Modelling the Performance of an Initial-LIGO Interferometer with *Realistically-Deformed* Optics

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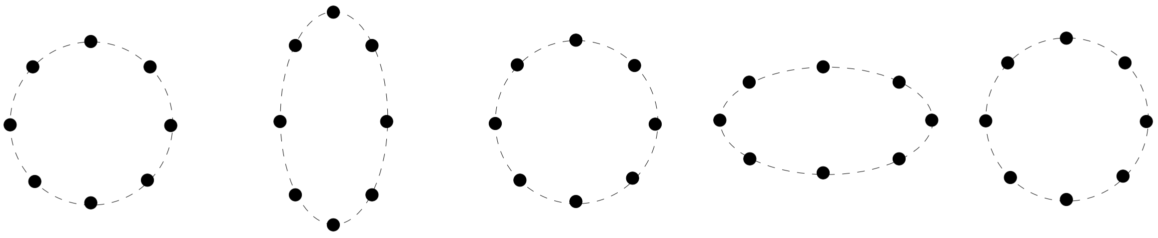
LIGO = “Laser Interferometer Gravitational-Wave Observatory”

LIGO is one of the new breed of interferometric detector systems designed to observe gravitational waves arriving from distant astrophysical sources.

A gravitational wave encountering a ring of free masses:

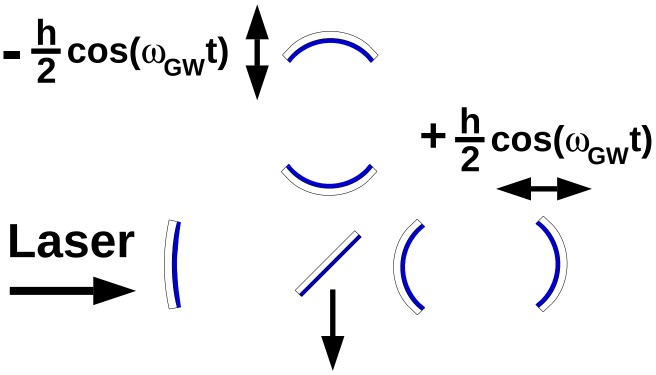
+ - polarization, propagation direction into page:



$T = 0$ $\quad T = \frac{P_{GW}}{4}$ $\quad T = \frac{P_{GW}}{2}$ $\quad T = \frac{3}{4}P_{GW}$ $\quad T = P_{GW}$

A LIGO interferometer:

(1 of 3 for LIGO;
A simplified schematic:
a Michelson interferometer
with cavities to amplify the
circulating laser power.)



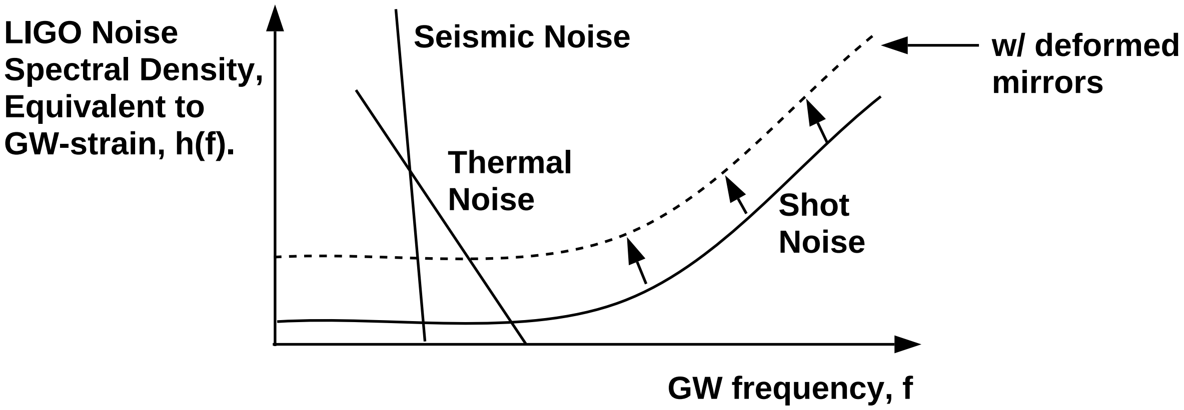
$-\frac{h}{2}\cos(\omega_{GW}t)$

$+\frac{h}{2}\cos(\omega_{GW}t)$

Laser

Dark-Fringe -or- GW-signal
+ Noise (Various Sources)

Question: Real interferometers have imperfect optics. How do these optical imperfections degrade LIGO's sensitivity to gravitational waves?



