Stochastic Background Search with ALLEGRO and LIGO Science Data

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on behalf of the LIGO Scientific Collaboration

XIth Marcel Grossman Meeting

2006 July 28

LIGO-G060371-00-Z

Outline

I Background/Motivation for LLO-ALLEGRO Search

- LLO-ALLEGRO Pair (proximity, overlap modulation)
- Technical Considerations (sampling, heterodyning)

II S4 Data Analysis

- Data Volume by Orientation; Projected Sensitivity
- Statistical Interpretation
- Software & Hardware Injections
- Cross-Corr Analysis Complete; Under Internal Review

Sensitivity to Stochastic GW Backgrounds

- Optimally filtered CC statistic

$$Y = \int df \underbrace{\tilde{s}_1^*(f)\, \tilde{Q}(f)\, \tilde{s}_2(f)}_{Y(f)}$$

- Optimal filter $\tilde{Q}(f) \propto \frac{S_{\text{gw}}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$
  (Initial analyses assume
  $S_{\text{gw}}(f)$ or $\Omega_{\text{gw}}(f) \propto f^3 S_{\text{gw}}(f)$ constant across band)
- Optimally filtered cross-correlation method has Ω_{gw} sensitivity

$$\sigma_\Omega \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f)P_2(f)} \right)^{-1/2}$$

- Significant contributions when
  - detector noise power spectra $P_1(f)$, $P_2(f)$ small
  - overlap reduction function $\gamma_{12}(f)$ (geom correction) near ± 1

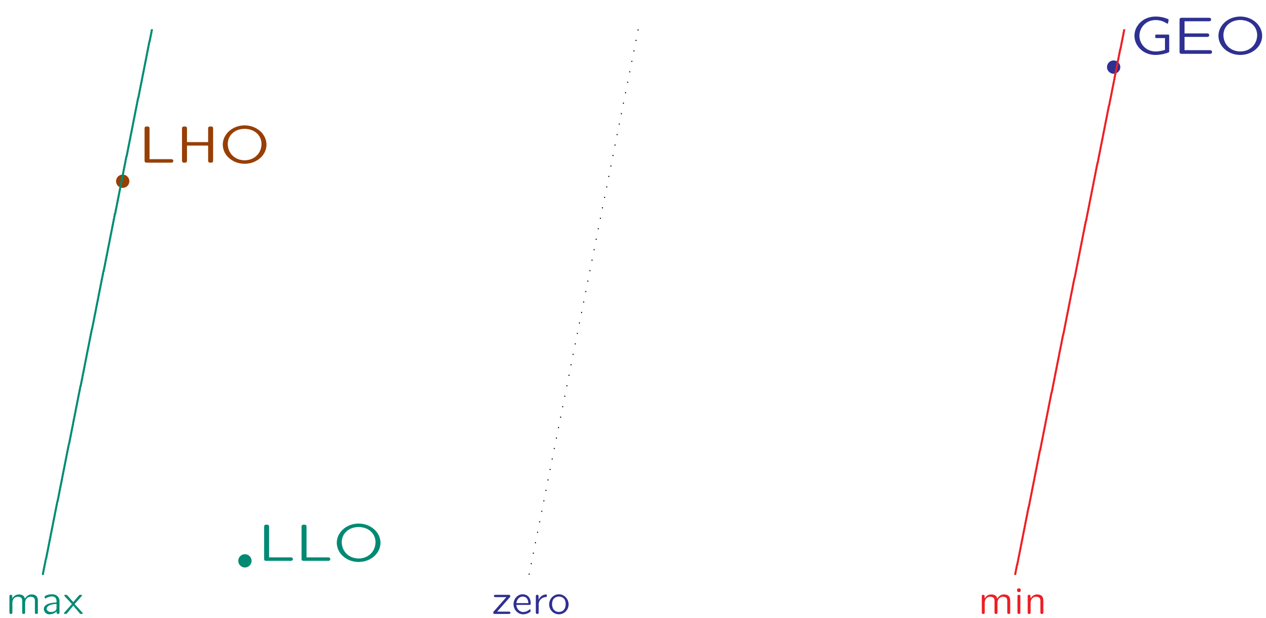
Overlap Reduction Function

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \iint_{S^2} d^2\Omega_{\hat{\mathbf{n}}} \; P^{\mathrm{TT}\,ab}{}_{cd}(\hat{\mathbf{n}}) e^{i2\pi f \hat{\mathbf{n}}\cdot\Delta\vec{\mathbf{r}}/c}$$

Depends on alignment of detectors (polarization sensitivity)

