Investigating a Parametric Instability in the LIGO Test Masses

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for the LIGO Scientific Collaboration

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Instabilities in Mirror Test-Masses

Problem

- Acoustic modes excited by radiation pressure
- Coupling of acoustic modes and optical modes

Solution

- Map instability behavior of interferometer, R values
- Use new FFT method for more complete R calculation

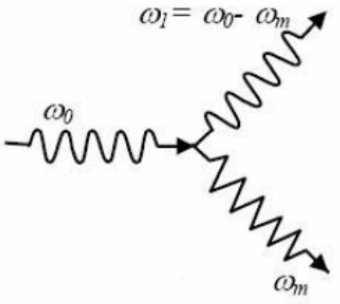
Outline

- Non-Linear Optical Interaction
- R values
- R Pipeline
- Results

Non-Linear Optical Interaction

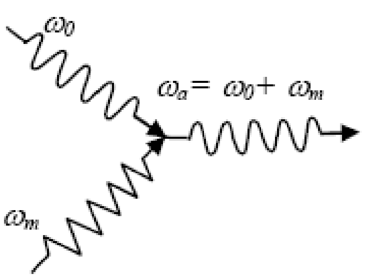
Mandelstam-Brillouin Scattering

- » Non-linear coupling of acoustic and optical waves



Stokes/anti-Stokes Process

- » Incident ω_0 optical wave excites phonons, releasing a ω_0 - ω_m optical sideband (destabilizing)
- » Incident ω_0 optical wave absorbs phonons, releasing a $\omega_0 + \omega_m$ optical sideband (damping)



Parametric Instability

- » Ponderomotive force on test-mass
- » Acoustic displacements in test-mass
- » Under certain conditions, instability

R Value

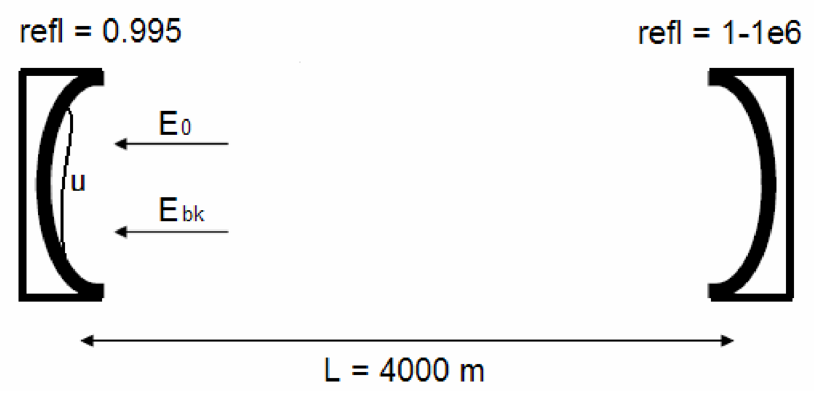
R Eigenvalue

- Real part of eigenvalue of system of equations describing
- Instability for R > 1
- Old “mode-pair” formulation
- New “total E field” formulation

$$R = \frac{4PQ_m}{m{\omega_m}^2 c}\left(\frac{V\int |E_0^s| Im(E_{bk}^s) u_z dA}{\int |E_0^s|^2 dA \int |\vec{u}|^2 dV} - \frac{V\int |E_0^{as}| Im(E_{bk}^{as}) u_z dA}{\int |E_0^{as}|^2 dA \int |\vec{u}|^2 dV}\right)$$

Configuration

- Advanced LIGO parameters
- Modelling one interferometer arm
- Considering acoustic mode deformation at one mirror

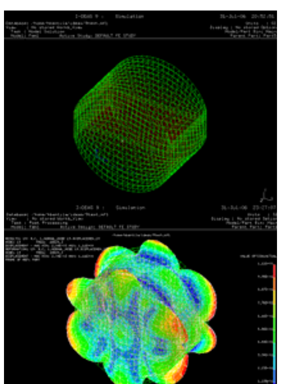


R Pipeline

Systematic calculation of R values

- FEM package to calculate acoustic modes
- FFT code to calculate optical modes
- Matlab code to process acoustic and optical data
- Calculate R values