Seismic & Thermal Noise } *Random Forces* on Mirrors

Photon Shot Noise } *Position-Sensing Error* for Mirrors

Mirror deformations, misalignments, etc., tend to:

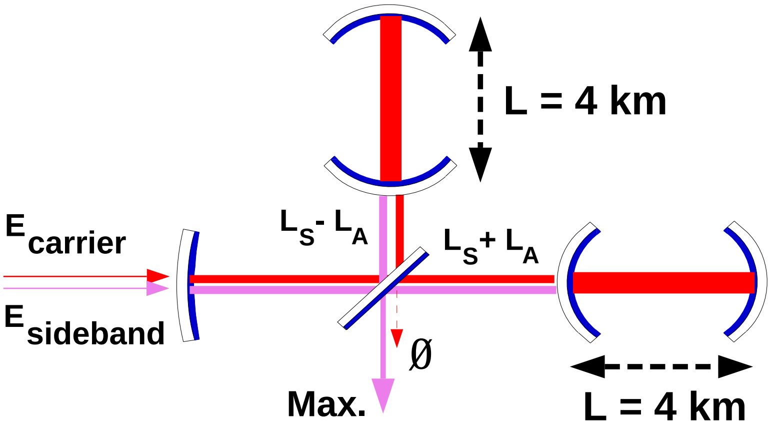
- Reduce the amount of resonating power available in the interferometer for GW detection => GW-*Signal is reduced*.
- Increase the amount of unmodulated (i.e. non-signal-bearing) light emerging from the beamsplitter exit port,
  => *Photon counting noise (i.e. Shot Noise) is increased*.

∴ The *shot-noise-limited sensitivity* to gravitational waves is reduced by the presence of optical imperfections.

To accurately estimate the effects of mirror imperfections, we perform detailed numerical simulations of LIGO interferometers, incorporating as much physical realism into our numerical model as possible.

The System We Model -- The Core Optics of a 1st-Generation LIGO Interferometer (IFO)

Features of the Simulated System:



- A carrier beam (Nd:YAG light, λ=1064 nm) built up in the IFO for sensing GW-induced fluctuations in mirror positions.
- 2 RF sideband beams (+/- 24 MHz), emerging from the beamsplitter exit port for use as a *local oscillator* for a heterodyne detection scheme.
- Long-baseline (4 km) Fabry-Perot arms for long storage of a *resonant* carrier beam for extended sensing of GW-effects.
- A Power-Recycling Mirror for *broadband amplification* of carrier & sideband fields in the interferometer (sensitivity ~ *square root* of power).
- A dark-fringe for the carrier at the beamsplitter exit port, to minimize carrier power losses & carrier-generated *shot noise*.
- A macroscopic (“Schnupp”) length asymmetry, L_A(~few cm), to maximally channel sideband “local-oscillator” power through the beamsplitter exit port, while maintaining a carrier dark-fringe.

Simulating the carrier & sidebands allows us to compute the full shot-noise-limited GW-sensitivity function, $h_{SN}(f)$, rather than just relative measures of performance degradation.

The Essentials of our LIGO Simulation Program

- A Fortran code, adapted for use on the *massively parallel* Paragon computers (Caltech).

- A Grid-Based program, representing *mirror profiles* & *transverse beam slices* on complex, 2-D maps.

- The Paraxial Approximation is assumed, allowing long-distance *beam propagations* to be done with *computationally fast methods* (FFT's, etc.)

- Reflection & Transmission operations, requiring *small-distance propagations*, are approximated with pixel-by-pixel map multiplications.

