Control of a Laser Interferometer

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January 12, 1994

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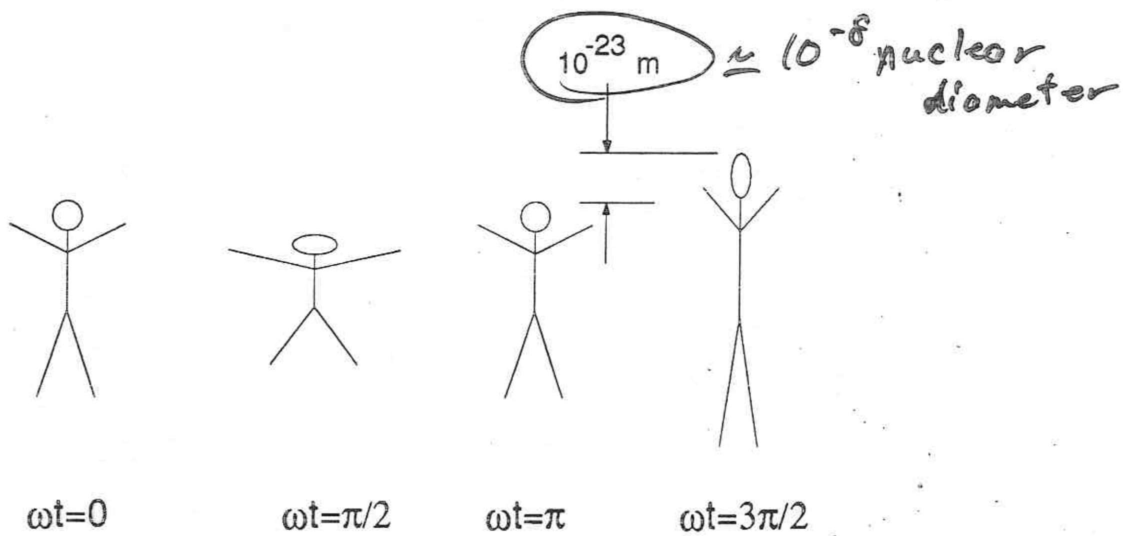
Gravitational Waves

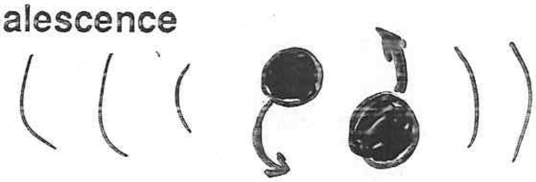
- Predicted by Einstein's General Theory of Relativity
- emitted by large, rapidly accelerating masses
- exceedingly small effect at earth

In plane perpendicular to direction of propagation, gravitational wave squeezes matter along one axis and compresses it along the orthogonal axis.



Why try to detect/observe Gravitational Waves?