Frequency dependence from cancellations when $\lambda \lesssim$ distance

$\rightarrow$ Widely separated detectors less sensitive at high frequencies



This wave drives LHO & GEO out of phase

Overlap Reduction Function

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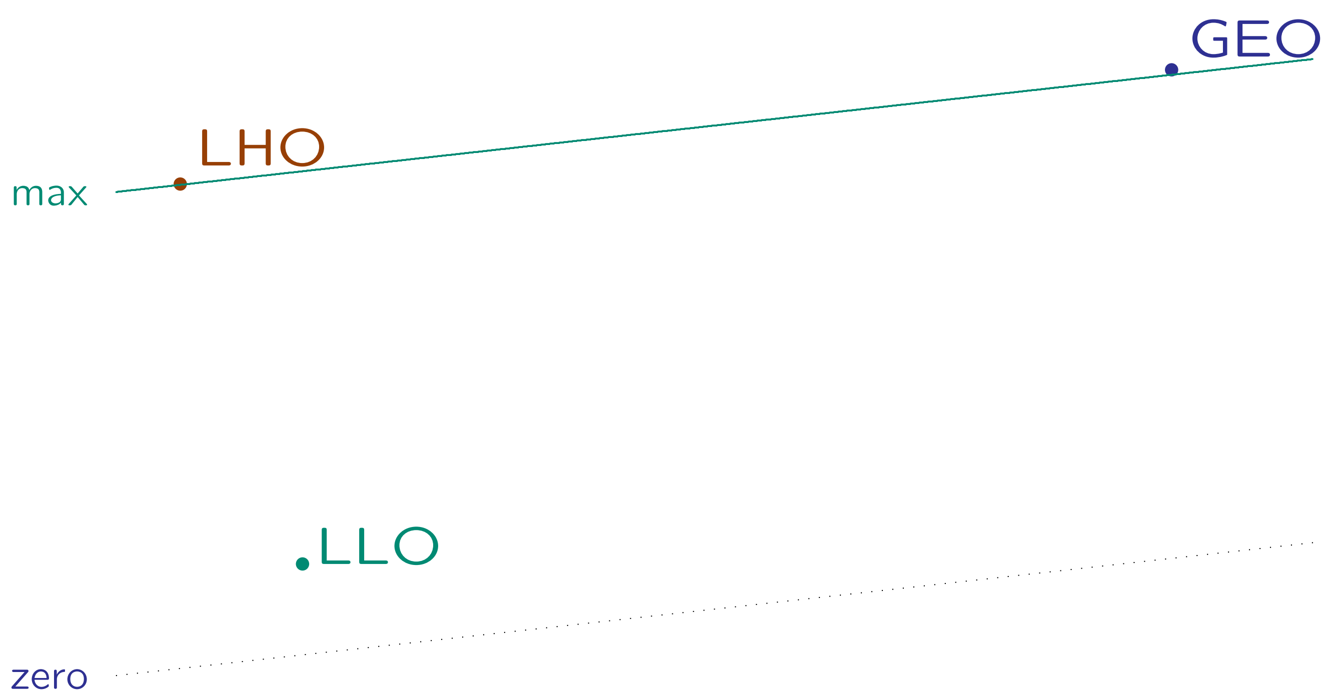
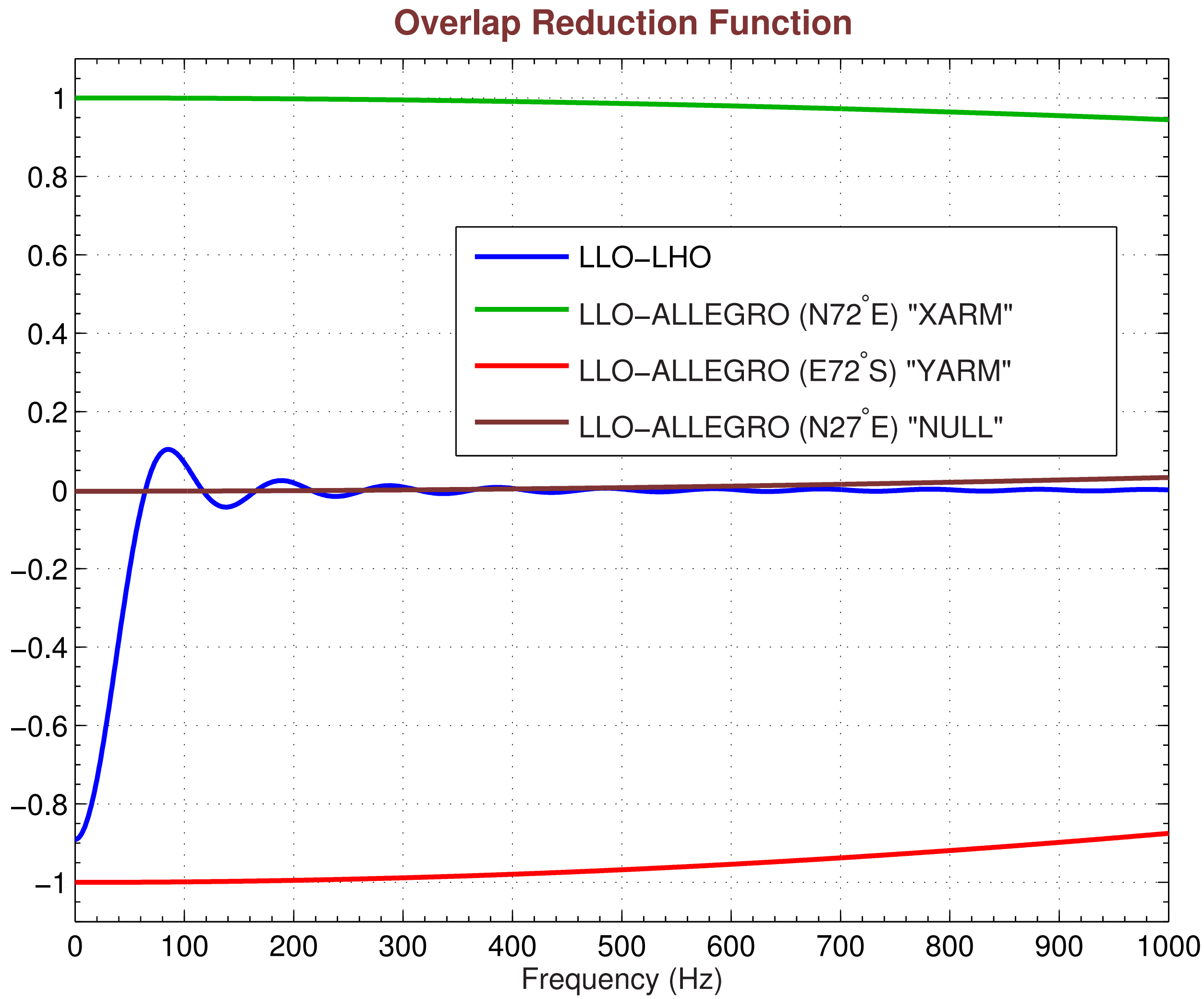


This wave (same λ) drives LHO & GEO in phase



LLO-ALLEGRO only $\sim 40\,\mathrm{km}$ apart $\longrightarrow$ still sensitive @ 900 Hz

Response different for XARM, YARM, NULL orientations

ALLEGRO ran in all 3 orientations during LIGO S4 Run (2005 Feb 22-Mar 23)

LLO-ALLEGRO: Technical Considerations

- LIGO data digitally downsampled 16384 Hz → 4096 Hz
  ALLEGRO data heterodyned at 904 Hz & sampled at 250 Hz

- Heterodyning means CC stat complex:

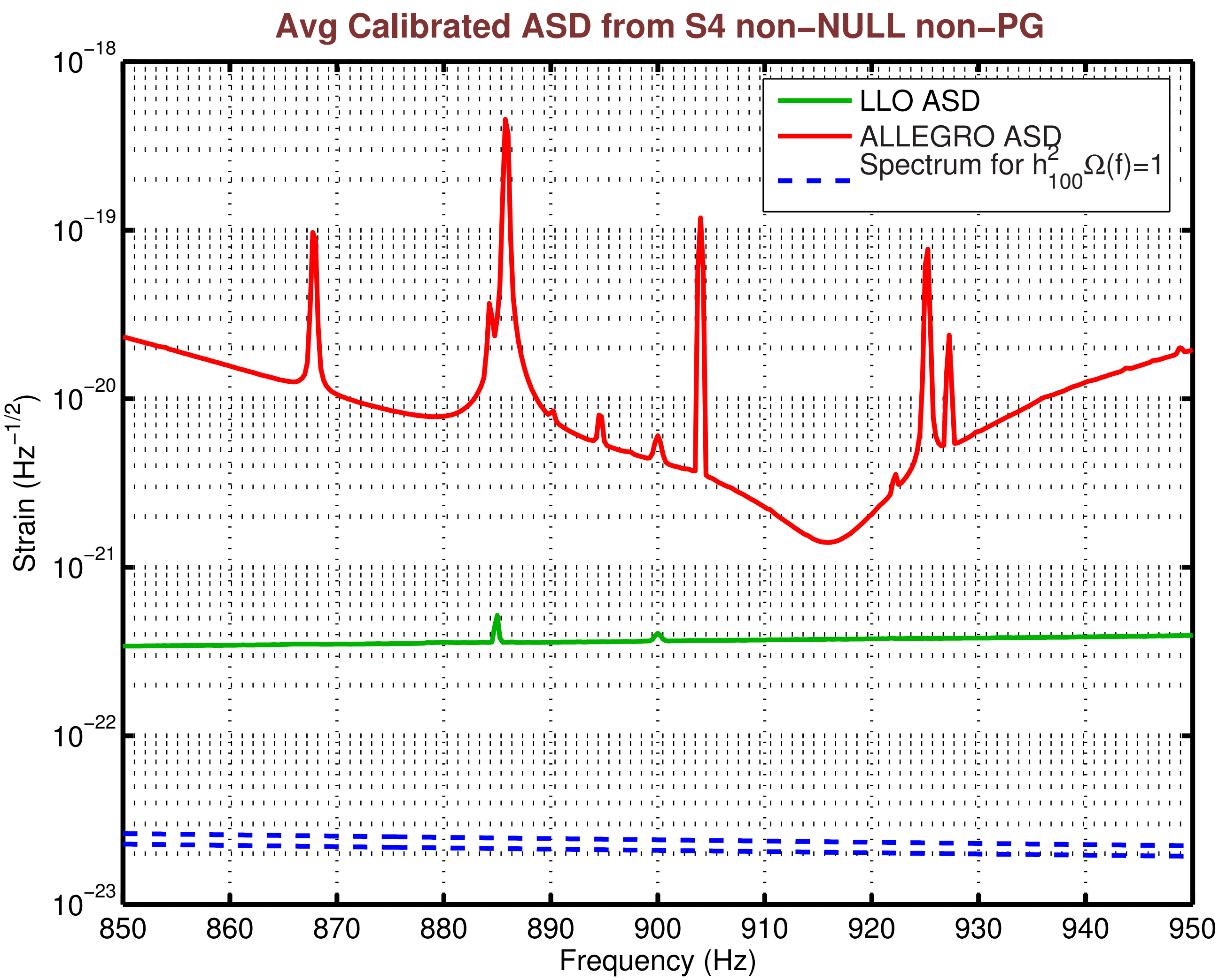
$$Y = \int_{f_{\text{min}}}^{f_{\text{max}}} df\, \tilde{s}_1^*(f)\, \tilde{Q}(f)\, \tilde{s}_2(f)$$

  real part Gaussian-distributed about SGWB strength;
  imag part Gaussian-distributed about 0.

