Macroscopic Quantum Measurement in AdLIGO

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presenting for

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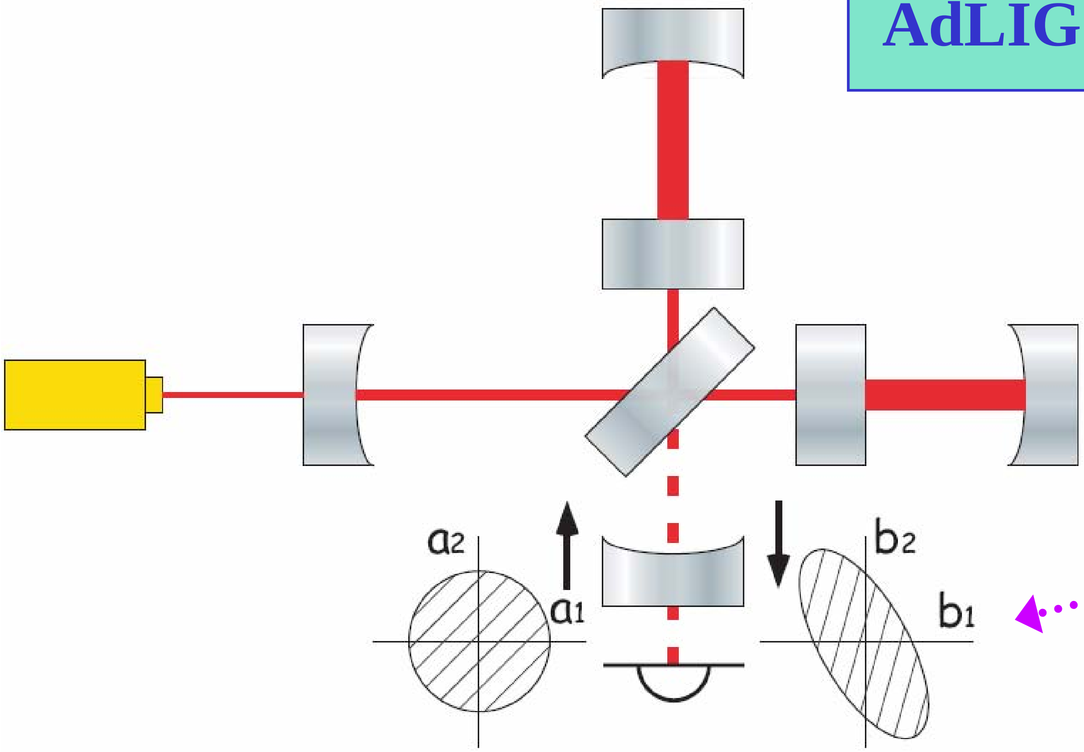
Quantum mechanics of 40kg masses

AdLIGO=high-power detuned RSE



Masses are locked by the optical spring

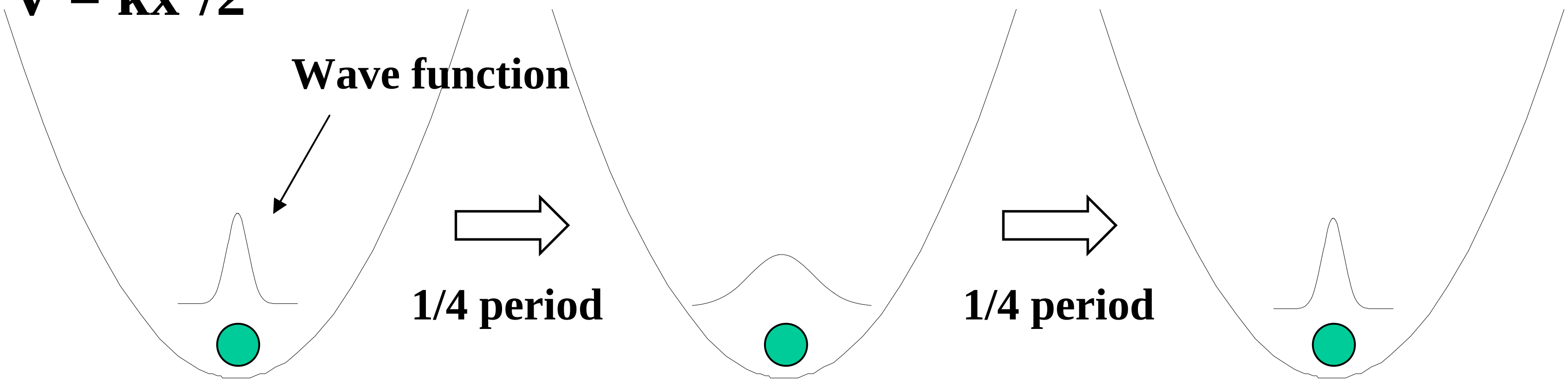
Light fields are ponderomotively squeezed

Can we see quantum behavior of test masses?

Test mass in a potential well

QM tells the wave function of the masses in a squeezed state should decay and recover in one period.

$V = kx^2/2$



To see this quantum behavior, we should

1. **Prepare a squeezed state of the test mass,**
2. **Wait for a while to let it grow,**
3. **And detect the distribution to see the result**

Points in each stage

1. **Preparation stage**

   Mass should better be close to a pure quantum state.
   Squeezing state should be prepared.