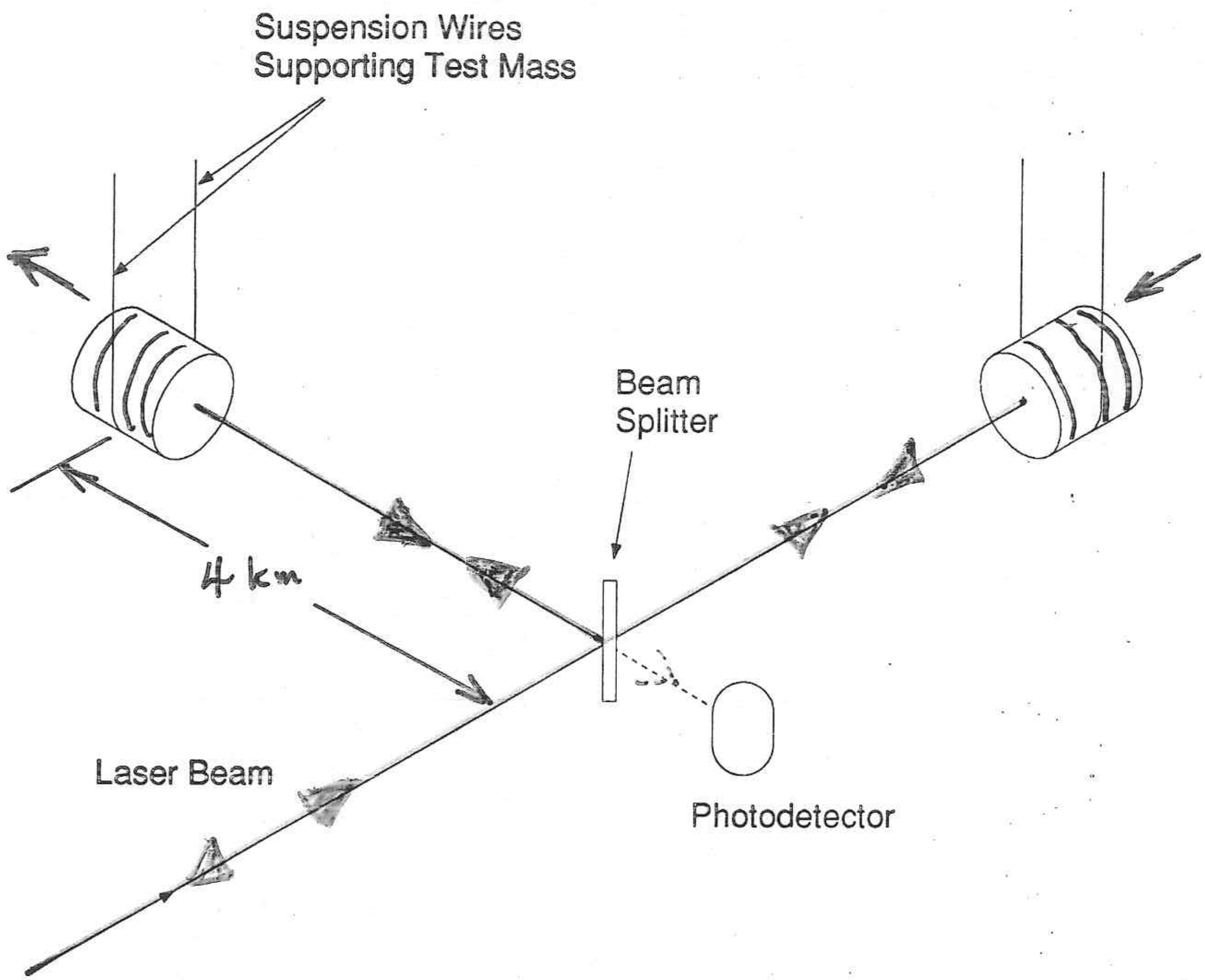
- Direct confirmation of Einstein's theory
- Can observe phenomena believed to exist but not observable optically
  - •Example: neutron-star-binary coalescence



- Many startling new discoveries in astronomy made soon after development of some new observational method

