Thermal noise limit to frequency stabilization of optical cavities

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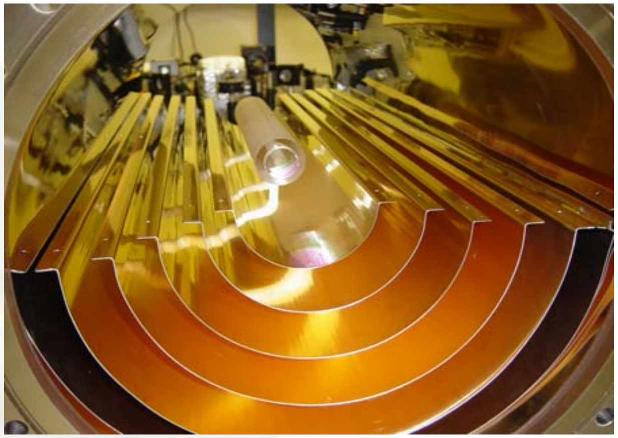
This talk

1. Frequency stability of laser locked to optical cavity
2. What is thermal noise?
3. Q measurements of cavity materials
4. Calculation of thermal noise and result
5. How to reduce thermal noise?
6. Summary

Frequency stability of rigid cavity

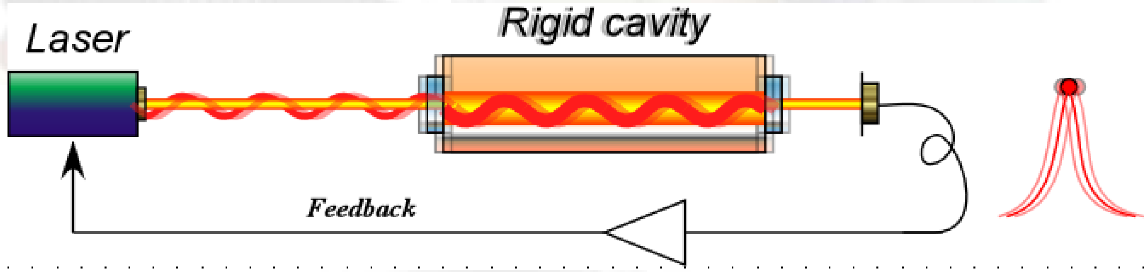
Rigid cavity as a frequency reference

- Laser light stored within the cavity



Possible noise sources

- Non-fundamental noise source
  - Length change due to temperature variation
  - Length change due to vibration (seismic noise)
  - Mirror heating, AF-RF conversion, pointing noise, circuit noise etc.
- Fundamental noise source
  - ***Thermal noise as a result of statistical physics***



Fluctuation-Dissipation Theorem

- Calculation of thermal noise spectrum *G(f)*
  - Based on **FDT** (Fluctuation-Dissipation Theorem)
    - Imaginary part of transfer function

$$G_x(f) = -\frac{4k_B T}{\omega}\,\mathrm{Im}[H(\omega)]$$

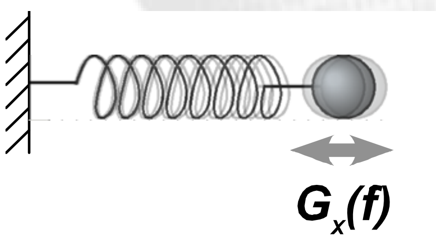
$$H(\omega) = \frac{X(\omega)}{F(\omega)}$$

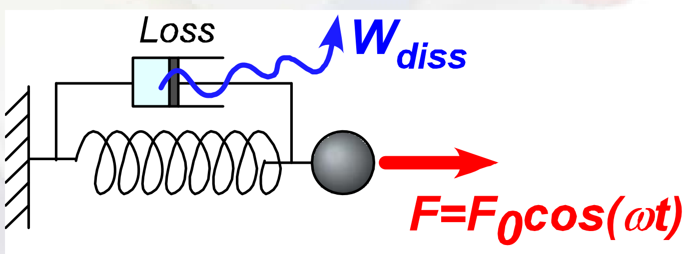
  - Useful form

$$G_x(f) = \frac{8k_B T}{\omega^2}\frac{W_{diss}(f)}{F_0^{\,2}}$$

W_{diss}: dissipated energy under cyclic force (averaged, per unit time)