2. **Waiting stage**

   Environment may destroy a quantum state.
   (=decoherence)

3. **Detection stage**

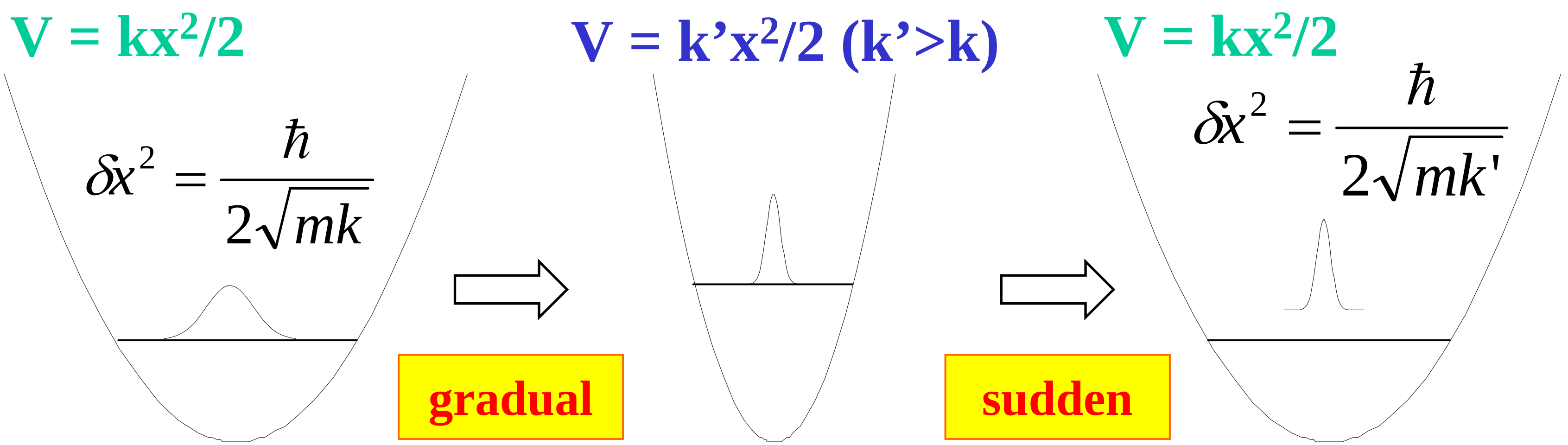
   We'll measure the mass position one by one and see the distribution. This will be easy.

How to prepare a squeezed state

Step 1: Initial state should better be close to a pure state.

→ We'll need feedback control.

Step 2: In AdLIGO, the center of mass position in differential mode (L-) is in a potential well of optical spring, then…



Changing the position of the SR mirror quickly will alter *k* and leave the mass in a squeezed state!

What is a pure state?

Let's say we have only states A and B: $|\psi\rangle = |\phi_A\rangle + |\phi_B\rangle$

Quantum objects ⟶ superposition of A and B

Classical objects ⟶ either A or B

The difference appears in the density matrix: $\rho = |\psi\rangle\langle\psi|$

Pure state ⟶ coefficient on $|\phi_A\rangle\langle\phi_B|$ doesn't decrease

Mixed state ⟶ coefficient on $|\phi_A\rangle\langle\phi_B|$ decreases

Decoherence

Non-diagonal term in the density matrix

$$\rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$ **pure state**

$$\longrightarrow \quad \rho = \begin{pmatrix} 1/2 & <1/2 \\ <1/2 & 1/2 \end{pmatrix}$$ **mixed state**

$$\longrightarrow \quad \rho = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$ **classical**

Decoherence

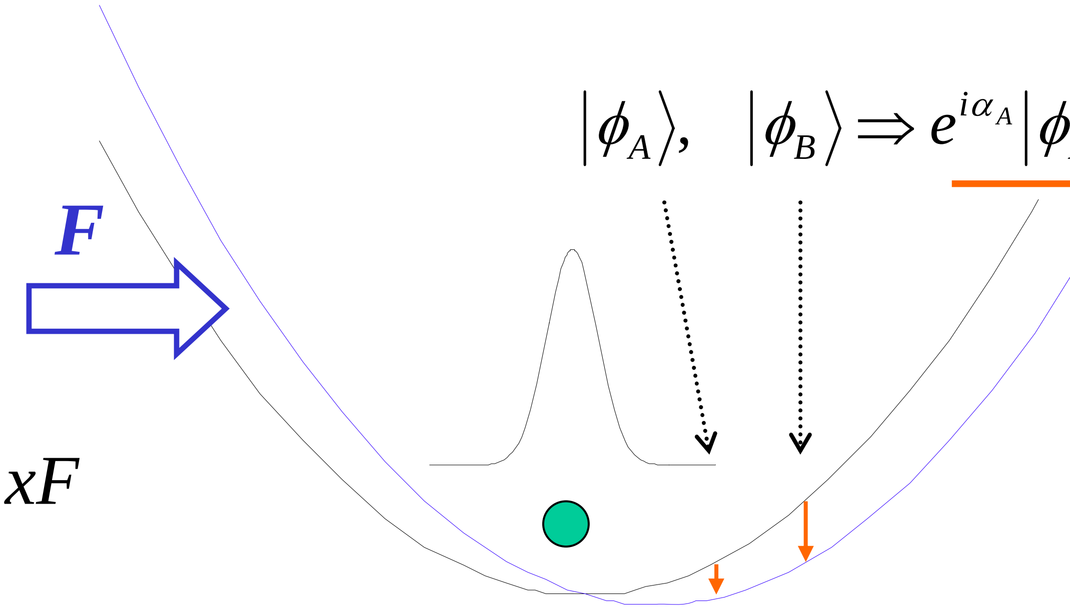
**ex.) thermal decoherence,
control-noise decoherence,
gravity decoherence, .. etc.**

