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Underground reduction of Gravity Gradient Noise.

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Introduction

In future advanced interferometers Newtonian (Gravity Gradient) Noise will be one of the fundamental limitations for the sensitivity in the low frequency region.

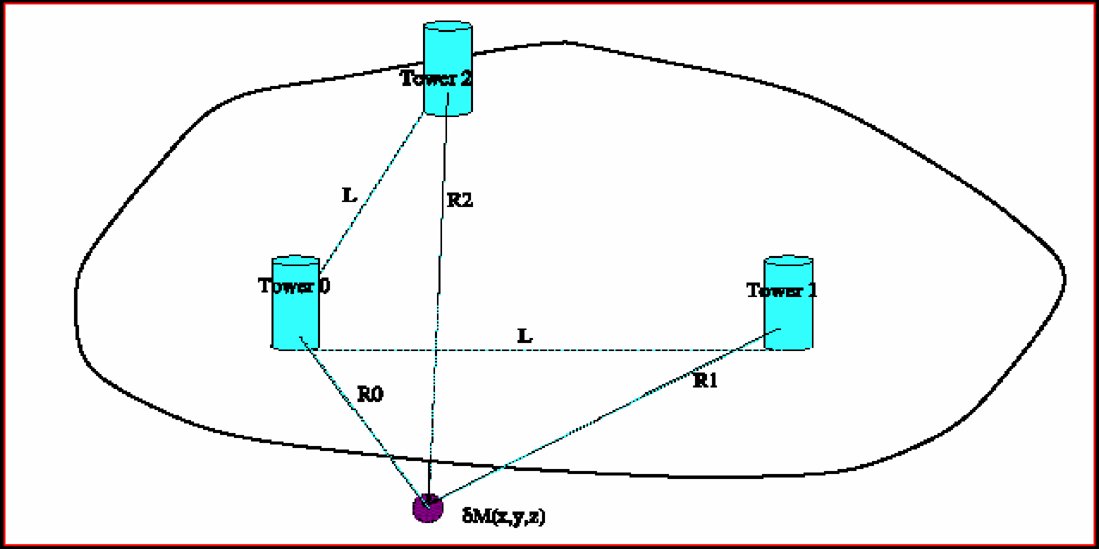
- Can it be estimated?
- What are the most important sources?
- Can it be reduced?
- What we gain going underground?

Outline

- Motivations
  - Seismic GGN
  - Atmospheric GGN
- Going underground
  - Underground GGN estimates
  - GGN reduction inside a cavity
- Other (but related) options
  - Monitoring and subtraction
  - Reference masses
- Conclusions

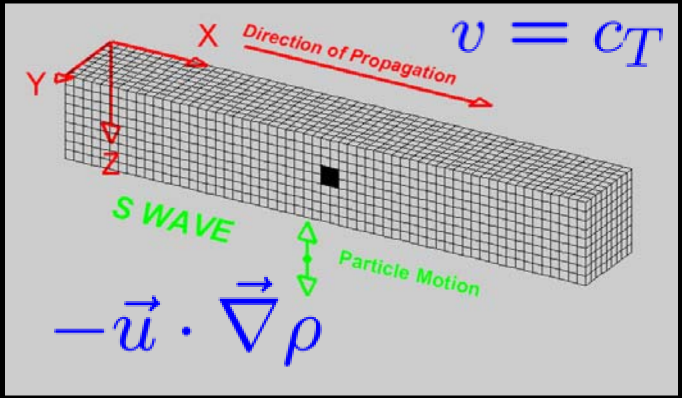
What is Gravity Gradient Noise

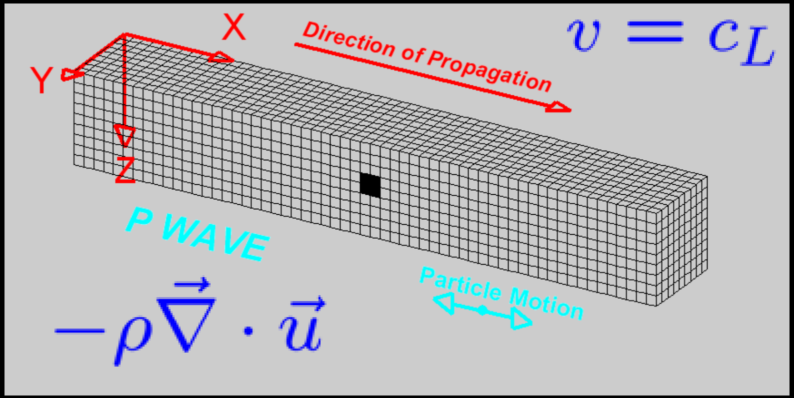
Mass density fluctuations couple directly to the test masses:



Example:
Elastic material

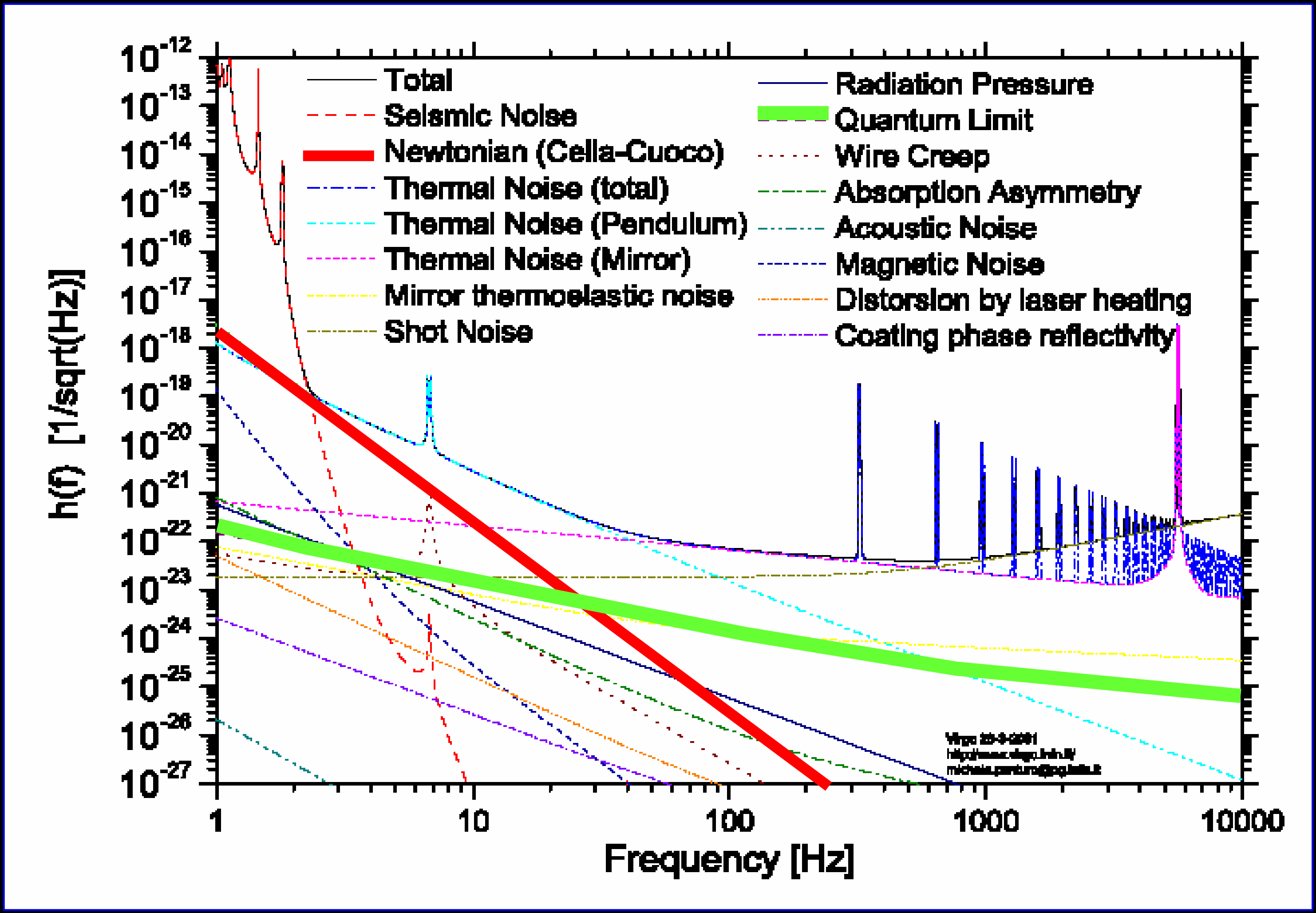
$$\delta\rho(x,t) = -\vec{\nabla}\left(\rho(x,t)\vec{u}(x,t)\right)$$





