LIGO G080082-00-R

Optimization of Coating Design for Reduced Thermal Noise

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TWG/Univ. of Sannio at Benevento & INFN

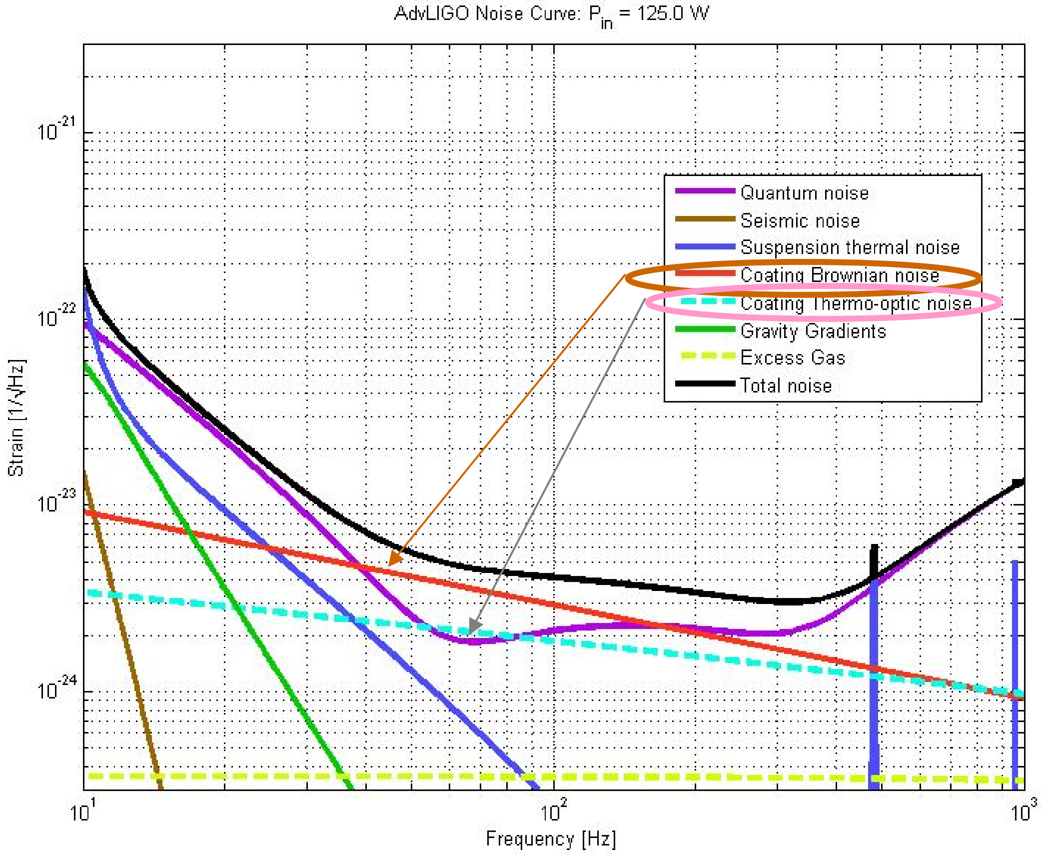
LSC / VIRGO Joint Meeting, 17-20 March, 2008, Caltech

Workshop on Optical Coatings in Precision Measurements

SUMMARY

Coating Optimization Method:	Stacked Doublet Coatings Barebone Optimization Code Coating Noise Model; How Did We Get Here - Genetic Selection Another Look at the Optimum
Results:	Prototype(s) Robustness Losses Event Rate Boost Usable Reflectance Windows
Perspectives:	M-ary Coatings Band-gap Engineered Coatings