Thermal decoherence

A hit of classical force doesn't make a decoherence

$$H_{\text{int}} = -xF$$

$$H = \frac{p^2}{2m} + kx^2 - xF$$

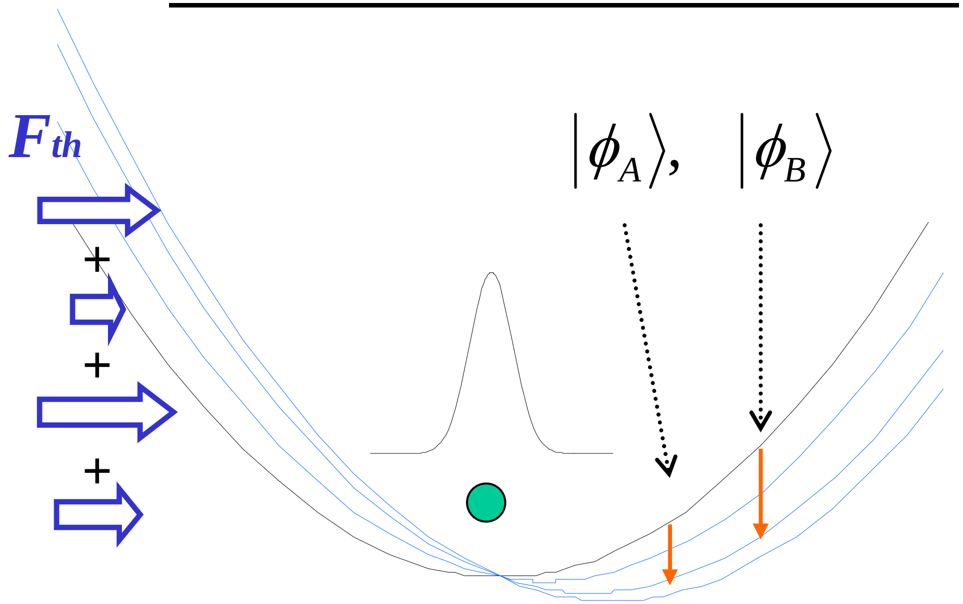


$$\left|\phi_A\right\rangle, \quad \left|\phi_B\right\rangle \Rightarrow e^{i\alpha_A}\left|\phi_A\right\rangle, e^{i\alpha_B}\left|\phi_B\right\rangle$$

The phase on $|\phi_A\rangle\langle\phi_B|$ changes by $\alpha=\alpha_A-\alpha_B$.
It's not decreasing ⟶ no decoherence

But the situation differs when F is a random force

Thermal decoherence



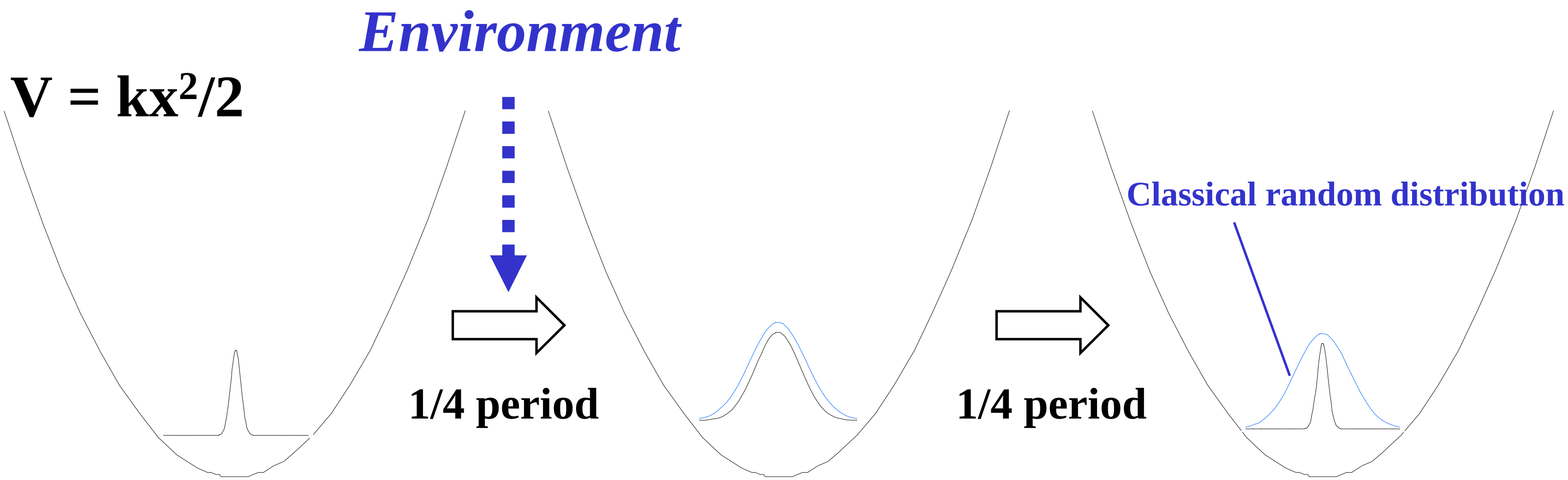
Random force makes a random phase shift.