Interferometric detectors

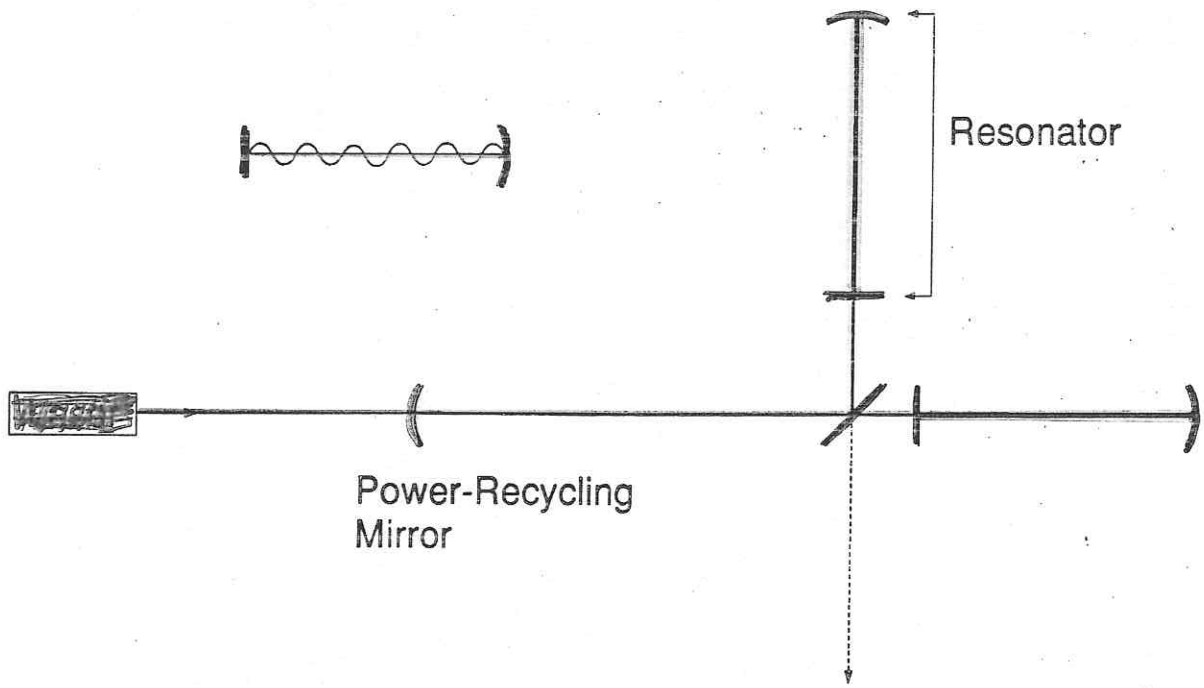
Simplest type: Michelson Interferometer



- Mirrors supended in wire loops to provide seismic isolation at high frequencies (> 100 Hz)
- Operated in vacuum to protect from noise associated with air

Gravitational Wave pushes two end mirrors in different directions, changing interference condition at beam splitter.

Increase sensitivity by replacing end mirrors with optical resonators; increase power by installing a "recycling mirror".



Recall: all mirrors are suspended from wires; swing around because of seismic ground motion (typically 4×10^{-7} m_{rms}).

Four resonance conditions need to be met:

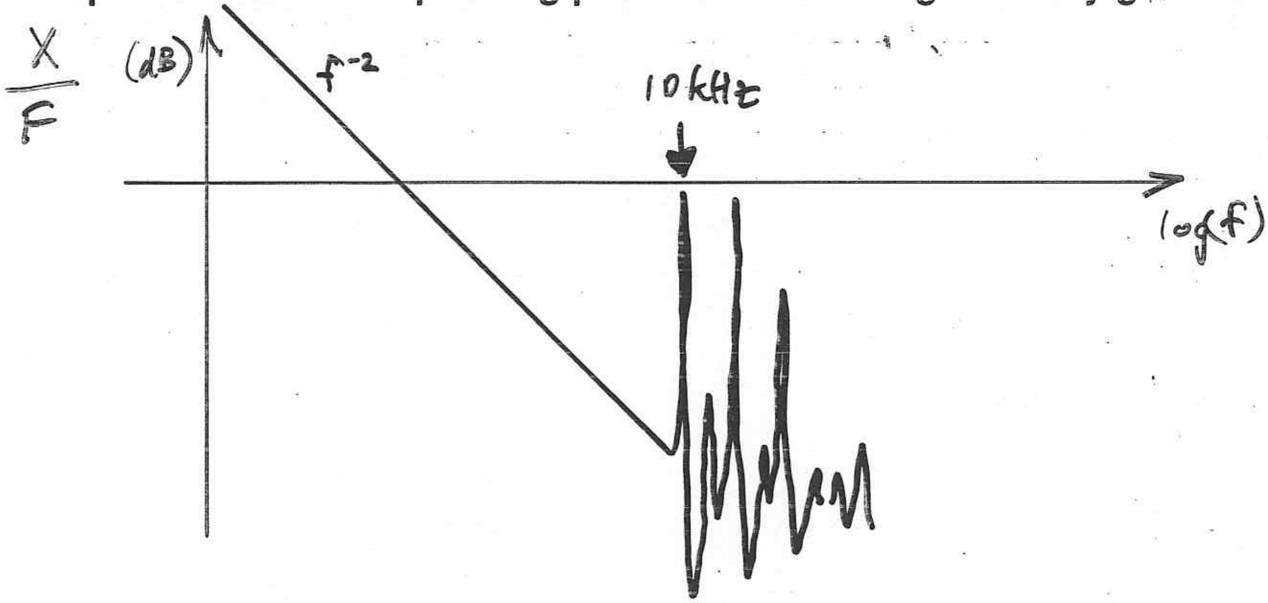
- one for each resonator (round trip phase of $2n\pi$)
- interferometer output dark
- light reflected back into interferometer by recycling mirror