- Many mirror imperfections can be modelled, including:
  - ›› Deformations in the surface height & substrate homogeneity profiles.
  - ›› Finite mirror apertures & realistic beam clipping.
  - ›› Mirror displacements, tilts, curvature errors & beam mismatch.
  - ›› "Pure losses" due to scattering & absorption.

- We assume a static, locked LIGO interferometer, and relax the *steady-state* IFO e-fields with a rapid convergence scheme.

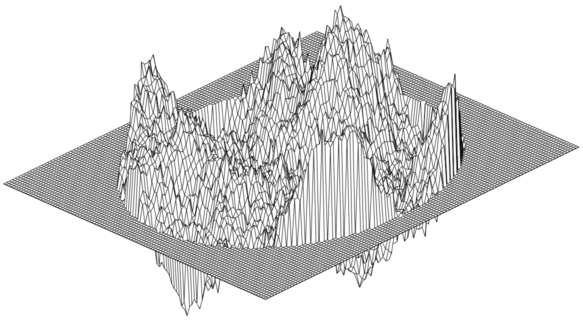
- Various optimizations are performed during program execution:
  - ›› *Cavity lengths* & the *RF sideband frequencies* are fine-tuned to achieve the specified resonance conditions.
  - ›› The *power recycling mirror reflectivity* is tuned to maximize IFO gain.
  - ›› The *Schnupp asymmetry length* is adjusted to maximize local oscillator power for heterodyne GW-detection.

Performing Runs with Realistically-Deformed MIrror Maps, Obtained from Measurements

We utilize two samples of real mirror *deformation maps*:

- A map of *surface figure deformations* of the polished calibration reference flat (“Calflat”) for the AXAF mirrors (courtesy HDOS).
- A map of *substrate inhomogeneities* for the finest grade of fused-silica mirror substrates (courtesy Corning).

To convert those 2 maps into enough surface & substrate maps to cover all the mirrors in the interferometer:



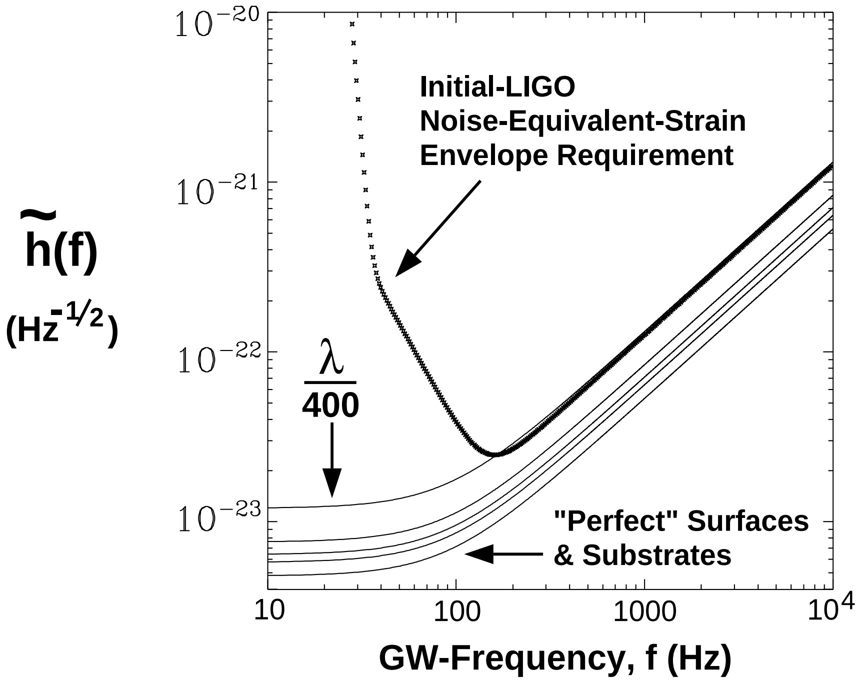
- Fourier-transform the mirror maps, and reconstruct with the same Fourier amplitudes but *randomized phases*.
- This produces a series of mirrors with identical power spectra but different (uncorrelated) structure.

