How to Build an Inertial Frame

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- Ideally LIGO's mirrors (test masses) should act like inertial frames, free to fall in response to gravitational fluctuations
- Job of seismic isolation system is to approximate this ideal, effectively removing ALL background forces on mirrors
- Outline:
  - general principles and consequences
  - sources of background forces
  - an attempted solution
  - latest results
  - optimism

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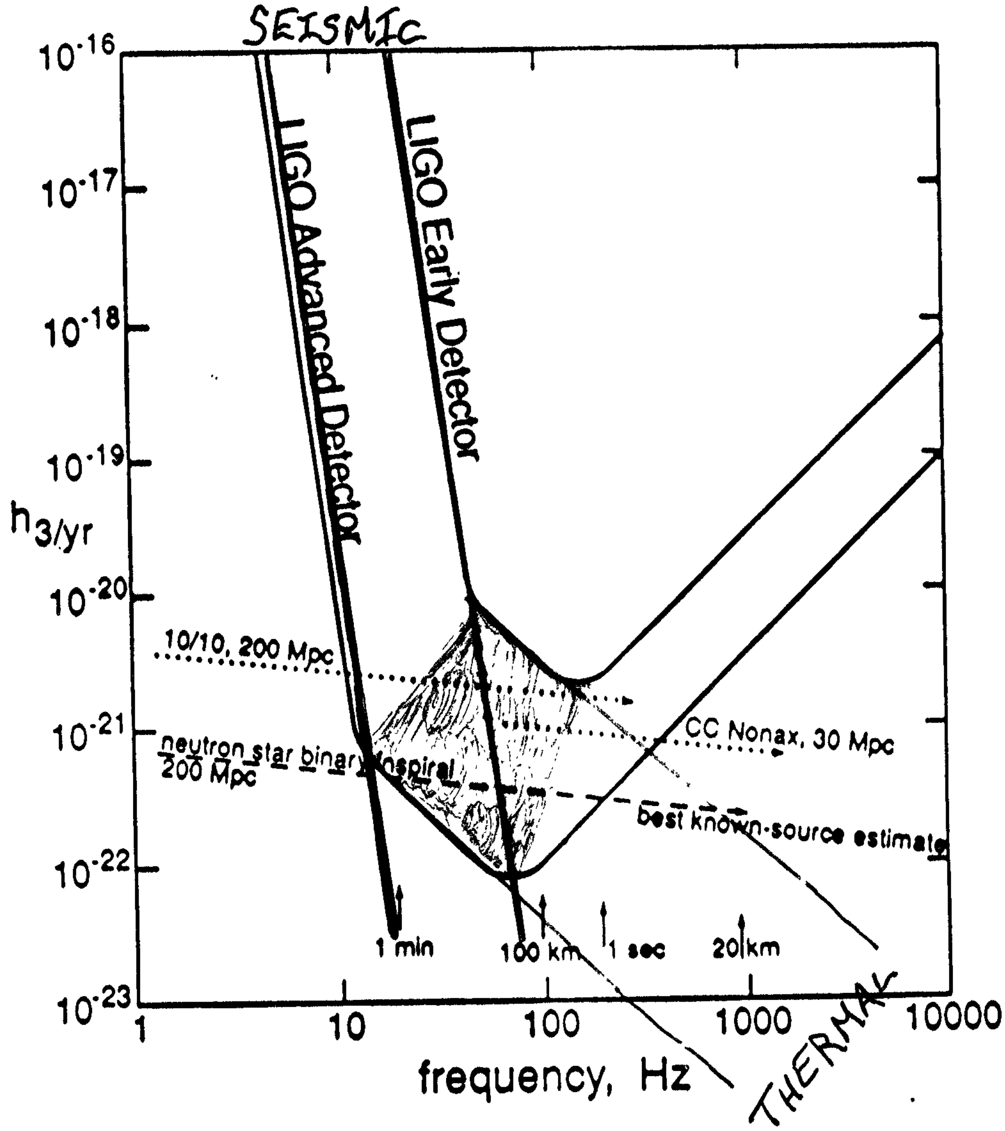
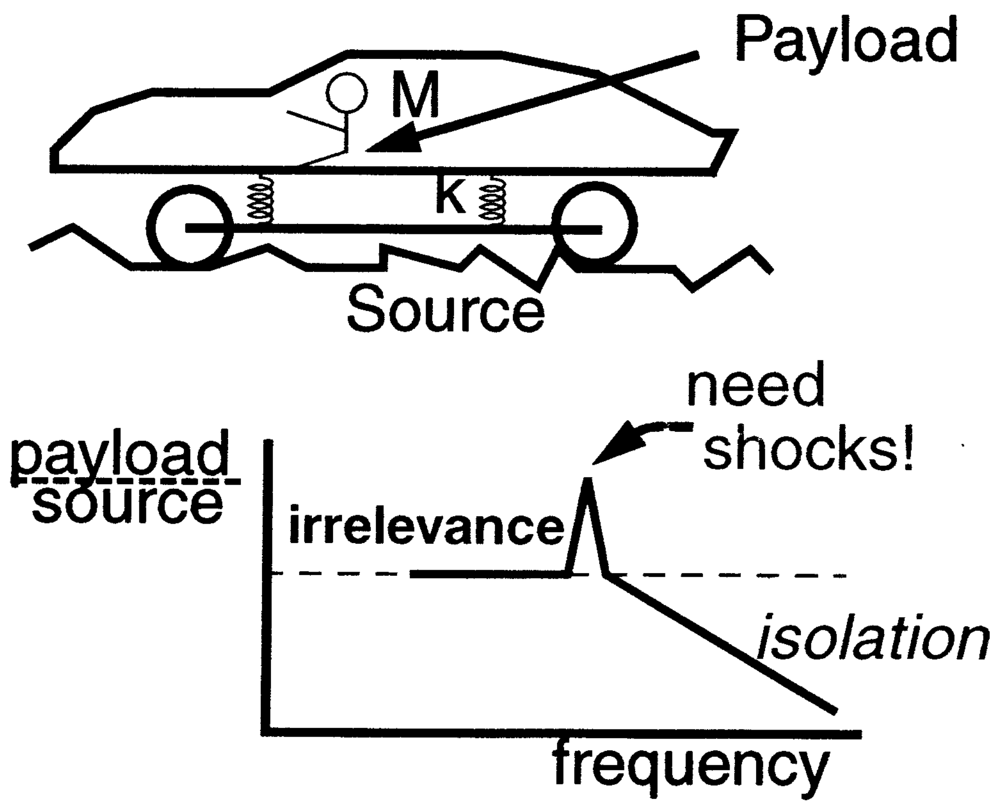