$$\nu = \left(\frac{c_T}{c_L}\right)^2$$

Seismic GGN



Estimate uses transfer function between seism and GGN

$$seism \sim f^{-2}$$

$$TF \sim f^{-2}$$

$$GGN \sim f^{-4}$$

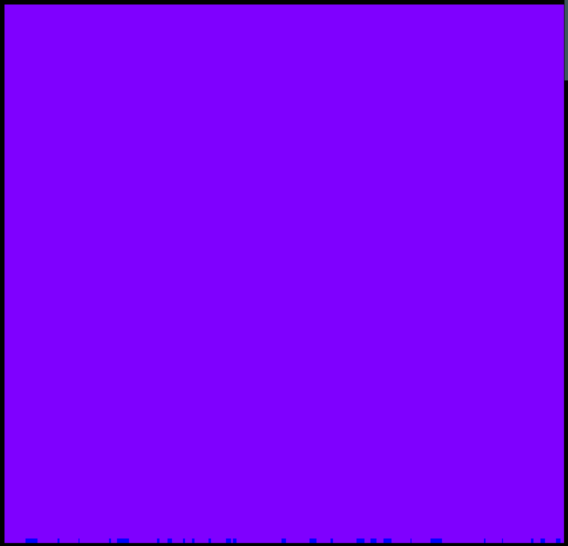
Atmospheric GGN

Rayleigh Bernard Scenarios (G.C., E. Cuoco, P. Tomassini)

$$\partial_t \vec{u} + \left(\vec{u}\cdot\vec{\nabla}\right)\vec{u} = -\frac{1}{\rho}\vec{\nabla}p + \nu\nabla^2\vec{u} + \alpha\vec{g}\theta$$

$$\partial_t \theta + \left(\vec{u}\cdot\vec{\nabla}\right)\theta = \chi\nabla^2\theta \qquad \vec{\nabla}\cdot\vec{u} = 0$$

Different possibilities accordingly with the intensity of thermal gradient.