$$|\phi_A\rangle\langle\phi_B| \rightarrow \left.\begin{matrix} e^{i\alpha_1}|\phi_A\rangle\langle\phi_B| \\ e^{i\alpha_2}|\phi_A\rangle\langle\phi_B| \\ e^{i\alpha_3}|\phi_A\rangle\langle\phi_B| \\ \vdots \end{matrix}\right\}$$

add them all $\longrightarrow$ **0**

decoherence!!

What does decoherence actually mean?



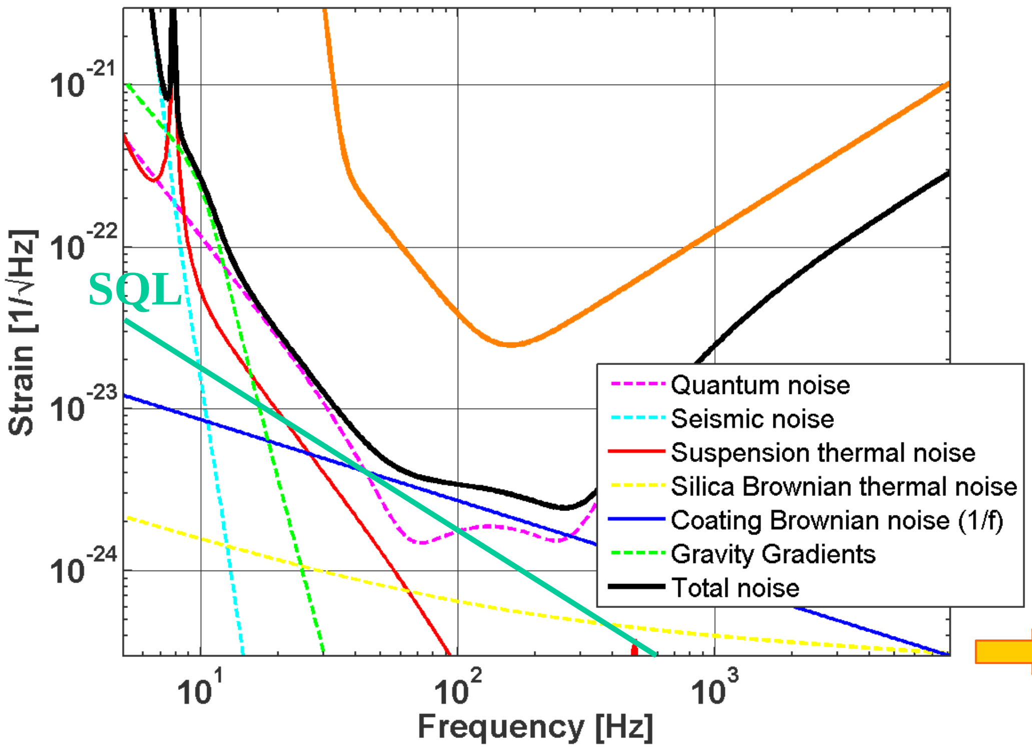
- Quantum wave function recovers to the initial squeezed state but classical noise doesn't recover.
- We cannot see the quantum behavior if decoherence is big.
- Decoherence effect accumulates in each one period.
(random walk)

Estimation of environmental decoherence

Assuming a Gaussian state,

$$\langle x^2 \rangle = \left(e^{-2q} + \frac{N\pi}{2} \frac{S_{cl}}{S_{SQL}} \right) \frac{\hbar}{2m\omega}$$

***N*: number of periods**
***q*: squeezed factor**
***S*$_{cl}$: spectrum of classical noise**



In the case of AdLIGO,

Optical spring frequency would be able to move between 50-150Hz.

→ e^{2q}**=sqrt[3]**

Classical noise could be about the same as the SQL at 100Hz.