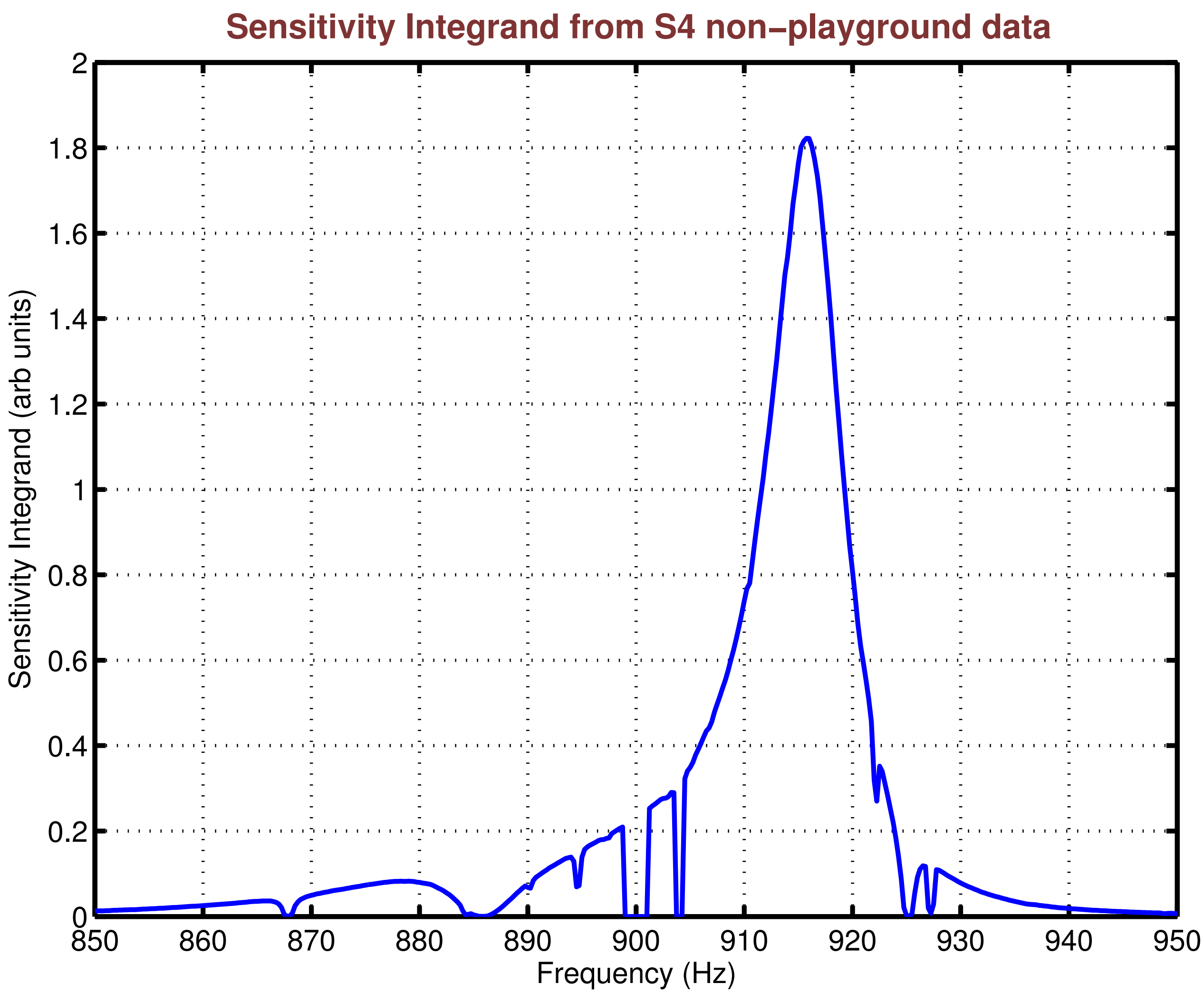
LLO-ALLEGRO data from LIGO S4 Run

- $\sim$ 10% of data set aside as "playground";
  Non-PG data divided into 60s segments; 3 orientations:
  - "NULL" ($0.023 < \gamma(f) < 0.029$): $\approx 5200\,\mathrm{min}$ after cuts
    "off-source" data useful for data quality & cross-checks
  - "YARM" ($-0.89 > \gamma(f) > -0.91$): $\approx 6600\,\mathrm{min}$ after cuts
  - "XARM" ($0.95 < \gamma(f) < 0.96$):] $\approx 10400\,\mathrm{min}$ after cuts

- Projected $h_{100}^2\Omega$ sensitivity using non-playground data: ~ 0.3–1;
  Corresponds to $\sqrt{S_{\mathrm{gw}}(f = 915\,\mathrm{Hz})}$ of $\sim 1\text{–}2 \times 10^{-23}\,\mathrm{Hz}^{-1/2}$



Frequency band determined by ALLEGRO noise curve



Most of sensitivity from 905–925 Hz

Constructing Posterior PDF

- Overall estimate $h_{100}^2\widehat{\Omega} = x + iy$ has likelihood function (for given actual $\Omega_R = h_{100}^2 \Omega_{\text{gw}}(915\,\text{Hz})$)

$$P(x, y|\Omega_R, \sigma_\Omega) \propto \exp\left(-\frac{|x + iy - \Omega_R|^2}{2{\sigma_\Omega}^2}\right)$$

- Bayes's theorem gives posterior PDF

$$P(\Omega_R|x, y, \sigma_\Omega) = \frac{P(x, y|\Omega_R, \sigma_\Omega)P(\Omega_R)}{P(x, y|\sigma_\Omega)}$$
$$\propto e^{-(x-\Omega_R)^2/2{\sigma_\Omega}^2} P(\Omega_R)$$

Note imag part y of pt est factors out

Marginalization Over Calibration Uncertainty

- Calibration of LLO & ALLEGRO uncertain in amp & phase
  Marginalize over unknown correction factor $e^{\Lambda+i\phi}$:

$$P(x,y|\Omega_R,\sigma_\Omega,\Lambda,\phi) \propto \exp\left(-\frac{\left|x+iy-\Omega_R e^{\Lambda+i\phi}\right|^2}{2{\sigma_\Omega}^2}\right)$$