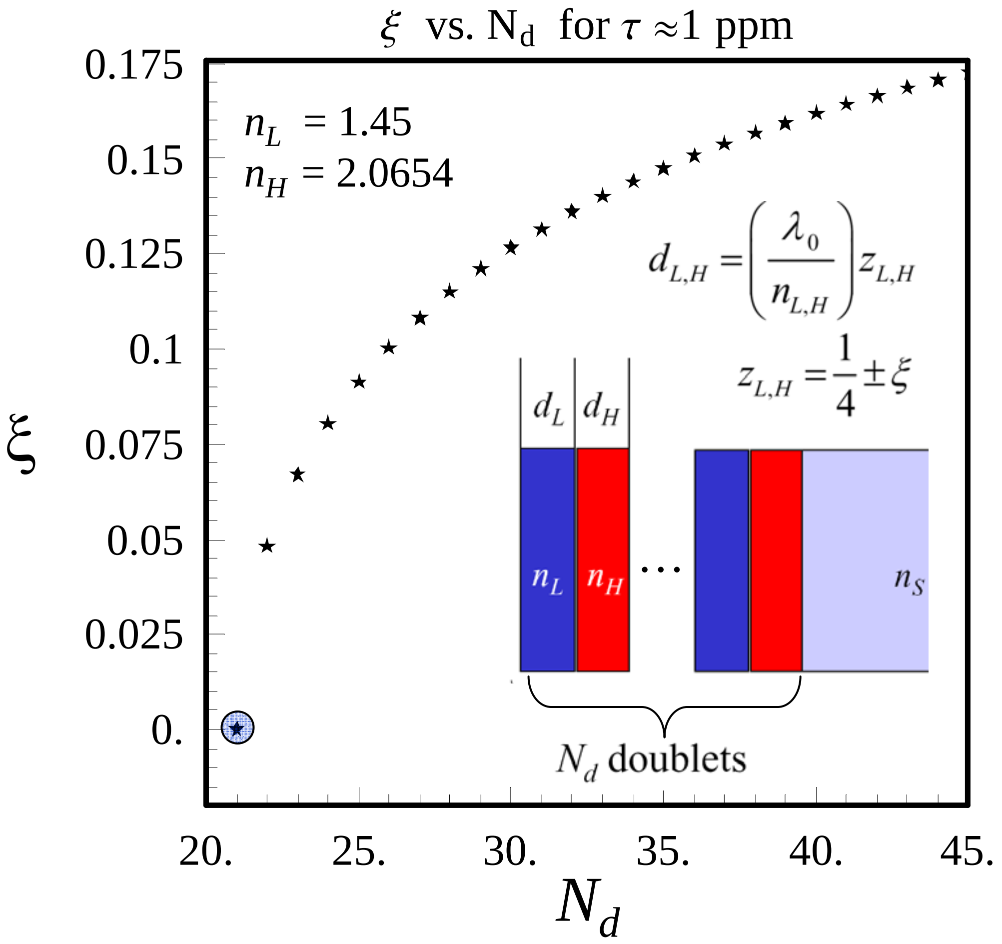
AdLIGO NOISE BUDGET (Quarter Wavelength Design)



LIGO G080082-00-R

BINARY COATING OPTIMIZATION IN A NUTSHELL, I

ISO-TRANSMITTIVE STACKED DOUBLET, “BRAGG” BINARY COATINGS



Reference QWL (ξ=0) design closest to 1ppm has

$$N_D = 21$$
$$\tau = 0.9727 \text{ ppm}$$

BINARY COATING OPTIMIZATION IN A NUTSHELL, II

AMONG ISO – TRANSMITTIVE ALTERNATIVE DESIGNS, PICK **OPTIMAL ONE**, IN TERMS OF THERMAL NOISE

WARNING : ***OPTIMAL SHOULD BE ALSO GOOD***, IN TERMS, OF

- DESIGN ROBUSTNESS [TOLERANCE W.R.T. DEPOSITION ERRORS & MATERIAL PARAMETERS.UNCERTAINTIES].
- OPTICAL LOSSES;
- REFLECTANCE ON OTHER USABLE WAVELENGTHS
- ANGULAR ACCEPTANCE, etc.

