DCC
LIGO - . 60176

Previous abstract Next abstract

Session E9 - Gravity Experiments and Theory.
MIXED session, Friday morning, May 03
Room 103,

[E9.06] Prediction of Test Mass Thermal Noise by Measurement of the Anelastic Aftereffect

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The thermal noise from the internal modes of the test masses of interferometric gravitational wave detectors depends on the dissipation at the frequencies of interest. To date, predictions have been based on the Q's of resonances, all at frequencies higher than the expected signals. We have developed a method to determine the dissipation of test masses in the signal band, using the anelastic aftereffect, the creep J(\tau) of a test mass after a compressive stress has been released. The loss angle \phi(ømega) is approximately given by the logarithmic derivative of J(\tau) evaluated at \tau = 1/ømega. For a transparent material such as fused silica, a convenient way to measure J(\tau) is via the photoelastic effect. We will describe the apparatus that we have constructed, present measurements of the losses in a dummy test mass made from BK7 glass, and discuss the application of this method to LIGO test masses.

Part E of program listing

PREDICTION OF TEST MASS THERMAL NOISE BY MEASUREMENT OF THE ANELASTIC AFTEREFFECT

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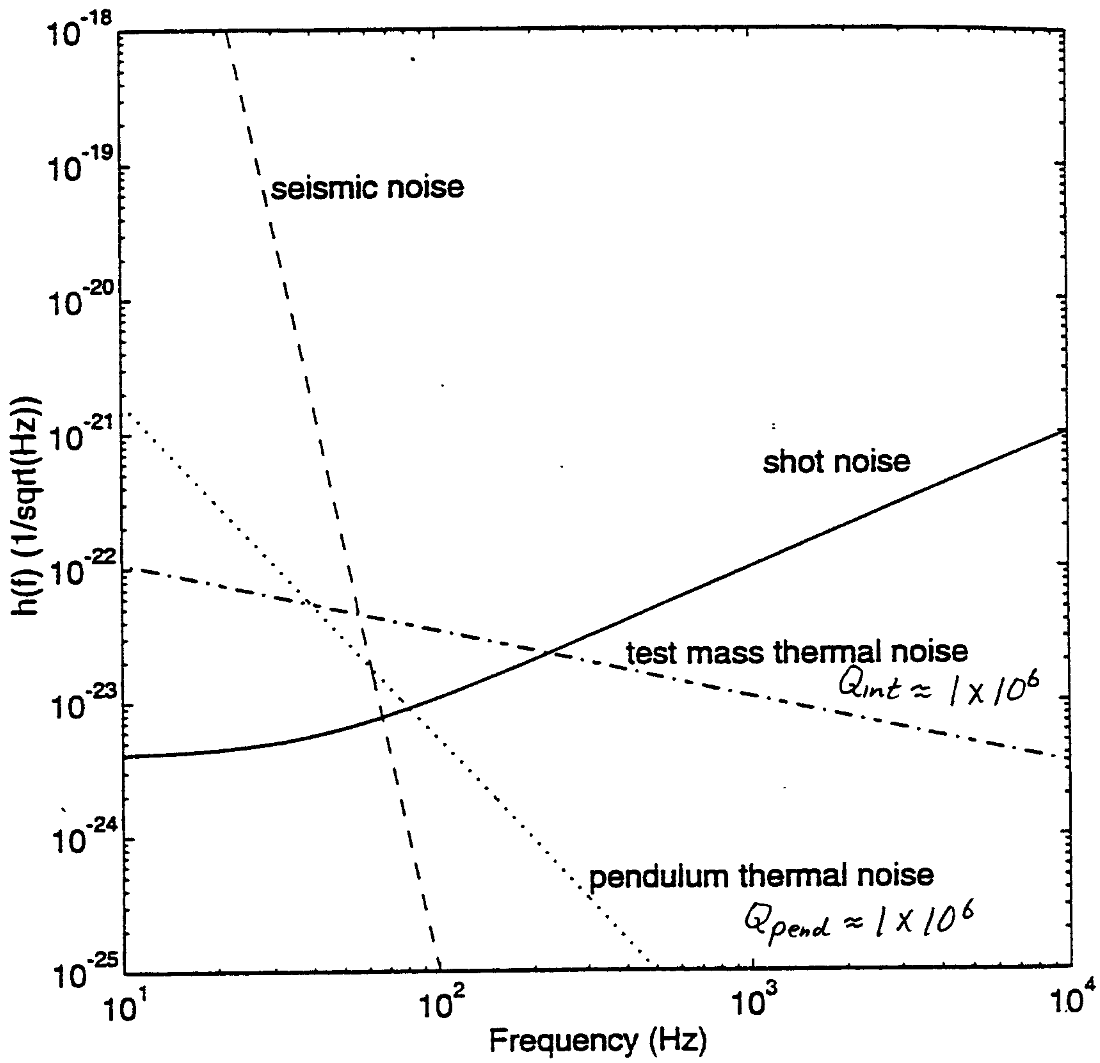
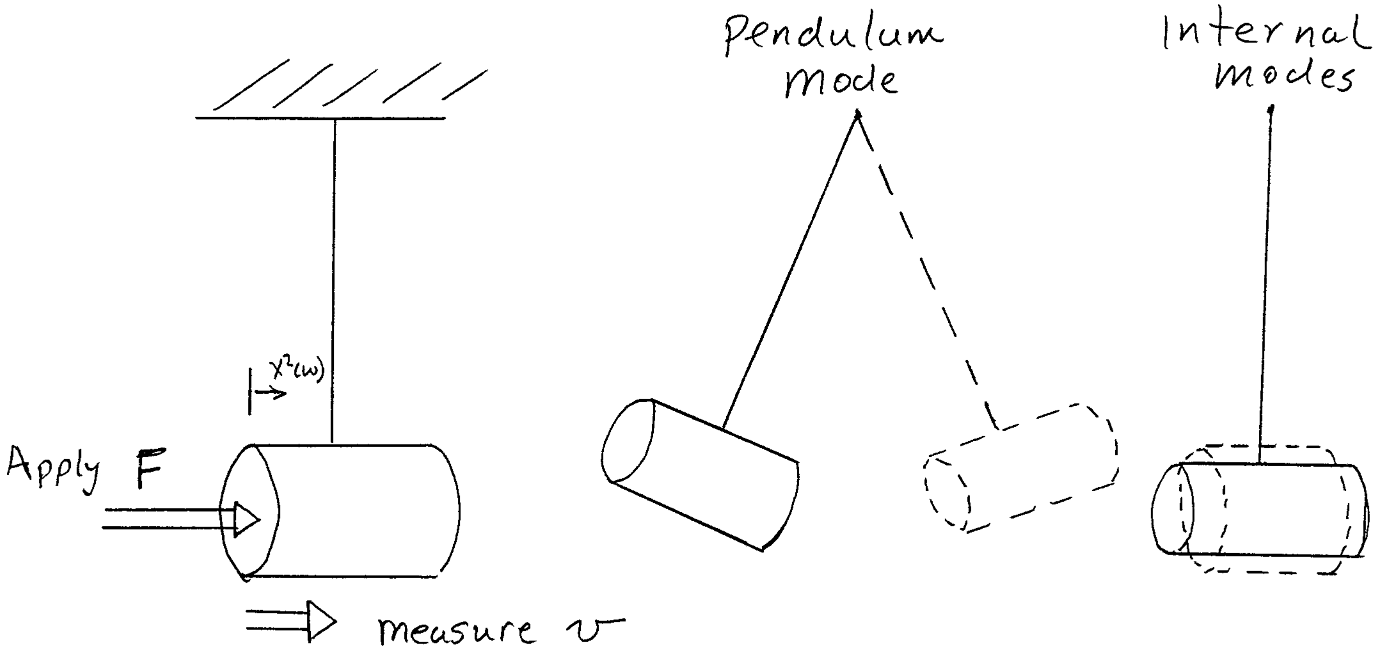


Figure 16.2 An estimate of the strain noise spectral density in an early LIGO interferometer.