F_0: force amplitude



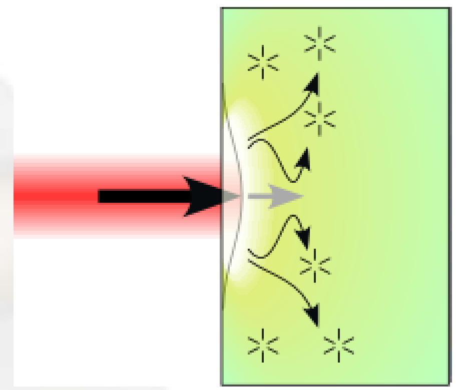


Brownian noise in mirror

Dissipated energy through mechanical loss

- ϕ : Loss angle
  - At resonance : ϕ=1/Q

$$W_{diss} \propto (\text{Stored strain energy}) \times \phi$$

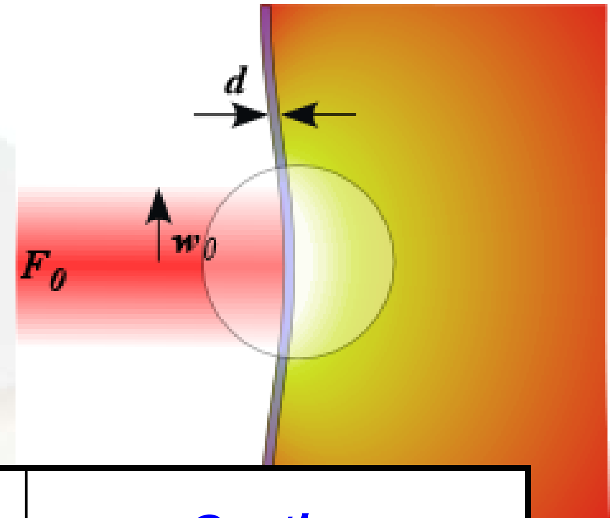


Noise observation with Gaussian weighting (beam)

– Gaussian force is applied in calculation

Thermal noise from substrate, coating

- Parameter dependencies
  - What happens if we apply a Gaussian force?



	Substrate	*Coating*
Stressed volume V	$\approx w_0^3$	$\approx d w_0^2$
Stress p	$\approx \frac{F_0}{w_0^2}$	
Dissipated energy $W_{diss} \approx \omega p^2 V \phi / E$	$\approx \frac{\omega F_0^2 \phi_{sub}}{w_0 E}$	$\approx \frac{\omega F_0^2 d \phi_{coat}}{w_0^2 E}$
Thermal noise G	$\approx \frac{8 k_B T}{\omega} \frac{\phi_{sub}}{w_0 E}$	$\approx \frac{8 k_B T}{\omega} \frac{d \phi_{coat}}{w_0^2 E}$