→ $S_{cl}/S_{SQL}=1$

N=0 N=1/2 N=1

$$\left(\quad \right)^{\frac{1}{2}} = 0.76 \to 1.59 \to 1.47 \to \ldots$$

One more thing we should be careful about

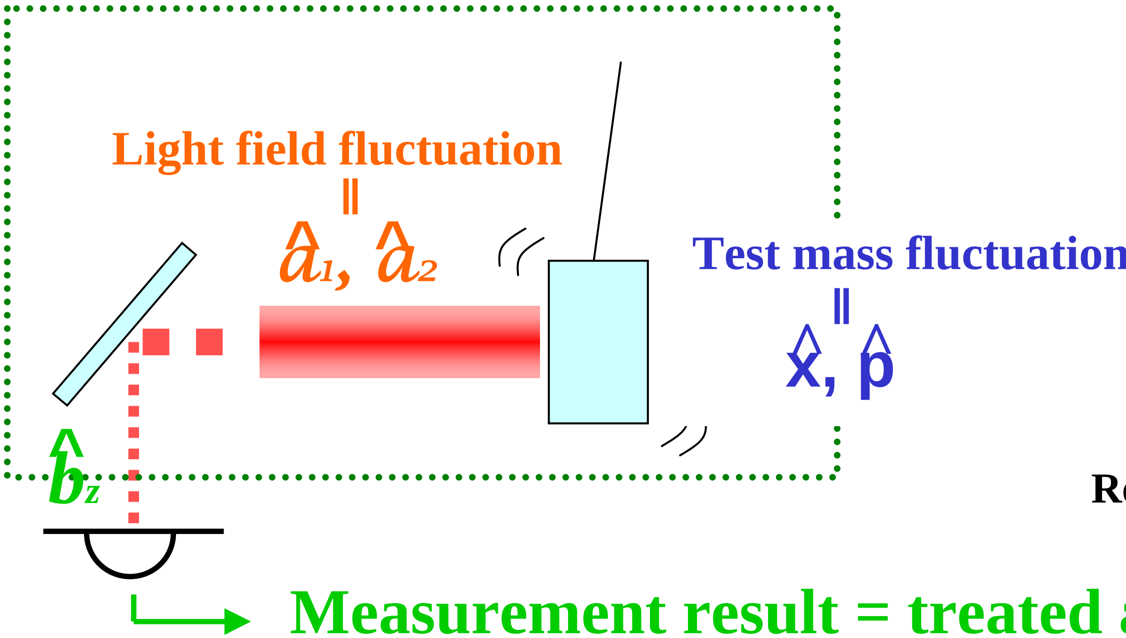
We will need feedback control to

- **prepare the initial vacuum state**
- **suppress the instability of the spring**

Here, one concern might be if the quantum state disappears after the photo-detection, but…

Measurement itself doesn't make state reduction.

Measurement and control



$$[x(t), x(t')] \neq 0$$

but

$$[b_\zeta(t), b_\zeta(t')] = 0$$

Ref: V.Braginsky et al, PRD 2003

Measurement result = treated as classical

but,

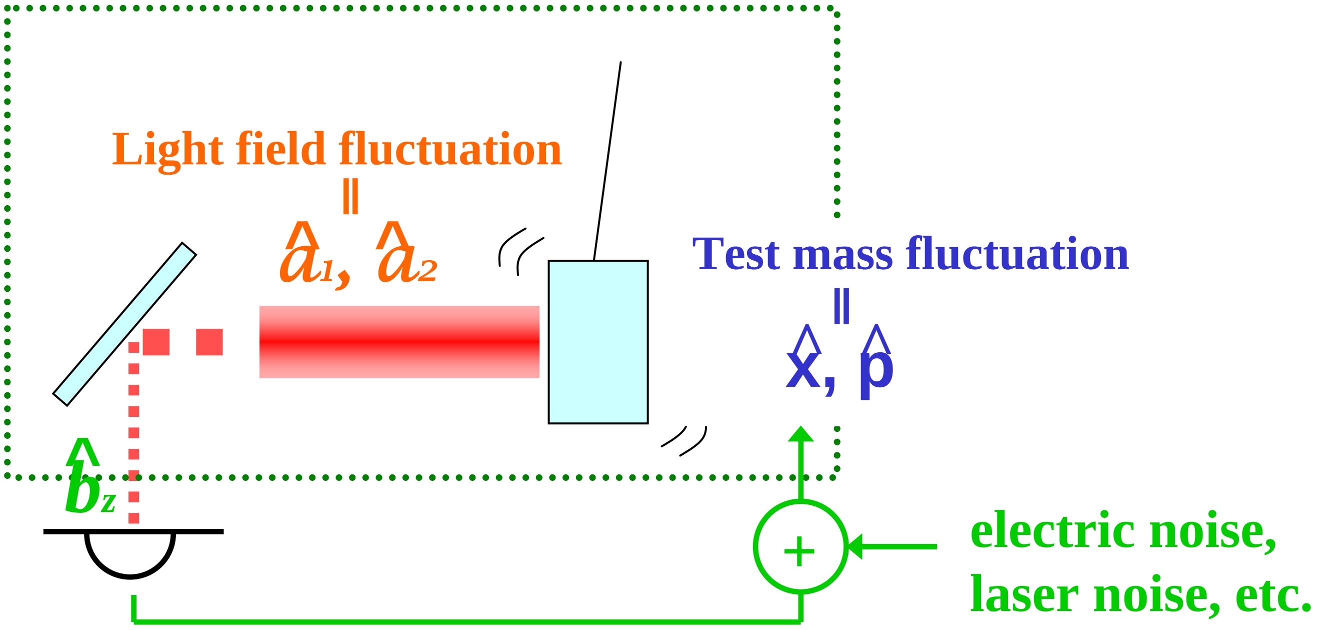
We don't know how much of x with p

and how much of a_2 with a_1 are included.