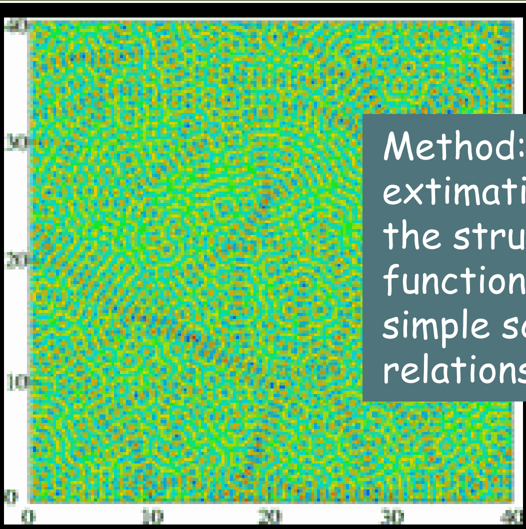
Thermal bubbles

- Nucleation phase (slow)
- Ascension phase

$$T_b(z) = T_b(0) + \gamma_{ad} z$$

$$\partial_{tt} z = g\left(\frac{T_b(z) - T(z)}{T(z)}\right) - 6\pi\nu r z$$

Well developed turbulence

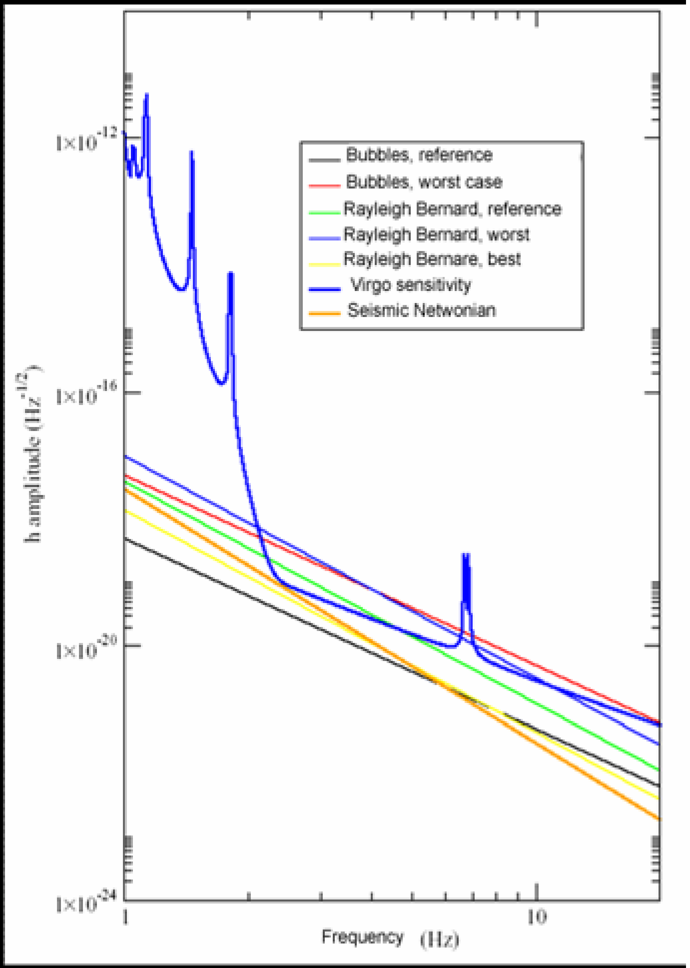


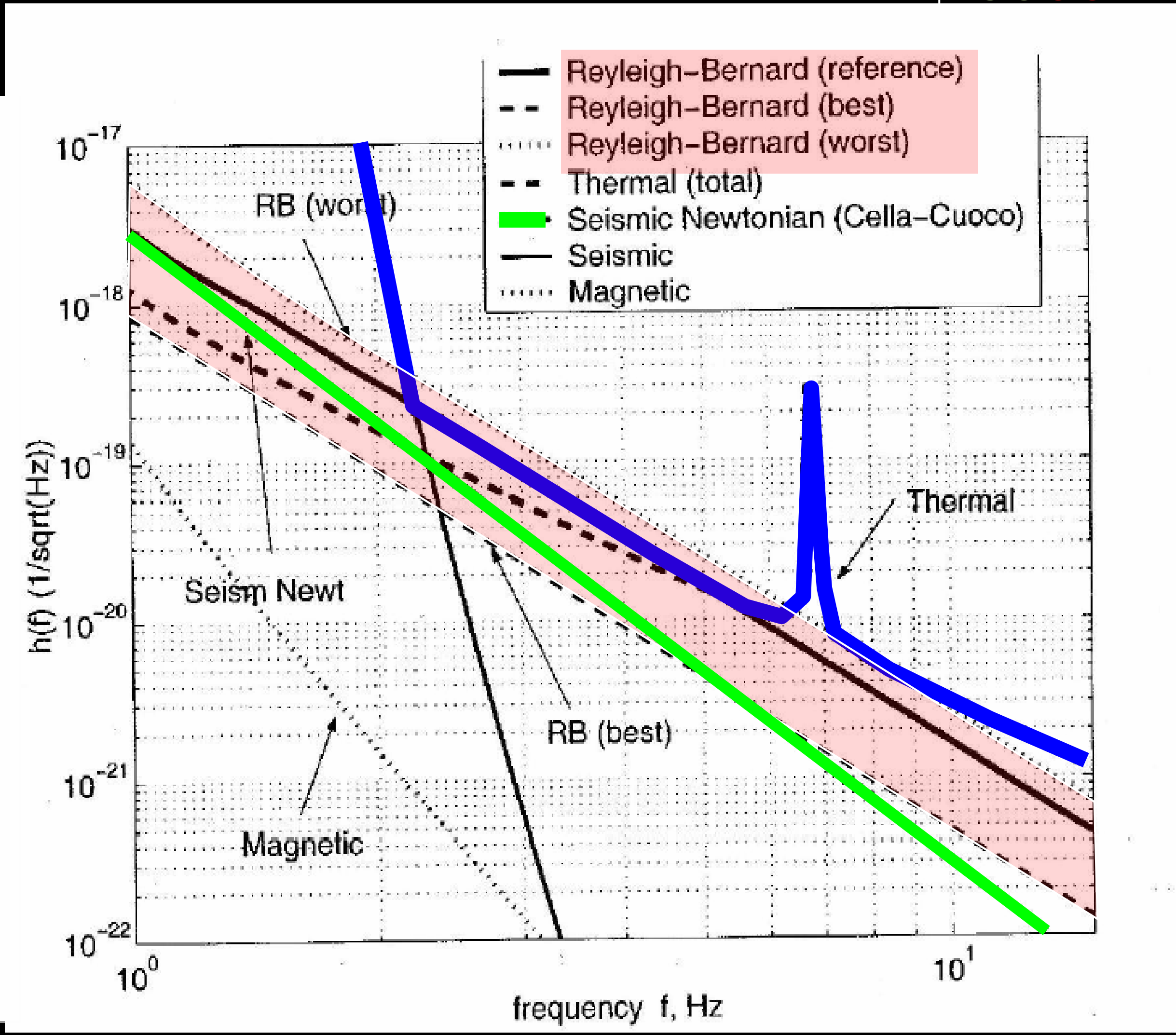
Method: extimation of the structure functions using simple scaling relations.

Other

- Effect of acoustic waves (Saulson) **Negligible**
- Airborne objects, sonic booms, advection, … (Creighton)
- Turbolent generation of acoustic waves (Lighthill process). (C. Cafaro, G. C.) **Negligible**