1



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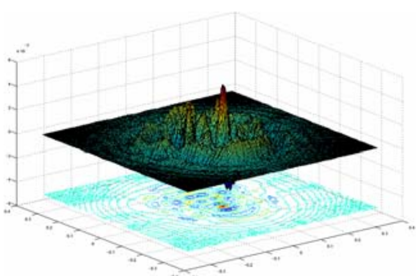

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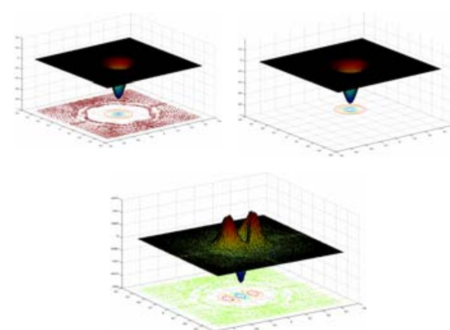


4

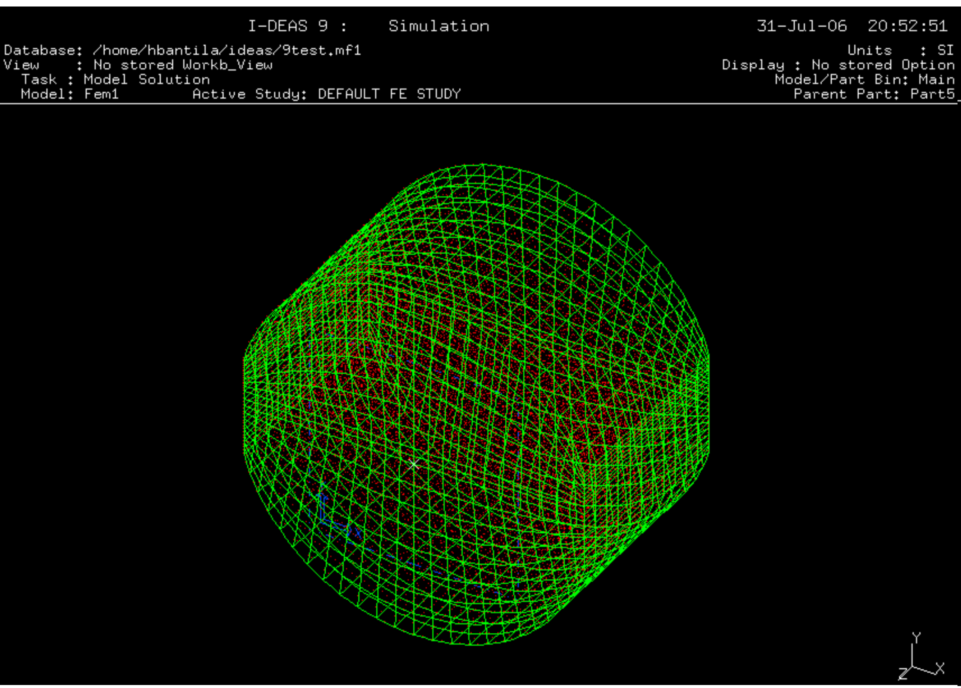
R

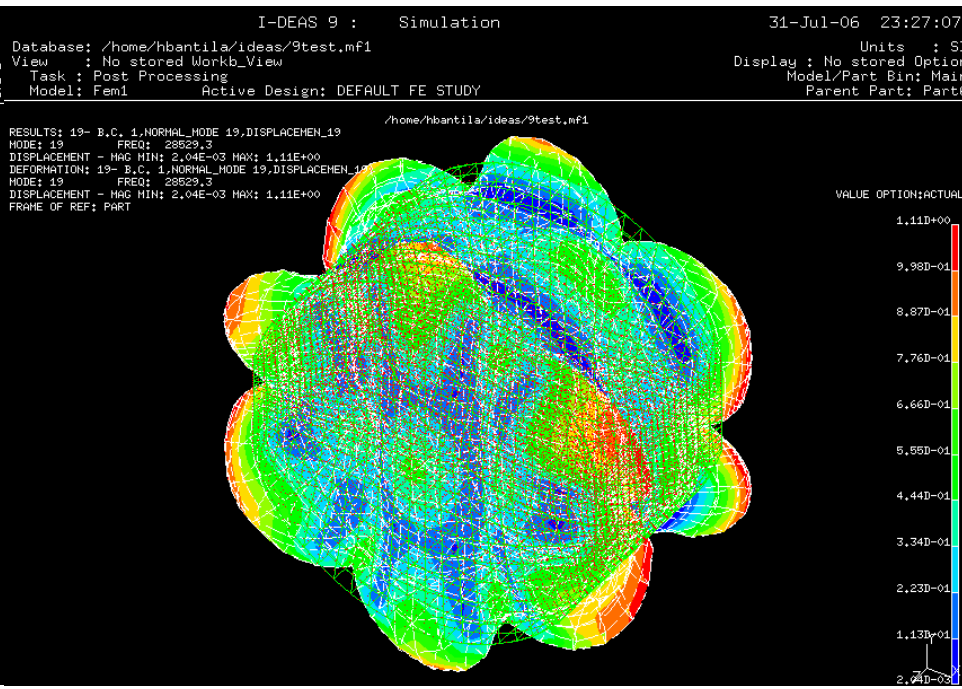
FEM Package

FEM Configuration

» 21797 nodes, 4752 elements

» 17 cm radius, 20 cm thickness, 95 mm flats