Geometry of rigid cavity

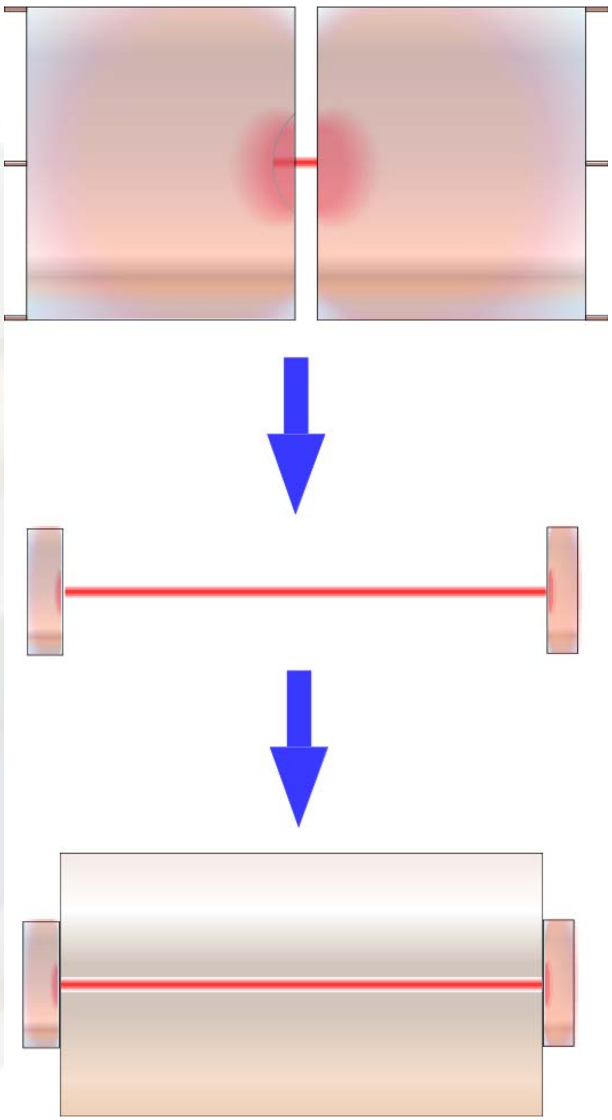
Short length FP cavity

$$w_0 \propto L^{1/2}$$

$$\sqrt{G} \propto w_0^{-1/2} \propto L^{-1/4}$$

Two mirrors connected by a spacer

- Spacer maintains fixed displacement between mirrors
  - Residual thermal noise can be observed
- Spacer has small contribution to thermal noise if mirrors are far apart



4. Calculation and result

- Calculation of strain energy under cyclic force
  - Done by solving EQuation of Motion (EQM) of the system
    - Finite element method adopted
  - Procedure
    - 1) Prepare rigid cavity mechanical model
    - 2) Apply cyclic force to the observing (beam-illuminating) points
    - 3) Calculate strain energy within the system based on EQM

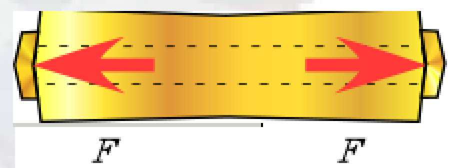
$$W_{diss}(f) = \int \varepsilon(\vec{r})dV \,/\, Q$$

