$\mathcal{F}$-Statistic Search for White Dwarf Binaries in the Third Mock LISA Data Challenge

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Abstract

We have performed a preliminary analysis of data in the third Mock LISA Data Challenge (MLDC3), using the open-source LALApps $\mathcal{F}$-statistic code to search for simulated LISA signals from galactic white-dwarf binaries (WDBs). Our search pipelines have now been extended to handle WDB frequency evolution, a feature not present in previous MLDCs. We recover amplitude parameters accurately in a targeted search. Our preliminary search over the full 4D Doppler parameter space finds many signals, but further refinements are needed to reduce false alarms and improve parameter recovery.

$\mathcal{F}$-Statistic Method

A white-dwarf binary GW signal $s(t)$ is characterized by its *Doppler parameters* θ, i.e. frequency f, frequency derivative f_t and sky-position (ecliptic latitude β, longitude λ), and its *amplitude parameters* $\{\mathcal{A}^\mu\}_{\mu=1}^4 = \mathcal{A}^\mu(h_0, \cos\iota, \psi, \phi_0)$, and can be written as

$$s(t; \mathcal{A}, \theta) = \mathcal{A}^\mu\, h_\mu(t; \theta)\,. \tag{1}$$

Maximizing the **likelihood ratio** statistic over the four amplitudes $\mathcal{A}^\mu$, results in maximum-likelihood estimators

$$\mathcal{A}^\mu_{\text{cand}}(x; \theta) = \mathcal{M}^{\mu\nu}\,(x\|h_\nu)\,, \tag{2}$$

where $\mathcal{M}^{\mu\nu}$ is the matrix inverse of $\mathcal{M}_{\mu\nu} \equiv (h_\mu\|h_\nu)$. Substituting the amplitude-estimator $\mathcal{A}^\mu_{\text{cand}}$ into the likelihood ratio, we obtain the $\mathcal{F}$-statistic:

$$2\mathcal{F}(x; \theta) \equiv |\mathcal{A}_{\text{cand}}|^2 \equiv \mathcal{A}^\mu_{\text{cand}}\,\mathcal{M}_{\mu\nu}\,\mathcal{A}^\nu_{\text{cand}}\,, \tag{3}$$

and so we only need to search over the Doppler-space $\theta = \{f, f_t, \beta, \lambda\}$. If exactly targeting a signal, the expectation value of $2\mathcal{F}$ is $E\left[2\mathcal{F}(x; \theta_{\text{key}})\right] = 4 + \left|\mathcal{A}_{\text{key}}\right|^2$.

Targeted Search

Challenge 3.1 of MLDC3 includes 20 **Verification Binaries** w/known Doppler parameters θ. As in MLDC1[1, 2] and MLDC2[3] we can calculate the $\mathcal{F}$-stat for those parameters (now including df/dt) and deduce the amplitude parameters $\mathcal{A}^\mu$. We can compare our results on the **Training data**, for which the true amplitude parameters are known. We find that our errors are consistent with the expected statistical errors due to noise (Fig.1), even at higher frequencies.