BARE-BONE OPTIMIZATION ALGO

```
Prescribe transmittance : $\tau_P = 1 - |\Gamma_p|^2$
Find smallest $N_d$ : $\tau_{QWL}(N_d) = \tau^* \leq \tau_P$
  Do $N_d = N_d + 1$,
      find $(z_L, z_H)$ :
          $\begin{cases} z_L + z_H = 1/2 \\ \tau(z_L, z_H, N_d) = \tau^* \end{cases}$
  while $PSD(z_L, z_H, N_d) \leq PSD(z_L, z_H, N_d - 1)$
```

End layer tweaking

Tweak (trim) topmost L-layer for maximum reflectance

Tweak (trim) bottom H-layer to bring back reflectance to design value (trims noise further)

COATING NOISE MODEL

Three basic components should be included:

BROWNIAN - Key references:

[G. Harry, LIGO-T040029-00-R, 2004]

THERMOELASTIC - Key references:

[V.B. Braginsky and S.A. Vyatchanin, ArXiv:cond-mat/0302617, post-print of Phys. Lett. A312 (2003) 244; contains important corrections; M.M. Fejer et al., PRD-70 (2004) 082003]

THERMOEFRACTIVE - Key references:

[V.B. Braginsky, et al., Phys. Lett. A 271 (2000) 303; I.M. Pinto et al., LIGO T-070159-00-Z (2007), A. Gretarsson, LIGO G-08151-00-R (2008).

COATING NOISE - i) BROWNIAN

[G. Harry, LIGO-T040029-00-R, 2004]

$$S_{\Delta x}^{(B)}(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{(1-\sigma^2)}{wY} \phi_{eff}$$