the corresponding mirror positions need to be controlled to about $5 \times 10^{-11} m_{rms}$.

Can enforce these resonance conditions by pushing on the mirrors (magnets glued to the mirrors, and nearby coils) and by changing laser frequency.

Suspended Mirror Resonances

The cylindrical fused quartz mirrors have very high-Q (typ. 10^6 to 10^7) resonances at 10 kHz and above. Servos driving a test mass must have a loop gain (loop broken at the test mass) sufficiently attenuated to prevent the corresponding peaks from crossing the unity-gain axis.



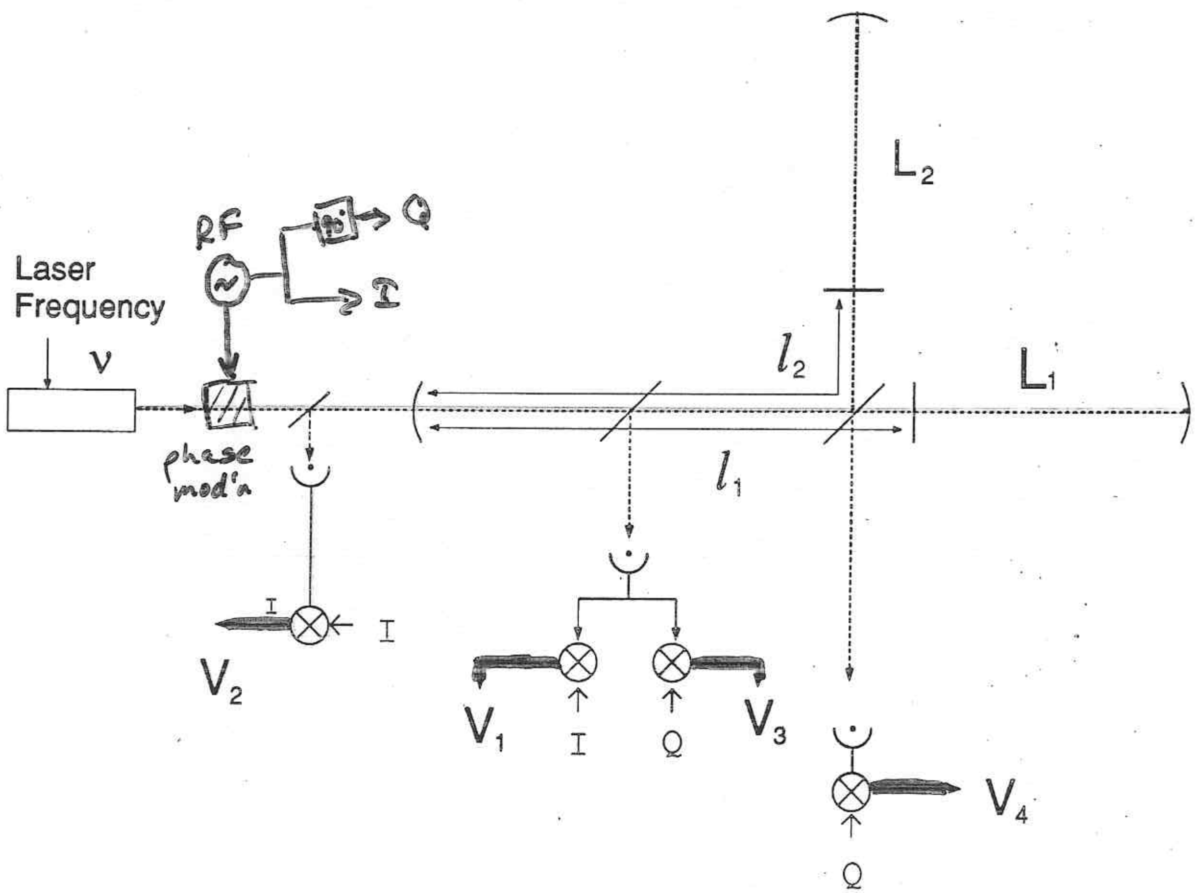
Optical Resonances

at frequencies above about ~~40~~ 20 kHz, optical resonances produce sharp features in some of the plant transfer functions. The ones of interest are the response to laser frequency changes (since the other plant inputs are already limited by mechanical resonances); these show sharp *notches.*

Figure: Response to laser frequency fluctuations at pickoff output