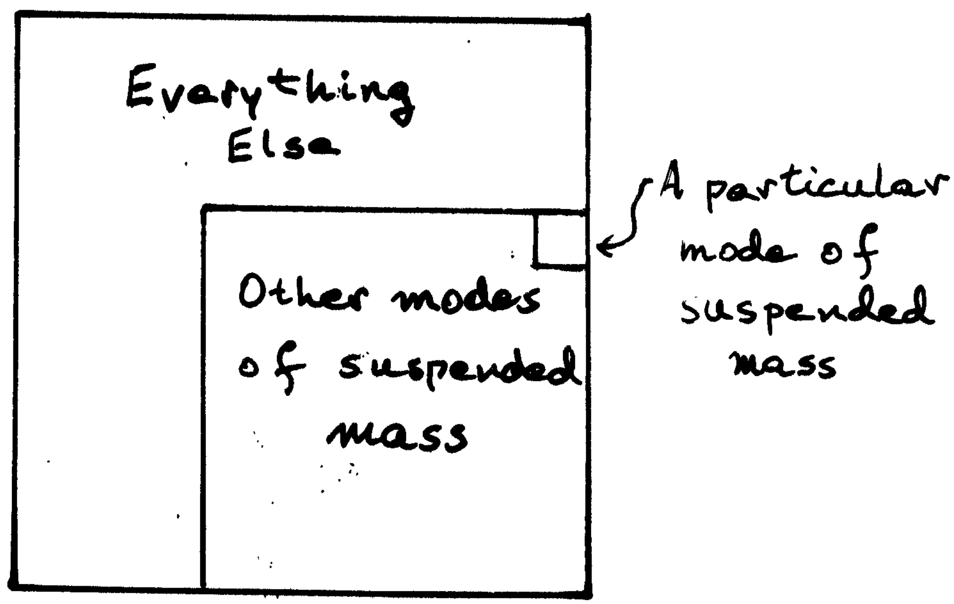
Why Random Forces Are Important

Everything to Know about Seismic Isolation Can Be Learned from Your Car



The Fluctuation-Dissipation Theorem

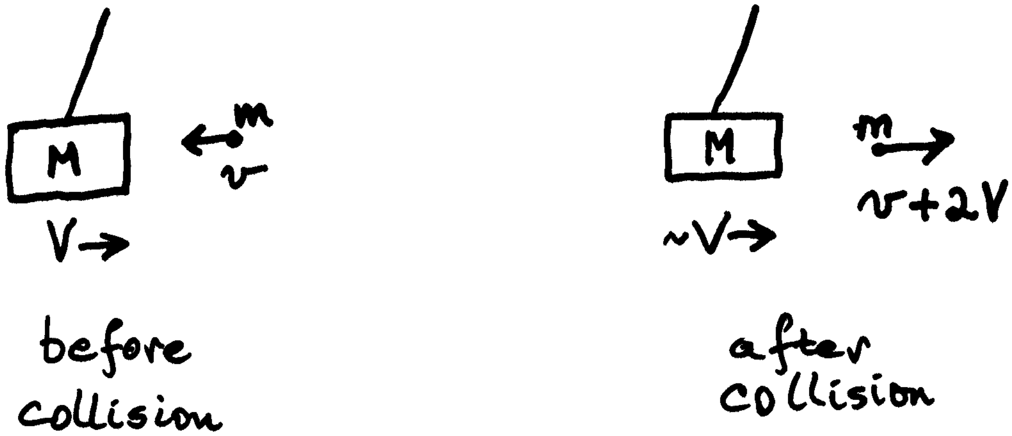
- Consider a particular mode of motion as an open system.
- Realize that this mode is part of a larger closed system in which energy is conserved.
- This particular mode exchanges energy with the rest of the closed system (a.k.a. "the reservoir").



- Fluctuation-Dissipation Theorem gives prescription for treating energy exchange between this particular normal mode and the surrounding reservoir, based on microscopic reversibility of energy flow:

$$\tilde{F}^2(f) = 4k_BT \cdot Re[Z(f)]$$

Example: Damping of a Pendulum by a Rarefied Gas



- **For the pendulum mass:**

$$F = \frac{dp}{dt} = \Delta p(per\ collision) \times R(collision\ rate)$$
$$= \quad 2mV \quad \times \quad \rho < v > \cdot 2A_\perp$$

- **Damping is "viscous", i.e.,**

$$\left(\frac{F}{M}\right) = \gamma V$$

$$\gamma = 4\left(\frac{m}{M}\right)\rho < v > A_\perp$$

- **Fluctuating force spectral density is:**

$$\tilde{F}^2(f) = 4k_BTM\gamma = 4k_BTM \cdot \frac{\omega_0^2\phi(f)}{\omega}$$

- **Thermal noise is:**

$$\tilde{x}^2(f) \approx \frac{4k_BT\gamma}{M\omega^4}, \quad f >> f_0$$

$$\phi(f) = \gamma \frac{\omega}{\omega_0^2}$$

Thermal Fluctuations of a Damped Harmonic Oscillator

- Thermal agitation results in random fluctuations:

$$\tilde{x}_T^2(f) = \frac{4k_BT}{\omega} \frac{k\phi(\omega)}{(k - m\omega^2)^2 + k^2\phi^2(\omega)}$$

complex spring constant = $k[1 + i\phi(\omega)]$

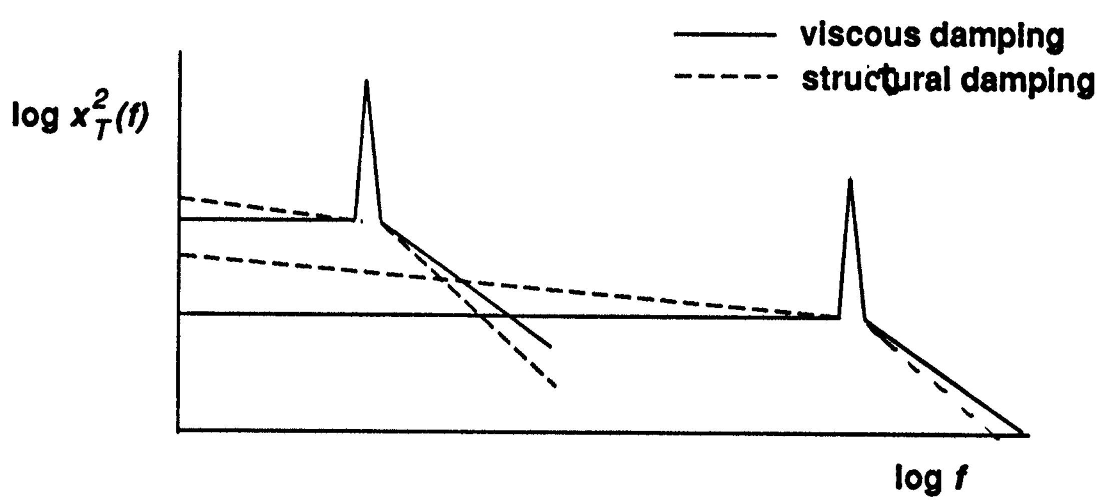
- Reasonable parameterizations for $\phi(\omega)$:

Viscous Damping

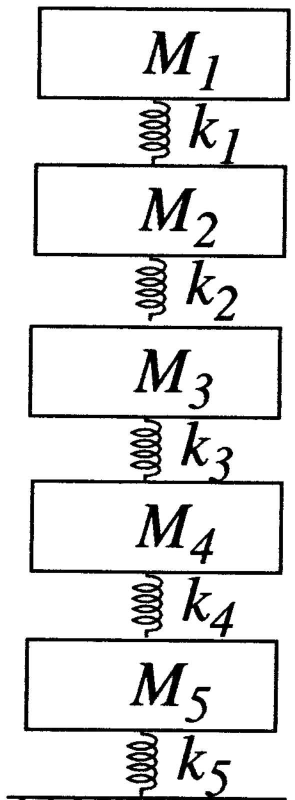
$$F_d = -2\gamma v \rightarrow \phi(\omega) = \frac{2\gamma\omega}{k}$$

Structural Damping

$$\phi(\omega) \;=\; \text{constant} \;=\; 1/Q$$



Generalized Cascaded Seismic Filter



Negligible Seismic Noise

↑

Smaller Seismic Noise

↑

Large Seismic Noise

Thermal Noise Crucial

↑

Thermal Noise a Possible Concern

↑

Thermal Noise Irrelevant

Passive damping will give good all-around performance on the coarser stages, but thermal noise will require high-Q final stage(s). Electronic damping servo allows resonance suppression with virtually no damping at higher frequencies.