Fluctuation-Dissipation Theorem

$$x^2(\omega) = \frac{4 k_B T}{\omega^2} \, \mathrm{Re}\left[-Y(\omega)\right]$$

admittance



$$Y(\omega) \equiv \frac{v(\omega)}{F(\omega)}$$

$$\Rightarrow \quad x^2(\omega) = \frac{4k_BT}{\omega k} \frac{\phi(\omega)}{\left[\left(1-\frac{\omega^2}{\omega_0^2}\right) + \phi^2(\omega)\right]}$$

ω_0 = resonant frequency

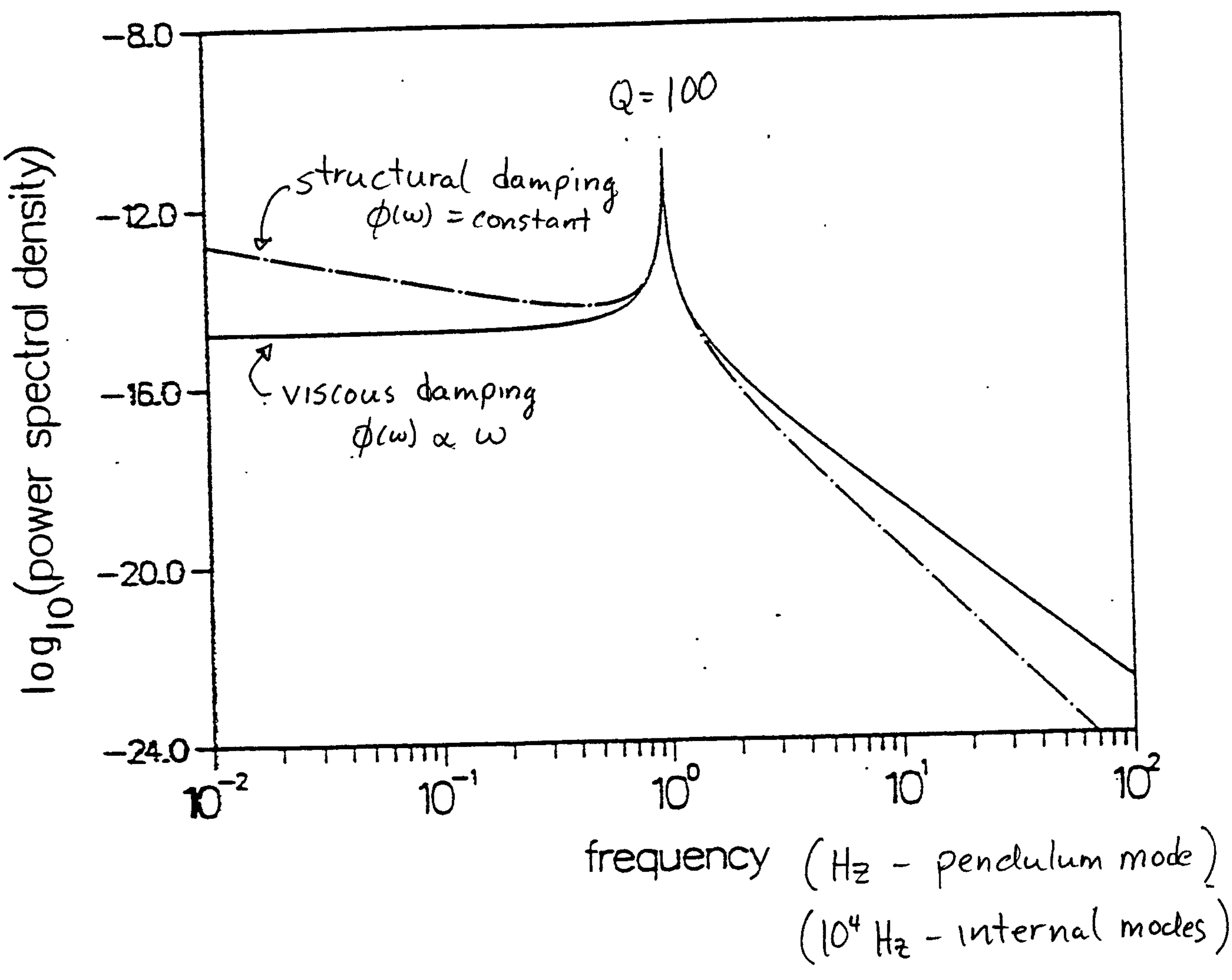
k = spring constant $= m\omega_0^2$

$\phi(\omega)$ represents the fractional imaginary part of the spring constant:

$$F = k(1 + i\phi(\omega))\, x$$

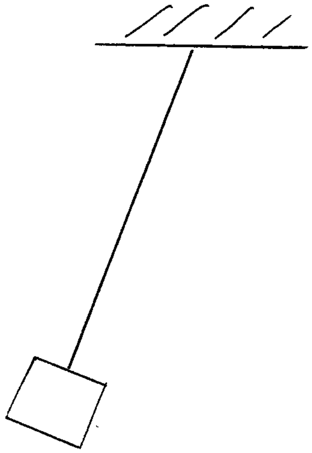
For sinusoidal F, the response x will lag the force by an angle $\phi(\omega)$

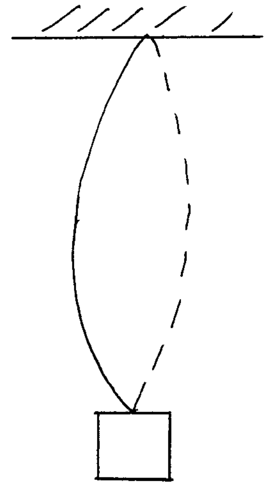
Also note that: $Q = \frac{1}{\phi(\omega_0)}$


Q = 100
structural damping
φ(ω) = constant
viscous damping
φ(ω) ∝ ω
-8.0
-12.0
-16.0
-20.0
-24.0
log10(power spectral density)
10^-2
10^-1
10^0
10^1
10^2
frequency (Hz - pendulum mode)
(10^4 Hz - internal modes)

$\phi(\omega)$ for Pendulum Mode
(Yinglie Huang - Syracuse University)

Can measure $\phi(\omega)$ in region of interest using <u>violin</u> modes



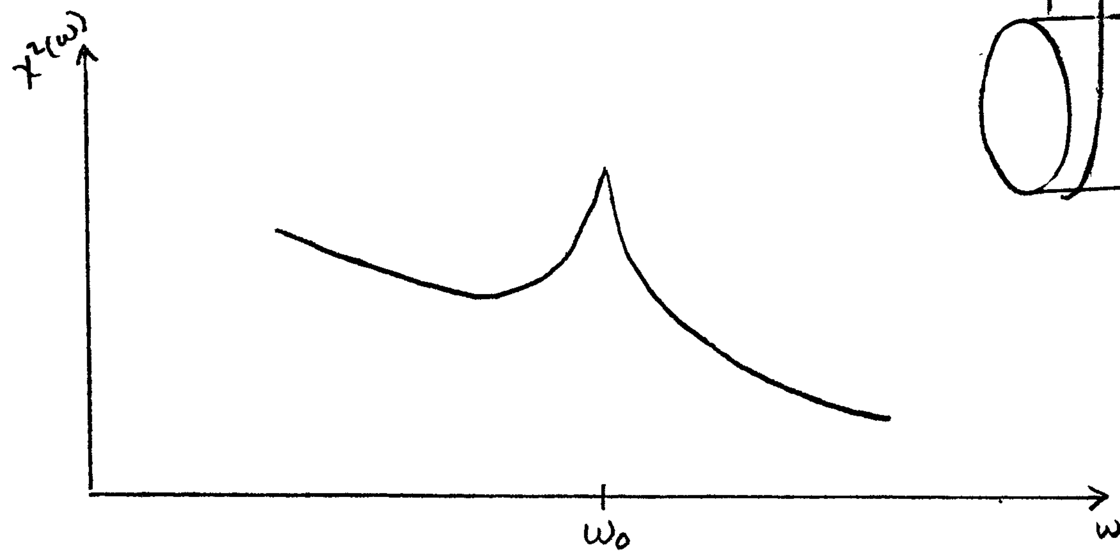


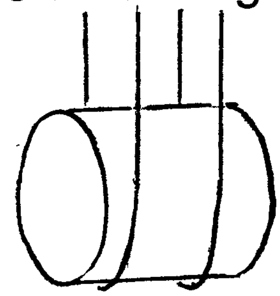
$f_n \approx 300 \sim 3000$ Hz

$$\phi(f_n) = \frac{1}{Q_n}$$

obtain Q_n by measuring ring down time

PROBLEM: Predict the thermal noise spectrum $x^2(\omega)$ for LIGO test masses in a frequency range 10 Hz to 1 kHz from measurement of internal friction (or loss angle) $\phi(\omega)$. This is well below the internal resonant modes which begin at about 10 kHz (by design).