Atmospheric GGN

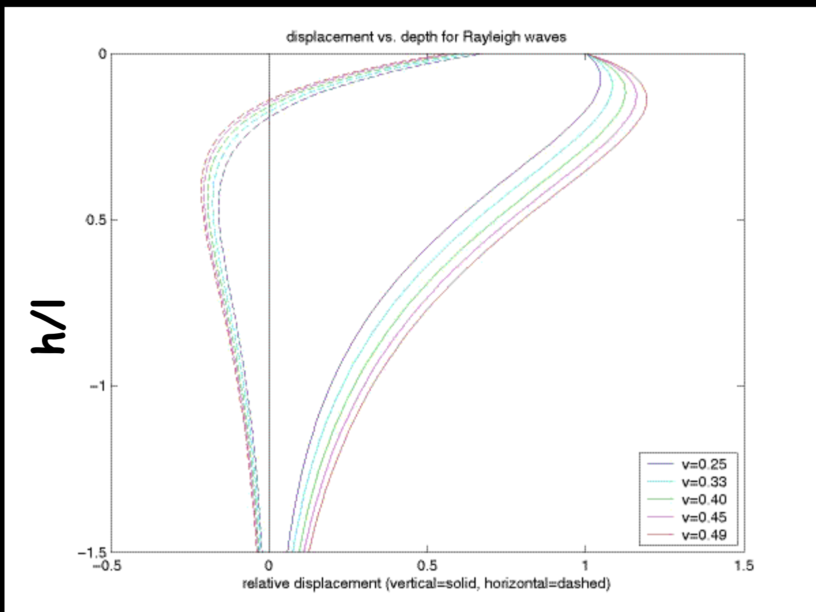




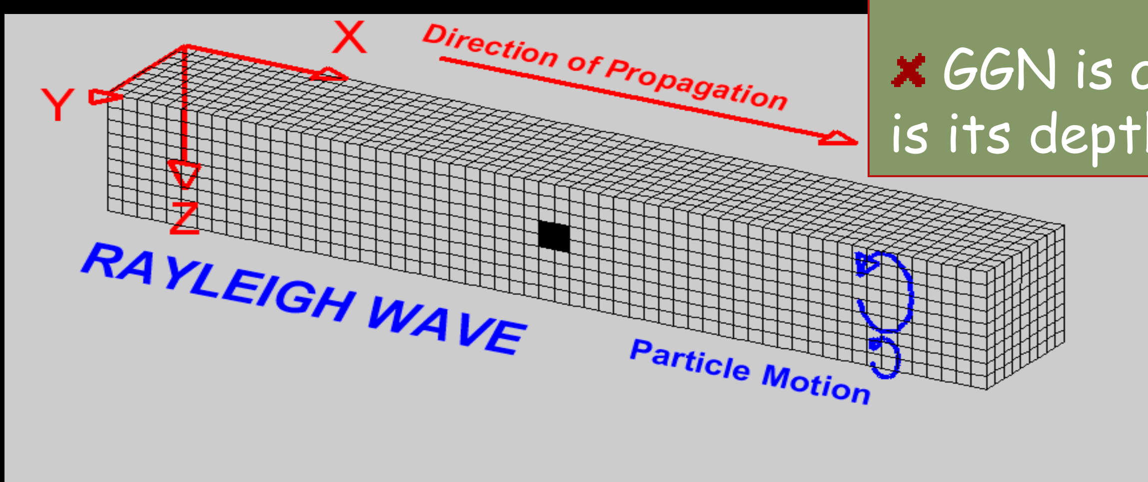
What we can expect by going underground

Going underground: seismic GGN reduction

A simple fact: surface waves die off exponentially with the depth

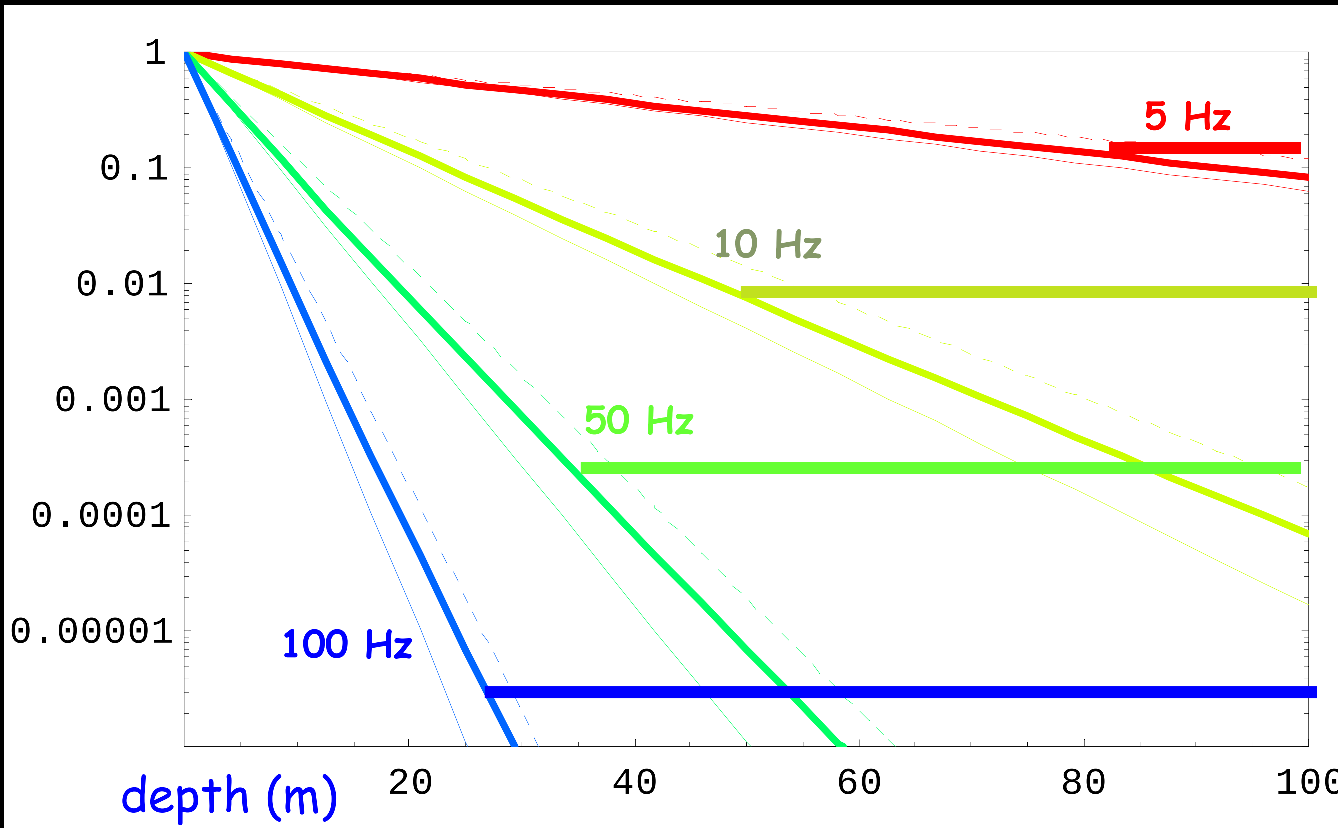


- Surface waves are probably the most important excitations for GGN
  - Surface movement dominate the bulk compression effect
  - Most efficient mechanism to transport energy from “far” sources
  - Significative coupling with “local” sources (human activity)
- GGN is a “long range” effect: what is its depth dependence?



“Smearing effect”

GGN vs. depth



Relative reduction of GGN with depth:

Bulk

Surface

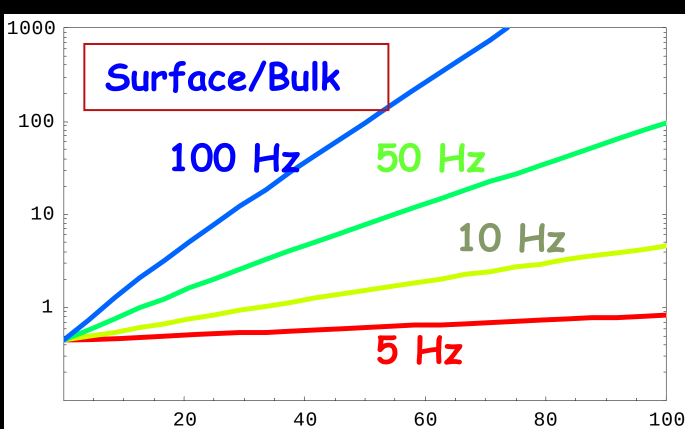
Total

$$bulk = g_1(f,\nu)e^{-h/\ell} + g_2(f,\nu)e^{-h/\ell_L}$$

$$surf = g_3(f,\nu)e^{-h/\ell}$$

$$\ell = \frac{c_T}{2\pi f}\sqrt{x}$$

$$\ell_L = \frac{c_T}{2\pi f}\sqrt{\frac{x}{1-x\nu}}$$



All this is pretty good, but

- Volume waves contributions will not share this fast decay
- Surface fluctuations in the depth?

