expansions (cont'd)

- Expansion coefficients [Sheppard & Saghafi, *Opt. Comm.* **132**, 144, 1996]

$$A_m^{(\pi)} = \frac{\sqrt{2}w_0^2}{R_0^2} P\left(m+1, \frac{R_0^2}{2w_0^2} \right)$$

P: incomplete Gamma function



- Easily computable
- Abrupt fall-off for $m \geq \frac{R_0^2}{2w_0^2}$

⇒ **Rapid convergence**

GL expansions (cont'd)

- Extension to ***generic* BT hyperboloidal beams** [Galdi *et al.*, *Phys. Rev. D* **73**, 127101, 2006]

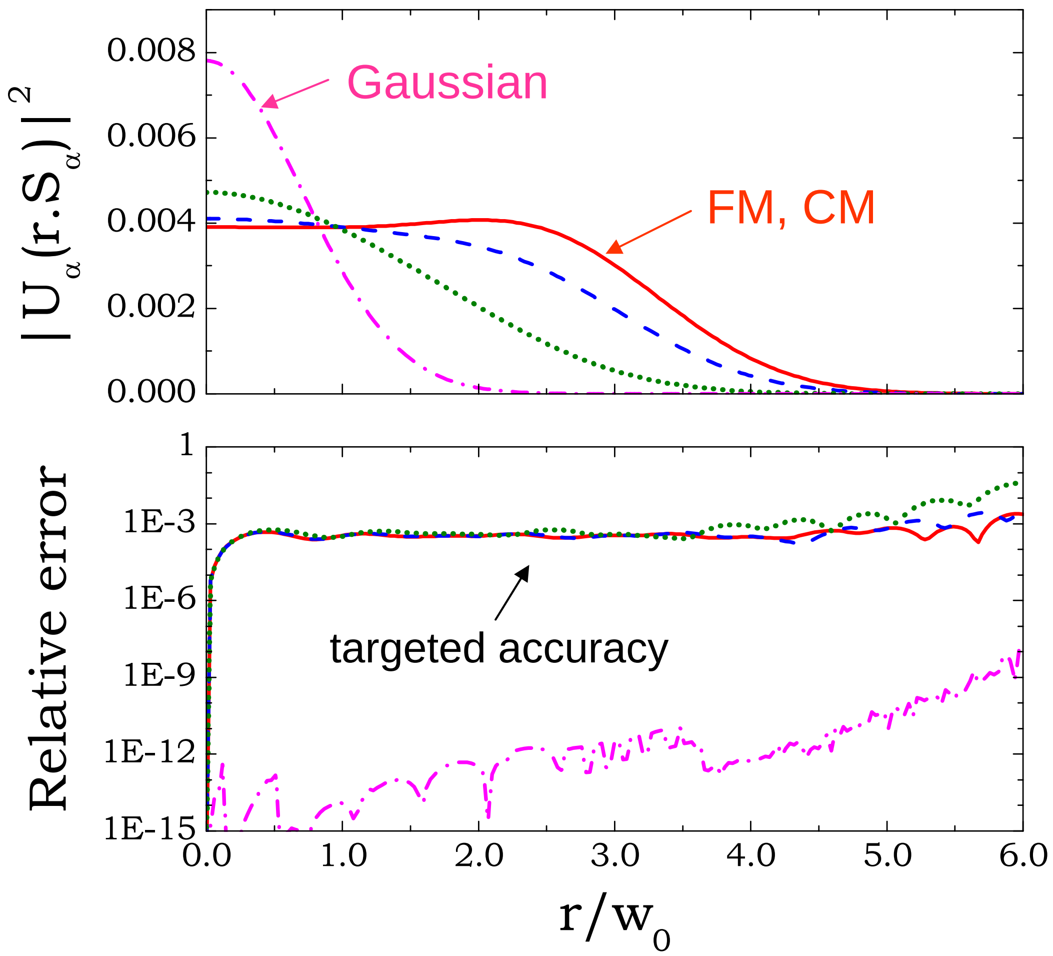
$$U_\alpha(r,0)=\sum_{m=0}^{\infty}A_m^{(\alpha)}\psi_m\left(\frac{\sqrt{2}r}{w_0}\right),\quad \boxed{A_m^{(\alpha)}=(-\cos\alpha)^m A_m^{(\pi)}}$$

- Allows field computation *at any point in space*
  - Use standard GL (paraxial) propagators
  - Field distribution on the fiducial spheroids

$$U_\alpha(r,S_\alpha)=\Omega\exp\left(ik_0\frac{r^2\cos\alpha}{2L}\right)\sum_{m=0}^{\infty}(-i)^m A_m^{(\alpha)}\psi_m\left(\frac{r}{w_0}\right)$$