Boltzmann constant

Absolute temperature

Poisson ratio of substrate

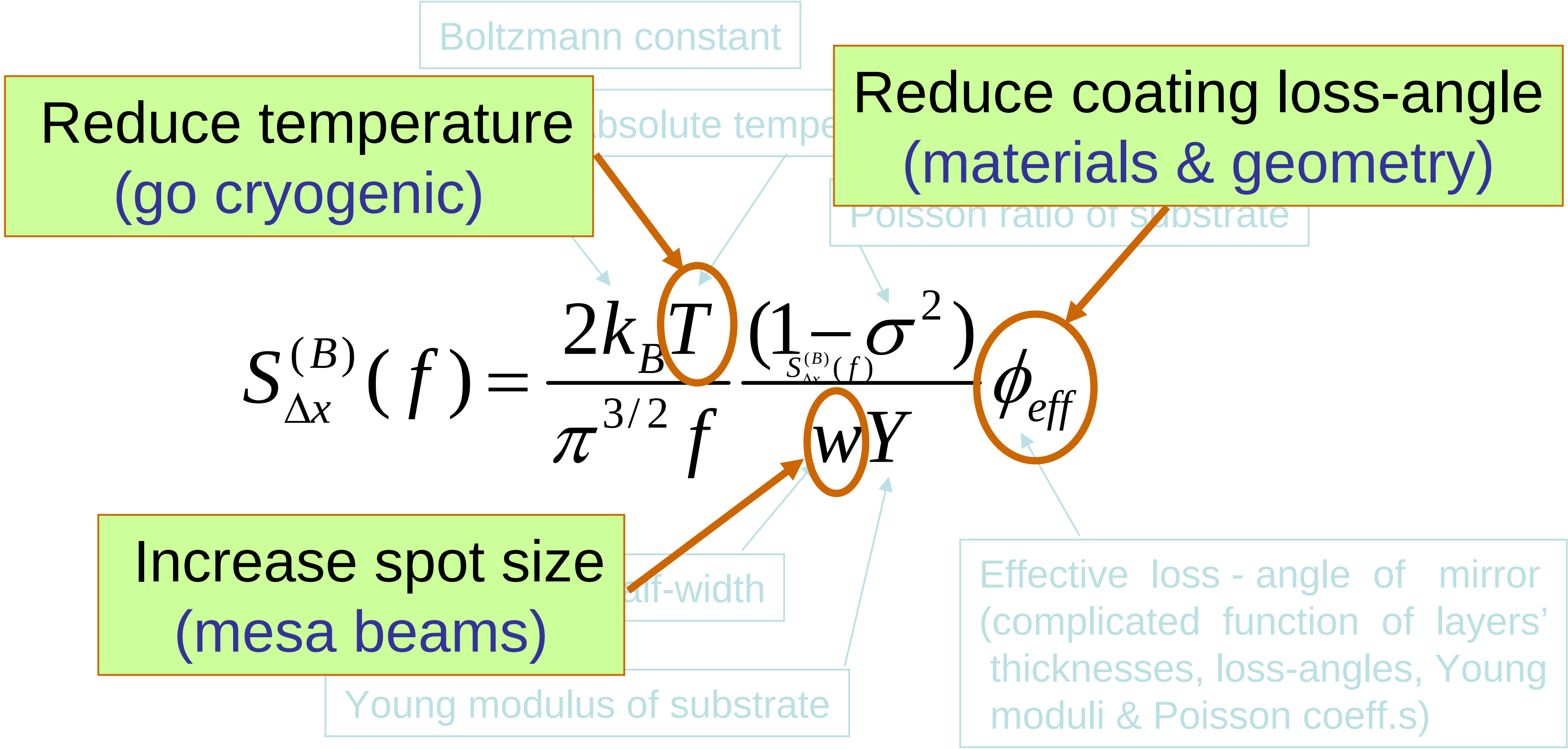
Beam half-width

Young modulus of substrate

Effective loss - angle of mirror (complicated function of layers' thicknesses, loss-angles, Young moduli & Poisson coeff.s)

COATING NOISES: i) BROWNIAN

[G. Harry, LIGO-T040029-00-R, 2004]



Boltzmann constant

bsolute tempe

Poisson ratio of substrate

Reduce temperature
(go cryogenic)

Reduce coating loss-angle
(materials & geometry)

$$S_{\Delta x}^{(B)}(f) = \frac{2k_B T}{\pi^{3/2} f}\frac{(1-\sigma^2)}{wY}\phi_{eff}$$

Increase spot size
(mesa beams)

alf-width

Young modulus of substrate

Effective loss - angle of mirror (complicated function of layers' thicknesses, loss-angles, Young moduli & Poisson coeff.s)

COATING LOSS ANGLE : APPROXIMATE

…can be boiled down to simplest formula

$(\nu_{L,H,S} \ll 1)$

$$\phi_{eff} = (b_L d_L + b_H d_H)$$

$$b_{L,H} = \frac{\lambda_0}{\pi^{1/2} w} \frac{\phi_{L,H}}{n_{L,H}} \left(\frac{E_{L,H}}{E_s} + \frac{E_s}{E_{L,H}} \right)$$

$\gamma = b_H / b_L \in [5,10]$ plain Ta_2O_5

defines implicitly $\sigma_\parallel$

quadratic equation; solvable in analytic form; one positive root

approximate solution : $\sigma_\parallel = \frac{\sigma_1 + \sigma_2}{2}$

[G. Harry, LIGO-T040029-00-R, 2004]

COATING NOISE: ii) THERMAL ELASTIC & THERMAL REFRACTIVE

Effective fluctuations of the test-mass (coated mirror) front - face position with respect to the mirror center of mass may occur as an effect of

-Thermal expansion of the coating layers (thermoelastic effect),

$$\Delta x^{(TE)} = \alpha_{eff} d_{tot} \Delta T$$

effective coating expansion coeff.