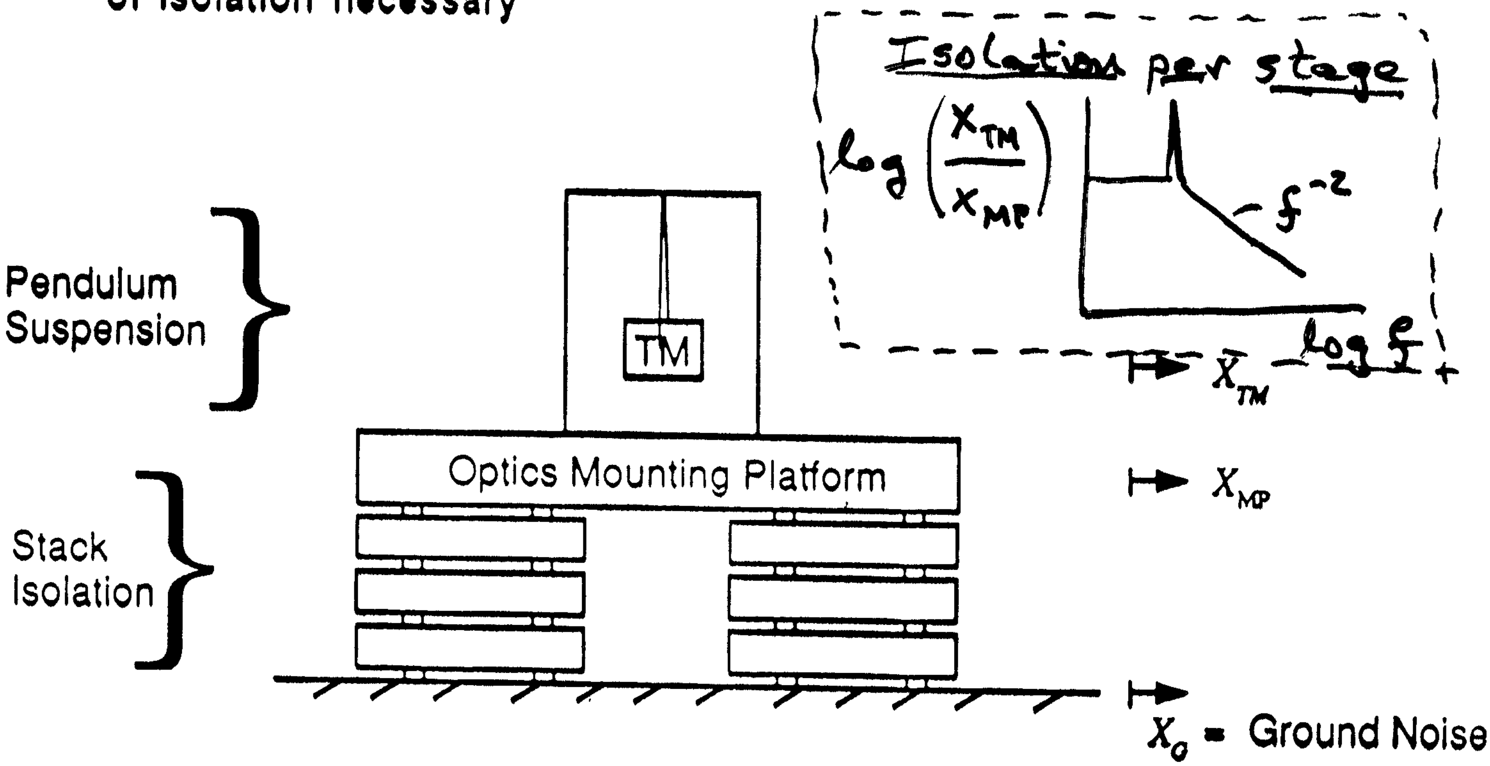
SEISMIC ISOLATION OF TEST MASS

Seismic Isolation of test mass is composed of 2 components:

- **Stack Isolation**
- **Pendulum Suspension**

} Multistage Resonant Filters

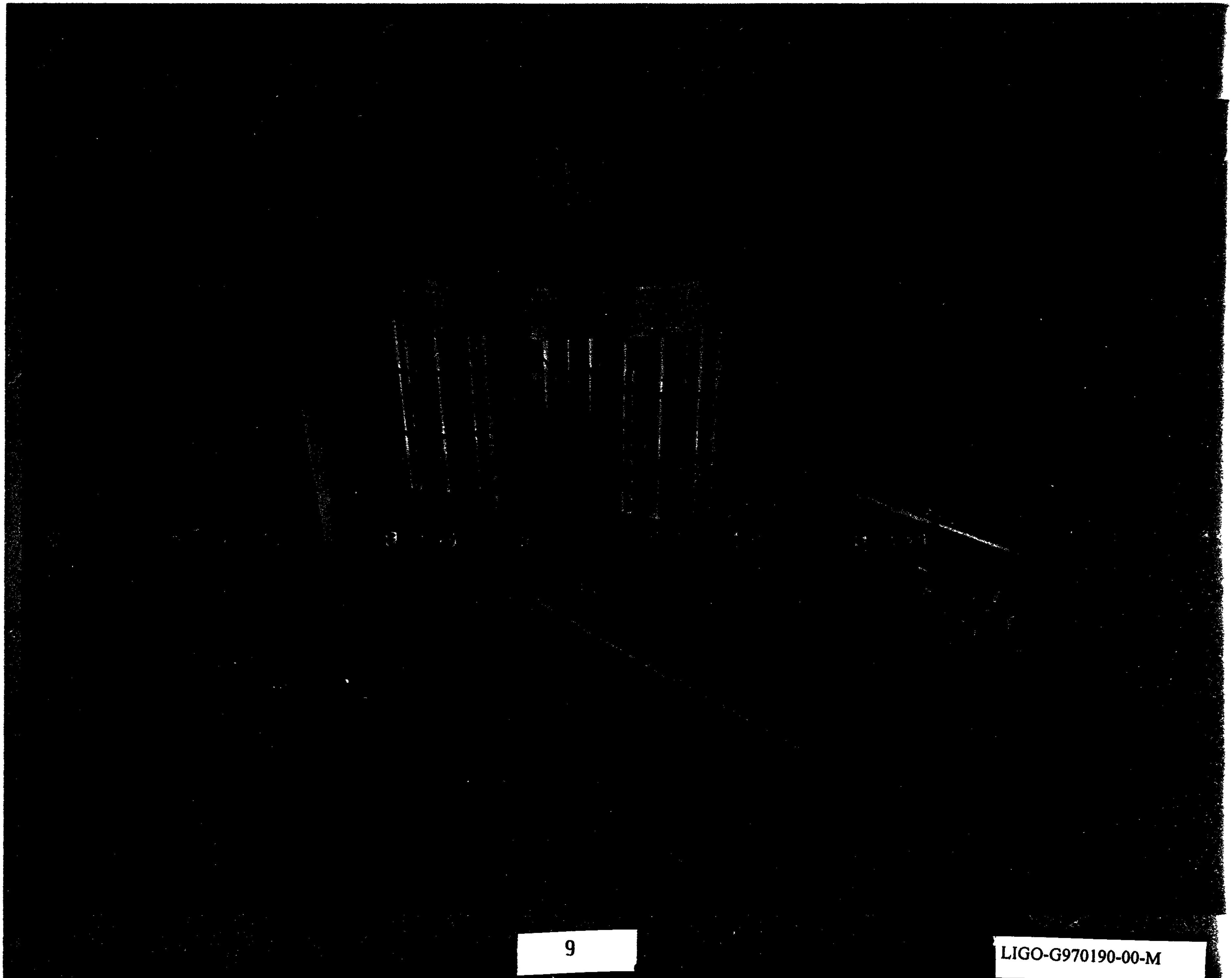
The ground noise at the sites drives the requirement on the amount of isolation necessary