so the posterior after marginalizing the likelihood function is

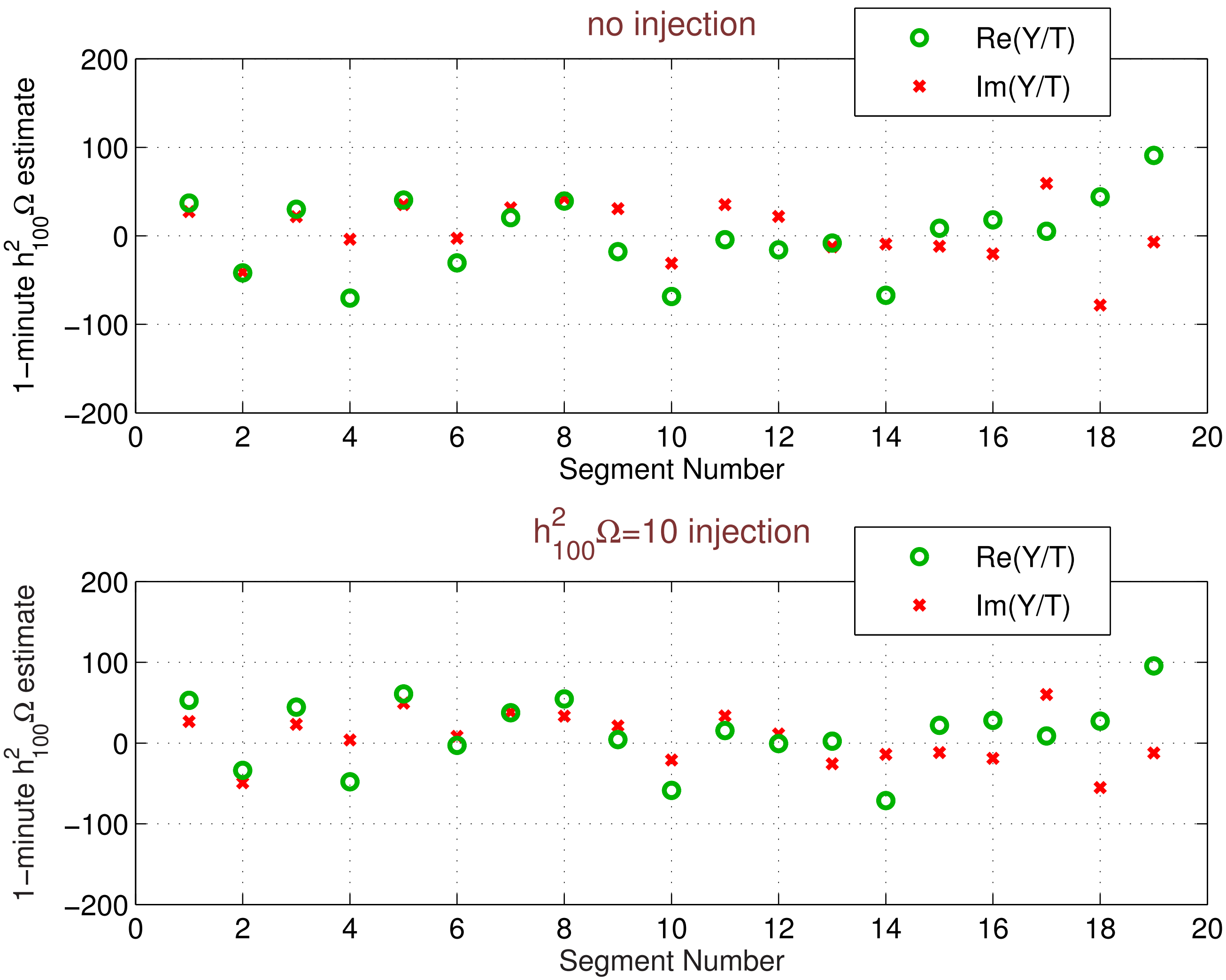
$$P(\Omega_R|x,y,\sigma_\Omega) \propto \int_{-\infty}^{\infty} d\Lambda \int_{-\pi}^{\pi} d\phi \, \exp\left(-\frac{\left|x+iy-\Omega_R e^{\Lambda+i\phi}\right|^2}{2{\sigma_\Omega}^2}\right) P(\Lambda,\phi)\, P(\Omega_R)$$

which does depend on imag part y

Software Injections into S4 Playground

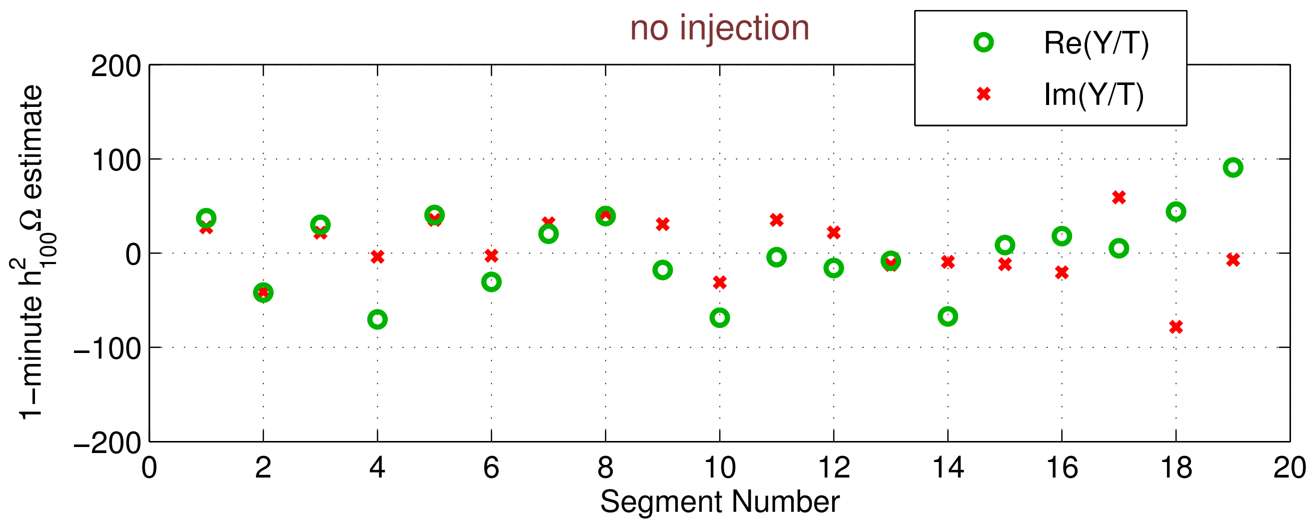
- Combined 90% error bar for all PG data: 0.8 (XARM-only: 0.9; YARM-only: 1.4; NULL-only 46)
- Inject $h_{100}^2\Omega(f) = 1, 2, 5, 10$.
- Note: individual jobs have error bars around 60. SW injections only detectable over time.

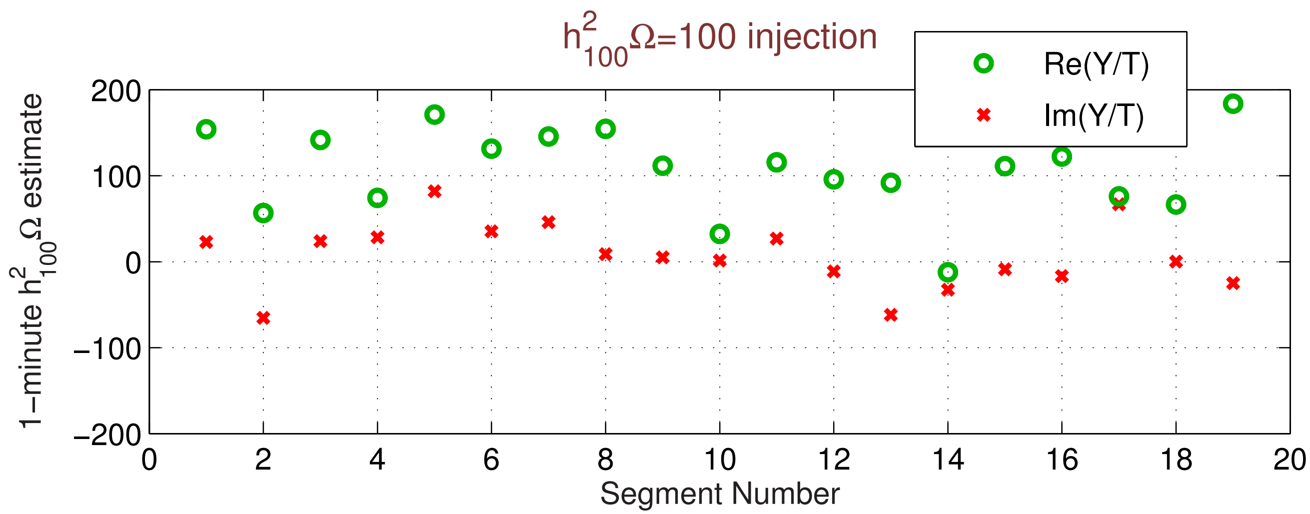
Stats w/ & w/o SW Inj (19 60-sec segs)