coating thickness

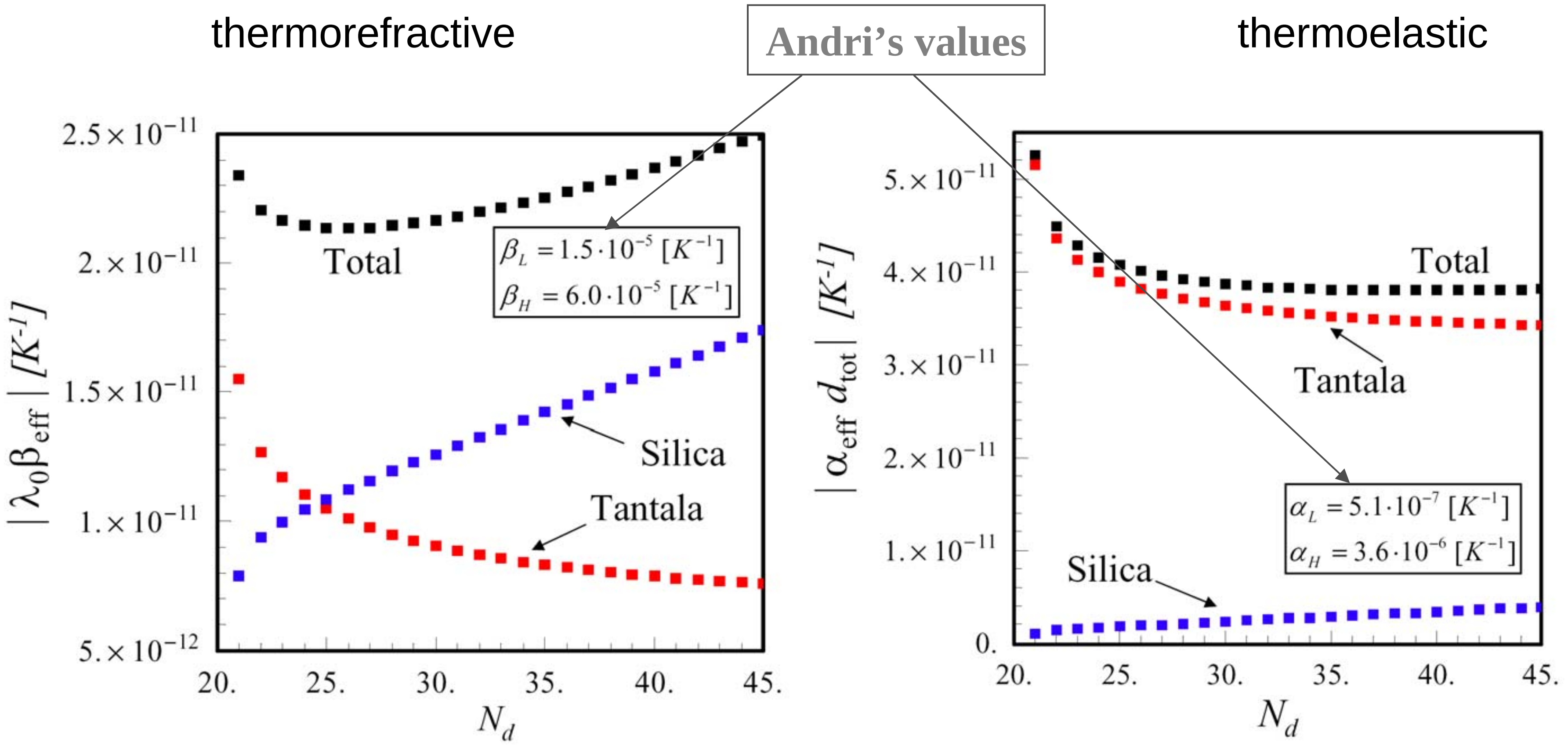
-Thermal variations of the refraction indexes $n_{H,L}$ of the coating materials (thermorefractive effect),

$$\Delta x^{(TR)} = \beta_{eff} \lambda_0 \Delta T$$

effective thermorefractive coefficient

optical wavelength (vacuum)

THERMO-OPTIC NOISE COEFFICIENTS
(PLAIN TANTALA)



SPECTRAL DENSITY FOR $\Delta x^{(TE,TR)}$

quasi-static

Wiener-Khinchin th.