COMPLICATION: Can NOT use the trick as used in measuring $\phi(\omega)$ for pendulum modes of the suspension, by measuring Q's of various violin modes, since test masses have no resonance in frequency range 10 Hz to 1 kHz.

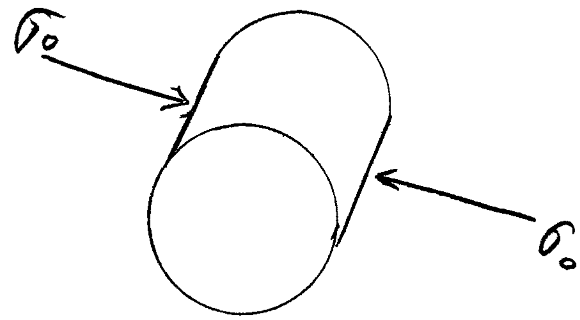
SOLUTION: Measure the anelastic aftereffect, after the sample is "instantaneously" relieved of stress.

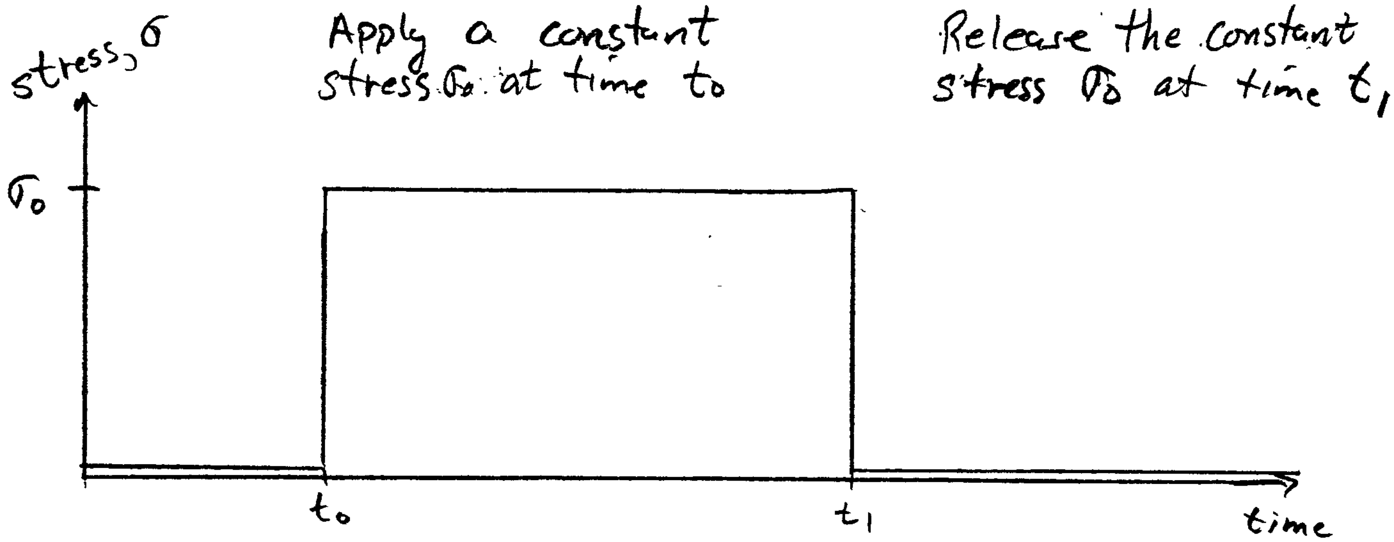
CREEP AND THE ANELASTIC AFTEREFFECT

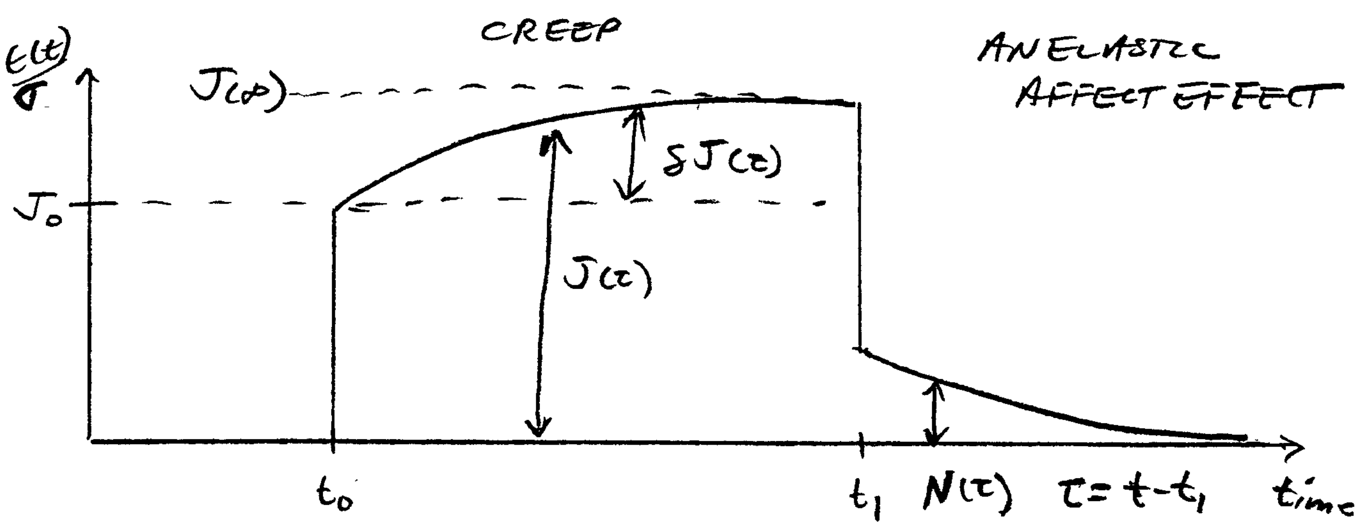
Hooke's Law:

$$\epsilon = J\,\sigma$$

strain — compliance — stress







The creep function $J(\tau)$ is defined:

$$J(\tau) \equiv \frac{\epsilon(\tau)}{\sigma_0} \qquad \tau = t - t_0 \qquad\qquad J(\tau) = J(\infty) - N(\tau)$$

RELATIONSHIP BETWEEN $\phi(\omega)$ AND $J(\tau)$:

$\phi(\omega)$ is well approximated by:

$$\phi(\omega) \approx \frac{\pi}{2} \frac{d(\ln J(\tau))}{d \ln \tau}\bigg|_{\tau = 1/\omega} \qquad (\phi << 1)$$

(A.S. Norwick & B.S. Berry, *Anelastic Relaxation in Crystalline Solids)*

The internal friction ϕ at some frequency ω is approximately equal to the fractional change in the strain of the sample on the corresponding time scale τ.

Transformation of *J(t)* from the time domain to the frequency domain:

$$J_1(\omega) - J_o = \omega \int_0^\infty (J(t) - J_o) \sin \omega t dt$$

$$J_2(\omega) = -\omega \int_0^\infty (J(t) - J_o) \cos \omega t dt$$

where $J^*(\omega) = J_1(\omega) - iJ_2(\omega)$

$$\phi(\omega) = \frac{J_2(\omega)}{J_1(\omega)} \qquad (\phi << 1)$$

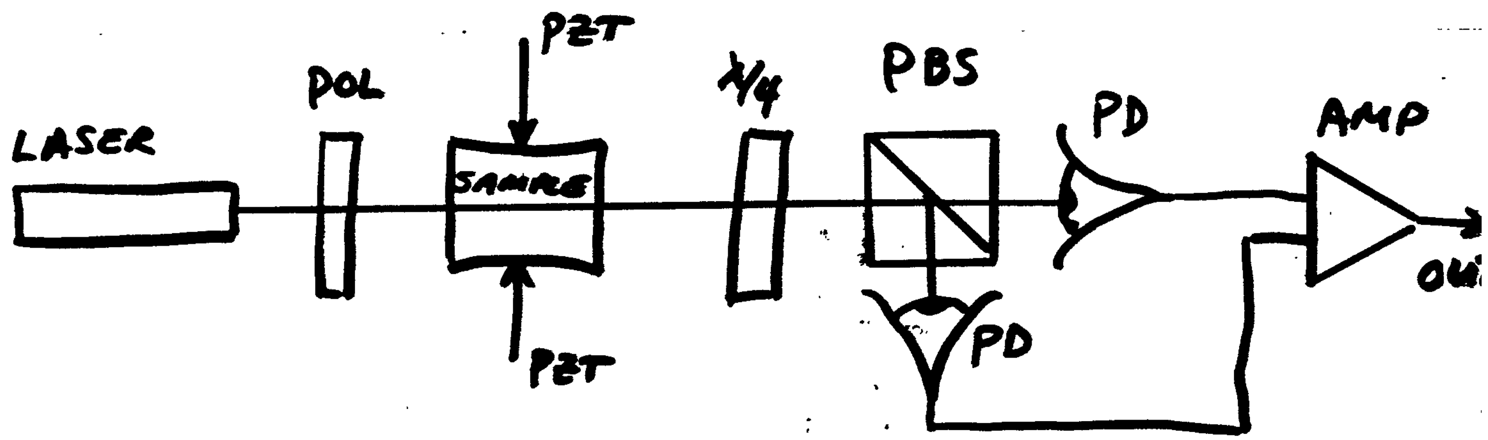
EXPERIMENTAL SET-UP

MEASURE $\epsilon(t)$ BY USE OF PHOTOELASTIC EFFECT (STRESS-DEPENDENT BIREFRINGENCE)