Scale up the *surface deformation maps* to obtain several “families” of polished mirrors with increasing levels of deformation:

- The original Calflat map has RMS deformations of ~ $\lambda_{YAG}/1800$ ~ 0.6 nm over an 8 cm diameter central mirror portion.
- Re-scaling leads to families of $\lambda/1200$, $\lambda/800$, & $\lambda/400$ mirrors.

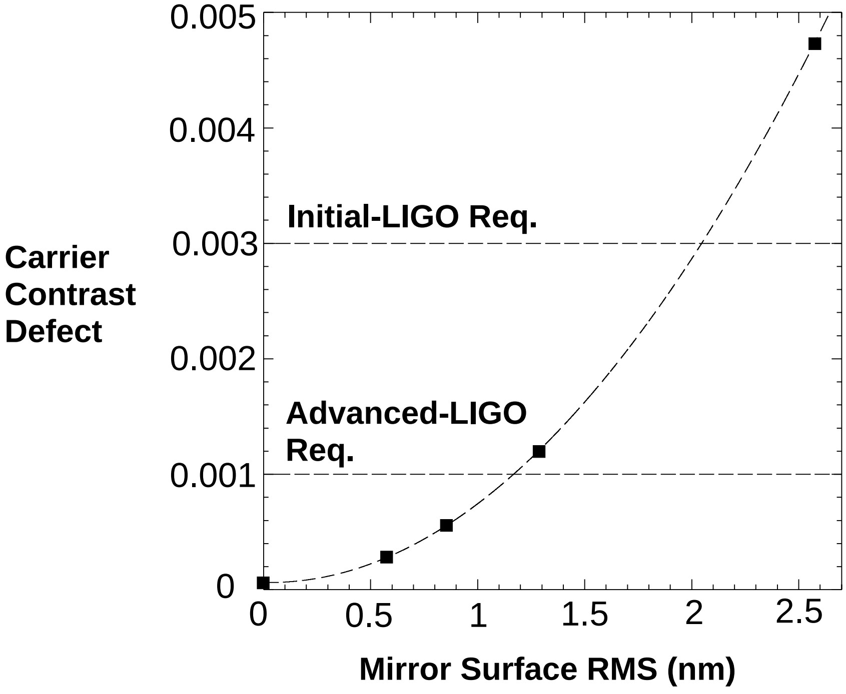
Producing mirror families of higher deformation levels allows us to estimate conservatively, and to make room for the effects of deformed mirror coatings, which we do not have good measurements of, but may be very significant.

A Selection of Results with Deformed Substrates & Each Family of Deformed Surfaces



Conclusion: All of the families of mirror surfaces, *except* the worst-case $\lambda/400$ mirrors, satisfy the Initial-LIGO GW-strain sensitivity requirement.