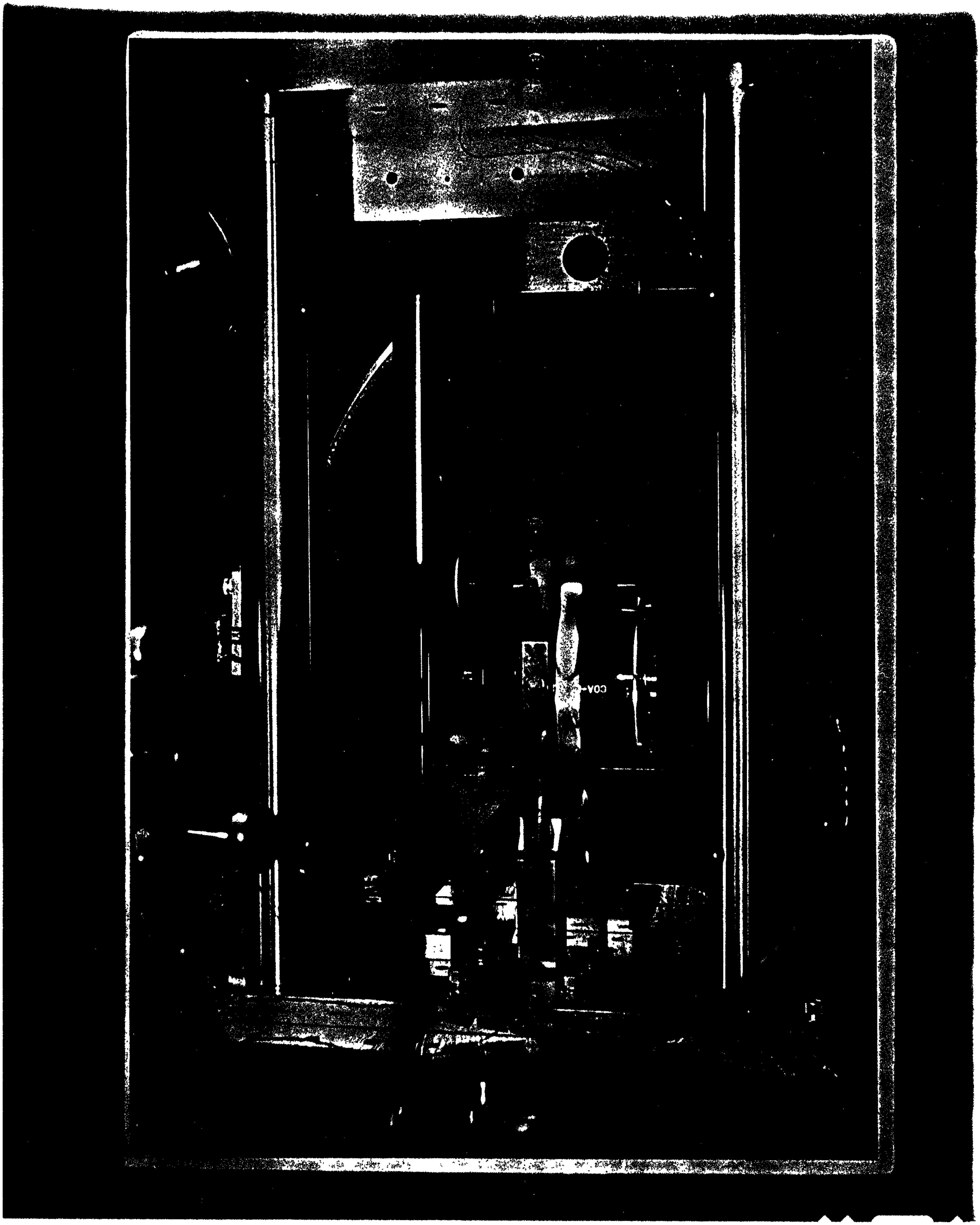


$$X_{TM} \sim \left[\frac{X_{TM}}{X_{MP}}\right]\left[\frac{X_{MP}}{X_G}\right] X_G$$