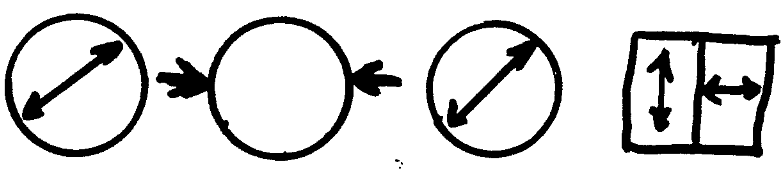
$$\Delta n_x = C \, \Delta \epsilon_x$$

constant

index of refraction

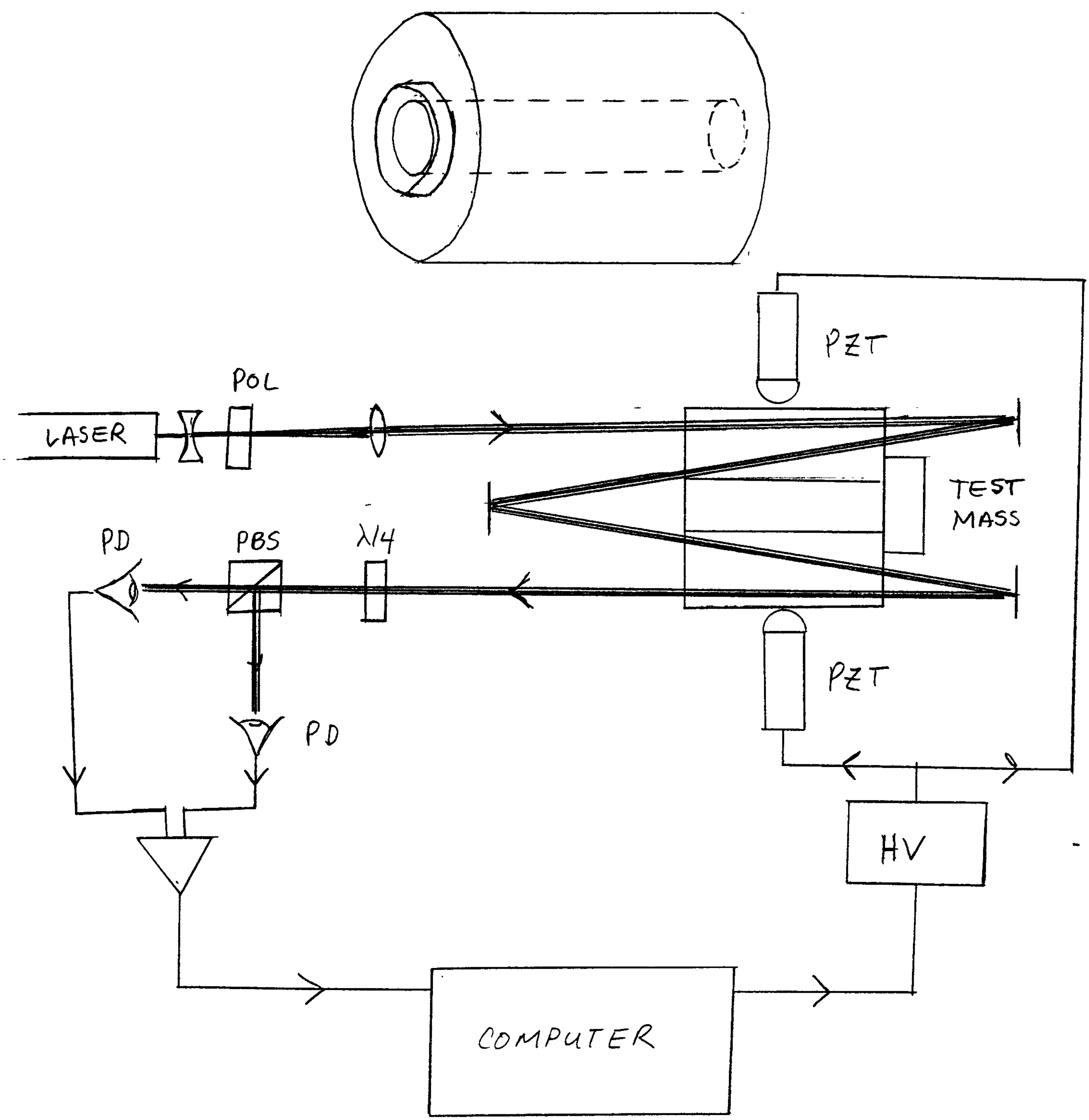


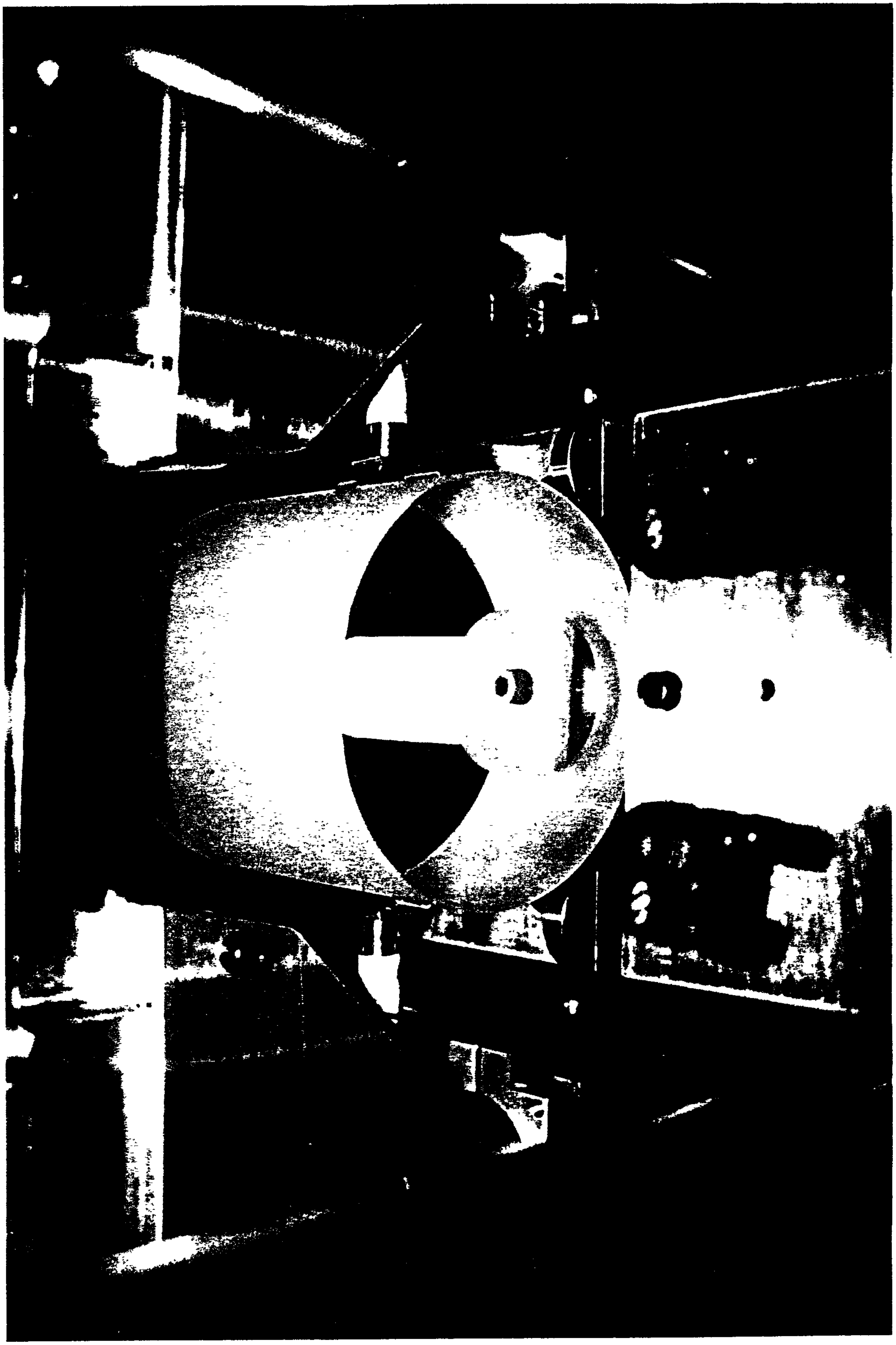