$$S_{\Delta x}(f) = \mathcal{F}_{\tau\to f}\langle \Delta x(t)\Delta x(t+\tau)\rangle_t = \left(\frac{\Delta x}{\Delta T}\right)^2 \mathcal{F}_{\tau\to f}\langle \Delta T(t)\Delta T(t+\tau)\rangle_t =$$

$$= \left(\frac{\Delta x}{\Delta T}\right)^2 S_{\Delta T}(f)$$

PSD of T - fluctuations
in the coating

$$S_{\Delta T}(f) = S^{(\Theta)}_{\Delta T}(f) + S^{(\Phi)}_{\Delta T}(f)$$

Intrinsic fluctuations of
thermodynamic origin

add
in-coherently

Photo-thermal fluctuactions
arising from laser shot noise
through optical absorption

COATING TEMPERATURE FLUCTUATIONS

$$S^{(\Theta)}_{\Delta T}(f) = \frac{k_B T^2}{\pi^{3/2} r_0^2 \sqrt{f \kappa_s C_s \rho_s}}$$

[V. Braginsky, Phys. Lett A264 (1999) 1]

mass density
specific heat capacity
thermal conducivity

} of substrate

single photon energy
power abs. in coating

$$S^{(\Phi)}_{\Delta T}(f) = \frac{P_{\text{abs}} E_\lambda}{4\pi^3 r_0^4 \kappa_s \rho_s C_s f}$$

[S. Rao, PhD Thesis, Caltech, 2003, etd-05092003-153759]

$E_\lambda \cong 1.867 \cdot 10^{-19} J \ @ \ \lambda = 1064 nm$

$P_{abs} = 0.4 W$ for Adv LIGO

(a *different* formula for $S^{(\Phi)}_{\Delta T}$ applies for sapphire substrates)

COATING THERMAL NOISE BUDGET

Total Coating Noise PSD

$$S_{\Delta x}^{(tot)}(f) = S_{\Delta x}^{(B)}(f) + \left(\frac{\Delta x^{(TE)}}{\Delta T} + \frac{\Delta x^{(TR)}}{\Delta T}\right)^2 S_{\Delta T}(f)$$

Brownian-structural;

Thermally - driven elastic and refractive fluctuations. *may likely add incoherently or :* indeed, the temperature in the coating does *not* vary

- on the space-scale (thickness) of the coating,
- on the time scales whereby the field in the coating builds up.

?

SD COATINGS: HOW DID WE GET HERE ? GENETIC SELECTION !

Educated ignorance attitude

- *no a-priori assumption* on structure;
- easy inclusion of heterogeneous design constraints;

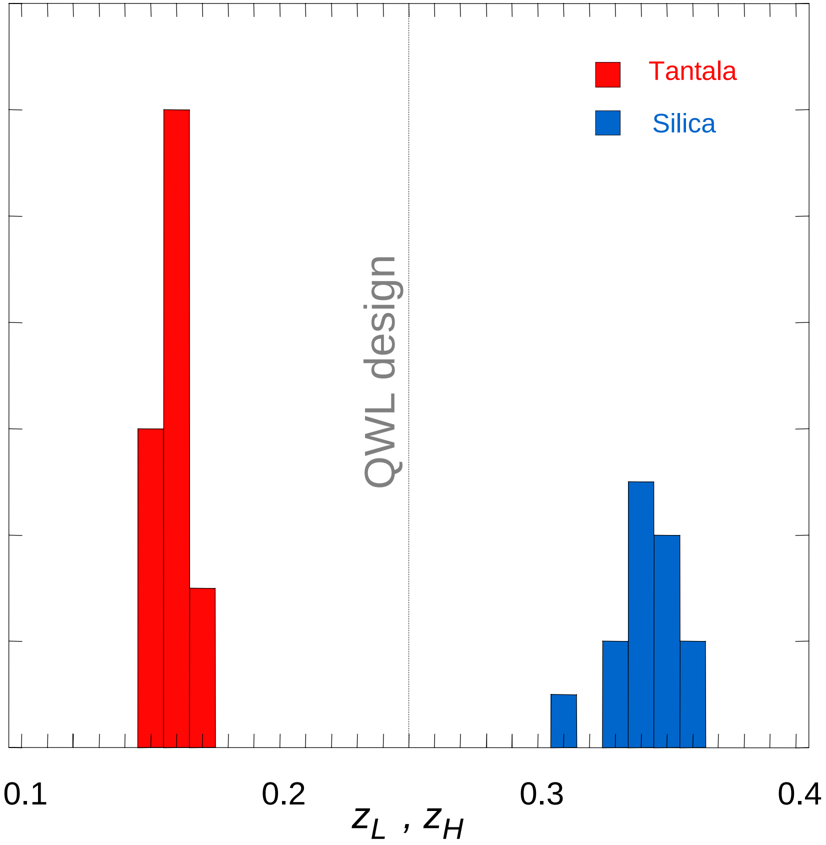
GA - engineered minimum noise *binary* coatings show trend toward non - QWL quasi - Bragg ($z_L + z_H = 0.5$) stacked-doublet (SD) configurations;