Reason to keep the quantum property

Control-noise decoherence

However, the feedback imposes force onto the mass.



Random force due to control noise makes decoherence just like the random thermal force.

Summary of the proposed experiment

1. **Preparation stage**
   - **Initial quantum state should be prepared by control**
   - **Altering the optical spring makes a squeezed state**
   - **Control noise should be investigated**

2. **Waiting stage**
   - **Thermal decoherence may be seen in AdLIGO**
   - **Quantum behavior may remain after the 1st period**

3. **Detection stage**
   - **This will be hopefully easy.**

A couple of more slides for further discussions

1. **Are there any fancy configurations to see quantum behavior more clearly?**

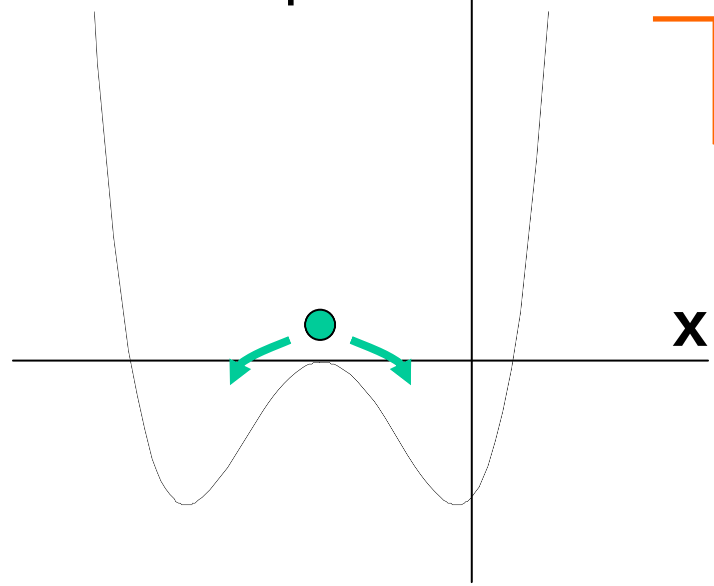
   ➡ **Proposal of using a Schroedinger's cat state**

2. **Is there a border of quantum and classical? Even if there is no influence from environment?**

   ➡ **Penrose's hypothesis of gravity decoherence**

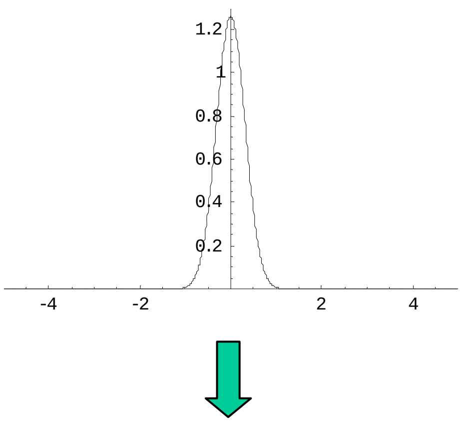
Proposal of using a Schroedinger's-cat state