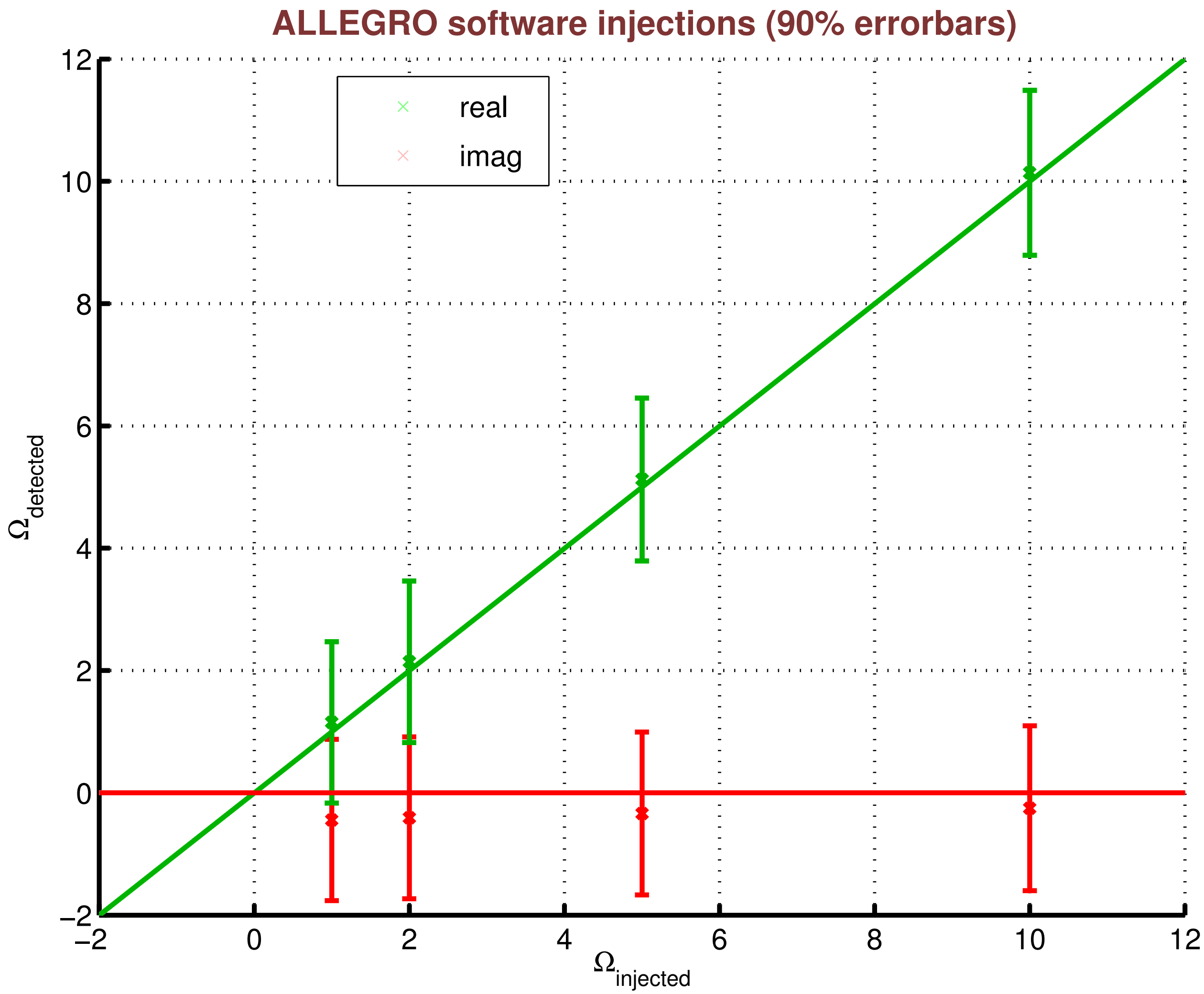
Injecting $h^2_{100}\Omega(f) = 10$ has negligible impact on minute-by-minute correlations

Stats w/ & w/o SW Inj (19 60-sec segs)





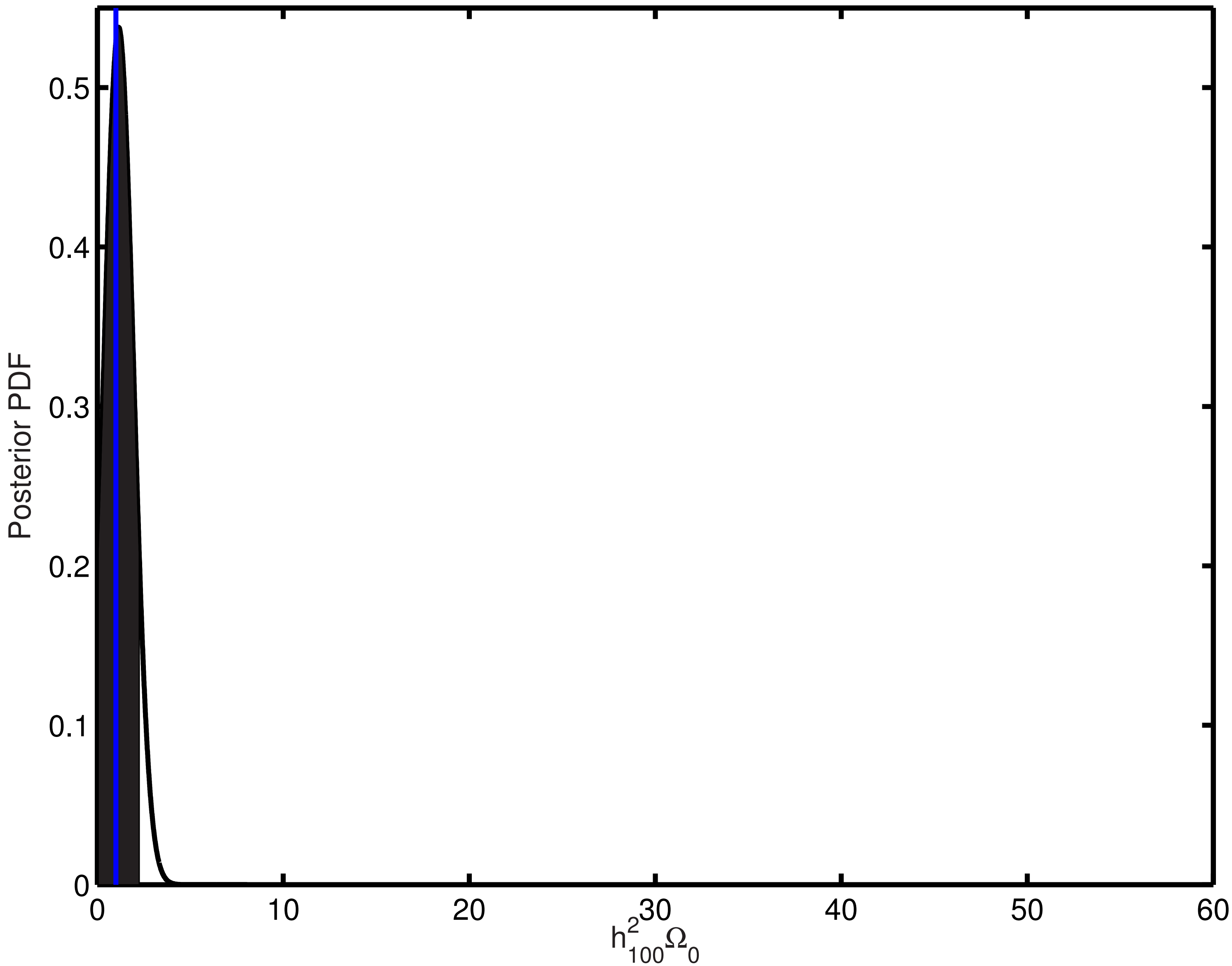
Compare $h^2_{100}\Omega(f) = 100$ injection, which is visible minute-by-minute