A rough model for an underground cavity

Spherical cavity in a homogeneous elastic medium:

- Elasticity eq. $-\rho\omega^2\vec{u} = \mu\nabla^2\vec{u} + (\lambda+\mu)\vec{\nabla}(\vec{\nabla}\cdot\vec{u})$
- Free boundaries $\sigma\cdot\hat{n} = 0$
- Mode Classification accordingly with rotation symmetry:

$$\vec{u}_{l,m} = \underbrace{\vec{\nabla}\xi_{l,m}}_{\text{Spheroidal longitudinal}} + \underbrace{\vec{\nabla}\times(\vec{x}\times\vec{\nabla})\eta_{l,m}}_{\text{Spheroidal transverse}} + \underbrace{(\vec{x}\times\vec{\nabla})\tau_{l,m}}_{\text{Toroidal}}$$

For each ω,l,m:

- 2 spheroidal modes (mixed transverse & longitudinal)
- 1 toroidal mode (transverse only)
- Incoming wave scattered to an outgoing one

$$\xi_{l,m} = Y_{lm}(\theta,\phi)\alpha^{(+)}_{l,m}h^{(+)}_l(qr) + \alpha^{(-)}_{l,m}h^{(-)}_l(qr)$$

$$\eta_{l,m} = Y_{lm}(\theta,\phi)\gamma^{(+)}_{l,m}h^{(+)}_l(kr) + \gamma^{(-)}_{l,m}h^{(-)}_l(kr)$$

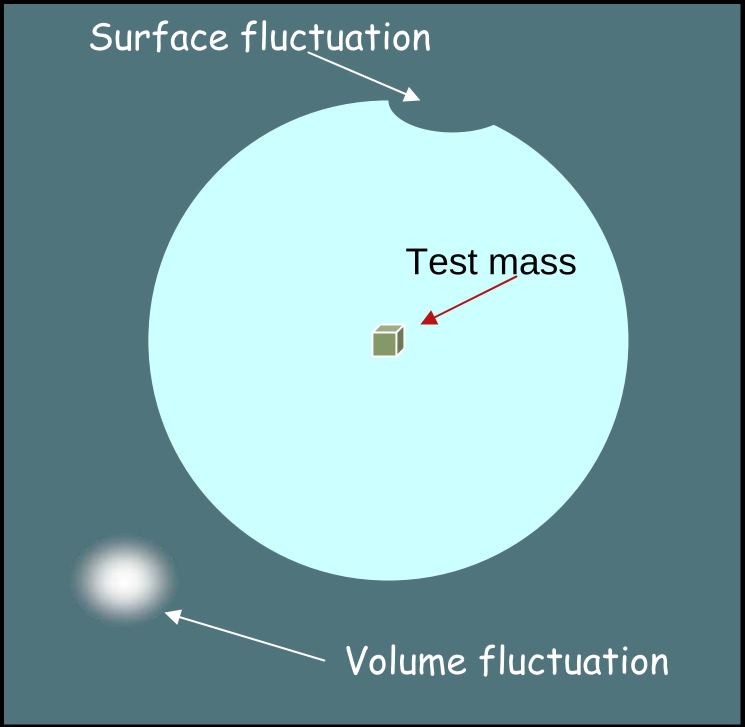
$$\tau_{l,m} = Y_{lm}(\theta,\phi)\beta^{(+)}_{l,m}h^{(+)}_l(kr) + \beta^{(-)}_{l,m}h^{(-)}_l(kr)$$

$$k^2 = \rho\omega^2/\mu \qquad q^2 = \rho\omega^2/(\lambda+2\mu)$$

What is the contribution of each mode to GGN?

A rough model for an underground cavity

$$\vec{u}_{l,m} = \vec{\nabla}\xi_{l,m} + \vec{\nabla}\times(\vec{x}\times\vec{\nabla})\eta_{l,m} + (\vec{x}\times\vec{\nabla})\tau_{l,m}$$



Bulk contribution to GGN:

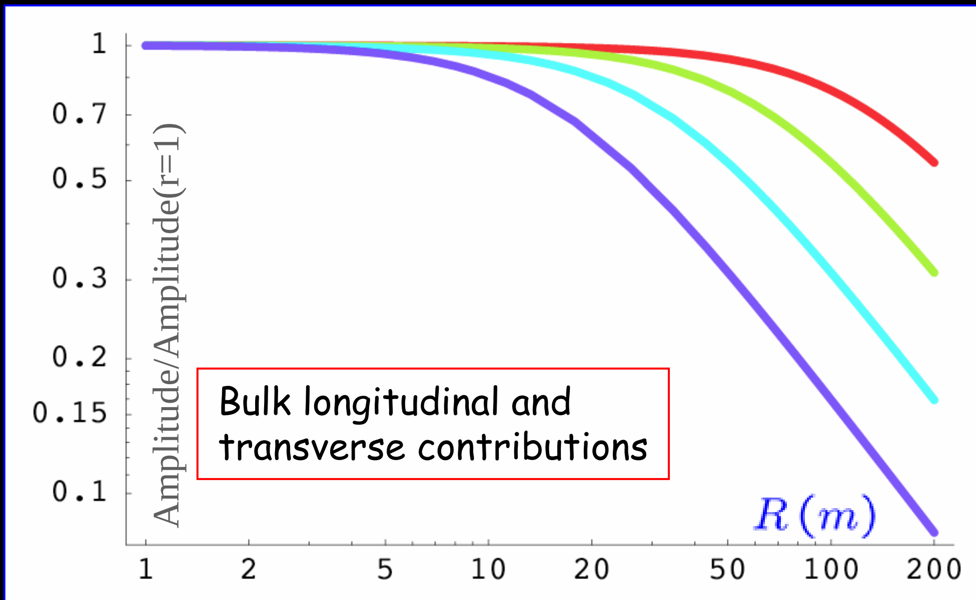
$$\vec{a}_{l,m} = -G\rho\int\vec{\nabla}\cdot\vec{u}_{l,m}\frac{\vec{r}}{r^3}dV$$

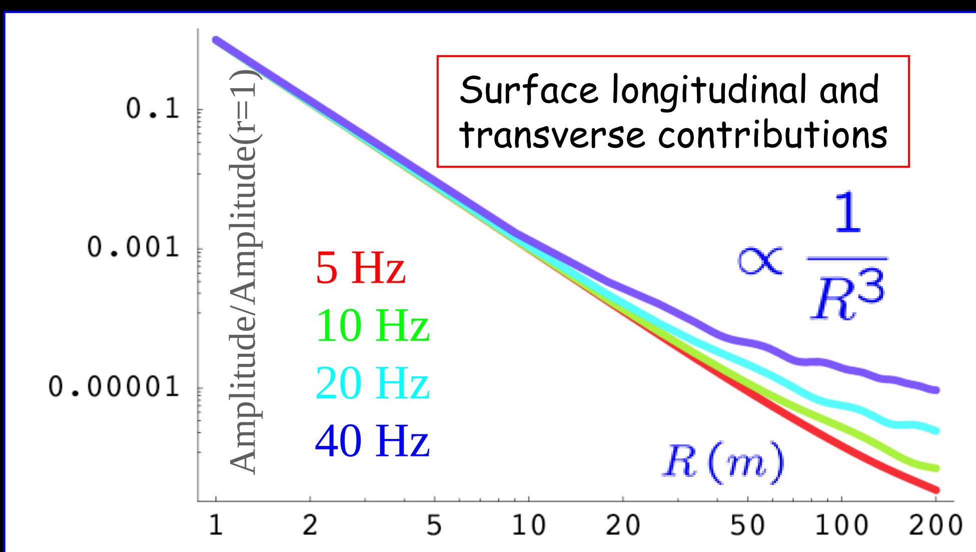
Surface contribution to GGN:

$$\vec{a}_{l,m} = -G\rho\int\vec{R}\cdot\vec{u}_{l,m}\frac{\vec{R}}{R^2}d\Omega$$