Double-well potential : $V=k_1x^4-k_2x^2$

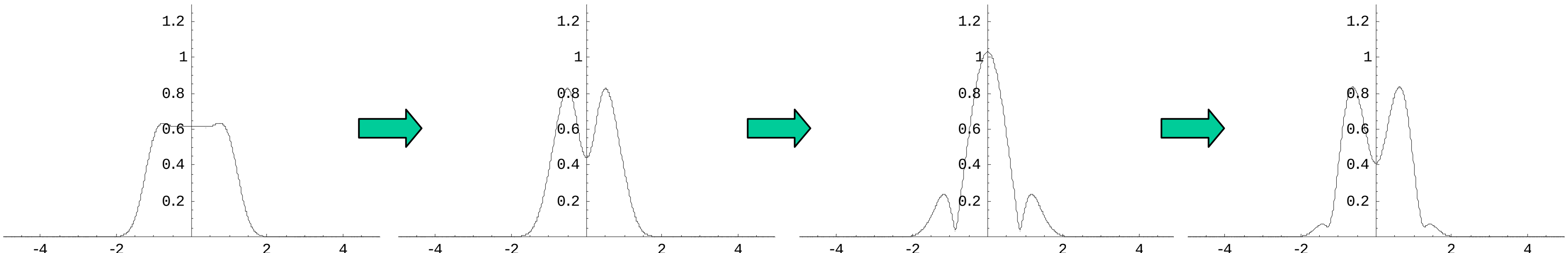


Non-linear feedback is required

Numerical calculation of Schroedinger equation

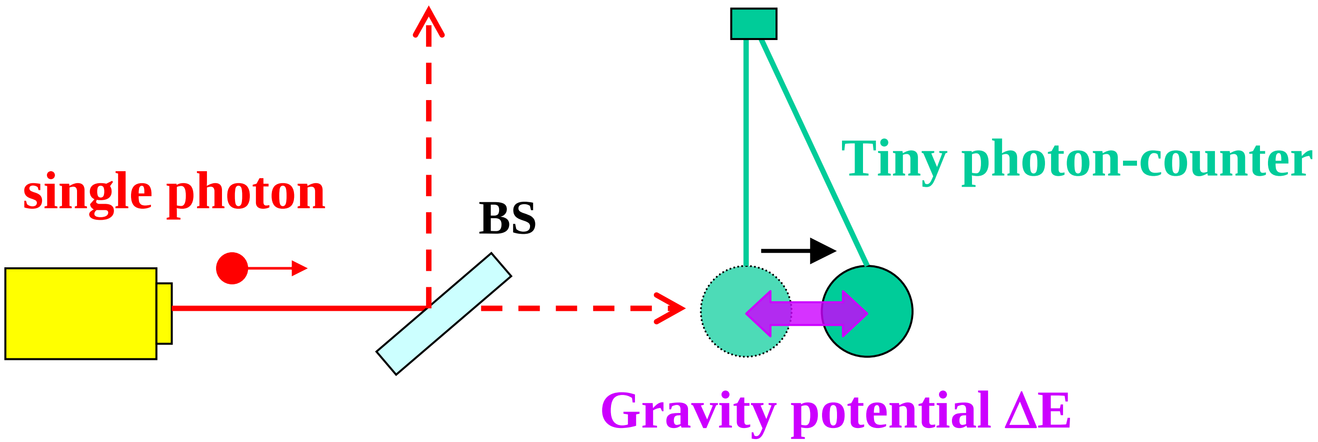


Quantum behavior will be more clearly seen.



Gravity decoherence

There is a gravity potential between two possible masses



Heisenberg's Uncertainty Principle allows instantaneous violation of energy conservation; $\Delta E \Delta t \sim \hbar$.
Heavy masses become a single mass after short time.

It's hard to test this as is easily hidden under thermal decoherence.

What we are doing now

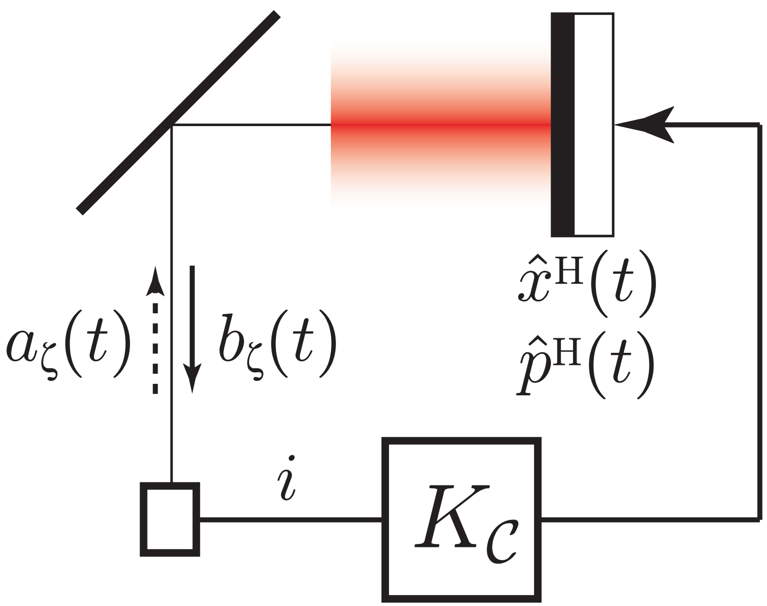
- **How well can we prepare an initial quantum state**
- **How can we make a Schroedinger's-cat state**
- **How can we calculate the time-evolution of density matrix with a non-linear feedback control**

Yanbei will lead further discussions…

Yanbei's talk & discussions

State Preparation via Linear Feedback Control

- Feeding **measurement results** of a **QND observable** back to the system: quantum dynamics **undisturbed** by measurement *[i.e., as if there were no measurements done.]*
- **Quantum Heisenberg Equations** equivalent to **Classical Equations of Motion**, ***including control***.



$$\frac{d\hat{x}(t)}{dt} = \frac{\hat{p}(t)}{m}$$

$$\frac{d\hat{p}(t)}{dt} = \underbrace{\sqrt{I}\hat{a}_1(t)}_{\text{rad. pres.}} + \underbrace{\int_0^t dt' K_C(t-t')\hat{b}_\zeta(t')}_{\text{feedback}}$$

$$\hat{b}_\zeta(t) = \underbrace{\hat{a}_\zeta(t)}_{\text{shot noise}} + \underbrace{\sqrt{I}\hat{x}(t)\cos\zeta}_{\text{"signal"}}$$

Techniques of linear control will apply!!

State Preparation via Linear Feedback Control

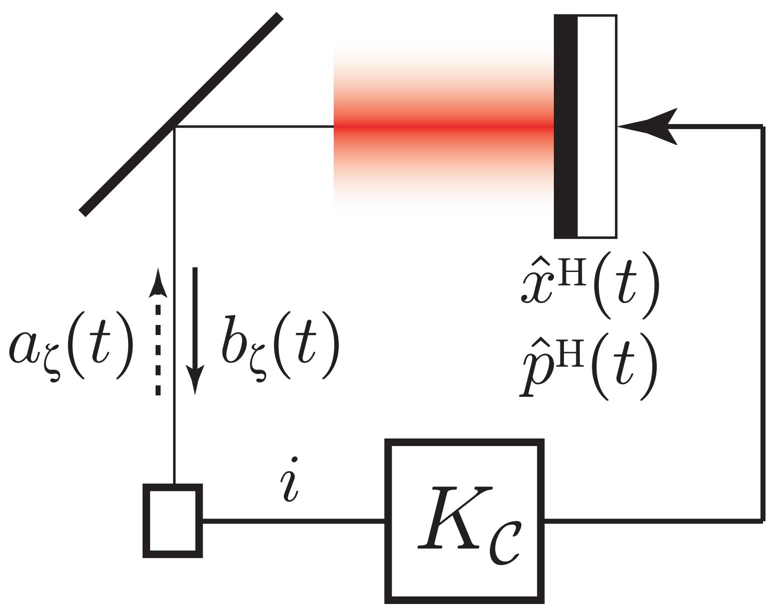
- A **stable** linear system: final "state" does not depend on initial "state".