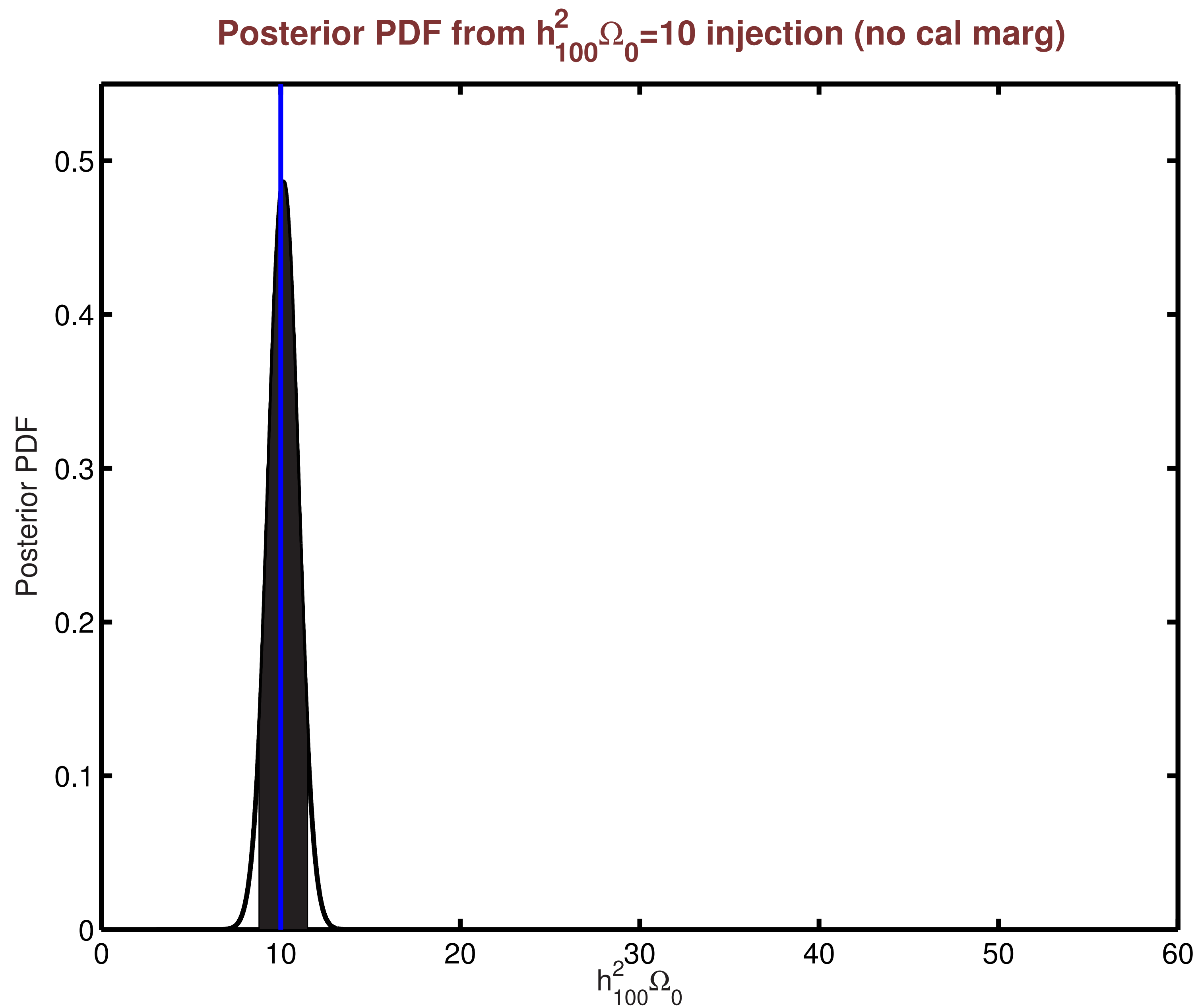


$h^2_{100}\Omega(f) = 2, 5, 10$ injections recovered from full PG

Note: injected same random signals w/different amplitudes into same noise

Posterior PDF from $h^2_{100}\Omega_0$=1 injection (no cal marg)




Posterior PDF from $h^2_{100}\Omega_0$=10 injection (no cal marg)
Posterior PDF
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50
60
$h^2_{100}\Omega_0$

S4 Hardware Injections

- 1024-second simulated signals injected into LLO & ALLEGRO hardware a total of nine times. Simulated all three orientations.

- "second" & "third" rounds injected const $h_{100}^2\Omega_{\text{gw}}(f) = 4195$
  Sensitivity of cross-correlation to injections simulating XARM ("plus") and YARM ("minus") is comparable
  "null" injection less correlated b/c of simulated misalignment

Time-Shift Analyses

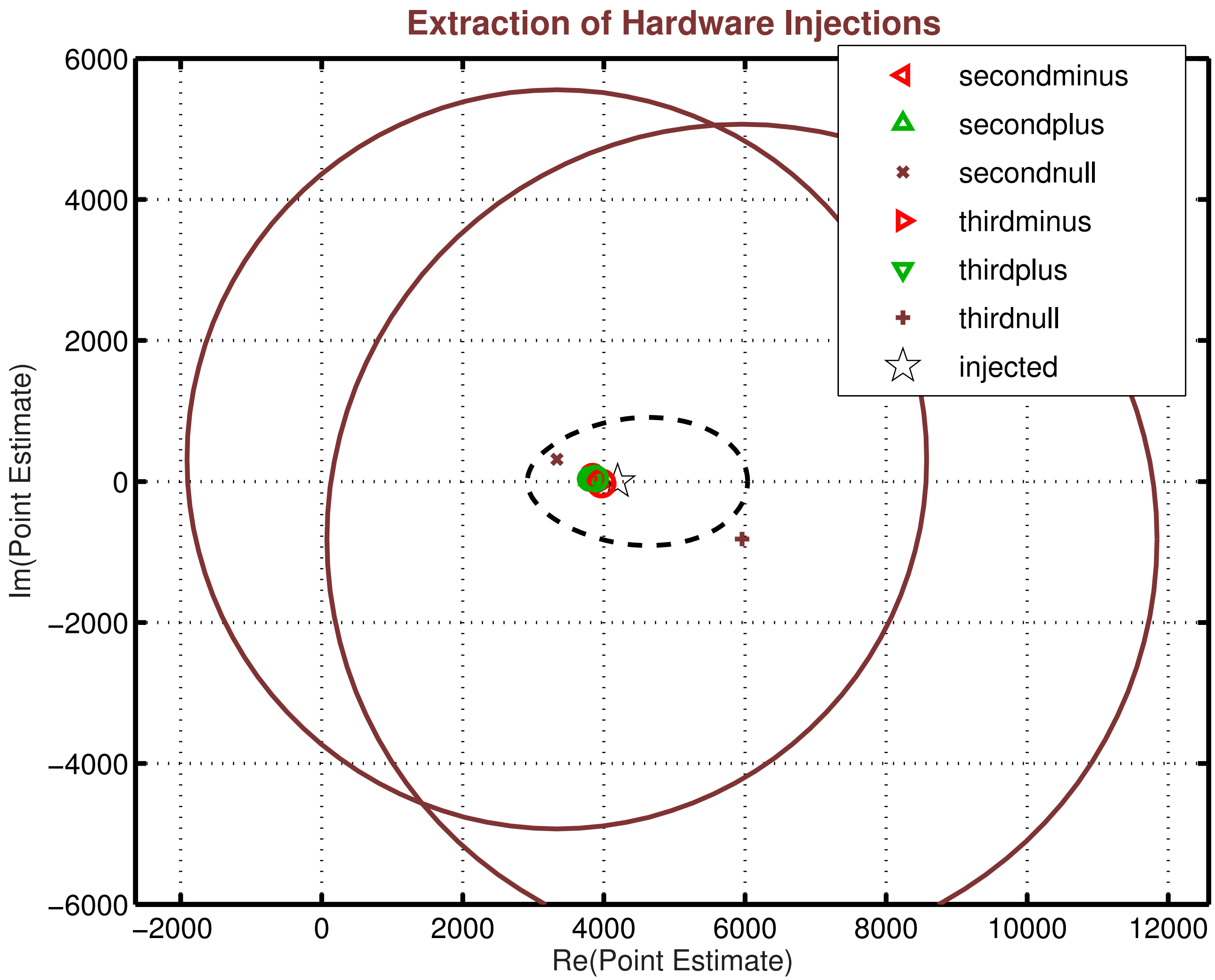
- Learned about timing issues via HW injections:
  Time-shift analysis helped resolve issues w/ALLEGRO timing
  Also revealed sample-and-hold & other digital effects
  in injection system which introduce relative time shift of
  $\frac{1}{2\times 4096\,\text{Hz}} - 18\,\mu\text{s} = 104\,\mu\text{s}$