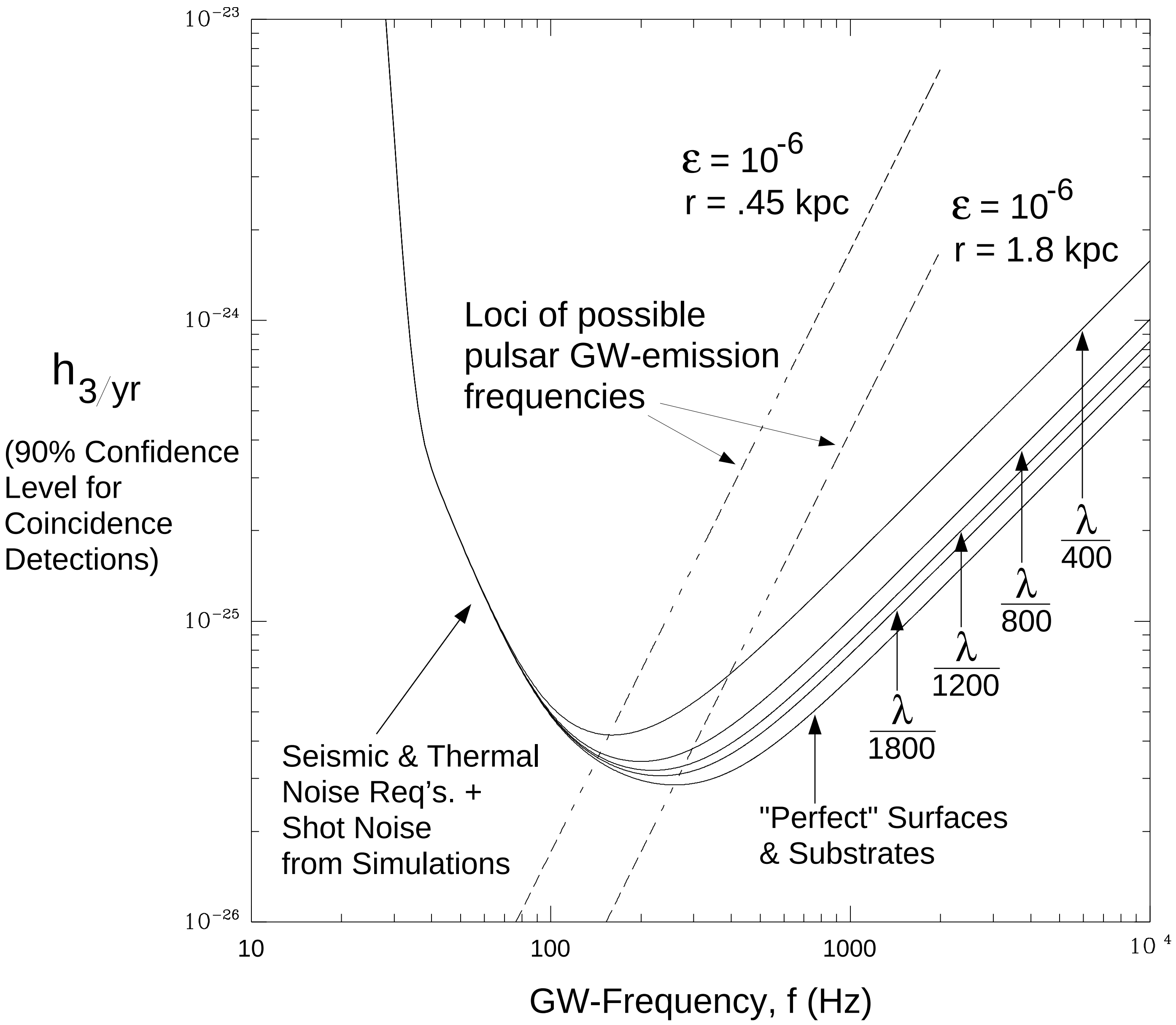
$$\text{Contrast Defect} \equiv 1 - \text{Contrast} \approx 2 \cdot \frac{P_{\text{splitter dark port}}}{P_{\text{splitter bright port}}}$$



Conclusions: All of the families of mirror surfaces, *except* for the worst-case $\lambda/400$ mirrors, satisfy the Initial-LIGO contrast-defect requirement.

In addition, all cases of $\lambda/1200$ or better satisfied the (tentative) contrast-defect requirement for the *“Advanced-Subsystem” LIGO* interferometers.

Effects of Real-Mirror Deformations on Detection of GW's from Non-Axisymmetric Pulsars