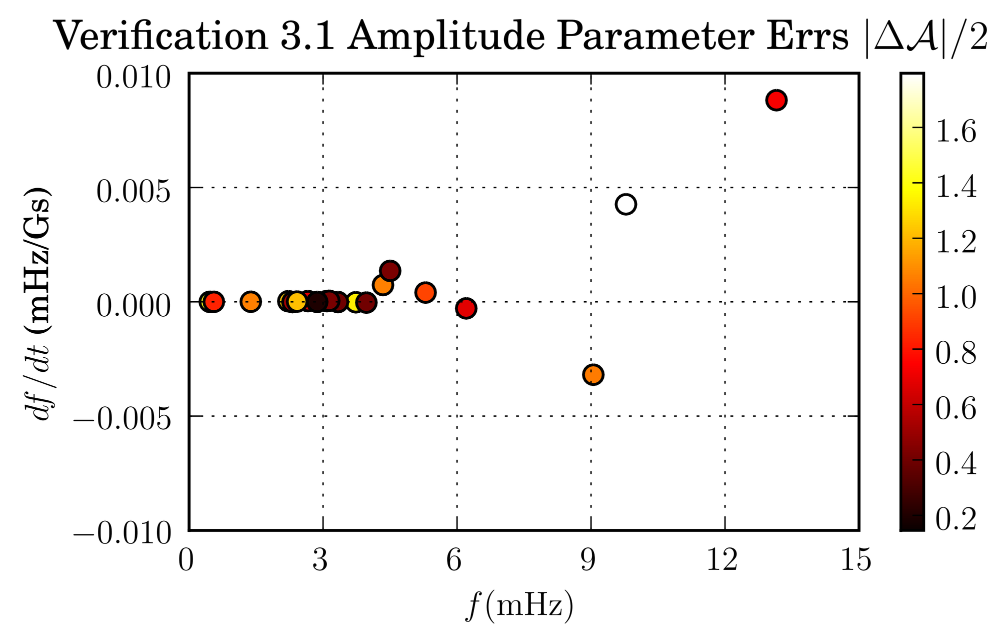

Figure 1: *Recovery of amplitude parameters for verification binaries in training data set. The quantity plotted on the color scale, $|\Delta\mathcal{A}|/2$, should have an RMS value of unity for Gaussian statistical errors.*

This can also be illustrated by using the inverse of the Fisher matrix $\mathcal{M}_{\mu\nu}$ to determine the error bars on the $\mathcal{A}^\mu$, e.g., $\sigma_{\mathcal{A}^1} = \sqrt{\mathcal{M}^{11}}$. The 20 verification binaries give 80 errors $\mathcal{A}^\mu/\sigma_{\mathcal{A}^\mu}$ which should be Gaussian distributed with zero mean and unit variance. (Fig.2)

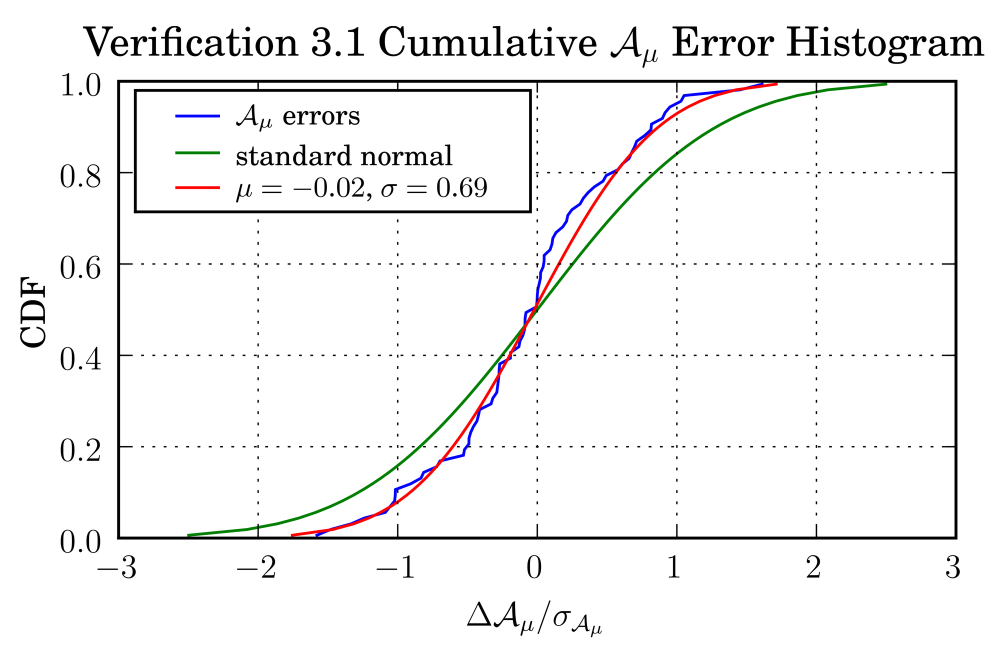

Figure 2: *Distribution of errors in verification binary amplitude parameters relative to Fisher matrix error bars. The errors are consistent with statistical fluctuations.*

Multiple Sources

In addition to verification binaries, challenge 3.1 contains 60 million galactic WD binaries, whose orbital frequency can increase or decrease due to evolution resulting from GW emission or mass transfer. Of those, 40784 were designated as "bright" sources, the norms of whose amplitude parameter vectors are shown in Fig. 3.