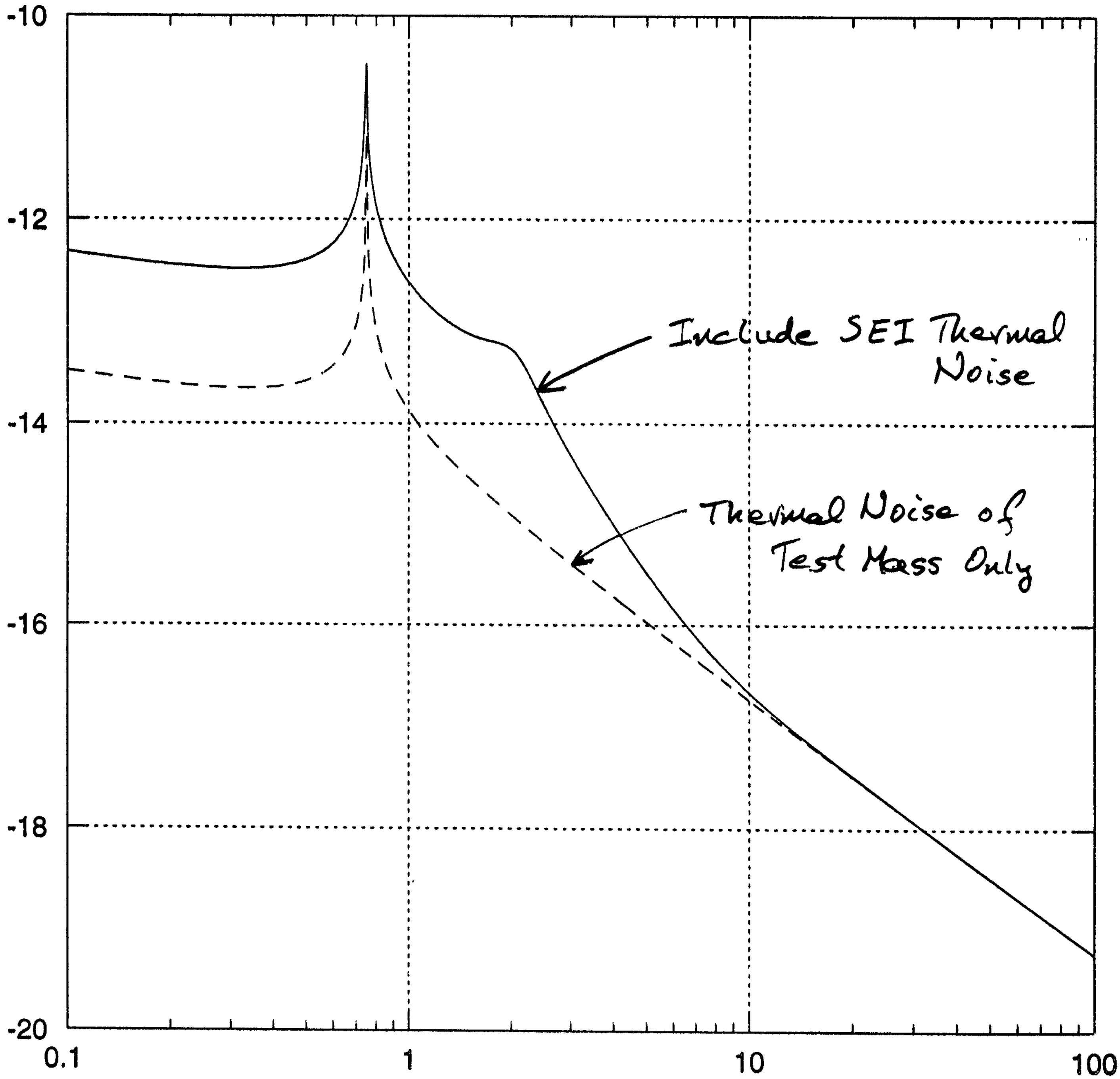
↑ Stack Xmission

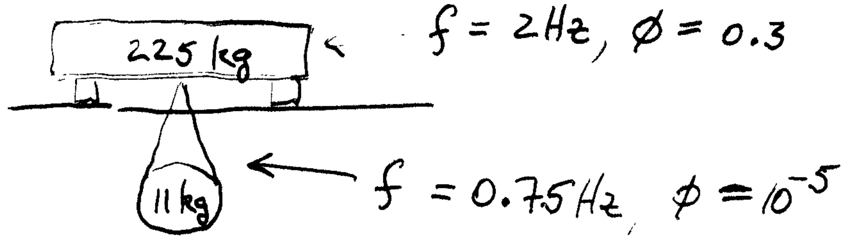
↑ Suspension Xmission



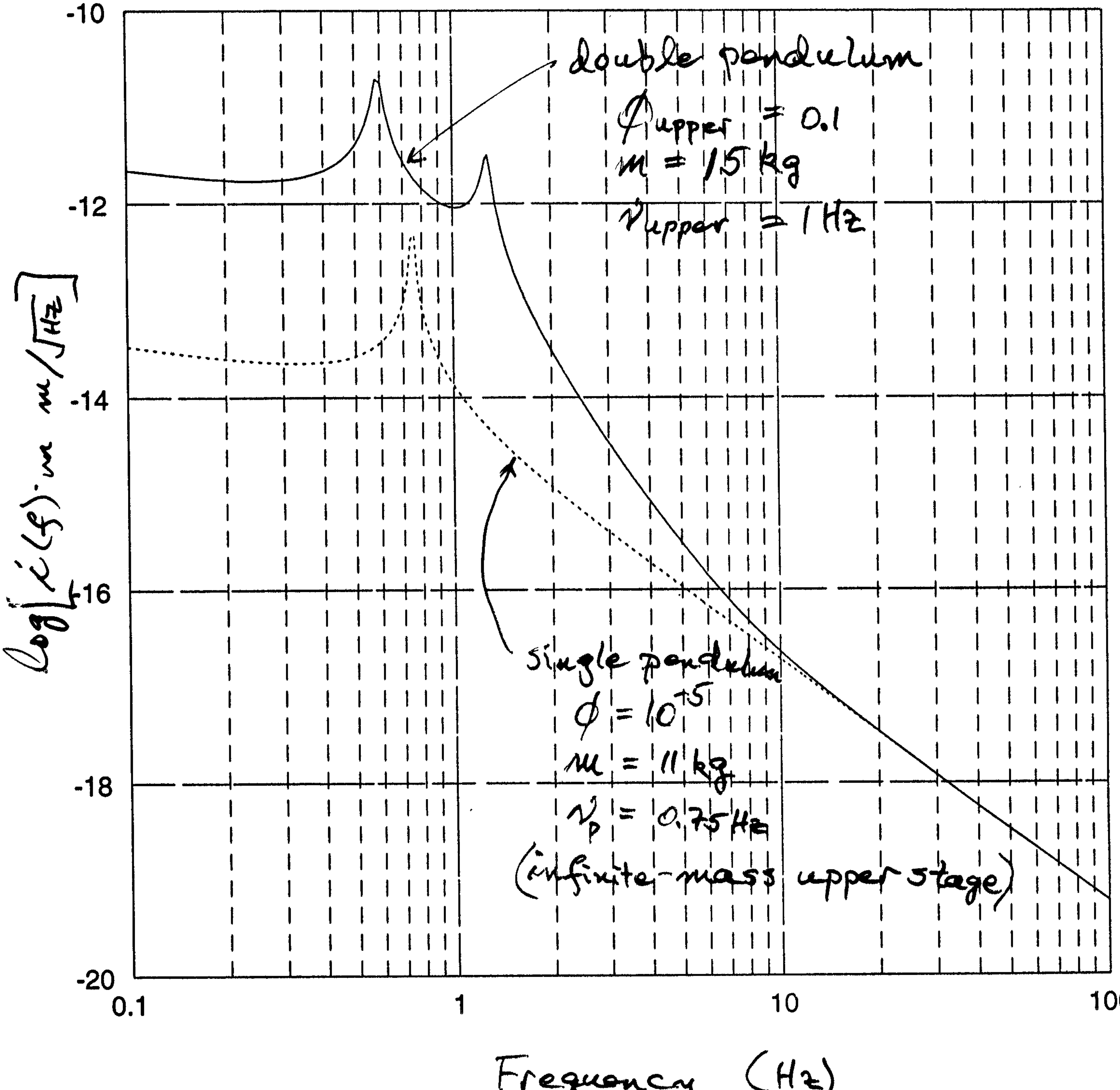


Effect of SEI Thermal Noise in Low Frequency Modes on Test Mass Motion





Thermal Noise in Coupled Pendula
Horizontal Displacement



Tides: a Coherent Background Force

- The orbital motion of the sun and the moon distort the earth's crust, causing as much as 0.4 mm changes in the 4-kilometer distance between end- and mid-station buildings

  ››Problem is peculiar to large size of LIGO; effect is totally negligible in 40-meter Interferometer

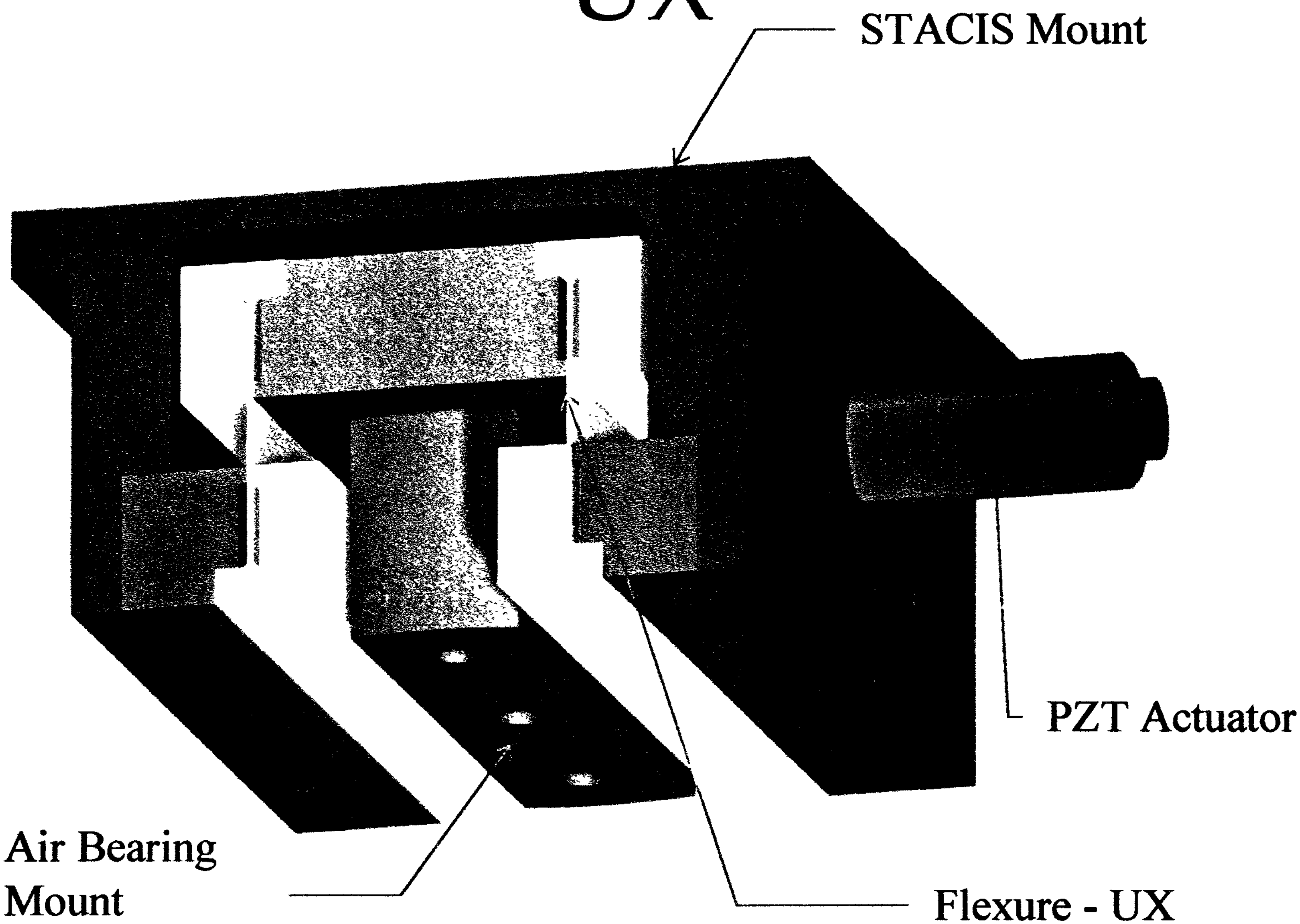
  ››Common-mode piece (affecting both arms equally) can be removed by small changes (100 ppb) in laser frequency

  ››Differential piece must be removed by moving seismic stacks on end masses relative to end buildings by up to 100 microns (peak-to-peak) to hold distances to corner building constant

  ››Fortunately, motion is slow

  ››Unfortunately, correction needs to be made with interferometer operating at full sensitivity

Line Actuator Sub-Assembly - UX



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Microseism: Effect of Ocean Wave Activity

- After the Earth Tide, the microseism is the largest feature of earth's seismic spectrum

  ››Driven by ocean wave activity at approximately 0.16 Hz

  ››Has Rayleigh distribution of peak heights on time scale of hours to days with standard deviation comparable to mean frequency