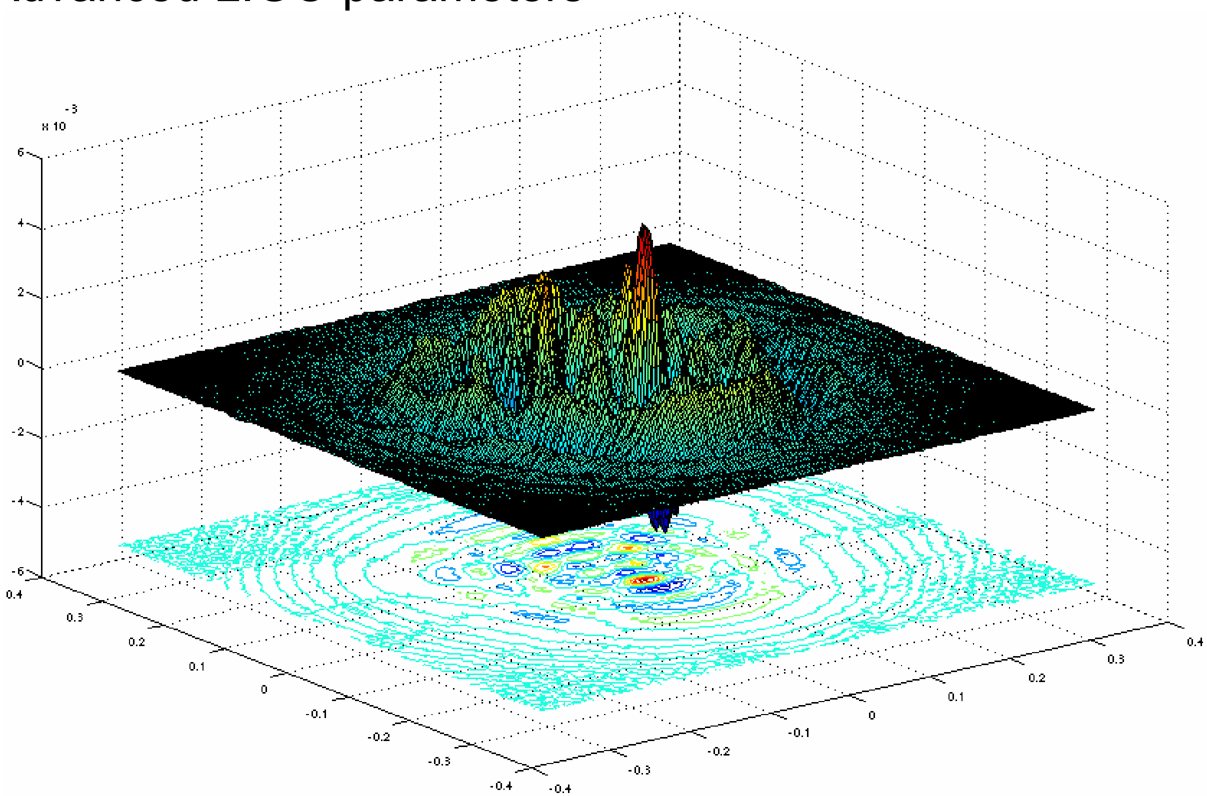




FFT Code

FFT Configuration

- » 256 x 256 grids
- » Advanced LIGO parameters



Matlab Code

Dynamical System, Static Model

- Dynamical system; scattering into different frequencies
- Static model; no concept of time or frequency

Phase "Trick"

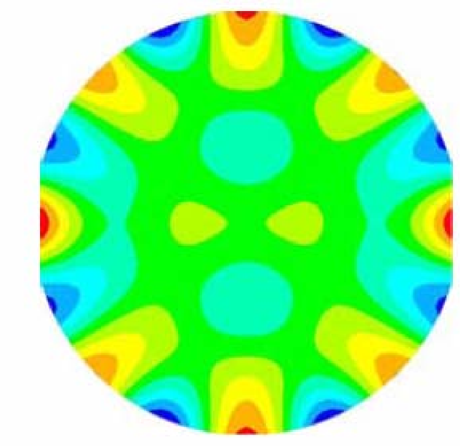
- Only concerned with round-trip phase
- Difference in frequency can be simulated by an appropriate change in cavity length
- Stokes field calculation: cavity length made shorter
- Anti-Stokes field calculation: cavity length made longer

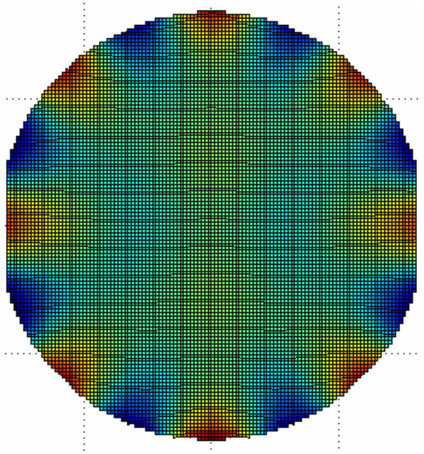
$$R = \frac{4PQ_m}{m{\omega_m}^2 c}\left(\frac{V\int |E_0^s| Im(E_{bk}^s) u_z dA}{\int |E_0^s|^2 dA \int |\vec{u}|^2 dV} - \frac{V\int |E_0^{as}| Im(E_{bk}^{as}) u_z dA}{\int |E_0^{as}|^2 dA \int |\vec{u}|^2 dV}\right)$$