Stored strain energy

Calculation

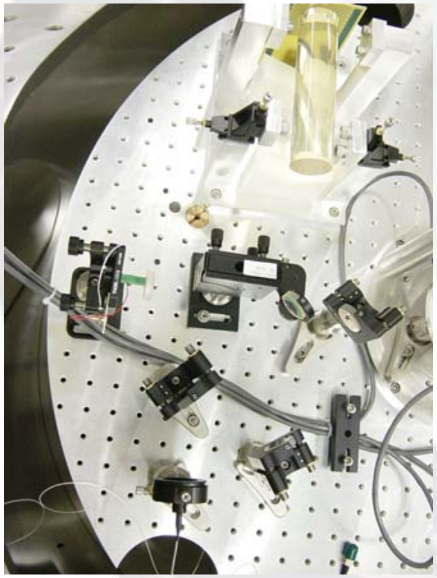
Quality factor

Experiment

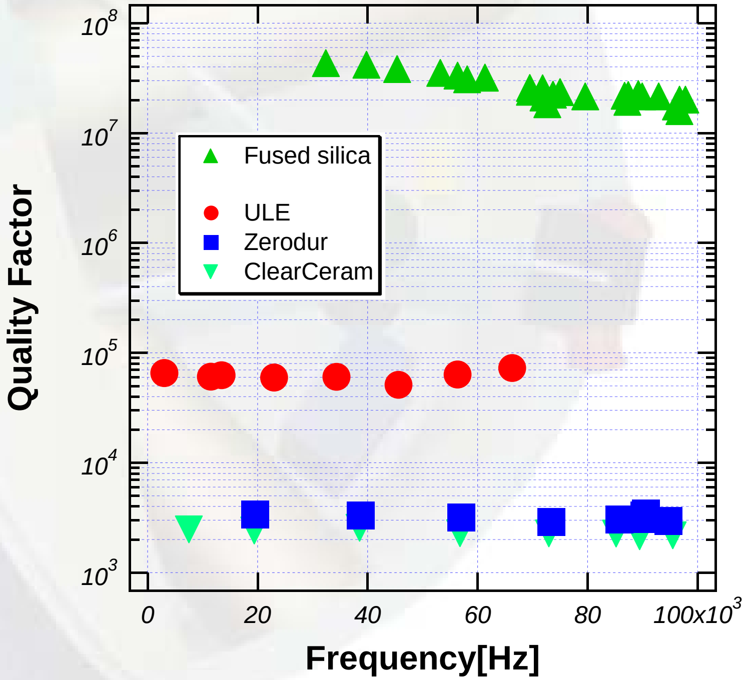


Q-measurement of cavity materials

- Mechanical quality factor (Q) measurement of cavity materials
  - Decay of free vibration measured by Michelson interferometer
  - Fairly low quality factor measured
    - ULE : Q~60000
    - Silica: $Q>10^7$

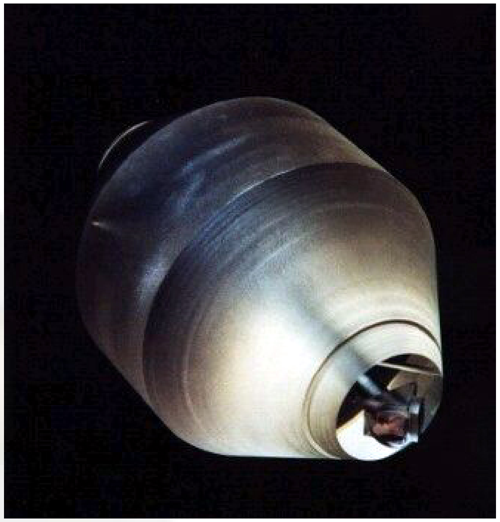


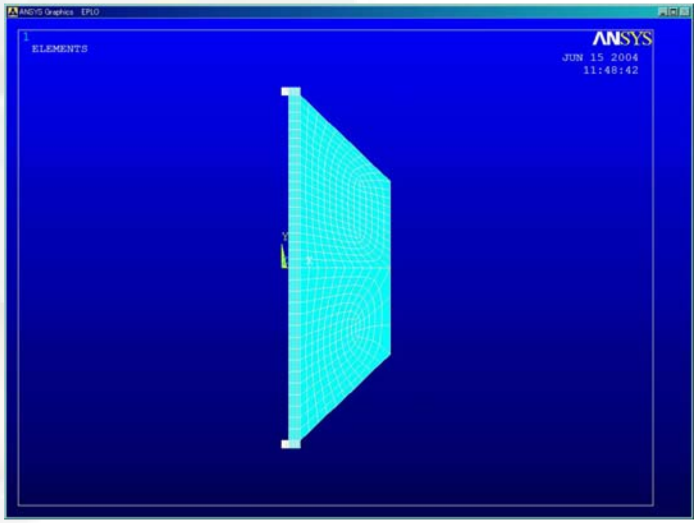
Measurement system



Calculation model

- Modeled noise of NIST and VIRGO cavities
  - World highest stabilities with rigid cavity
  - Results limited by unknown noise sources
- Calculation assumptions
  - Spacer
    - Material: ULE (Q=60000)
  - Mirror (optically contacted)
    - Material: ULE
  - Coating
    - phi(1/Q)=$4x10^{-4}$
- FEM model
  - ANSYS
  - Semi-3D model (2-D axisymmetric)