- Only "dipole" contribution $\sim Y_{1m}(\theta,\phi)$ to bulk GGN (cavity displacements)
- Both transverse & longitudinal contributions to surface GGN
- Toroidal modes: transverse, no surface motion, no Newtonian

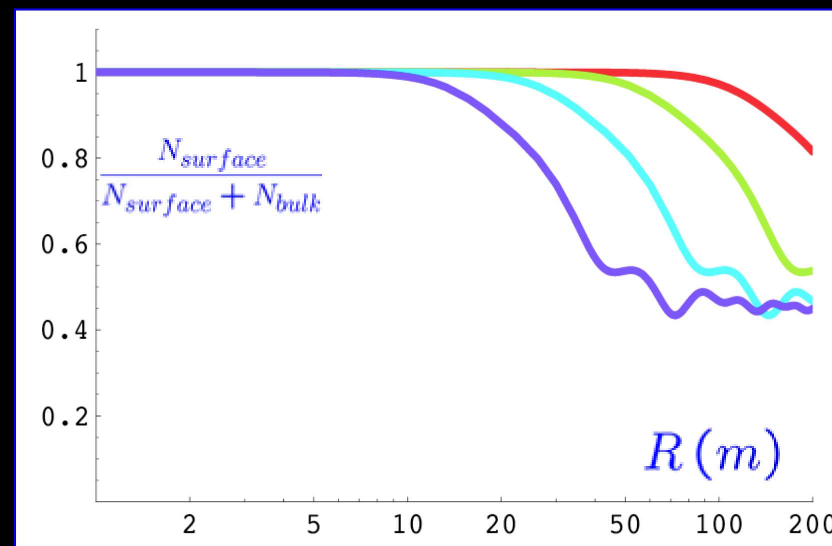
R dependence of GGN inside the cavity





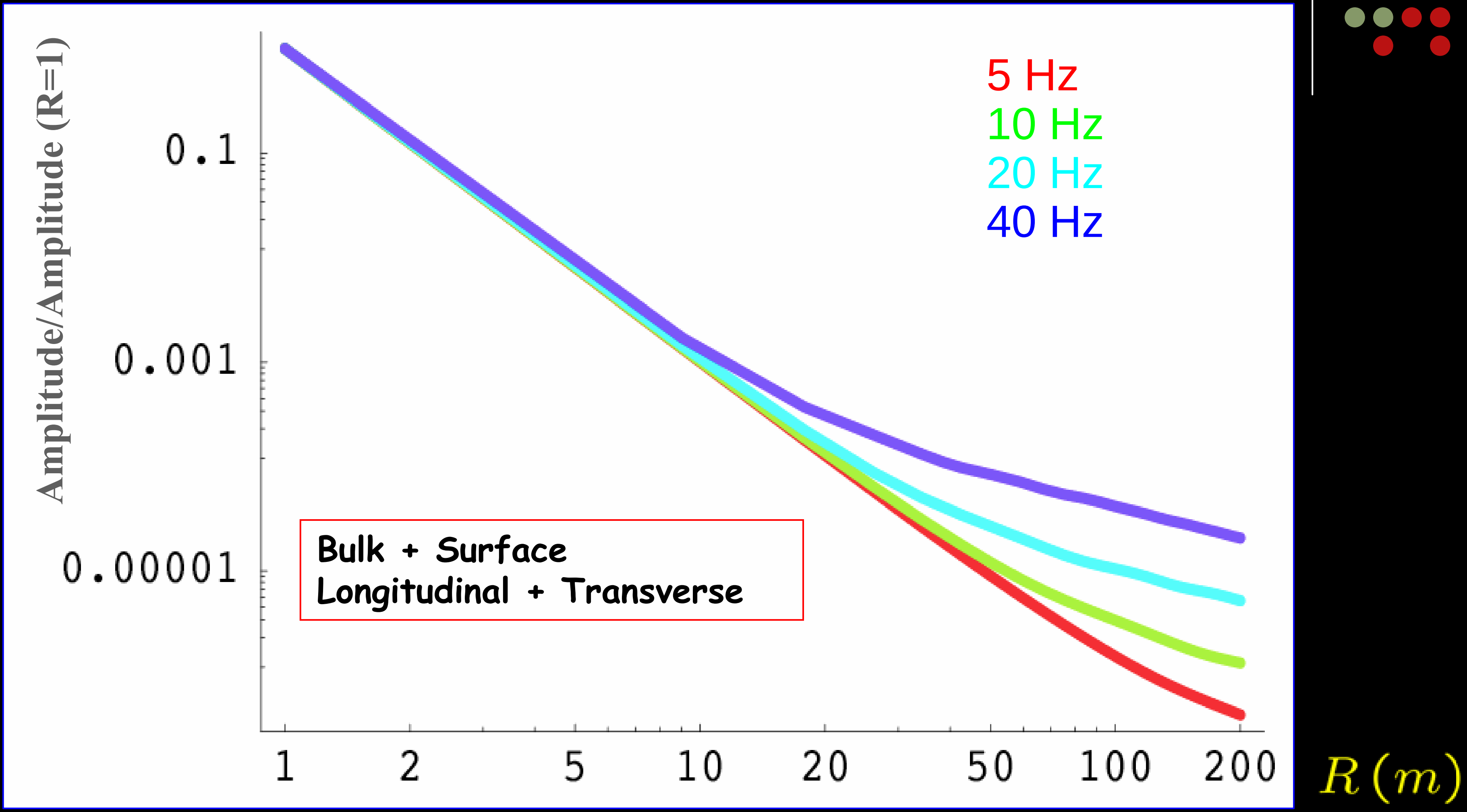
Method:

- Normalization: fixed incoming flux from infinity of elastic energy
- Statistical sum over modes
- Assumed absence of correlations between modes (weak dependence)



Surface contributions always dominant in the relevant frequency range

R dependence: final result



Good reduction with a reasonable cavity's size.