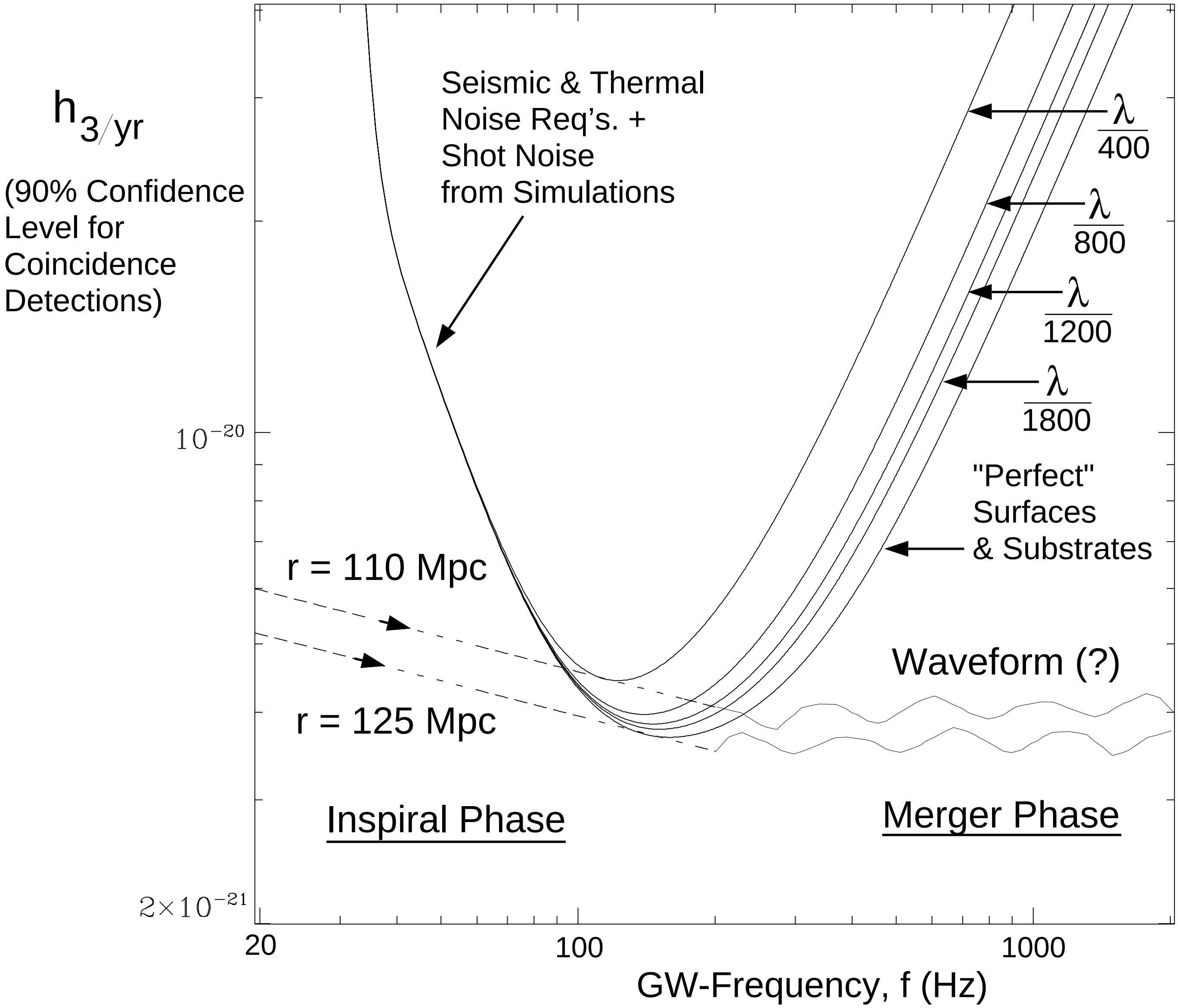


By improving from the worst mirror surfaces (& deformed substrates) to perfectly smooth (or nearly so) surface & substrate profiles, we gain either:

- A factor of 4 increase in sensitivity to the *ellipticity*, $\varepsilon \equiv (Q_{xx} - Q_{yy})/I_{zz}$, or:
- An *event rate increase* of ~ R^2 (for pulsars in galactic disk) ~ 16-fold!

Effects of Real-Mirror Deformations on Detection of GW's from Black Hole Binary Coalescences

$M_{BH\ \#1} = M_{BH\ \#2} = 10\ M_{\odot}$



The increase in "Lookout Distance" is small for worst vs. best mirror surfaces (~110 --> 125 Mpc), but the Event Rate ~ R^3 => ~50% increase.

››Could make the difference in enabling the Initial-LIGO detector to detect one or two of these exotic BH-BH coalescence events.

Conclusions:

- Numerical Simulations can be (& have been) used to drive specifications for LIGO optics.

- The Sensitivity Goals of the Initial-LIGO detector can be met with feasibly obtainable optics (assuming adequate mirror coatings).

- Substantial benefits to LIGO science can be obtained by procuring extremely-high-quality mirrors.