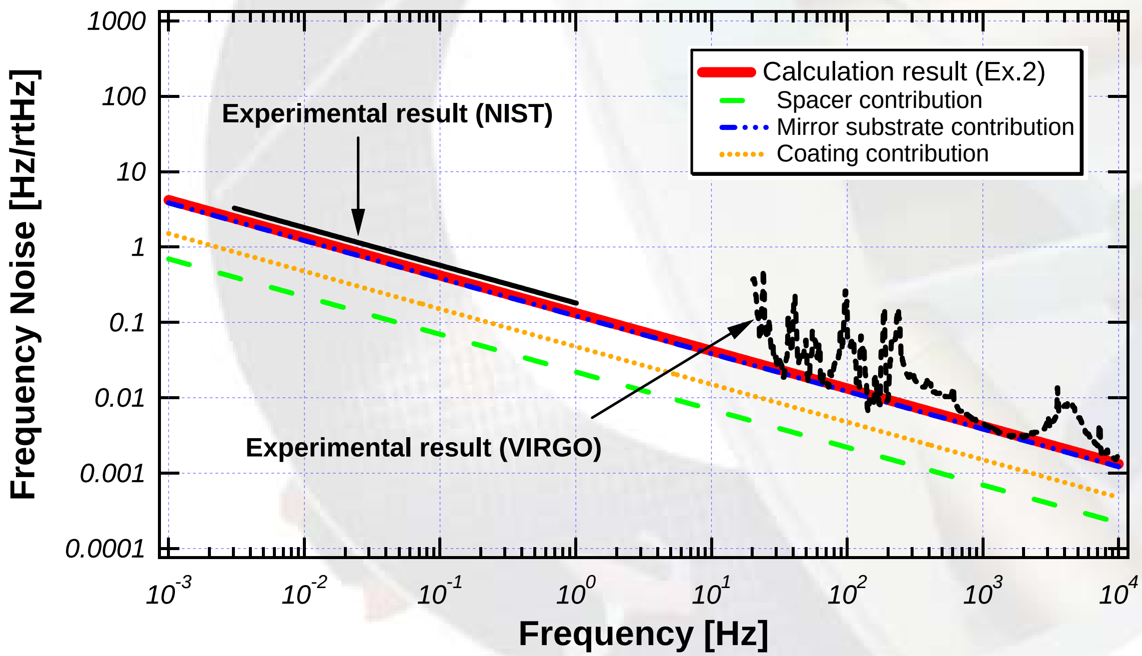


Half of the cross section model of the cavity

Comparison with experiments

Calculations agreed well with measurements

- ~0.01Hz/rtHz @100 Hz level (563nm wavelength)
  - We cannot neglect thermal noise (Brownian motion) anymore!
- Spacer shape and/or material have marginal effects.



Notcutt, Hall, **Phys Rev A 73 (2006)**

"We compare frequency noise of lasers locked to cavities of differing lengths and materials…results are in agreement with Numata, Kemery, and Camp"

5. How to reduce thermal noise?

Research directions in the ground-based GW community

- Smaller loss in substrate and coating
  - Fused silica mirror substrate
  - New lower-loss coating materials
  - **Factor 3 - 5 improvement in frequency stability possible**
- Use of a larger beam
  - Large mirror, large *g* factor
  - ***Gain small in a rigid cavity***
- Cooling
  - LCGT project in Japan
  - ***Applicable to rigid cavities if carefully designed***
- Other techniques
  - Thermal noise subtraction, folded cavity, flat-topped beam etc...
  - ***Possibly usable***

Other possible causes of losses

- Other possible causes of loss
  - Thermoelastic noise
    - Roughly, thermoelastic noise is proportional to CTE.
      - Still smaller than Brownian if silica/ULE is used for substrate.
  - Surface loss
    - Rough surface has larger mechanical loss.
  - Support loss
    - Isolation from external world
  - Contact loss etc…

6.Summary

Thermal noise limit found to frequency stability of optical cavity

- Experiment: Q measurement of cavity materials
- Calculation: Numerical analysis of strain energy
- Agreement with the world-highest level stabilization results
  - Frequency noise of 0.01 Hz / $Hz^{1/2}$ at 100 Hz

Suggestions to lower noise

- Use higher Q silica mirror substrate
  - Gain of factor 3 in noise
- Lower mechanical loss of coatings
  - Additional gain of ~ factor 2 may be possible

References

K Numata, A. Kemery, J. Camp, “Thermal noise limit to frequency stabilization of rigid cavities”, PRL 93, 250602 (2004)

M Notcutt, J Hall et al “Contribution of thermal noise to frequency stability of optical cavities via Hz linewidth lasers”, Phys Rev A 73, 031804(R) (2006)