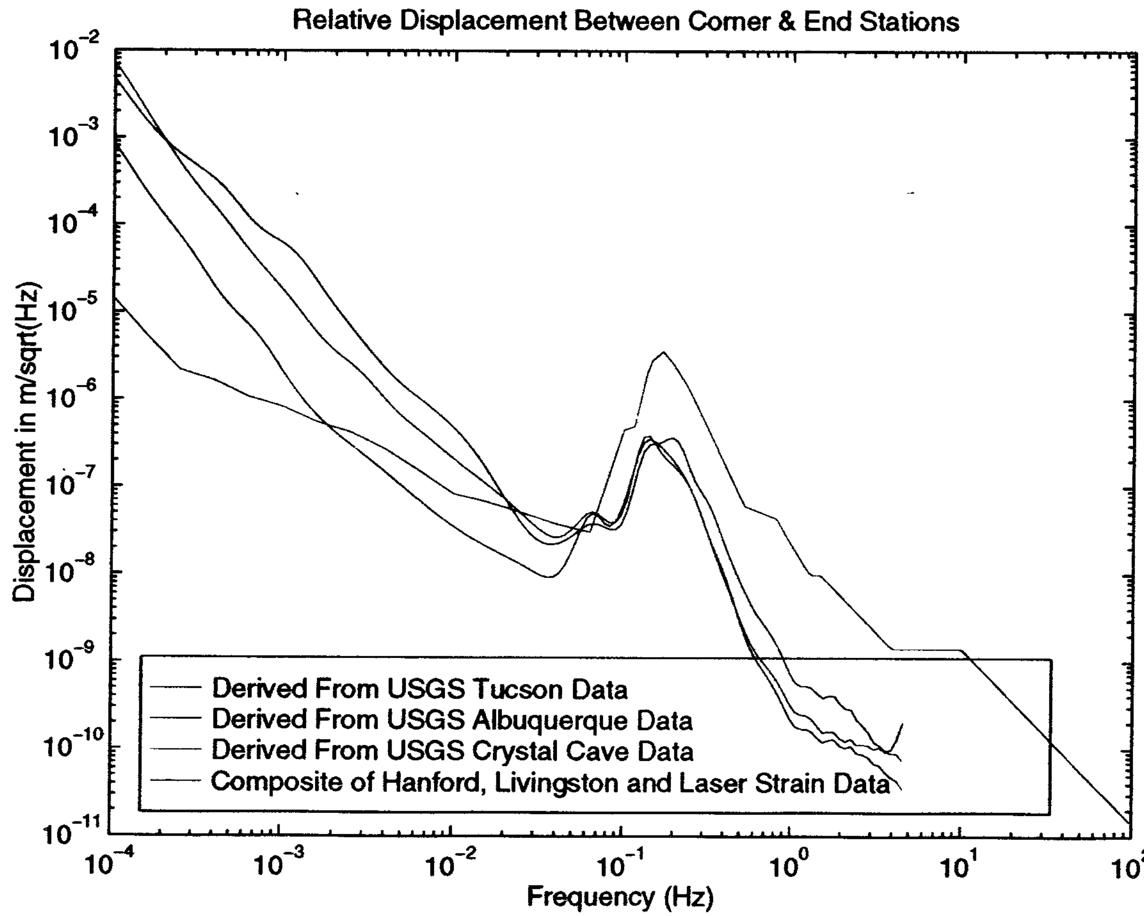
  ››Problem is peculiar to large size of LIGO; effect is totally negligible in 40-meter interferometer or any apparatus smaller than a kilometer

  ››Highly variable with weather conditions over oceans and seas; factor of ten variations with severe weather are common; seasonally variable

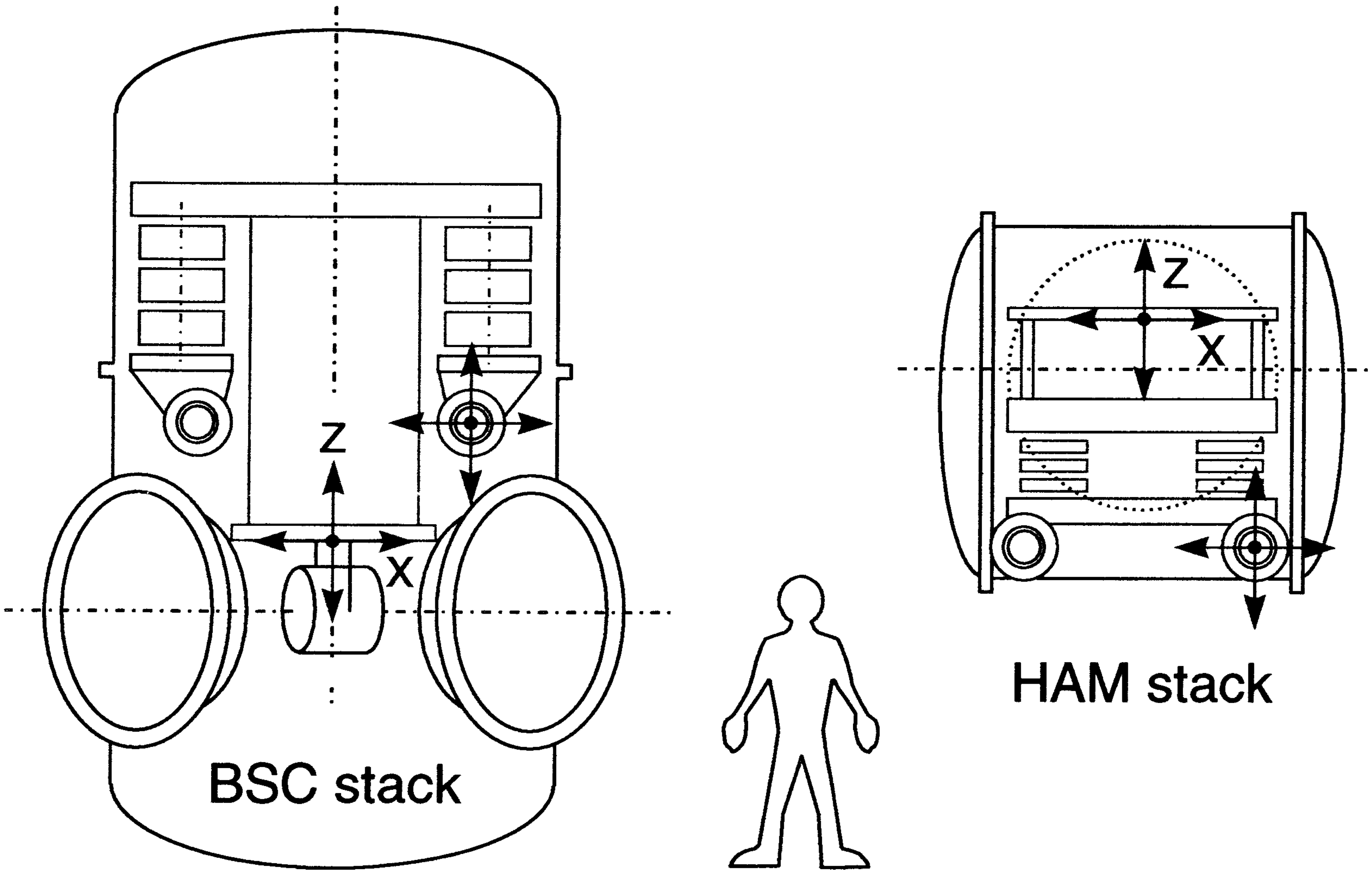
  ››Fortunately, the corner station (containing most of LIGO's optics) will respond to microseism like a rigid body

  ››Fine actuators on end-mass stacks need to track corner station with 14-40 microns of actuation range to control microseism

Microseismic Motion at Different Locations



LIGO Stacks



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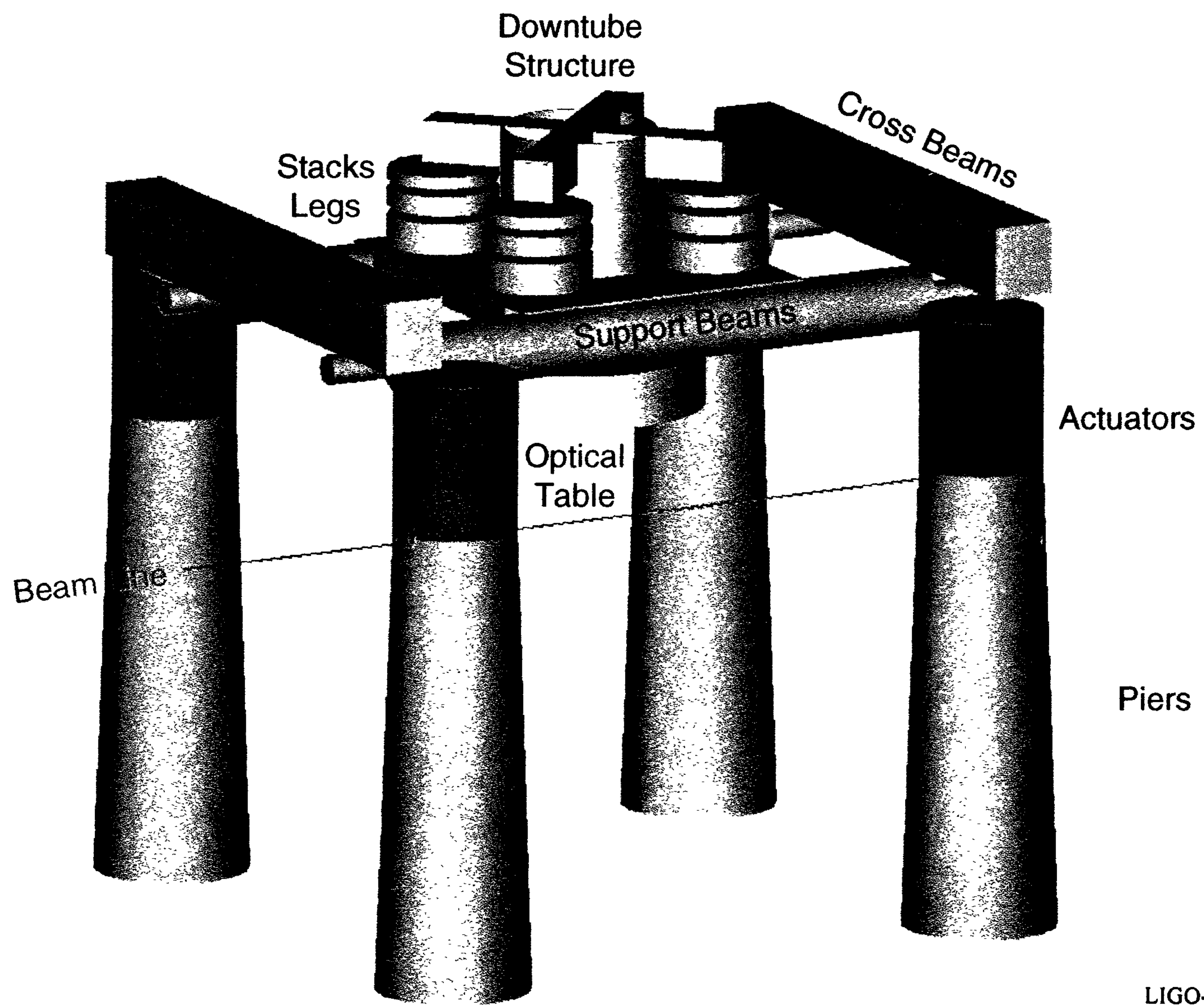
BSC Stack Layout