Sensing Scheme

Need to sense deviations from resonance in order to be able to correct them. Phase modulate input laser beam at high frequency (typically about 10 MHz); demodulate photocurrents at certain optical outputs of interferometer.



$$V_1 = \delta\nu + \varepsilon_1(\delta l_1 + \delta l_2)$$
$$V_2 = \delta\nu + \varepsilon_2(\delta l_1 + \delta l_2)$$
$$V_3 = (\delta l_1 - \delta l_2) + \varepsilon_3(\delta L_1 - \delta L_2)$$
$$V_4 = (\delta L_1 - \delta L_2) + \varepsilon_4(\delta l_1 - \delta l_2)$$

$$|\varepsilon_i| \simeq \frac{1}{100}$$

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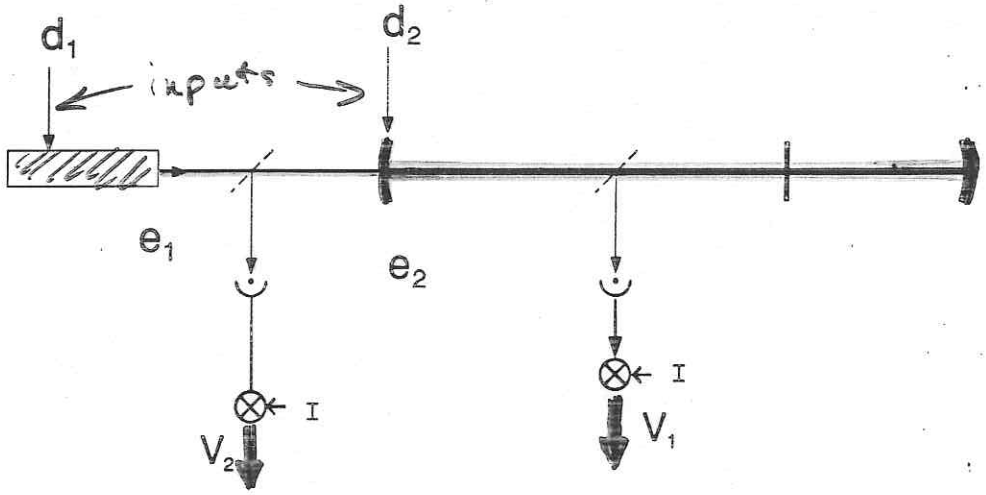
For

$$\vec{V} = P \begin{bmatrix} \delta\nu \\ \delta l_1 + \delta l_2 \\ \delta l_1 - \delta l_2 \\ \delta L_1 + \delta L_2 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & \varepsilon_1 & 0 & 0 \\ 1 & \varepsilon_2 & 0 & 0 \\ 0 & 0 & 1 & \varepsilon_3 \\ 0 & 0 & \varepsilon_4 & 1 \end{bmatrix}$$