Verification

“Australian” Case

- Acoustic mode at 28.34 kHz
- R values closely correspond



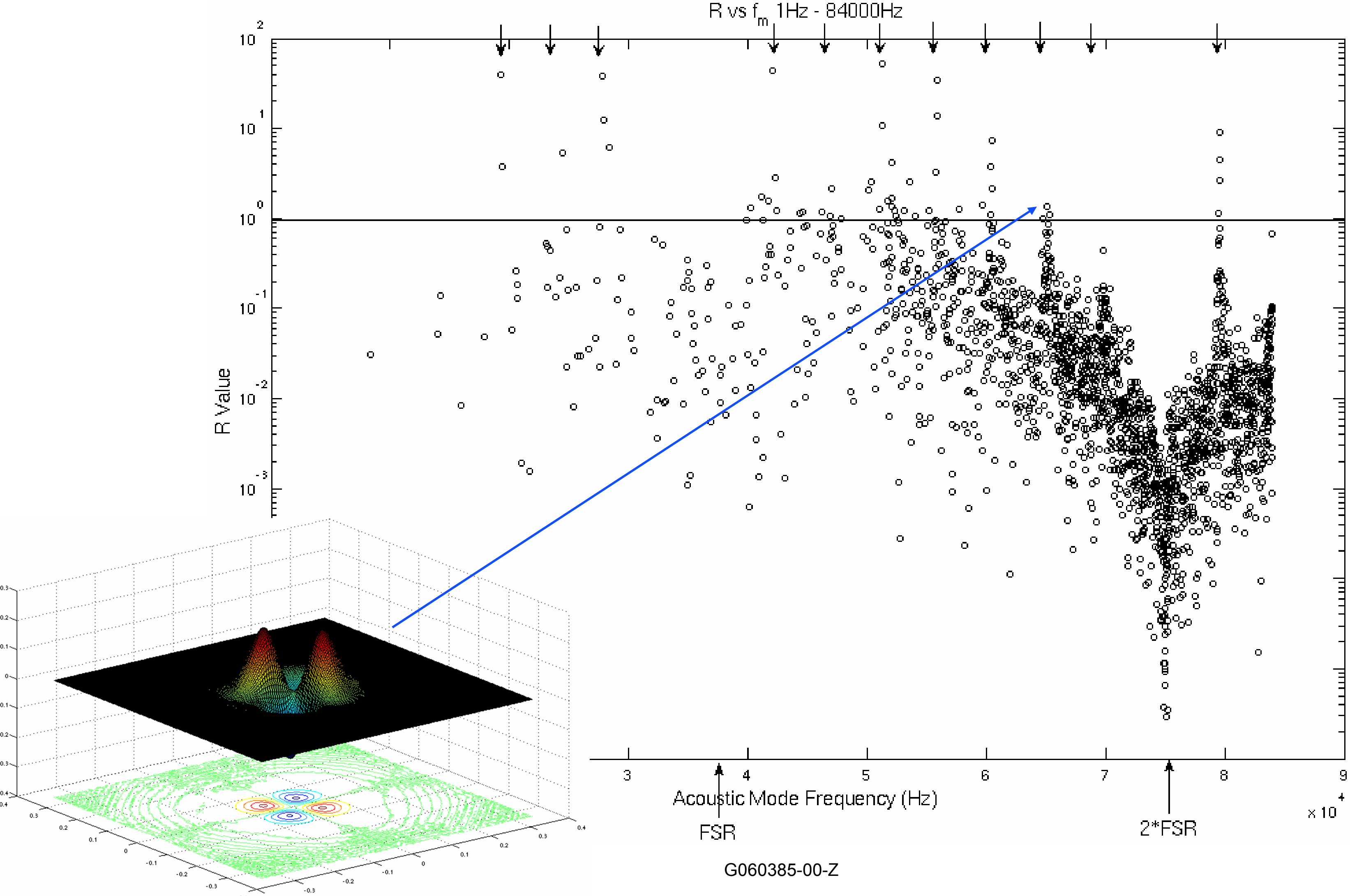


R = 3.63 R = 3.71

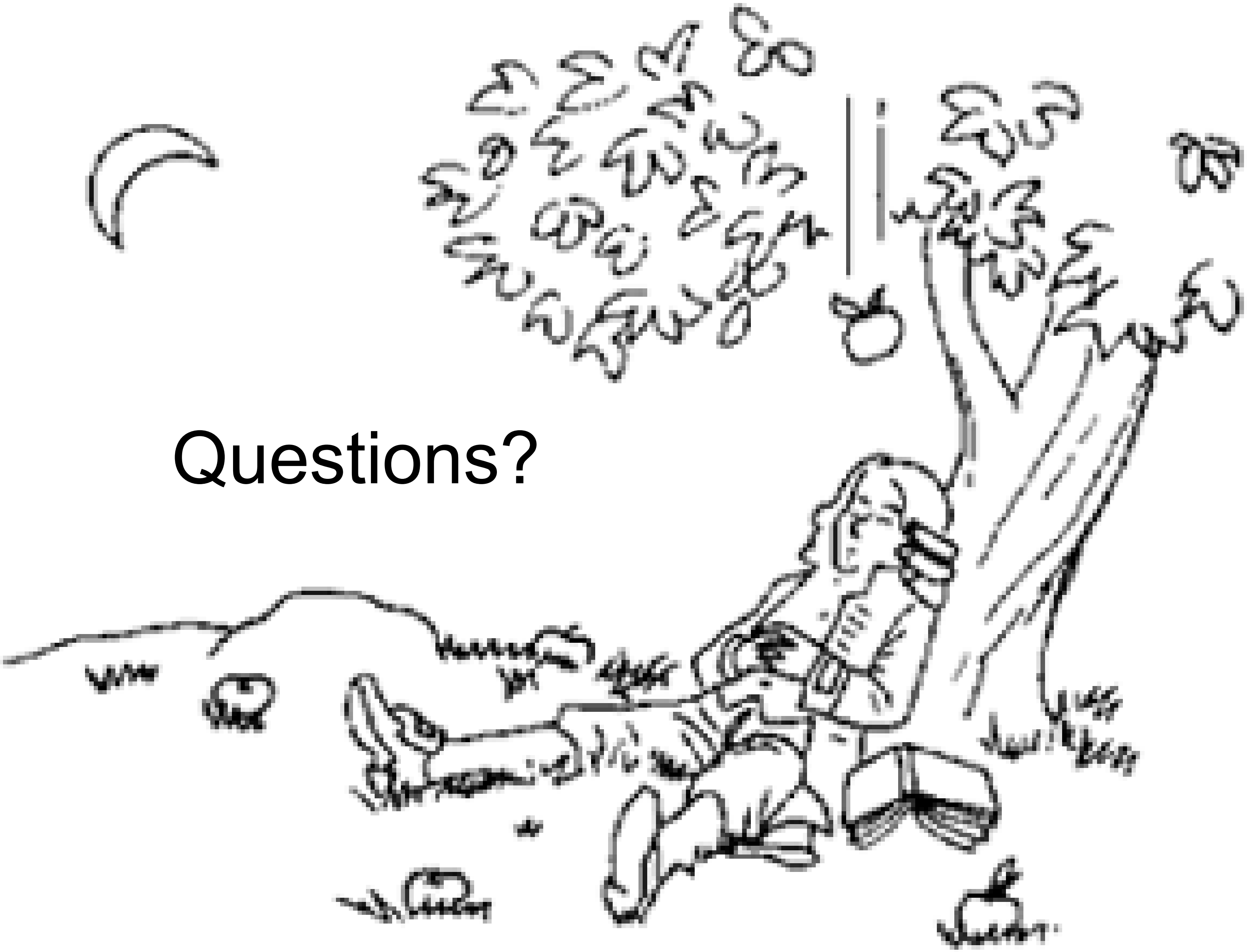
Synthetic LG10 Case

- Generalized Laguerre polynomial “acoustic mode”