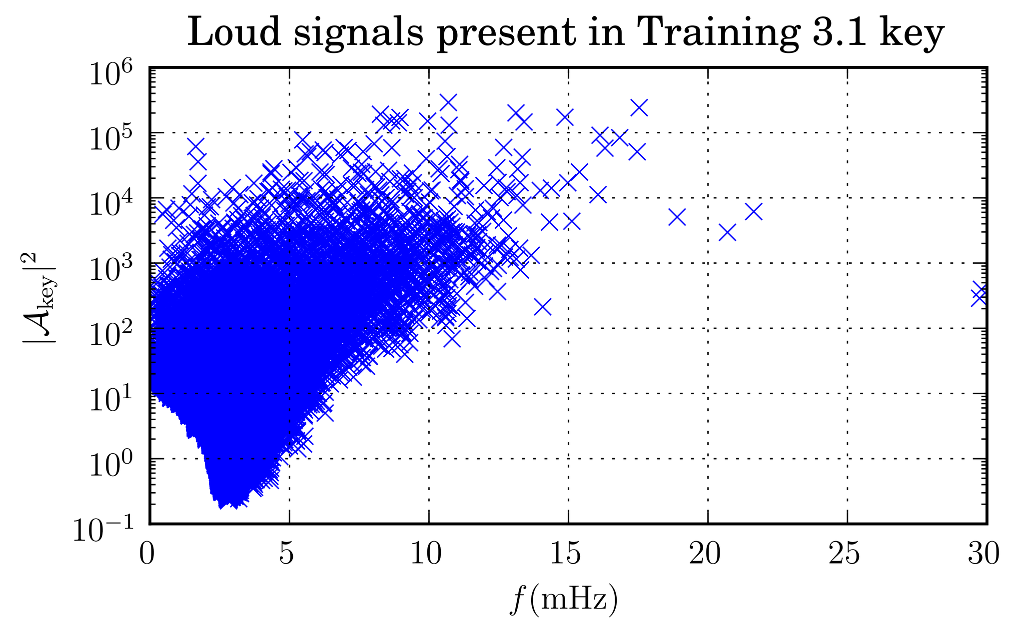

Figure 3: *Sources in Challenge 3.1 (training dataset) "loudest" key file: norms $\left|\mathcal{A}_{\text{key}}\right|^2$ (corresponding to SNR^2) of injected amplitude-vectors as function of frequency f.*

The large number of detectable sources makes it difficult to distinguish the actual ("primary") from "secondary" maxima of the detection statistic $2\mathcal{F}(x;\theta)$ in Doppler parameter space. Our pipeline is based on the empirical observation that primary maxima show better coïncidence between different TDI variables X, Y, Z than secondary maxima. The coïncidence criterion is based on the **metric** g_{ij} in Doppler parameter space, namely

$$m = g_{ij}\, d\theta^i\, d\theta^j + \mathcal{O}(d\theta^3)\,, \tag{4}$$

which attributes a "distance" m to the Doppler offsets $d\theta = \{df,\, df_t,\, d\beta,\, d\lambda\}$.