Deviations from trend are confined to fewest end - layers (first, last);

Suggests sequential design recipe :

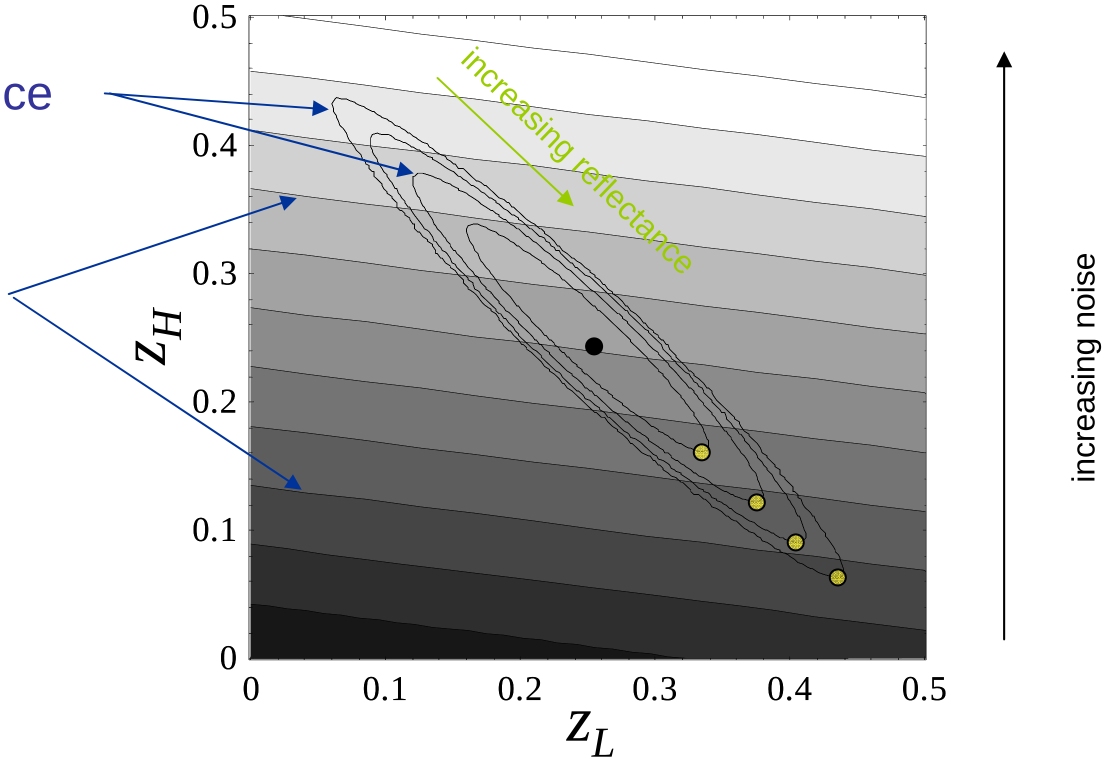
a) Design minimum-noise SD;
b) Tweak terminal layers;

GA-optimized 20ppm transmittance prototype.
z_L and z_H histograms after 10^4 generations.