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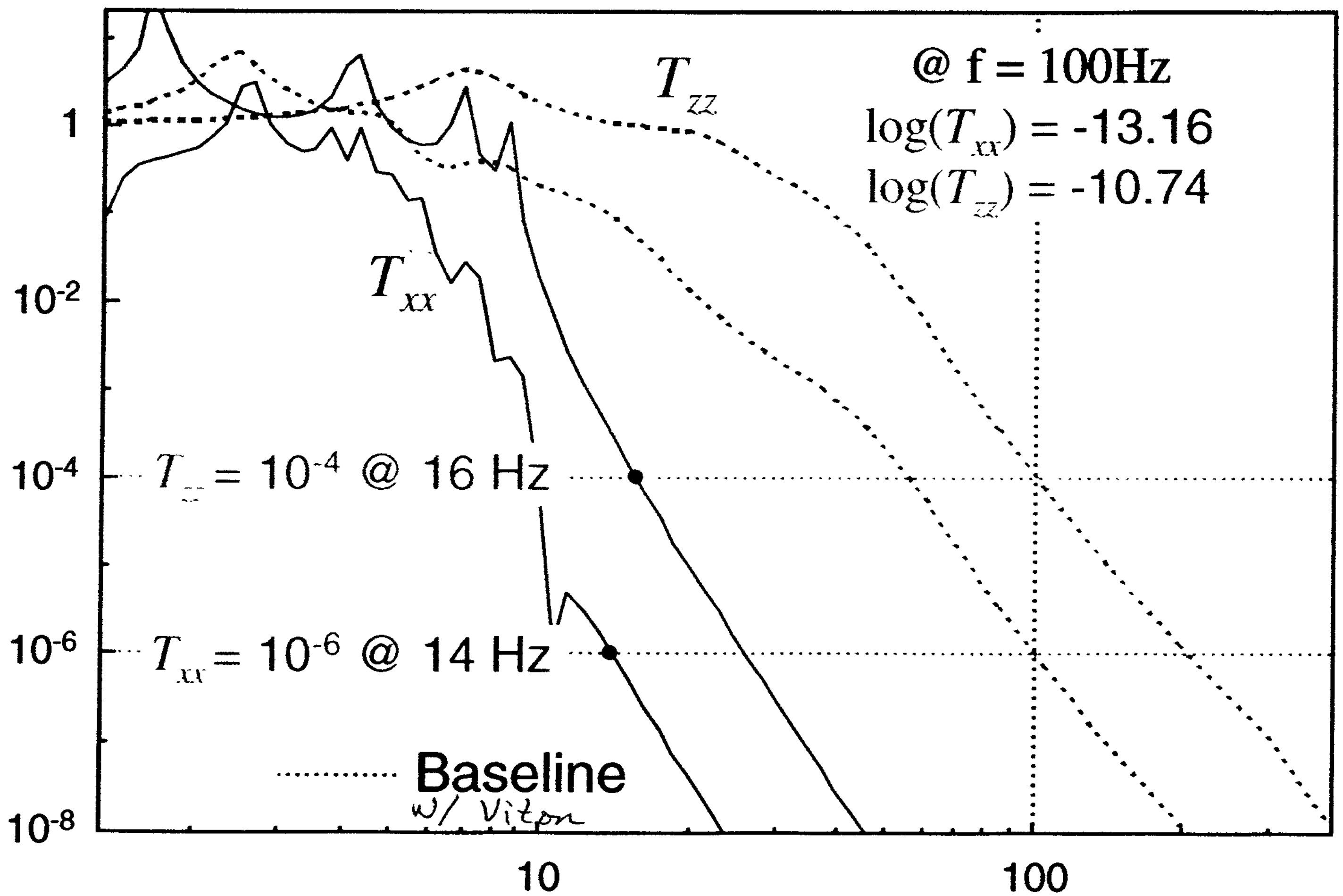
Limit Case: Undamped Coil Spring



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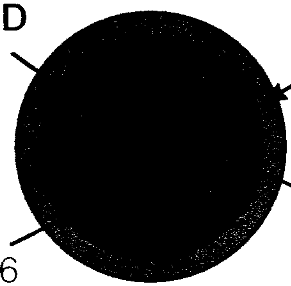
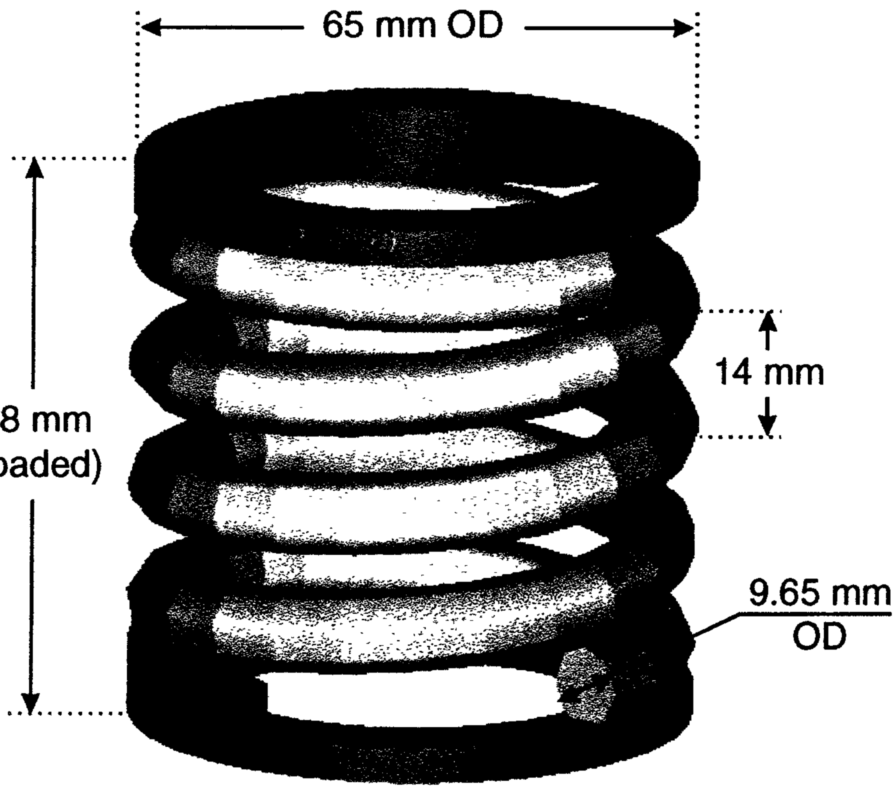
Design





- Outer tube: Ph Bronze
  (high yield, low relief t°)
- Inner tubes: Aluminum
  (pliable)
- Viscoelastic Layer: Soundcoat DYAD 606
  (thick but stiff, high loss 105%)

- Design Optimization:
  - adjust cross section & coil geometry
  - maximize loss factor