- Scattering into only LG10 optical mode; exact R expression
- R value correctly predicted

Results

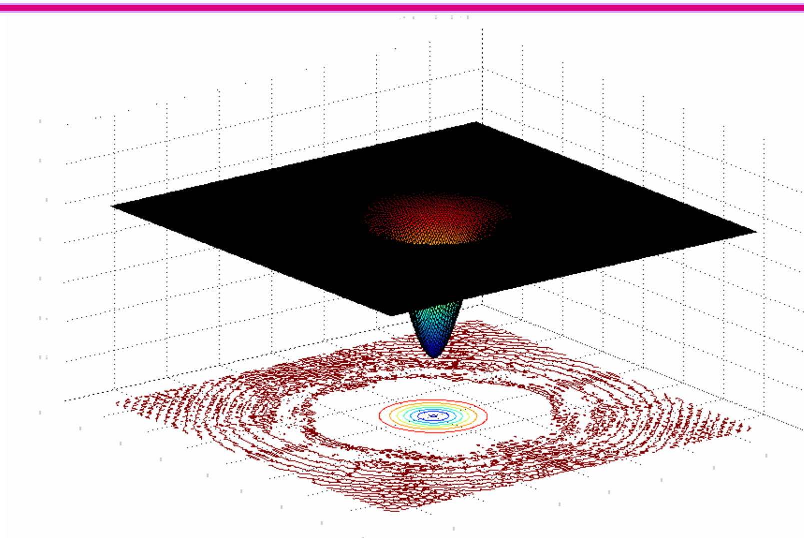

R vs f_m 1Hz - 84000Hz
R Value
10^2
10^1
10^0
10^-1
10^-2
10^-3
Acoustic Mode Frequency (Hz)
3
4
5
6
7
8
9
x 10^4
FSR
2*FSR