FRONT VIEW OF COMPONENTS

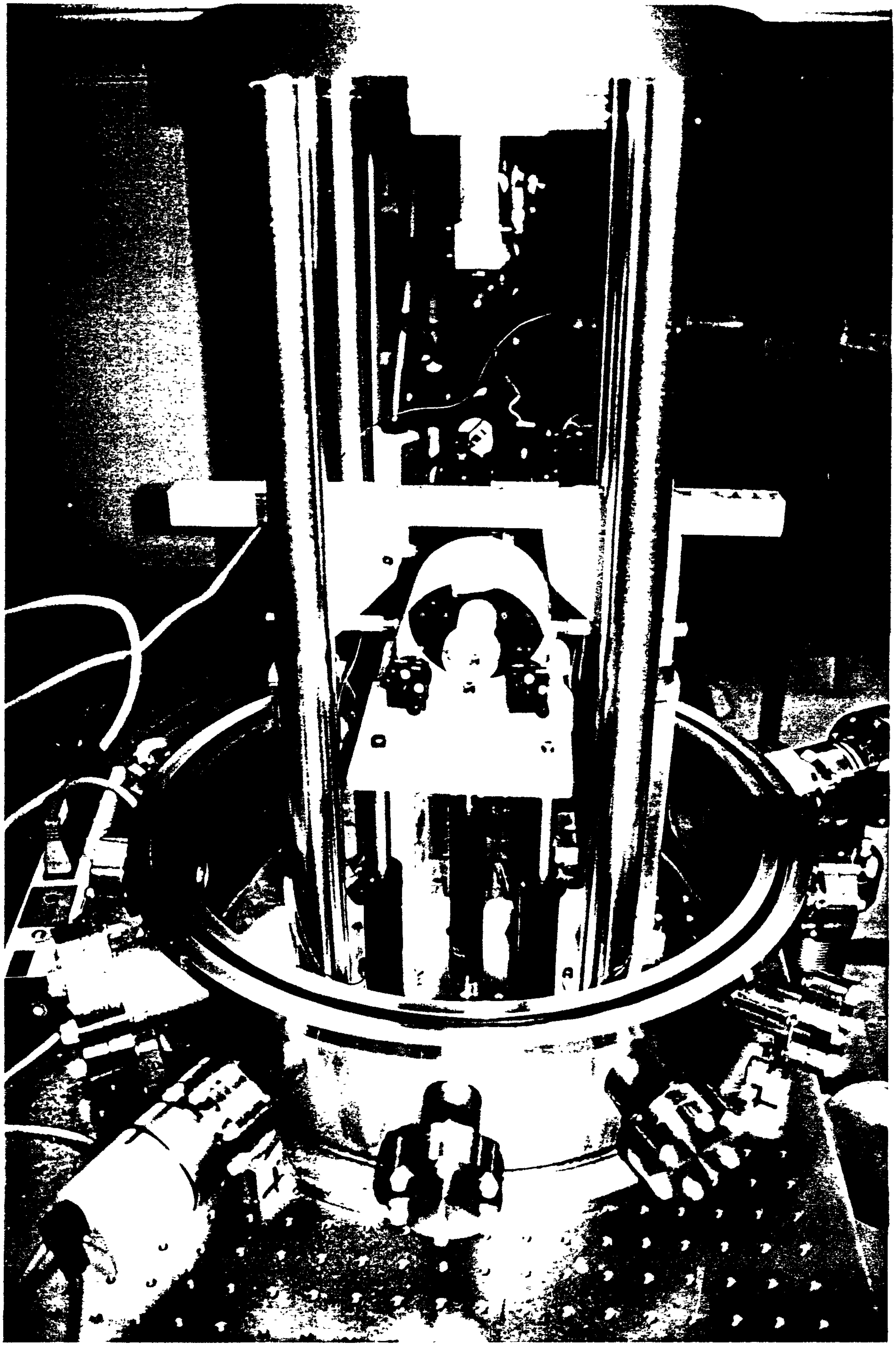


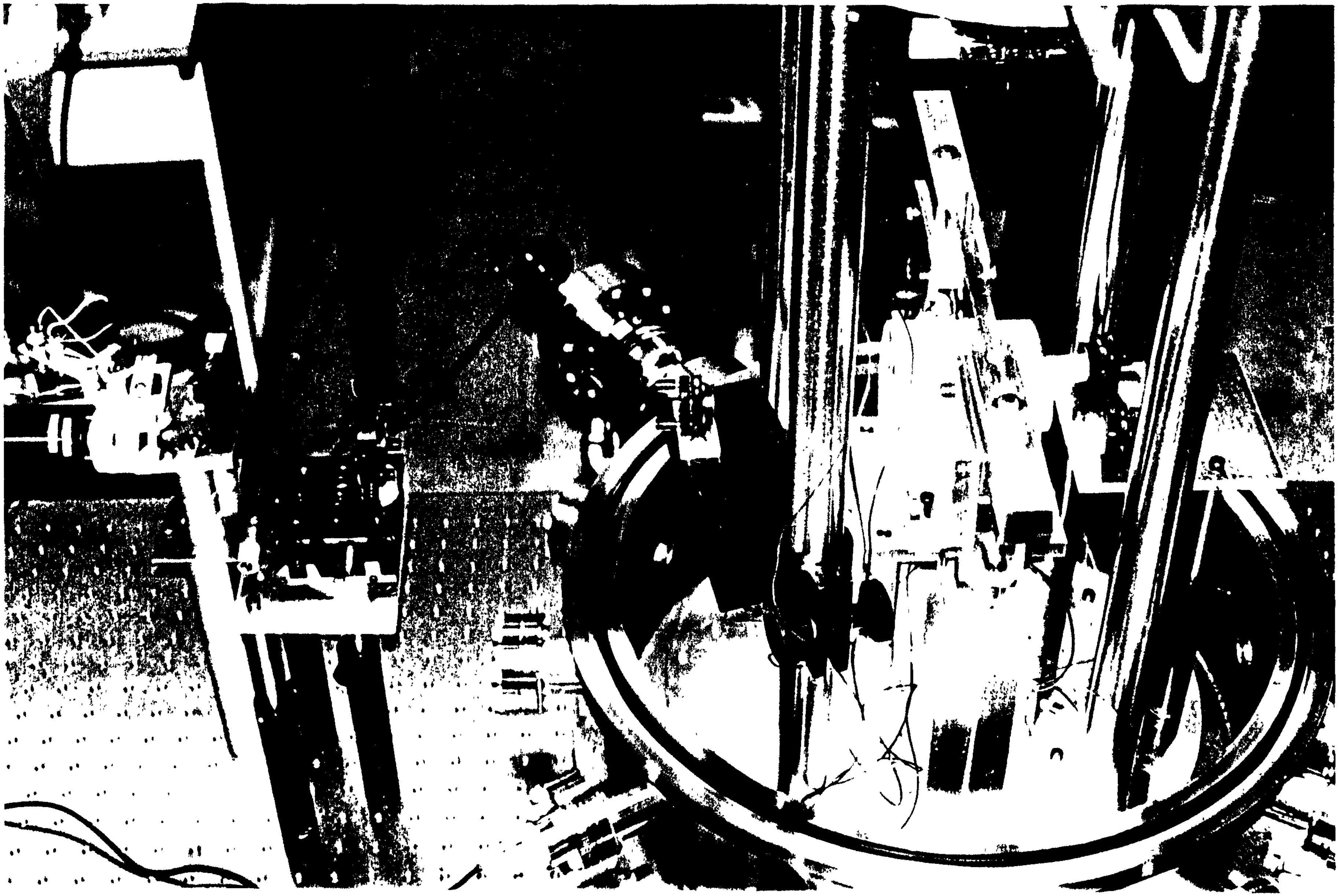
null output when birefrigence = 0
first order sensitive to birefringence