The cost of poor conditioning

Look at upper 2x2 sub-plant. Corresponds to common-mode sub-system



$$P = \begin{bmatrix} 1 & \varepsilon_1 \\ 1 & \varepsilon_2 \end{bmatrix}$$

$$C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}$$

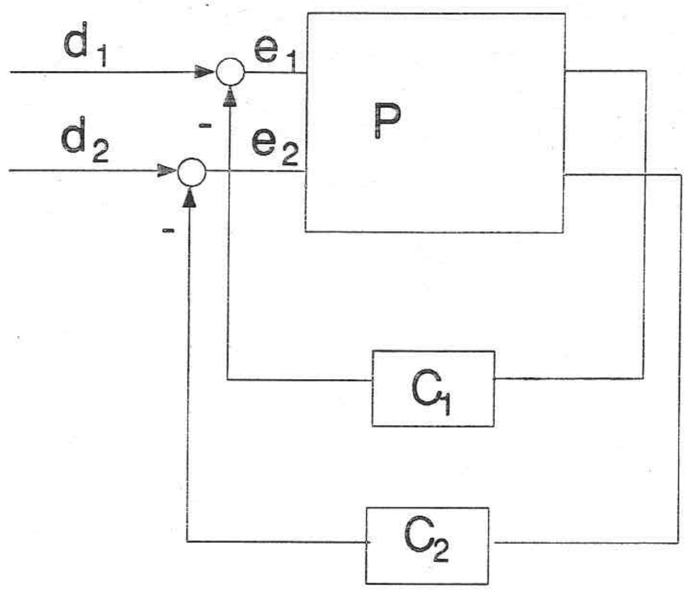
$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = [1 + CP]^{-1} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

Assume

$$d_1 \simeq d_2$$, specs on $\overline{e_1^2}, \overline{e_2^2}$ about the same

Question: what would be advantage of installing sensor system providing a better-conditioned plant?

Look at suppression of seismic noise in loop 2. (more critical loop)



Notation:

let

$$S = [1 + CP]^{-1}$$

Then

$$\vec{e} = S\vec{d}$$

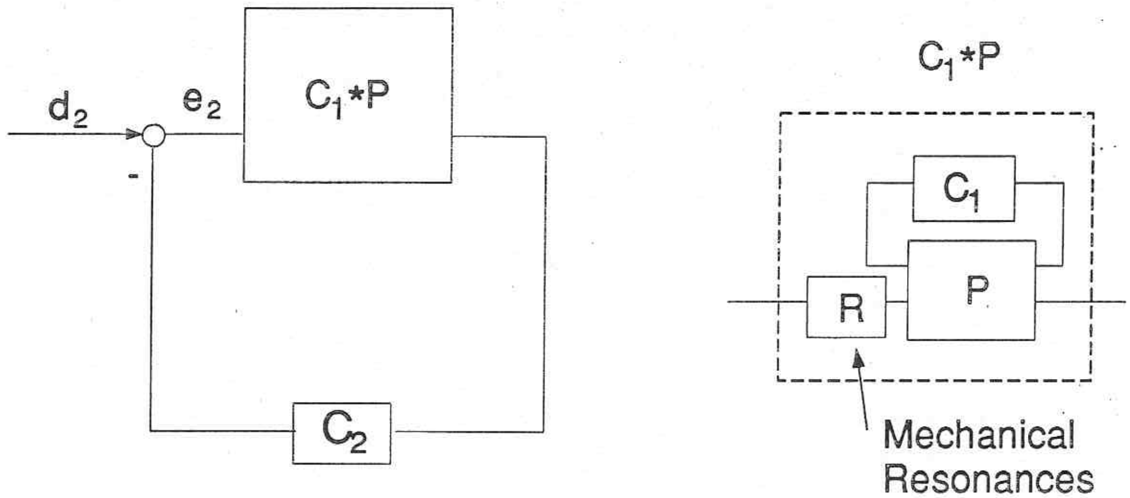
In particular,

$$e_2 = S_{21}d_1 + S_{22}d_2$$

Now

$$S_{22}$$

is limited by actuator dynamic uncertainties (mechanical resonances):