$\mathbf{X}(t)$ = **integral of driving fields over finite history** + $e^{\mathbf{A}t}\mathbf{X}(0)$

usually stationary, like vacuum optical fields, thermal force, etc.

A only has negative eigenvalues, decay to 0 very soon

- [Heisenberg Picture] Mirror Position Operator *driven by* noise operators



$$\hat{x}^H(t) = \int_{-\infty}^{t} [A_{1,2}(t-t')a_{1,2}(t')]\, dt'$$

only finite history matters, due to stability

Test-mass state will be Gaussian, fully characterized by

$$\langle \hat{x}^H \hat{x}^H \rangle,\ \langle \hat{x}^H \hat{p}^H + \hat{p}^H \hat{x}^H \rangle,\ \langle \hat{p}^H \hat{p}^H \rangle$$

State Preparation via Linear Feedback Control

- **Preliminary example:** Signal-Recycling Interferometer, slight modification of the controller in Buonanno & Chen, PRD **67**, 062002 (2003)

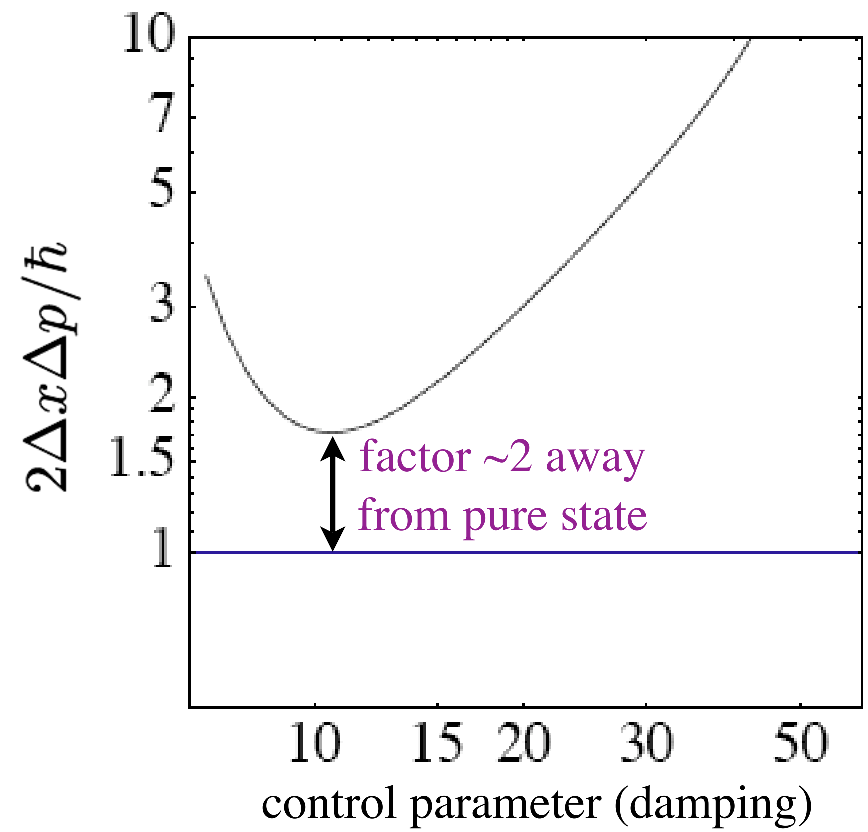
$$K_C^{\mathrm{BC}}(\Omega) = \frac{\Lambda(\Lambda - 2i\Omega)\left[\lambda^2 + (\epsilon - i\Omega)^2\right]}{\left[\lambda\cos\zeta - (\epsilon - i\Omega)\sin\zeta\right]}\sqrt{\frac{\hbar M}{8\epsilon\Theta^3}}$$

$$K_C(\Omega) = \frac{\Lambda(\Lambda - 2i\Omega)\left[\lambda^2 + \epsilon^2\right]}{\left[\lambda\cos\zeta - (\epsilon - i\Omega)\sin\zeta\right]}\sqrt{\frac{\hbar M}{8\epsilon\Theta^3}}$$

from photocurrent to force on mirror

~ optical response

Very simple controller!!



- **Pre-preliminary result:** Can also prepare pure state, *at least with the help of input squeezing*. *[Without using momentum feedback, as in Caves & Milburn 1987.]*

Summary of Current Understanding

	State Preparation	Evolution	Detection
Classical Environment	?	○	?
Quantum Mechanics	○	?	?