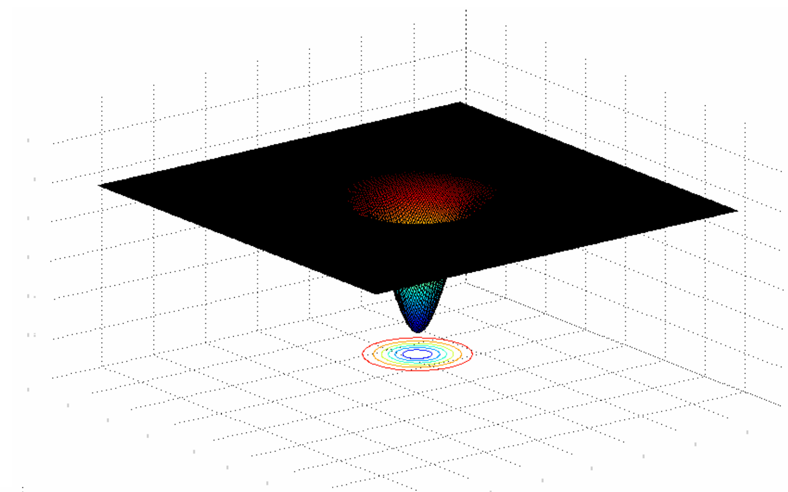
Questions?

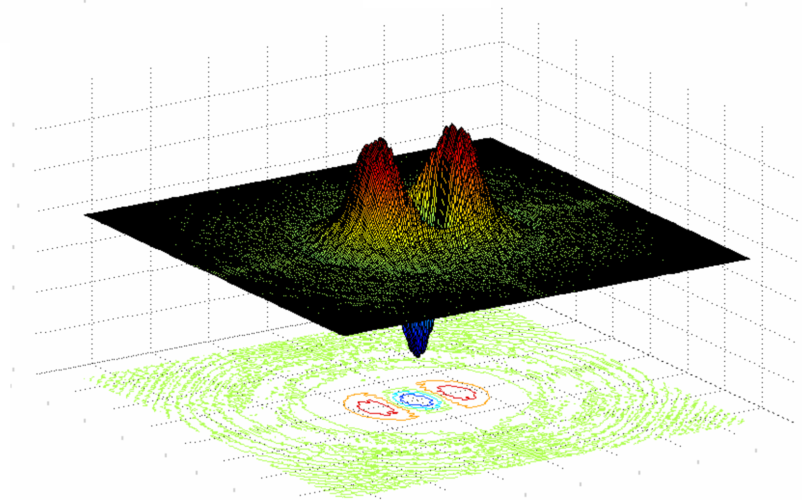


$$R = \frac{4P_0Q_m}{mcL\omega_m{}^2}\left(\sum_i \frac{Q_{1i}\Lambda_{1i}}{1+(\Delta\omega_{1i}/\delta_{1i})^2} - \sum_j \frac{Q_{1aj}\Lambda_{1aj}}{1+(\Delta\omega_{1aj}/\delta_{1aj})^2}\right)$$

$$R = \frac{4PQ_m}{m\omega_m{}^2c}\left(\frac{V\int |E_0^s| Im(E_{bk}^s)u_z dA}{\int |E_0^s|^2 dA \int |\vec{u}|^2 dV} - \frac{V\int |E_0^{as}| Im(E_{bk}^{as})u_z dA}{\int |E_0^{as}|^2 dA \int |\vec{u}|^2 dV}\right)$$







$$\varphi_{HTM} \pm \omega_m T = 2\pi n \quad \text{(resonance)}$$

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