Better sensing scheme will not reduce S_{22} (assuming $C_1 * P$ is well-behaved within the bandwidth of the C_2 loop). S_{21} can be reduced by using a better conditioned sensing scheme. This will improve performance if

$$S_{21} \gtrsim S_{22}$$

Let's look at ratio

$$\frac{S_{21}}{S_{22}}$$

We know that

$$[1+CP]^{-1} = \frac{1}{\Delta}\begin{bmatrix} 1+C_2\varepsilon_2 & -C_1\varepsilon_1 \\ -C_2 & 1+C_1 \end{bmatrix}$$

where

$$\begin{aligned}\Delta &\equiv \det\,[1+CP]^{-1} \\ &= 1+C_1+\varepsilon_2 C_2 + C_1 C_2(\varepsilon_2-\varepsilon_1)\end{aligned}$$

Now the ratio of the residual motion which could be reduced by a different sensing scheme to residual motion which can not be reduced in this manner is:

$$\frac{S_{21}}{S_{22}} = \frac{-C_2}{1+C_1}$$

$$\frac{S_{21}}{S_{22}} = \frac{-C_2}{1 + C_1}$$

Not very illuminating. Don't know what values to give C_1 and C_2. Would like to write in terms of the loop gains. We have a better idea of what the loop gains will be because they are limited only by the bandwidth-limiting plant uncertainties.

Let

$$\begin{aligned} G_1 &= C_1(P * C_2) \\ &= C_1\left(1 - \frac{\varepsilon_1 C_2}{1 + C_2\varepsilon_2}\right) \\ &\simeq C_1\left(1 - \frac{\varepsilon_1}{\varepsilon_2}\right) \end{aligned}$$

and

$$\begin{aligned} G_2 &= C_2(C_1 * P) \\ &= C_2\left(\varepsilon_2 - \frac{\varepsilon_1 C_1}{1 + C_1}\right) \\ &\simeq C_2(\varepsilon_2 - \varepsilon_1) \end{aligned}$$

Note

$$\begin{aligned} G_1 &\simeq 1 \quad @10\text{kHz} \\ G_2 &\simeq 1 \quad @100\text{Hz} \end{aligned}$$

← limited by optical resonances

↑ limited by mechanical resonances

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LIGO

Then

$$\left|\frac{S_{21}}{S_{22}}\right| = \frac{C_2}{1+C_1}$$

$$\simeq \frac{G_2}{G_1\varepsilon_2}$$

Performance is not compromised if

$$\boxed{G_1 > \frac{G_2}{\varepsilon_2}}$$

Similarly, can show that

$$\left|\frac{S_{12}}{S_{11}}\right| = \frac{C_1}{1 + C_2\varepsilon_2}$$

$$\simeq \frac{G_1\varepsilon_1}{G_2}$$

(when

$$G_1 \gtrsim \frac{G_2}{\varepsilon_1},$$

performance is degraded in loop 1)

and

$$\left|\frac{S_{12}}{S_{22}}\right| = \frac{C_1\varepsilon_1}{1 + C_1}$$

$$\simeq \varepsilon_1$$

(when G_1 is large, performance in loop 1 is nonetheless better than performance in loop 2 by a factor of ε_1).