- Post-processing correction:
  Simulate small timeshift w/freq-dependent phase shift

$$Y(f) \longrightarrow Y(f)\, e^{i2\pi f\tau}$$

  inv FT of CC integrand gives CC values as fcn of time-shift:

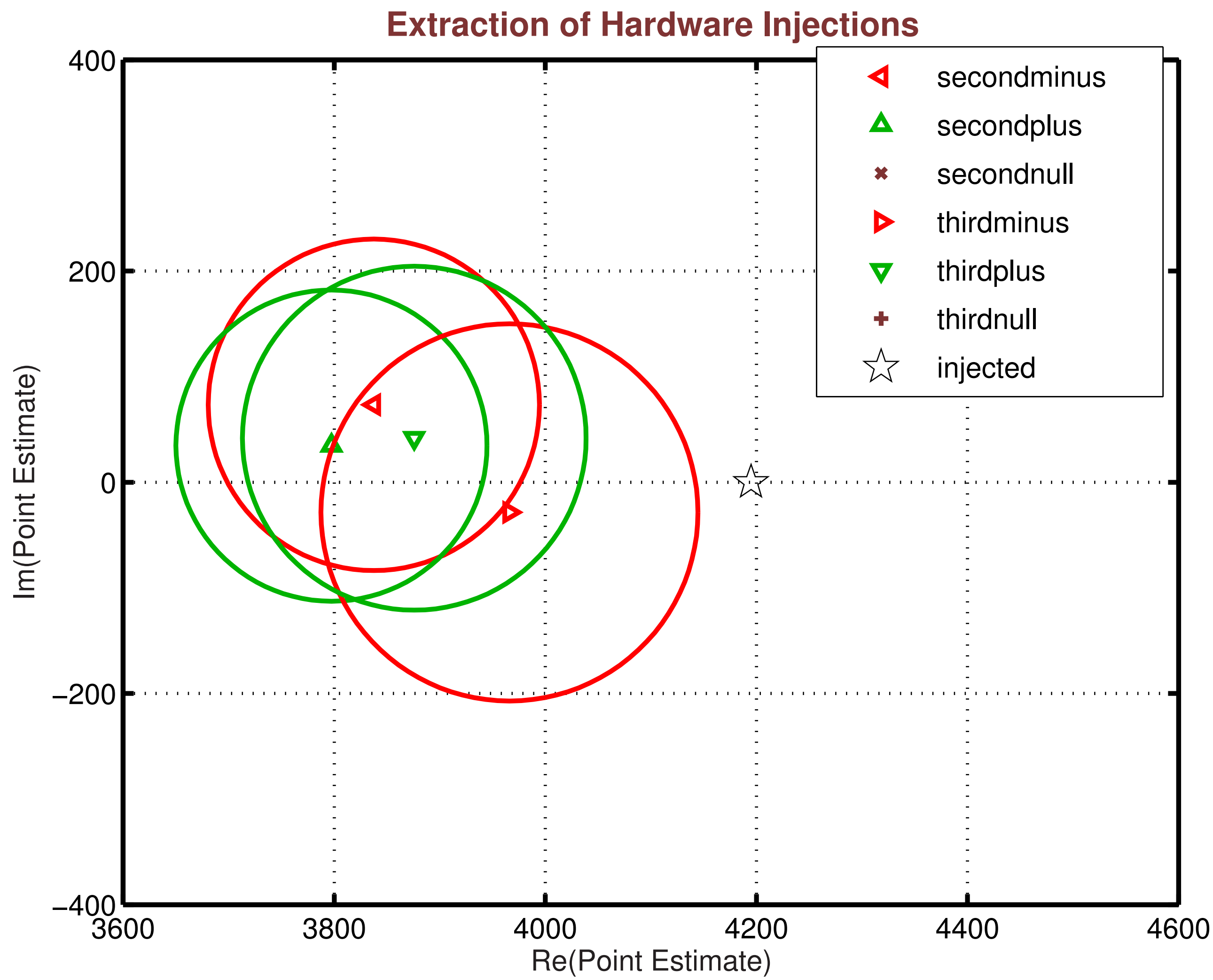
$$Y(\tau) = \int_{f_{\text{min}}}^{f_{\text{max}}} df\, Y(f)\, e^{i2\pi f\tau}$$



Circles: 90% statistical uncertainty (null measurements less sensitive)

90% calib uncertainty "teardrop" around $h_{100}^2\Omega_{\mathrm{gw}}(f) = 4195$