Multi-path set-up to measure $\phi(\omega)$ of pre-Oct 1994 LIGO prototype test mass

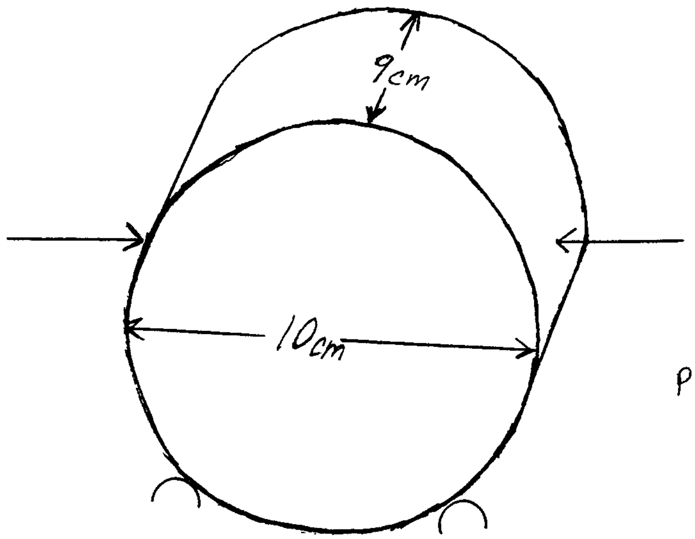




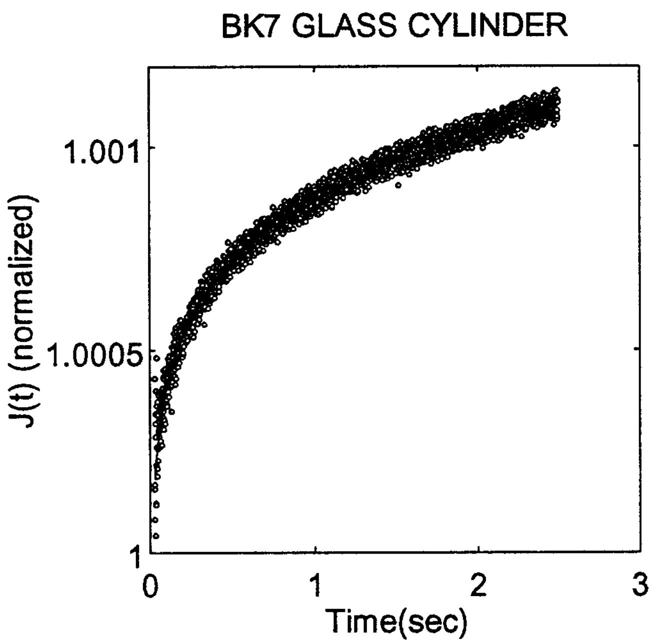


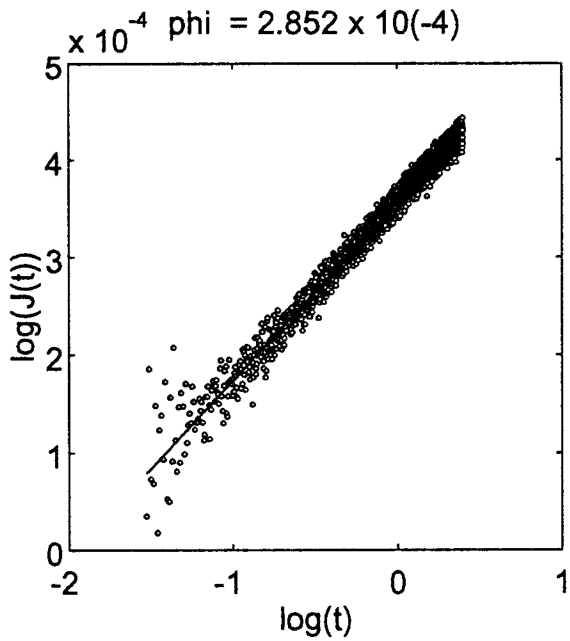



Newport
Newport

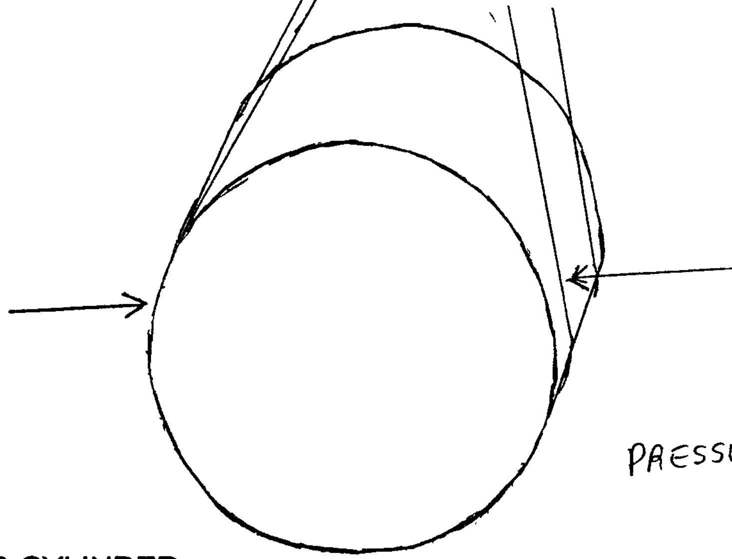


PRESSURE = ATM

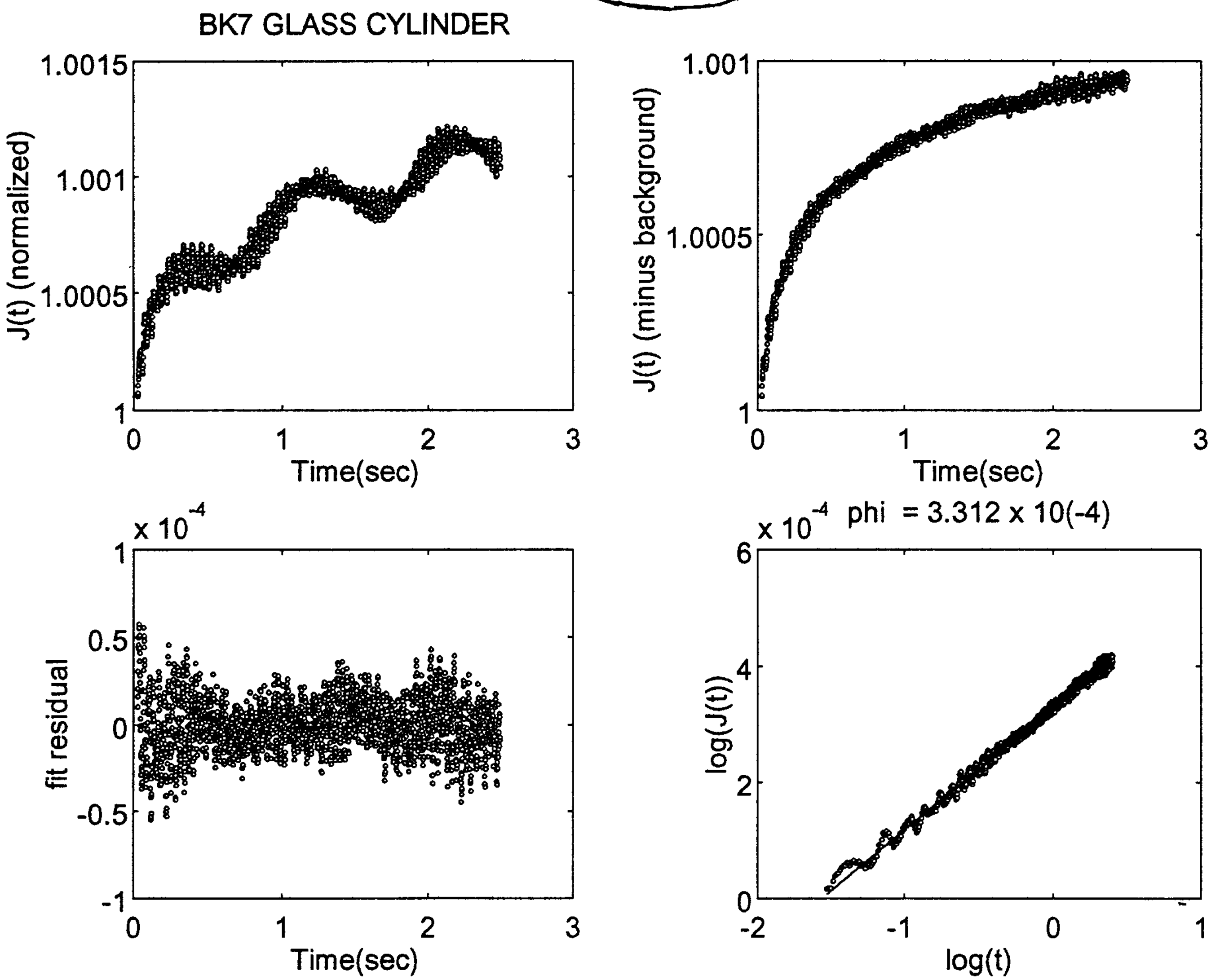




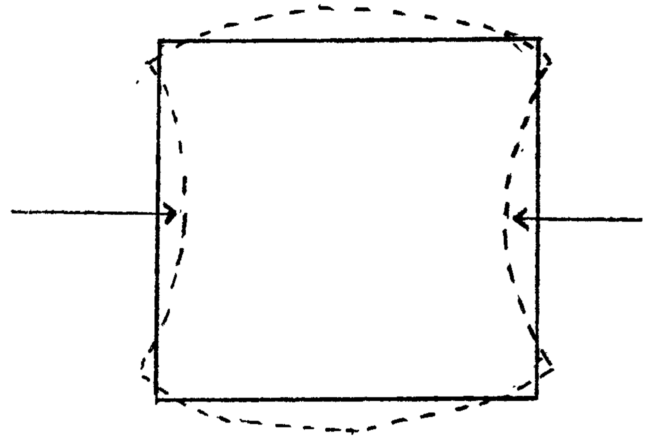
$\phi = (2.94 \pm 0.16) \times 10^{-4}$ 5 Hz → 0.06 Hz