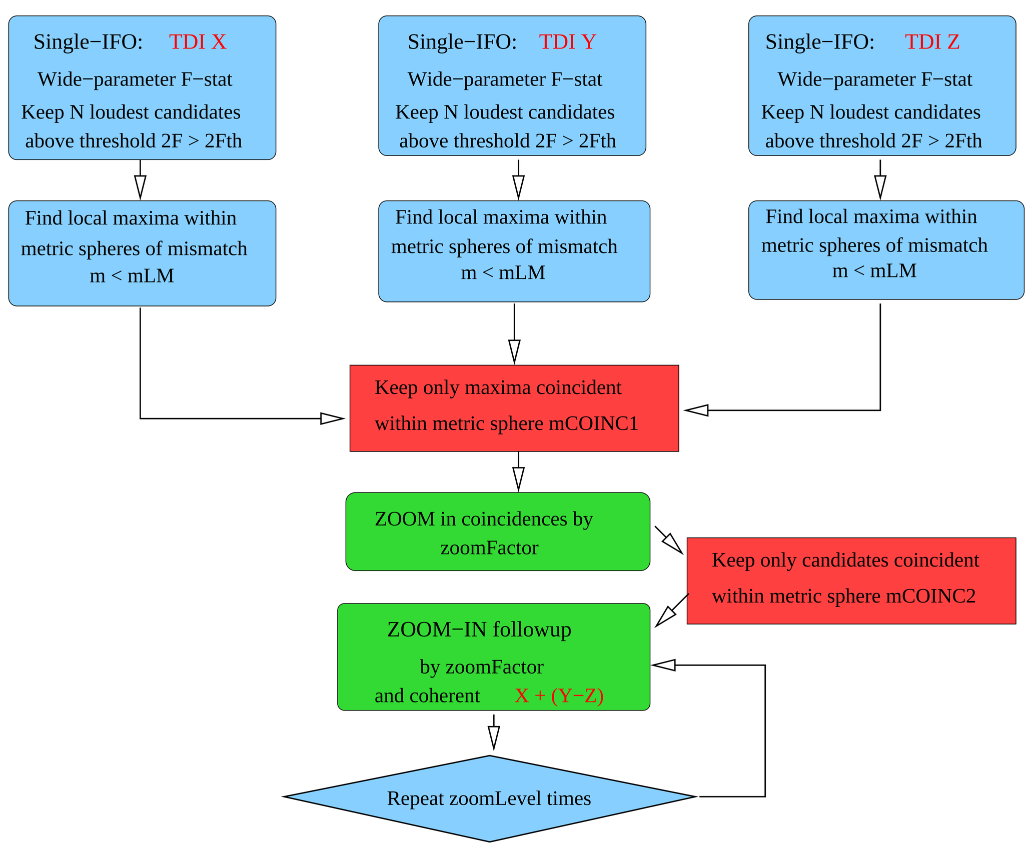

Figure 4: *Hierarchical coïncidence pipeline. True signals are identified by looking for coïncidences between single-observable searches, after which their parameters are refined using a full multi-detector search.*

Preliminary Results from Untuned Pipeline

As an initial application of our method to MLDC3, we performed a search for signals across the **4D** Doppler parameter space, using the same tunings that were applied to the **3D** search for monochromatic sources in MLDC2. The search was only conducted up to 16 mHz because so few signals were present between 16 and 30 mHz.

Freqs	Found	Missed	False
0–4 mHz	489	2779	39
4–8 mHz	871	2005	79
8–12 mHz	298	194	24
12–16 mHz	23	19	7
Total	1681	4997	149

Table 1: *Signals found in an untuned search of the Challenge 3.1 training data, along with missed sources and false alarms. Only sources with $\left|\mathcal{A}_{\text{key}}\right|^2 > 40$ are included in the "missed" category.*

Our pipeline identified 1830 candidate signals in the training data, with $2\mathcal{F}$ values ranging from 43.4 to 2.01×10^5. Of the 40784 "bright" sources in the key, 6678 had $\left|\mathcal{A}_{\text{key}}\right|^2 > 40$ and $f < 16\,\text{mHz}$. To evaluate our results, we identified each candidate with the loudest "bright" source within a Doppler mismatch of $m \le 1$. If there was no "bright" source within that Doppler window, we considered the candidate to be a false alarm. The results of this identification are summarized in Table 1. While this untuned search recovers a comparable number of signals to our MLDC2 search[3], it has a much higher false alarm rate. There is also a rather poor recovery of amplitude parameters relative to the verification binary search:

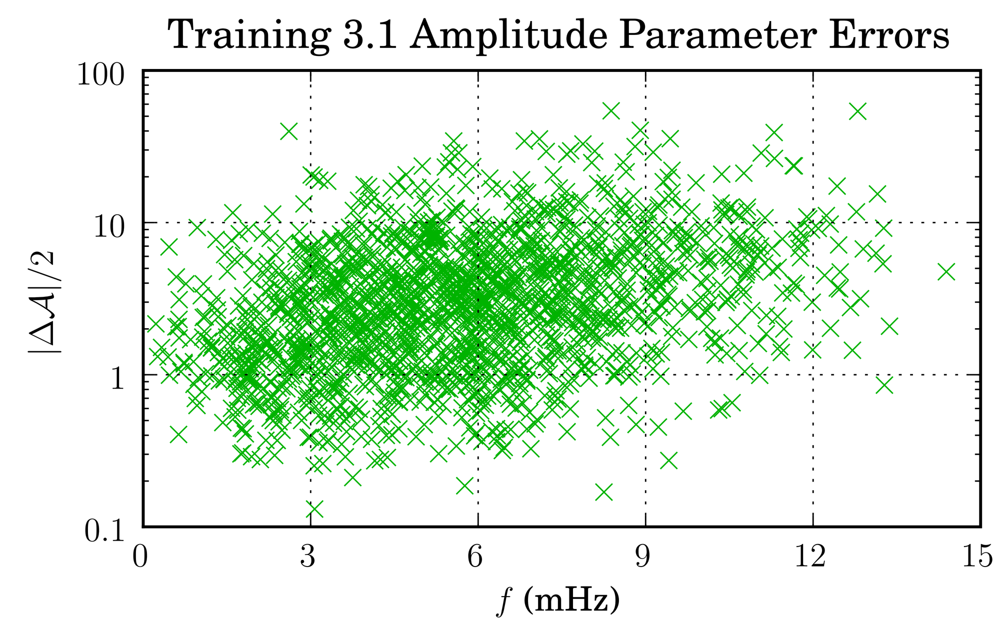

Figure 5: *Amplitude parameter errors for untuned search on Challenge 3.1 training data. The quantity plotted, $|\Delta\mathcal{A}|/2$, would have an RMS value of 1 due to Gaussian statistical error.*

Quantitatively, we see that the amplitude parameter errors are about 4 times as large as the Fisher matrix estimates. This is much larger than in the MLDC3 verification binary search and in a tuned MLDC2 search[3].