Conclusion

If

$$G_1 \gtrsim \frac{G_2}{\varepsilon_2}$$

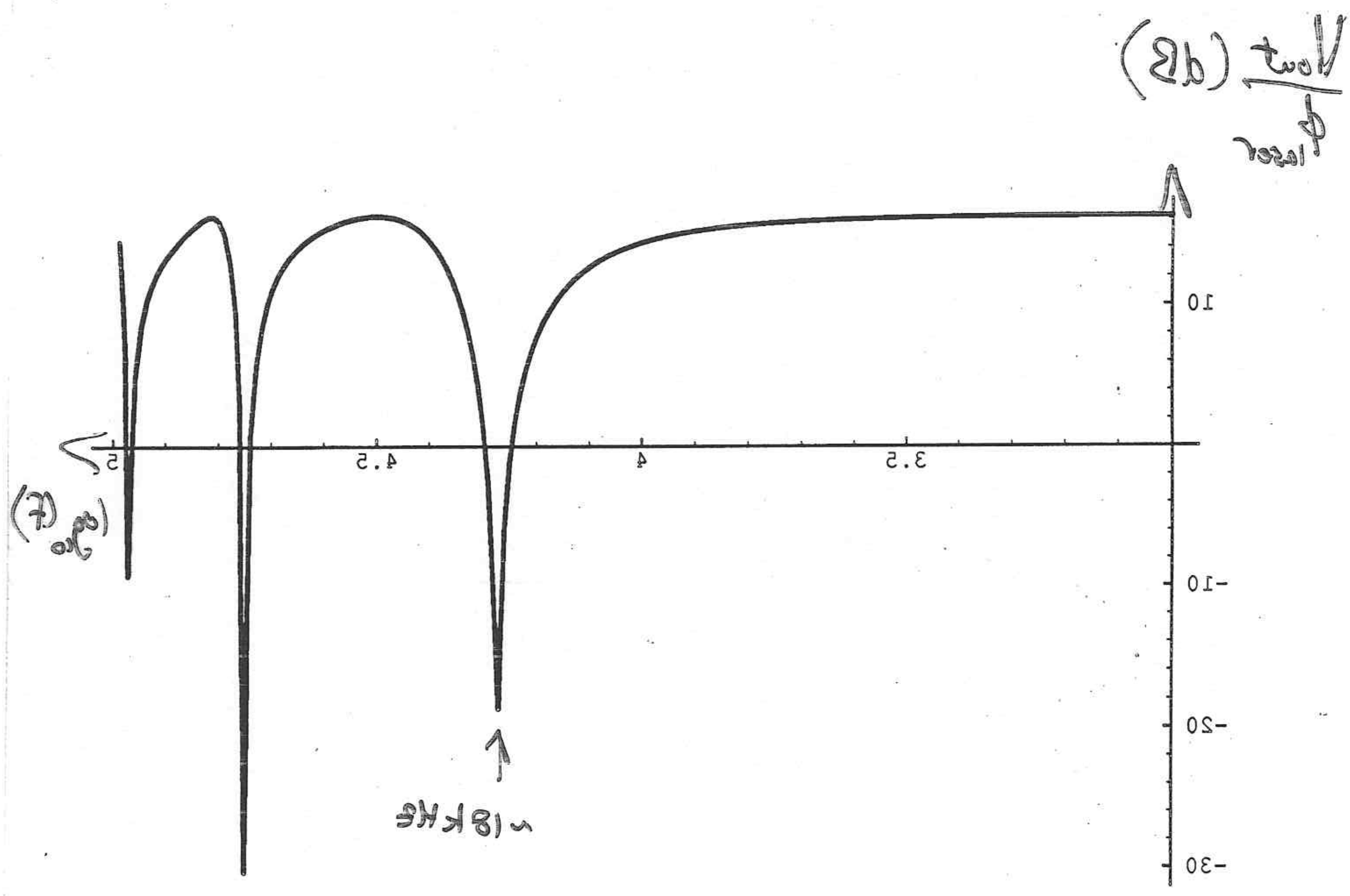
then performance at e_2 is not degraded by poor plant conditioning.

Performance in loop 1 in this case is limited to being better than performance in loop 2 by a factor of

$$\frac{1}{\varepsilon_1}$$

•Adequate performance for LIGO without "inverting" plant, because of high gain in one loop.

Would like to show a similar result for a more general problem, that of a more mixed plant (e.g., imperfectly set demodulator phase).



Vout / φlaser (dB)
10
-10
-20
-30
3.5
4
4.5
5
log10(f)
~18kHz

$$P = \begin{bmatrix} 1 & \epsilon_1 \\ 1 & \epsilon_2 \end{bmatrix} \qquad \epsilon_1 = \frac{1}{400} \qquad \epsilon_2 = \frac{1}{200}$$