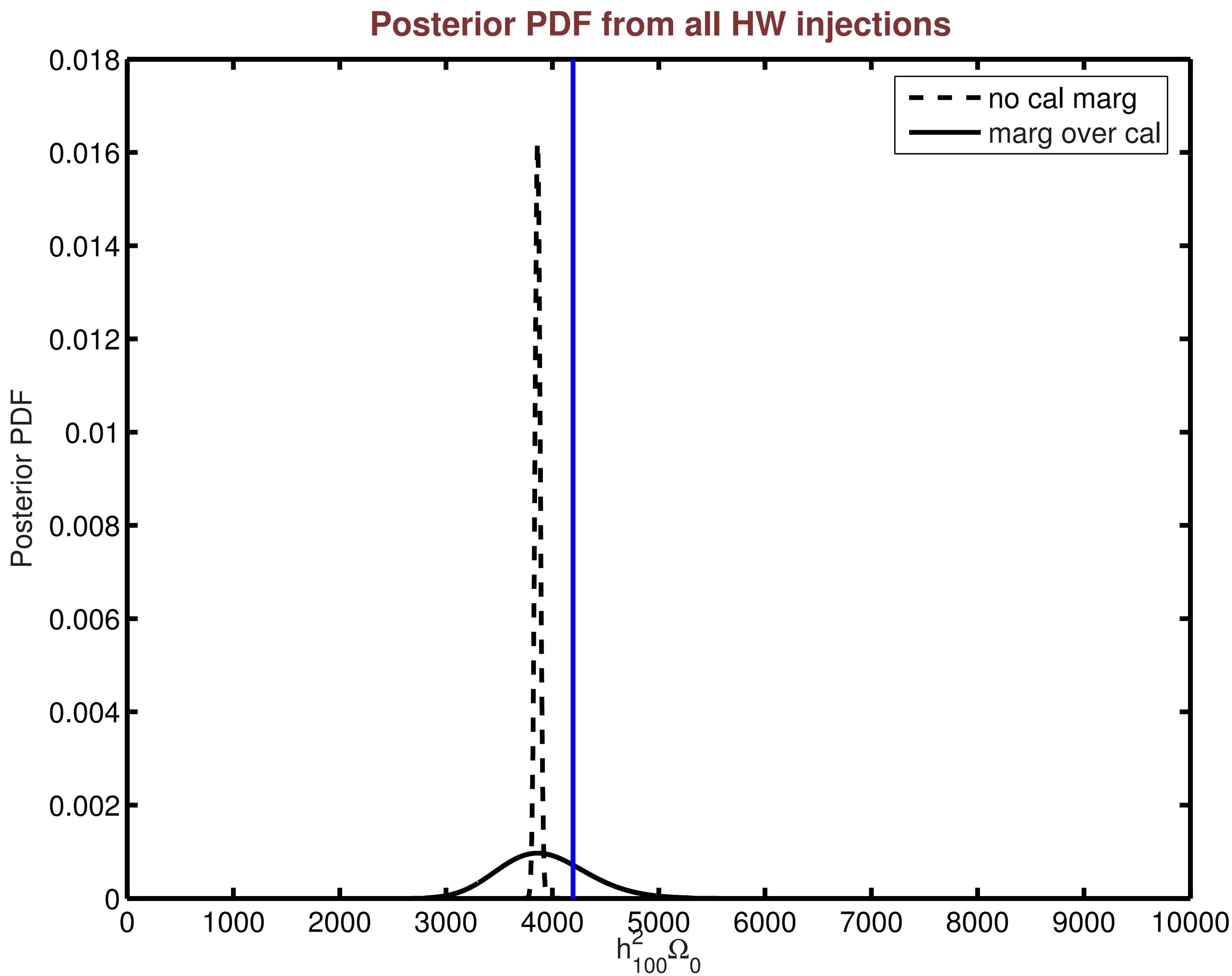
HW injections recovered consistent w/cal uncertainty

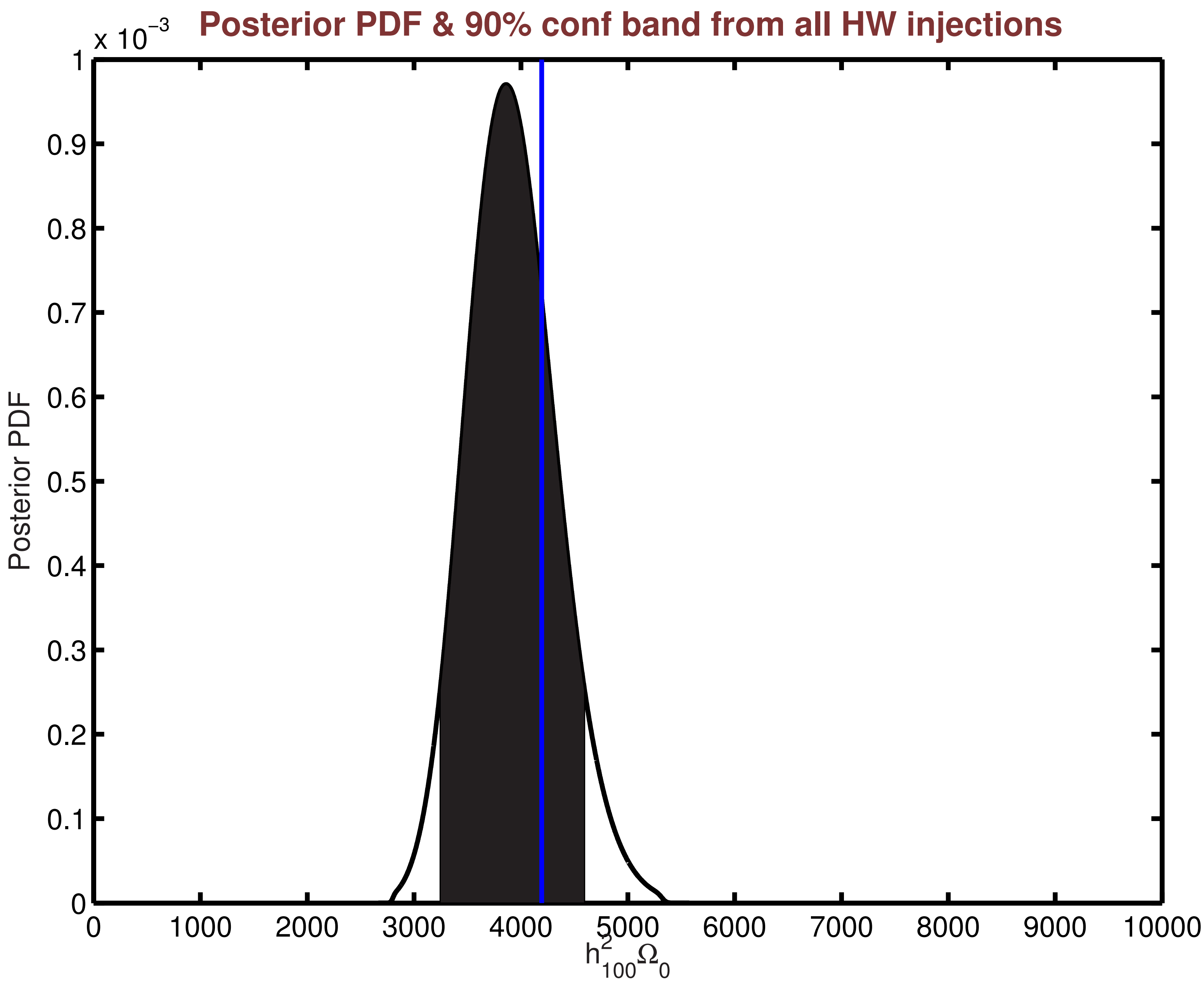


Circles: 90% statistical uncertainty

90% calib uncertainty "teardrop" around $h_{100}^2\Omega_{\rm gw}(f) = 4195$

Systematic offset < cal uncertainty


Posterior PDF from all HW injections
Posterior PDF
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
$h^2_{100}\Omega_0$
no cal marg
marg over cal


Posterior PDF & 90% conf band from all HW injections
x 10^-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Posterior PDF
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
h^2_100 Ω_0

LLO-ALLEGRO: Summary

- First stochastic measurement correlating bar w/ifo data; Probes higher frequency band than LLO-LHO: $\sim 850 - 950\,\text{Hz}$
- Rotation of ALLEGRO modulates stochastic response (data taken in 3 orientations during S4)
- Projected S4 sensitivity from $\sim 370\,\text{hrs}$ of data: $h_{100}^2\Omega_{\text{gw}}(f) \sim 0.3\text{–}1$ or $\sqrt{S_{\text{gw}}(f)} \sim 1\text{–}2 \times 10^{-23}\,\text{Hz}^{-1/2}$
- Analysis extracts long-time, low-amplitude simulated signals (software injections)
- Hardware inj extracted consistent w/calibration uncertainty
- Analysis complete; Under internal review. Stay tuned . . .