Transfer function

There is a relation between GGN in the cavity and seismic motion measured on the surface?

$$\hat{n}\cdot\vec{u}_{l,m} = \frac{\partial}{\partial r}\xi_{l,m} + \frac{l(l+1)}{R}\eta_{l,m}$$

Motion normal to the surface (as an example)

$$< |\hat{n}\cdot\vec{u}|^2 > \sim \sum_{\ell} g_\ell^2 \frac{2\ell+1}{4\pi}$$

Symmetries are not constraining enough……

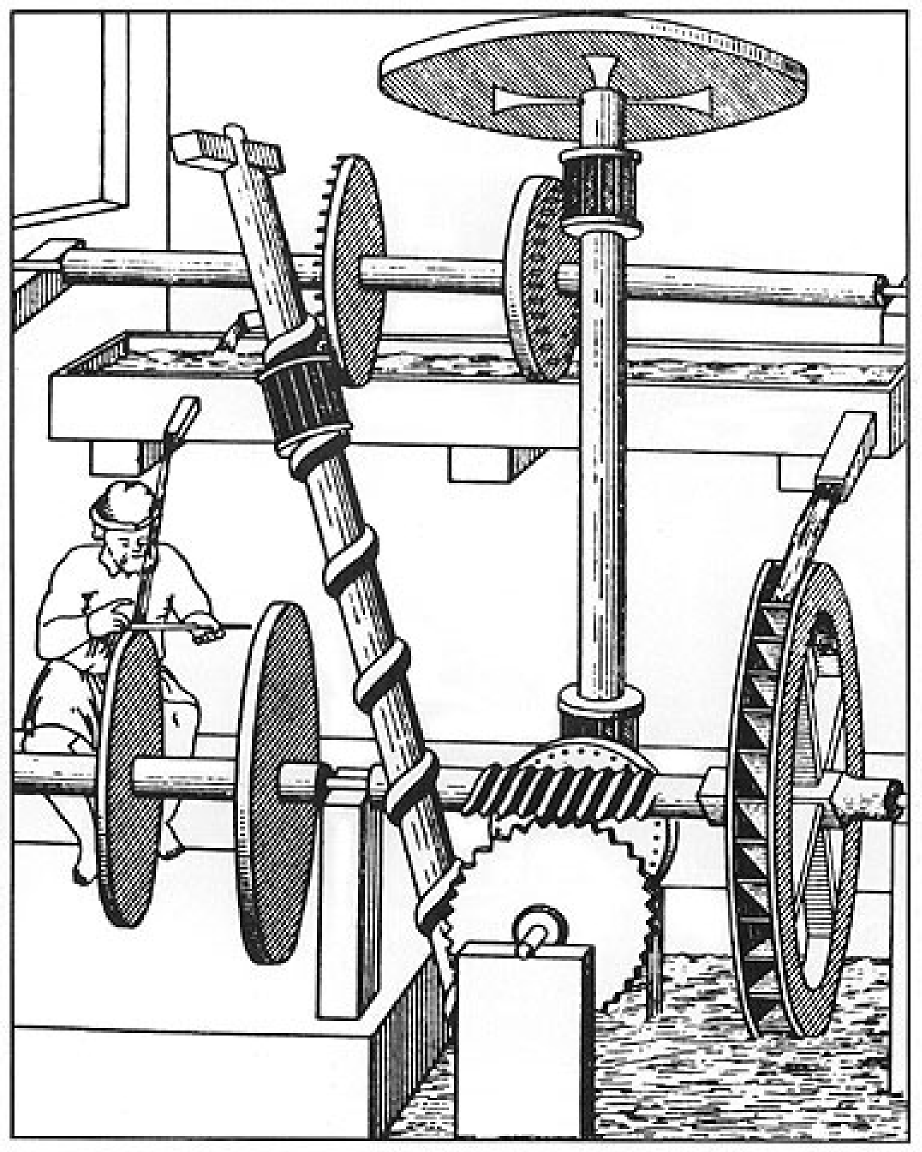
- Seismic motion get contributions from all g_ℓ
- GGN is controlled by $\ell = 1$ modes only

In other words: measuring the dipole mode is a difficult issue, without additional informations about the importance of each g_ℓ .

In principle: measure the correlations

$$< (\hat{n}\cdot\vec{u})\,(\hat{n}\cdot\vec{u}') > \sim \sum_{\ell} g_\ell^2 \frac{2\ell+1}{4\pi} P_\ell(\cos\gamma)$$