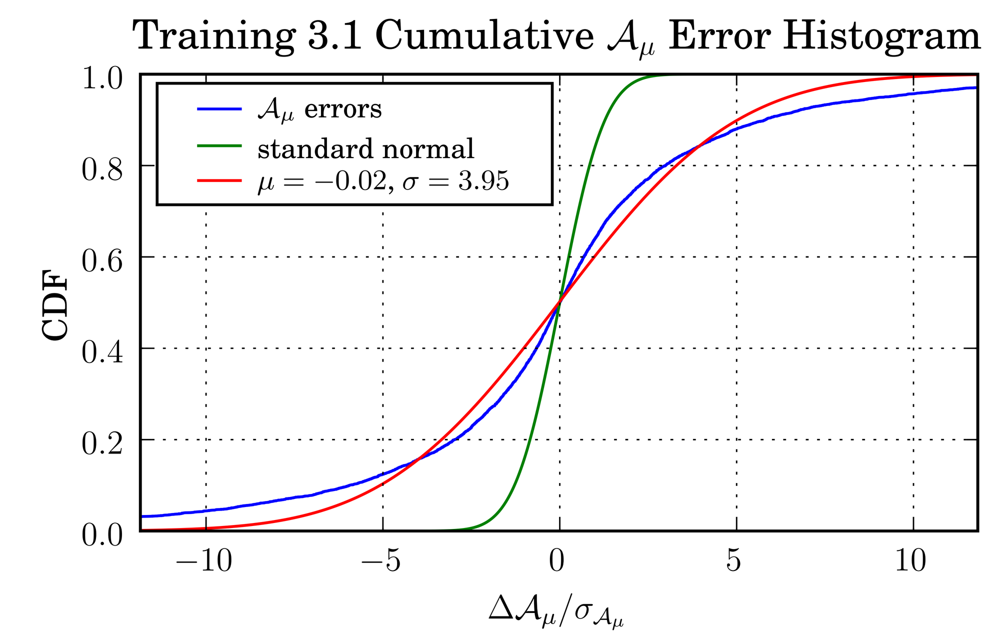

Figure 6: *Distribution of errors in amplitude parameters relative to Fisher matrix error bars. The errors are larger than would be expected due to statistical errors from noise.*

There are also errors in the Doppler parameters, as shown in Table 2. This is not surprising given the large number of false alarms, since it indicates the presence of signals with poorly-matched Doppler parameters.

	$\Delta f/\sigma_f$	$\Delta f_t/\sigma_{f_t}$	$\Delta\beta/\sigma_\beta$	$\Delta\lambda/\sigma_\lambda$	$\Delta\mathcal{A}^\mu/\sigma_{\mathcal{A}^\mu}$
mean	-0.27	0.29	0.09	-0.04	-0.02
std dev	3.53	3.60	1.02	1.09	3.95

Table 2: *Parameter errors for untuned search. Note that the sky position accuracy is consistent with statistical fluctuation, but the frequency and its derivative have not only large systematic errors but also an apparent bias.*

Summary and Outlook

We have extended our pipeline from previous MLDCs to handle the WDBs with **frequency evolution** in MLDC3. The program works well at recovering the amplitude parameters of verification binaries. There is still considerable room for improvement in the multiple-signal pipeline, in terms of signal recovery, false alarm rate, and parameter estimation. At present, in addition to using parameters tuned for the MLDC2 search, uses a suboptimal coïncidence condition to identify signals, relying on the mismatch arising from only the Doppler parameters. Possible improvements include moving from the amplitude-independent mismatch to the Fisher matrix, and including consistency of amplitude parameters in the coïncidence criteria.

References

[1] Prix & Whelan CQG**24**, S565 (2007)

[2] Whelan, Prix & Khurana CQG**25**, 184029 (2008)

[3] Whelan, Prix & Khurana, LIGO-P080087-Z