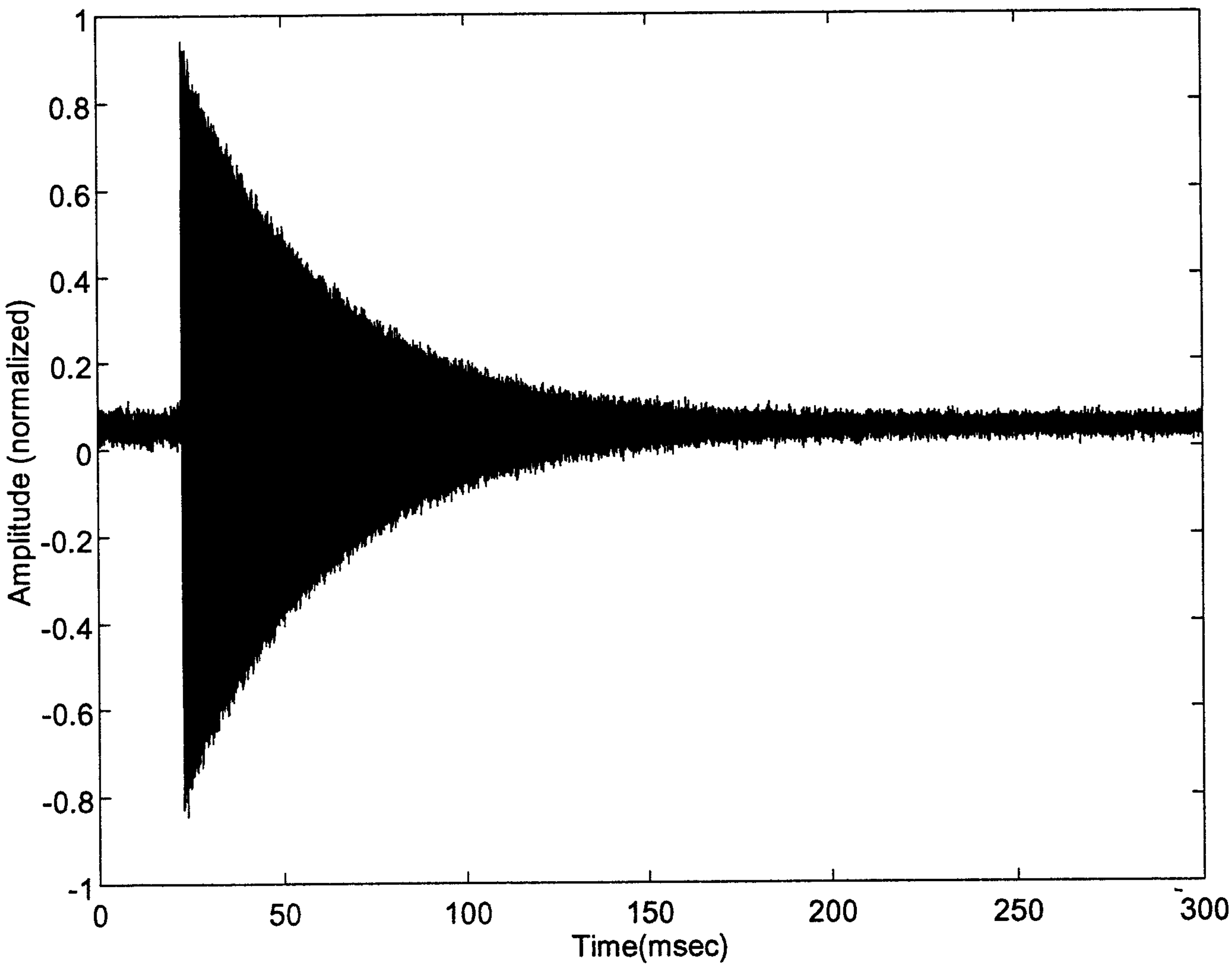
PRESSURE = ATM



$\phi = (3.22 \pm 0.09) \times 10^{-4}$ 5 Hz $\rightarrow$ 0.06 Hz



BK7 GLASS CYLINDER

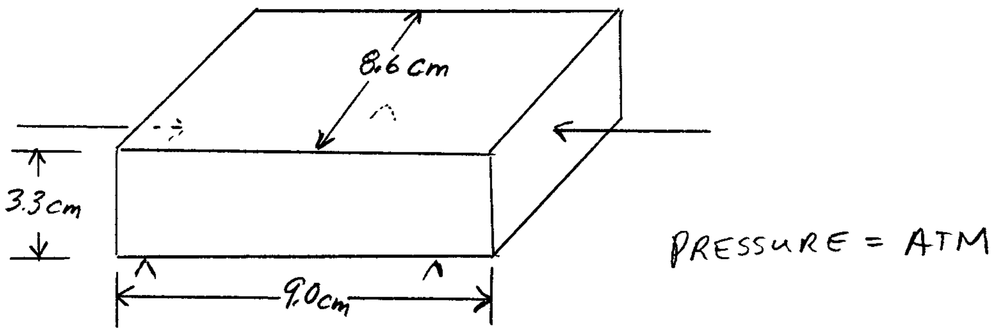


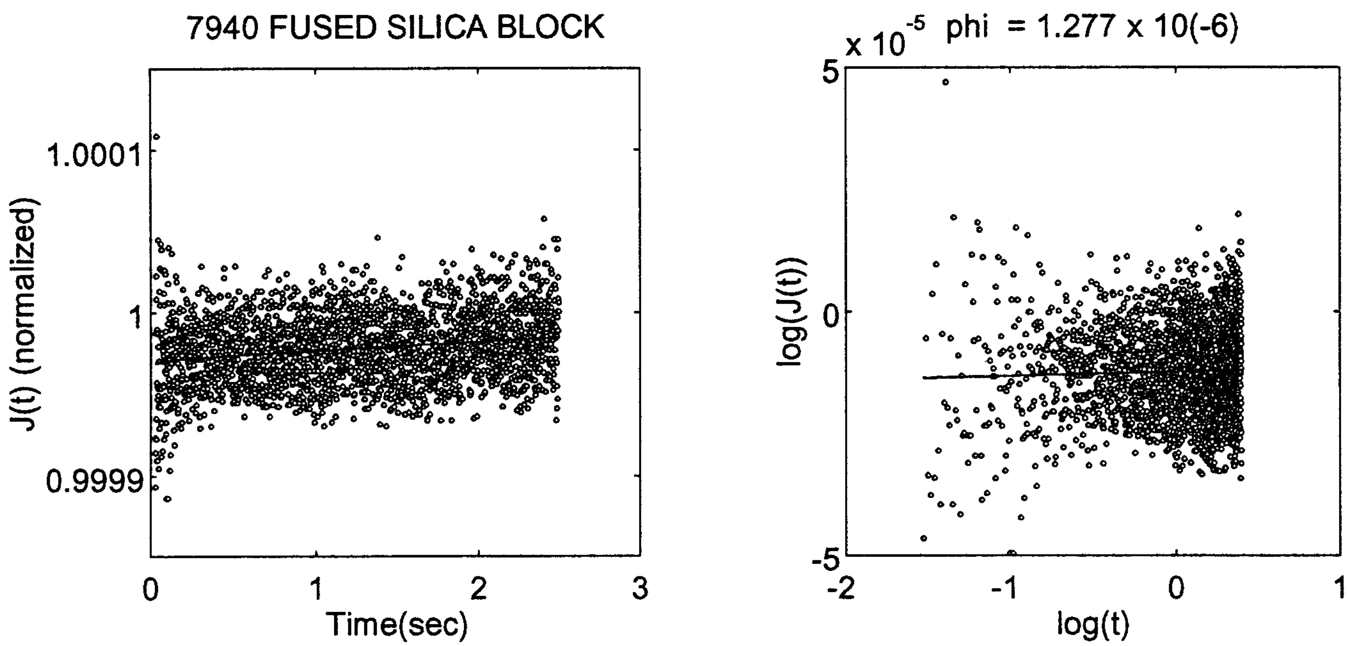
$$\tau_{damp} \approx 0.040 \text{ sec}$$

$$f_{resonance} = 26,900 \text{ Hz}$$

$$Q = 3.4 \times 10^3$$

$$\frac{1}{Q} = \phi = 2.9 \times 10^{-4}$$





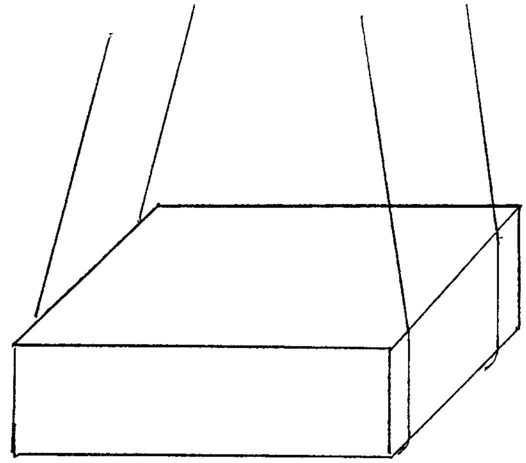
$\phi = (5.4 \pm 4.7) \times 10^{-6}$ (on 3 point mount in ATM)

5Hz $\rightarrow$ 0.06 Hz

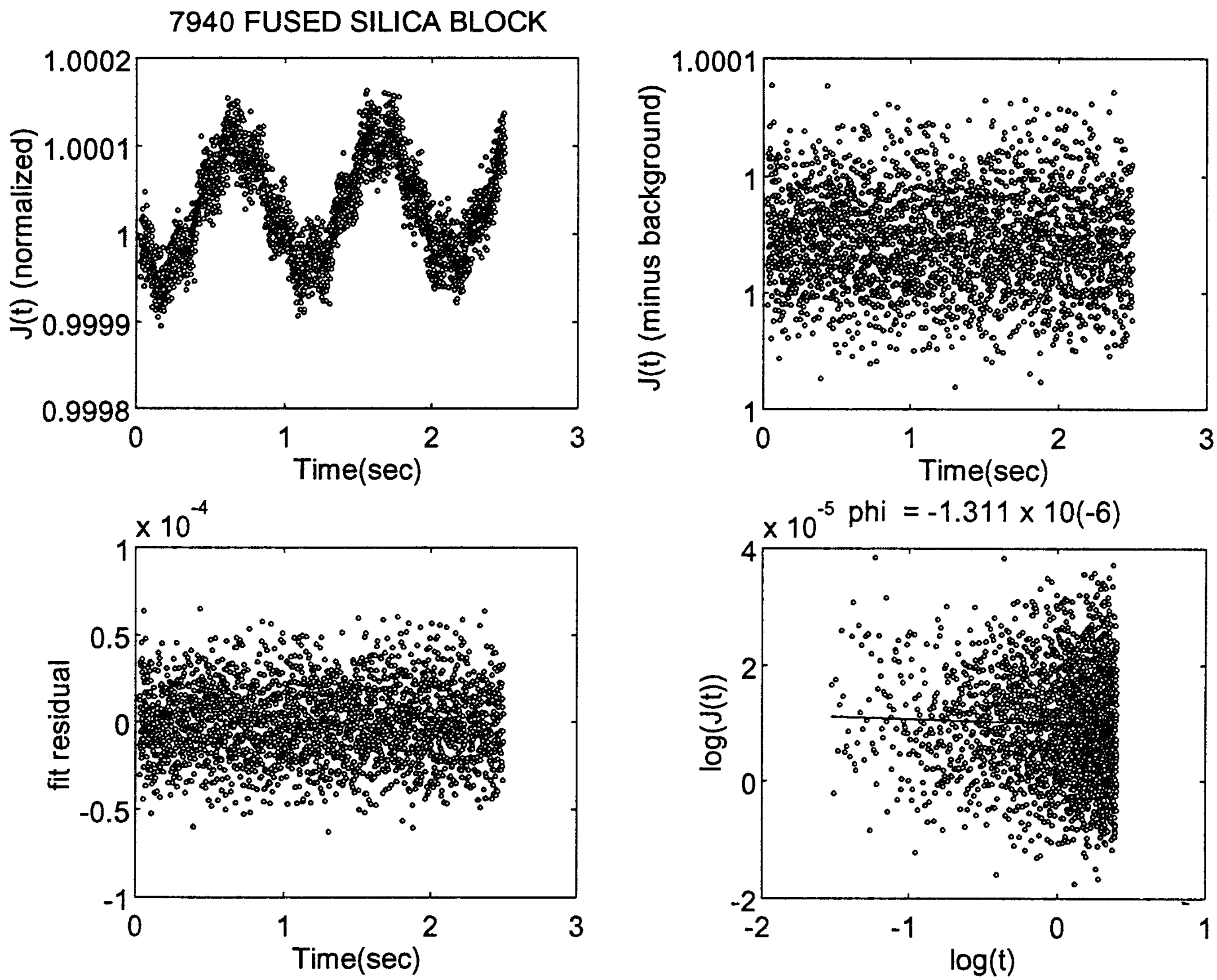
Q @ $f_0 \approx 6 \times 10^3$ (on 3 point mount in ATM)

Q @ $f_0 \gtrsim 5 \times 10^5$ (wire suspension @ 10^{-4} torr)

$f_0 \approx 3 \times 10^4$ Hz



PRESSURE = ATM



$\phi = (-0.6 \pm 3.6) \times 10^{-6}$ (wire suspension in ATM)

5Hz → 0.06 Hz

$Q @ f_0 \approx 2 \times 10^4$ (wire suspension in ATM)

$Q @ f_0 \gtrsim 5 \times 10^5$ (wire suspension @ 10^{-4} torr)

$f_0 \approx 3 \times 10^4$ Hz

SOURCES OF EXPERIMENTAL ERROR:

- excitation of mechanical modes of sample (internal, pendulum, torsional, bounce) and of measurement system (laser, optics, detector)

- electronic noise, and systematic effects from electronics

- seismic noise

- laser noise

- shot noise

NEAR FUTURE PLANS (next several months)

- improved rigid mounts for laser and optics, and installation of larger surface area photo-diodes.

- measure $\phi(\omega)$ for pre - Oct. 1994 LIGO test mass.

- test system with a fused silica grade OA cylinder.

FAR FUTURE PLANS

- Improve the measurement technique of $\phi(\omega)$ to 10^{-6} or better at frequencies up to 100 Hz, so $\phi(\omega)$ of other low internal friction optically transparent samples can be measured.

- design an anelastic aftereffect measuring apparatus that can be used to test actual test masses as suspended on the full scale LIGO.