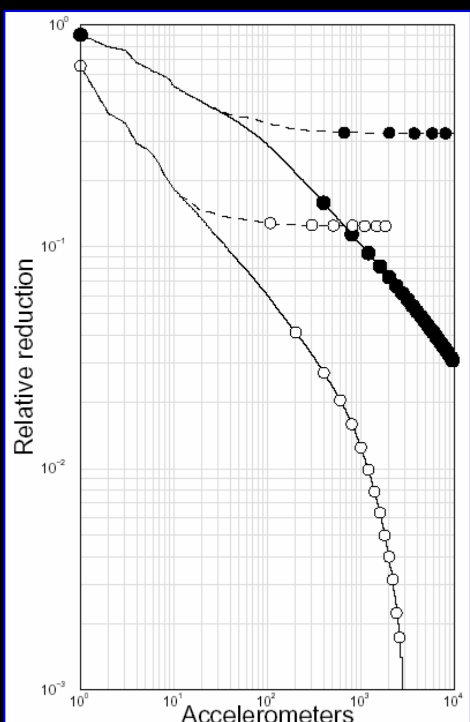
Other (related) options

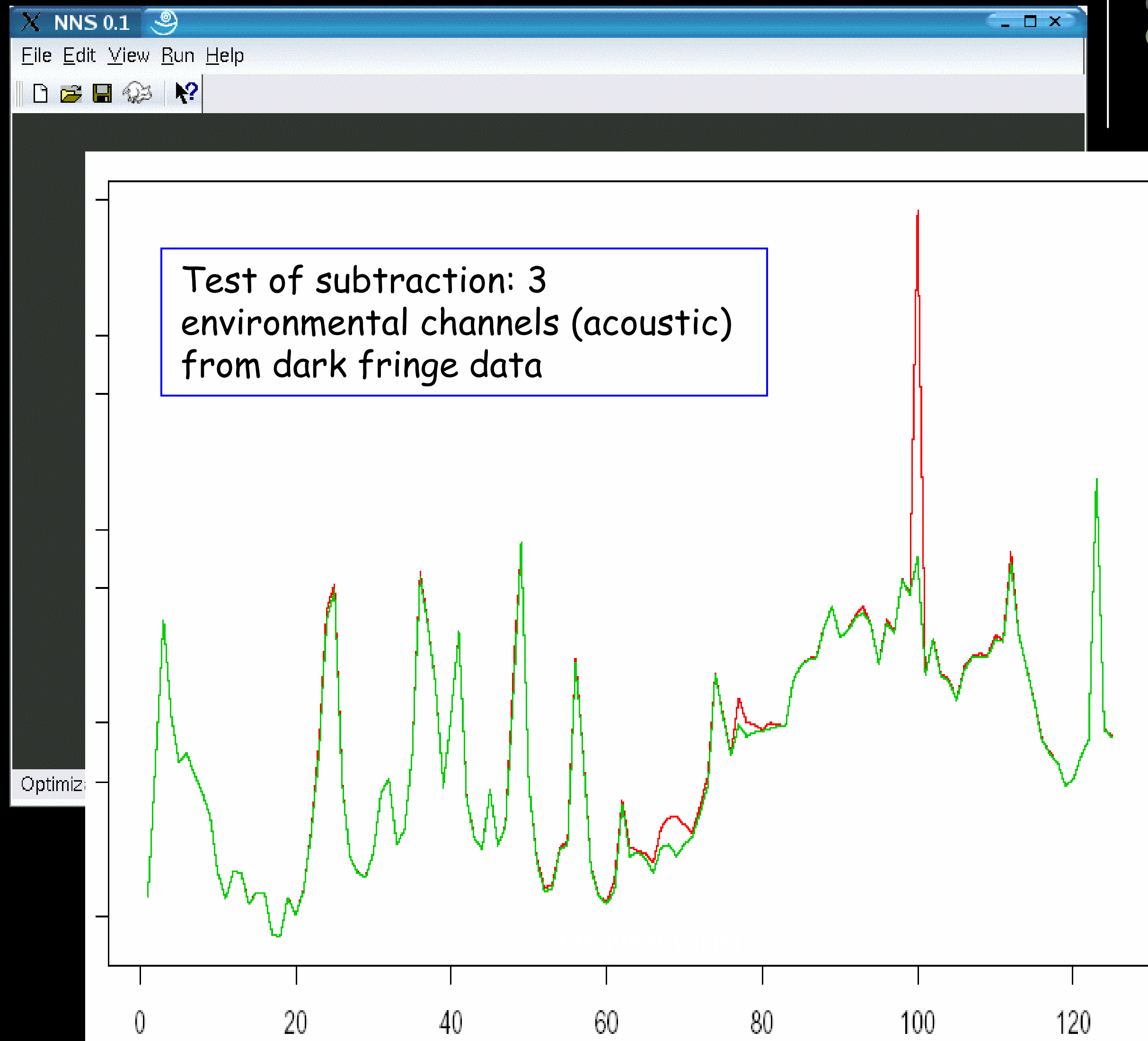


GGN subtraction

Example: a set of 10 accelerometers with random initial position....

…and after the optimization





Noise PSD (log arbitrary units)

Subtraction in the cavity?

We can apply the subtraction method to seismic measurements inside the cavity.

$$X_{sub}(\omega) = X(\omega) - \sum_{i,j} \langle X(\omega)\hat{n}_i \cdot \vec{u}(x_i,\omega)^\star \rangle \left[C^{-1}(\omega)\right]_{ij} \hat{n}_j \cdot \vec{u}(x_j,\omega)$$

Subtracted signal

$$\eta = \frac{\sum_{i,j} \langle X(\omega)^\star \hat{n}_i \cdot \vec{u}(x_i,\omega) \rangle \left[C^{-1}(\omega)\right]_{ij} \langle \hat{n}_i \cdot \vec{u}(x_i,\omega)^\star X(\omega) \rangle}{\langle X(\omega)^\star X(\omega) \rangle}$$

Subtraction efficiency

- The method can be applied
- We can't anticipate its efficiency
- Performances and number of sensors will depend on the number of relevant modes

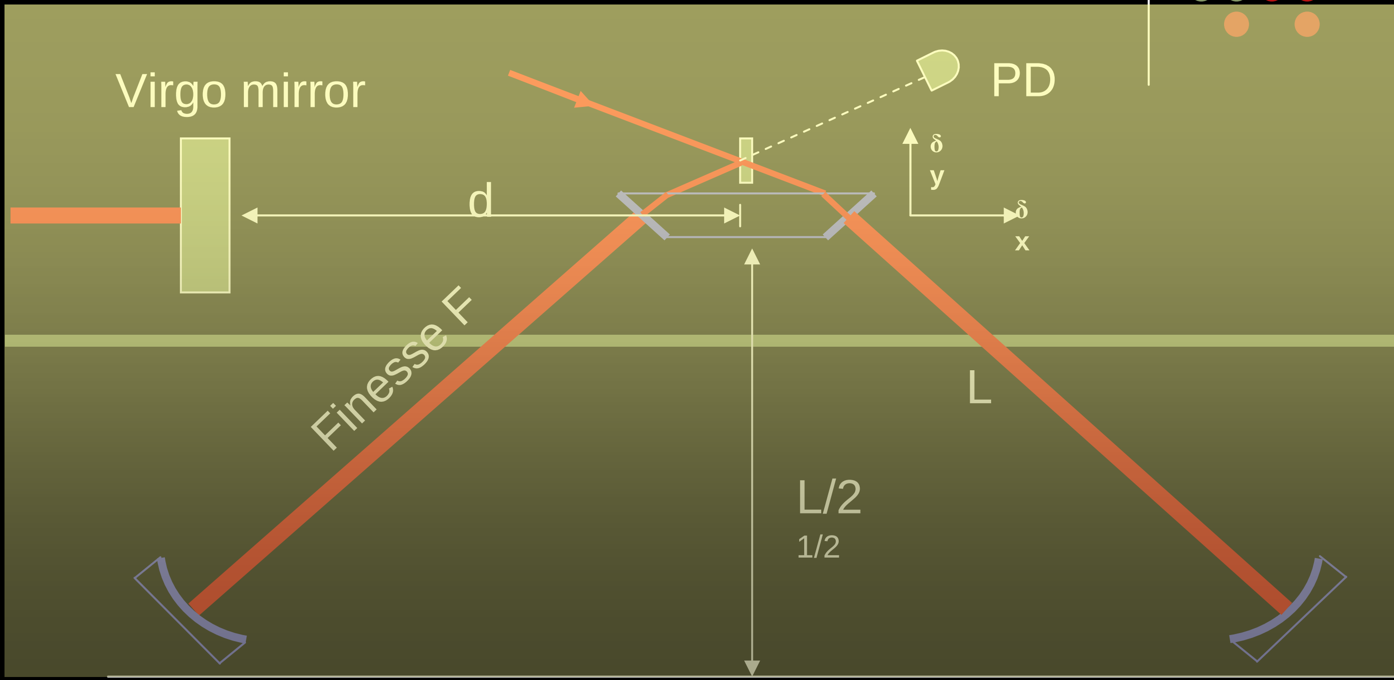
Measuring horizontal GGN

Preliminary idea:

1. Measure the GGN using as a reference "quiet" masses.
2. Subtract

Features:

1. Decoupled from vertical GGN
2. L large compared with
3. L small compared with interferometer arms



Problems:

1. "Quiet" masses must be dominated by GGN
2. "Quiet masses coupled to vertical seism (more refined schemes can cure this problem)

Conclusions:

- The underground option seems promising
  - Seismic surface waves contributions to GGN exponentially damped
  - Atmospheric contributions should be damped also exponentially
- A cavity can be used to further reduce GGN

Problems:

- Localized seismic waves on the gallery
  - Small masses involved
  - Monitorable
- Acoustic (pressure waves) resonances
  - Could be reduced
  - Monitorable
- Volume seismic waves

Measurements in realistic scenarios are mandatory!

Thank you for your attention!