    - Symmetry/duality relations verified

Results: Beam shapes



- Parameters

  $w_0 = 2.603cm,\ R_0 = 10.4cm,$
  $\lambda_0 = 1064nm$

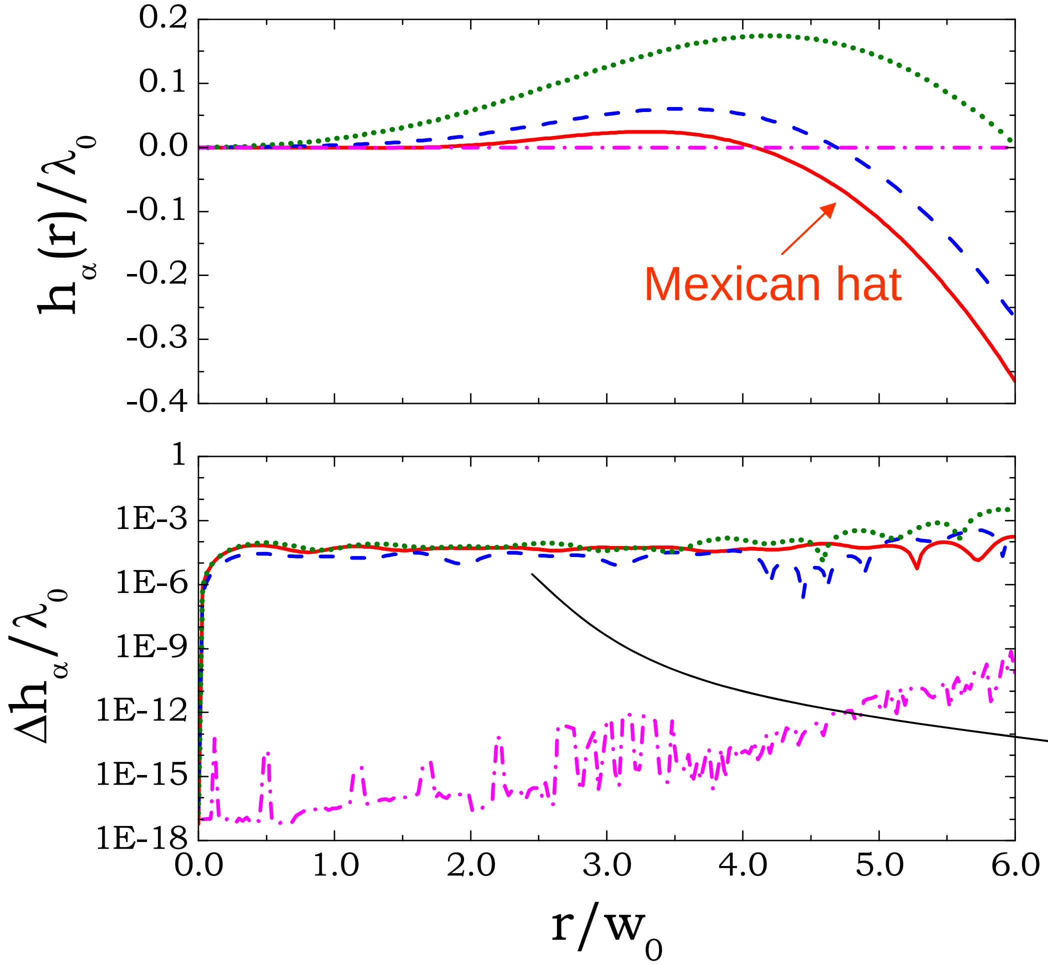
- Truncation criterion

  $m \le M, \quad \left| \frac{A_M^{(\alpha)}}{A_0^{(\alpha)}} \right| < 10^{-3}$

- Reference solution from [Bondarescu & Thorne, gr-qc/0409083]

$\alpha = 0, \pi \ (M = 18)$
$\alpha = 0.1\pi, 0.9\pi \ (M = 17)$
$\alpha = 0.2\pi, 0.8\pi \ (M = 14)$
$\alpha = 0.5\pi \ (M = 0)$

Results: Mirror corrections



- Same parameters

$\alpha = \pi \left(M = 18\right)$

$\alpha = 0.9\pi \left(M = 17\right)$

$\alpha = 0.8\pi \left(M = 14\right)$

$\alpha = 0.5\pi \left(M = 0\right)$

- For $0 \le \alpha < \pi/2$

  $h_{\pi-\alpha}\left(r\right) = -h_\alpha\left(r\right)$

- Typical errors: ~0.1nm
  - Well within fabrication tolerances

Generalized duality relations

- Two *arbitrary* (α_1,α_2)-indexed BT hyperboloidal beams related by:

$$U_{\alpha_2}\left(r,0\right)\xleftrightarrow{H_{w_0}^{(\sigma)}}U_{\alpha_2}\left(r,0\right),\quad \sigma=-\frac{\cos\alpha_2}{\cos\alpha_1}$$