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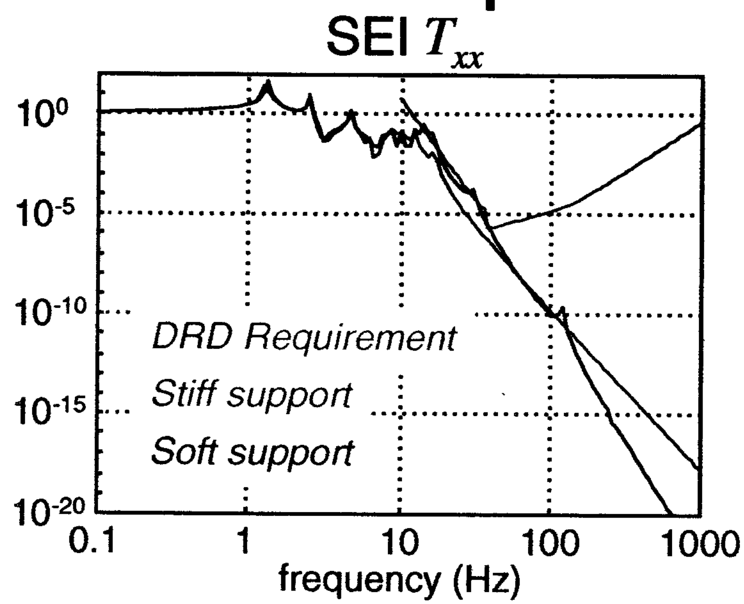
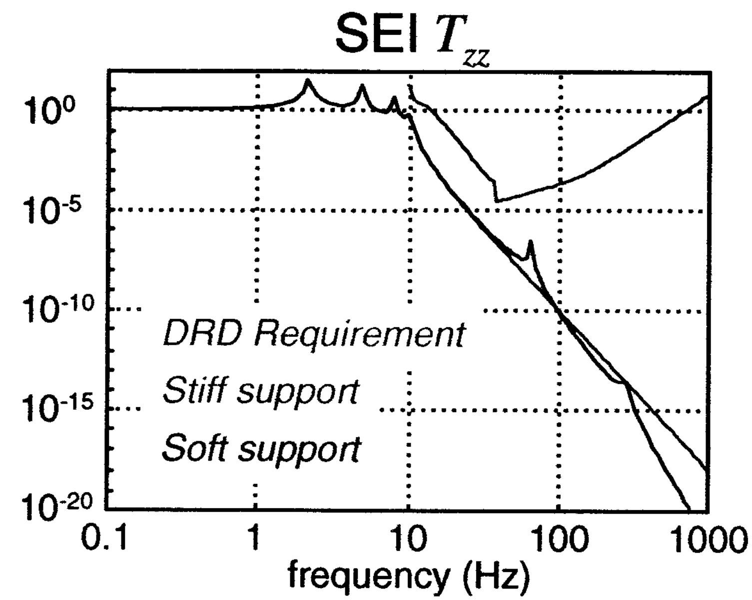
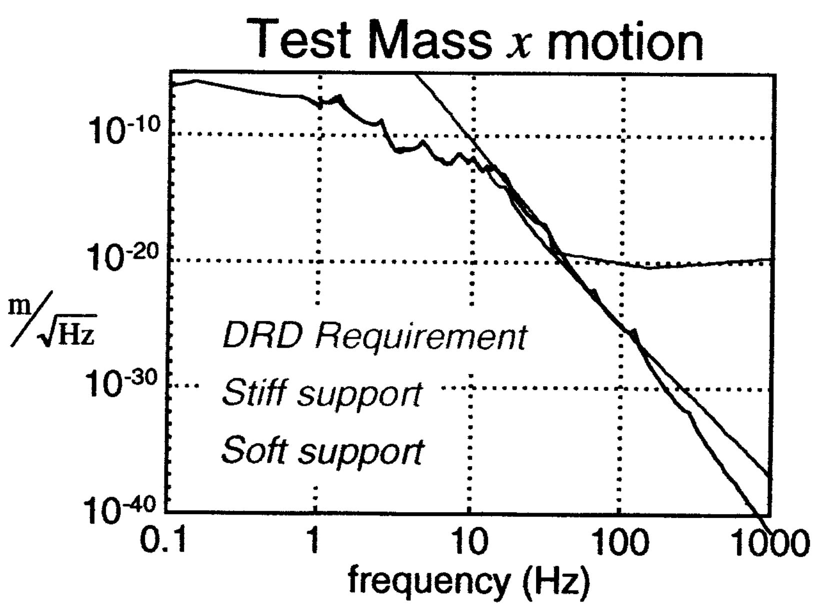
Expected Performance







Lock Acquisition

(test mass kinetic energy)

$v_{RMS} = 1.257\ \mu m/sec \sim 1\ \mu m/sec$

Lock Maintenance

(SUS actuator force rating)

$\chi_{RMS} = 2.287\ \mu m < 2.7\ \mu m$

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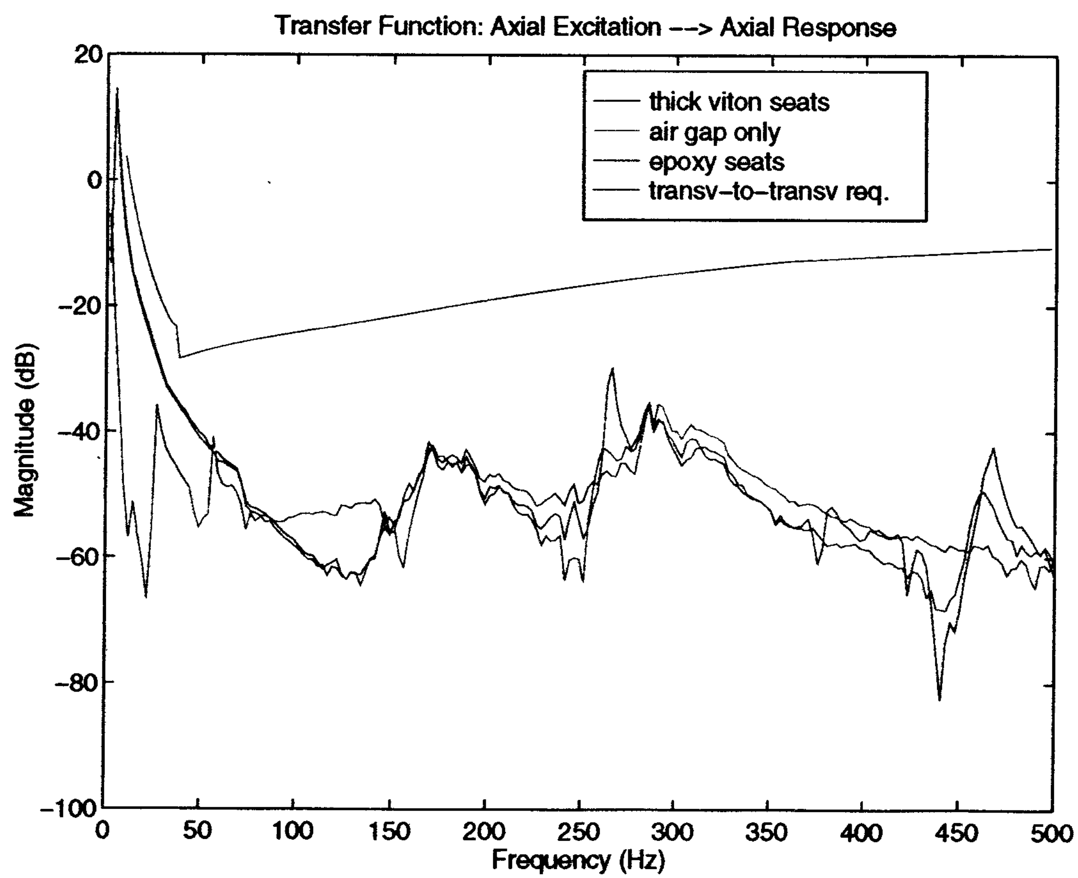
Test for Acoustic Transmission Modes



Wrap Up

- Seismic Goals look attainable, although design is severely constrained; end-to-end engineering model has proven to be important design guide
- Constrained-layer-damped, coil springs look like a suitable design for initial LIGO with Q's ~ 30 and relative freedom from acoustic transmission
- Light-weight, egg-crate design for mass elements and optical tables makes fine actuation possible and makes active seismic isolation of stack possible
- Actuation systems appear capable of performing "feed-forward" compensation of large tidal and microseismic perturbations; unfortunately, safety factor is not large