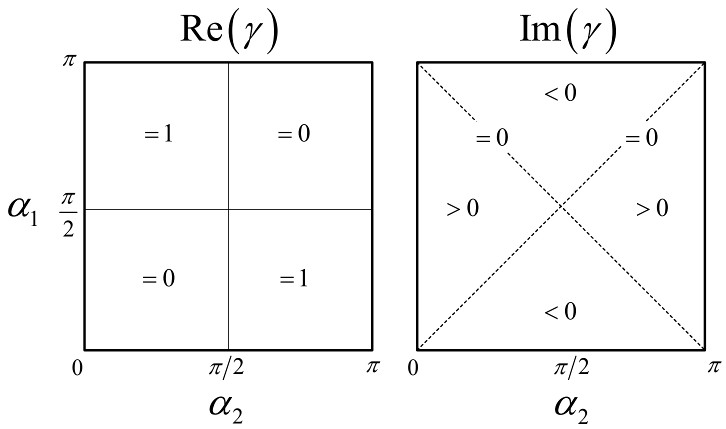
$$H_{w_0}^{(\sigma)}\left[F\left(r\right)\right]\equiv\frac{4}{w_0^2\left(1+\sigma\right)}\int_0^\infty r_0 dr_0 F\left(r_0\right)J_0\left[\frac{4rr_0\sqrt{\sigma}}{w_0^2\left(1+\sigma\right)}\right]$$
$$\times\exp\left[-\frac{\left(r^2+r_0^2\right)\left(1-\sigma\right)}{w_0^2\left(1+\sigma\right)}\right],\quad \sigma\geq-1$$

$$H_{w_0}^{(\sigma)}\left[F\left(r\right)\right]\equiv H_{w_0}^{(1)}\left\{H_{w_0}^{(-\sigma)}\left[F\left(r\right)\right]\right\}=H_{w_0}^{(-\sigma)}\left\{H_{w_0}^{(1)}\left[F\left(r\right)\right]\right\},\quad \sigma<-1$$

Generalized duality relations

- *Fractional* FT operators of *complex order*

$$\gamma = 1 + \frac{i}{\pi} \log\left(-\frac{\cos\alpha_2}{\cos\alpha_1} \right)$$



Special cases

$\alpha_1 = \alpha_2 \quad \Rightarrow \quad \gamma = 0$ (identity operator)

$\alpha_1 = \pi - \alpha_2 \quad \Rightarrow \quad \gamma = 1$ (standard FT)

- Generalizes (for lowest-order mode) the FM-CM duality relations in [Agresti *et al.*, gr-qc/0511062]
  - Numerically checked to work for lowest-order radial modes, at any azimuthal order

Conclusions and perspectives

- Summary
  - Focus on the analytic structure of a class of hyperboloidal beams of interest for future GW interferometers
  - Rapidly-converging, physically-insightful GL expansions for *generic* BT hyperboloidal beams
    - Field computation at any point in space
    - Validation/calibration against reference solution
  - Generalized duality relations
    - *Complex-order* FT
- Current/future research
  - Thorough parametric analysis
    - Implications for Adv-LIGO
    - Beam-shape optimization
  - Extension to higher-order modes (HOM)
    - Techniques to depress HOM (parametric instabilities)

For more details …

PHYSICAL REVIEW D **73**, 127101 (2006)

Analytic structure of a family of hyperboloidal beams of potential interest for advanced LIGO

Vincenzo Galdi,* Giuseppe Castaldi, Vincenzo Pierro, and Innocenzo M. Pinto
Waves Group, Department of Engineering, University of Sannio, I-82100 Benevento, Italy

Juri Agresti, Erika D'Ambrosio, and Riccardo DeSalvo
LIGO Laboratory, California Institute of Technology, Pasadena, California 91125, USA
(Received 20 February 2006; published 22 June 2006)

This paper is concerned with a study of the analytic structure of a family of hyperboloidal beams introduced by Bondarescu and Thorne which generalizes the *nearly-flat* and *nearly-concentric* mesa beam configurations of interest for advanced LIGO. Capitalizing on certain results from the applied optics literature on flat-top beams, a physically-insightful and computationally-effective representation is derived in terms of rapidly-converging Gauss-Laguerre expansions. A generalization (involving *fractional* Fourier transform operators of *complex* order) of some recently discovered *duality* relations between the nearly-flat and nearly-concentric mesa configurations is obtained. Possible implications for the advanced-LIGO optical cavity design are discussed.

DOI: 10.1103/PhysRevD.73.127101 PACS numbers: 07.60.Ly, 04.80.Cc, 41.85.Ew, 42.55.−f

I. INTRODUCTION

Fabry-Perot optical cavities with *nonspherical* mirrors capable of supporting *flat-top* (“mesa”) beams are being actively investigated [1] to be used in the baseline design of the advanced Laser Interferometer Gravitational-wave Observatory (LIGO) [2]. These configurations may reduce the thermal noise of the mirrors through better averaging over the beam profile of the thermally-induced surface fluctuations [3,4]. In this framework, *nearly-flat*, “Mexican-hat-shaped” mirror configurations were found capable of providing a reduction by a factor three in the

concerning flat-top beams, which have most likely not come to the attention of the gravitational-wave community. Specifically, the FM and CM beams belong to the class of flattened beams introduced in [10], and can therefore be represented in terms of the *rapidly-converging* Gauss-Laguerre (GL) beam expansions derived therein. Based on this observation, we extend the approach in [10] to accommodate the more general family of BT hyperboloidal beams [9]. This leads to a generalization (we limit the analysis here to the dominant eigenmode) of the FM-CM duality relations in [7,8], which involves *fractional* Fourier transforms of *complex* order.