ANOTHER LOOK AT THE OPTIMUM

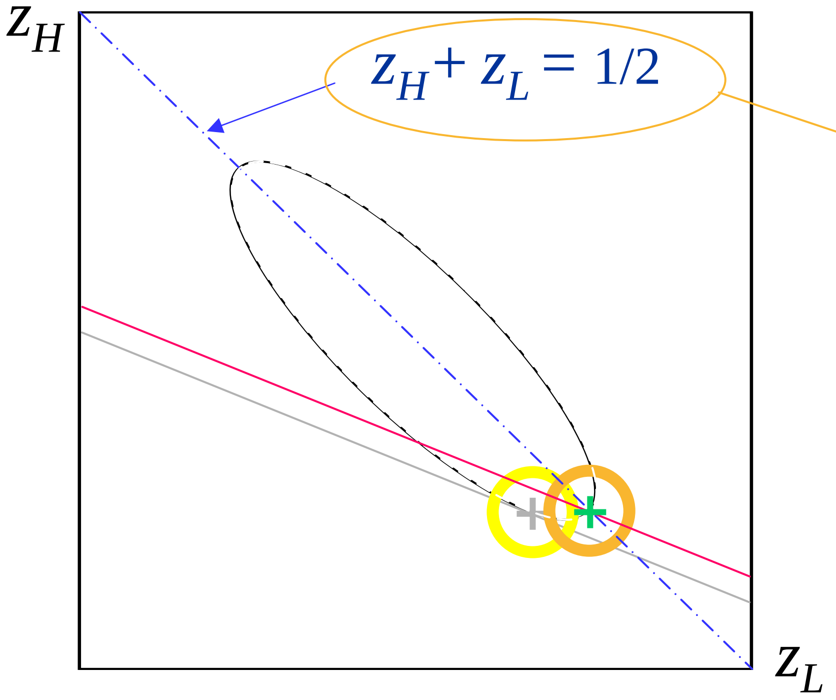
BINARY COATINGS MINIMUM - NOISE BRAGG SD SYNTHESIS



z_L = thickness of low-index (SiO_2) layer
z_H = thickness of high-index (Ta_2O_5) layer

In units of local wavelength $\lambda_0/n_{H,L}$

QUASI - OPTIMAL BINARY COATINGS SD SYNTHESIS : BRAGG DOUBLETS



Bragg condition

Iso-reflectance contour lines *very thin*: difference between optimal and quasi – optimal synthesis negligible (absorbed by material uncertainties)

SUMMARY

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Barebone Optimization Code
Coating Noise Model;
How Did We Get Here:
Genetic Selection
Another Look at the Optimum

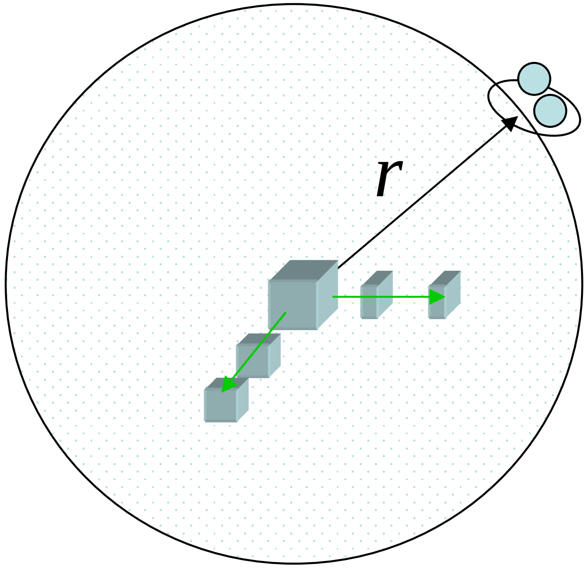
Results:

Event Rate Boost
Prototype(s)
Robustness
Losses
Usable Reflectance Windows

Perspectives:

M-ary Coatings
Band-gap Engineered Coatings

NOISE FLOOR VS. EVENT RATE



$$h \propto r^{-1}$$

$$h_{min} \propto PSD_{floor}^{1/2}$$

$$r_{max} \propto PSD_{floor}^{-1/2}$$

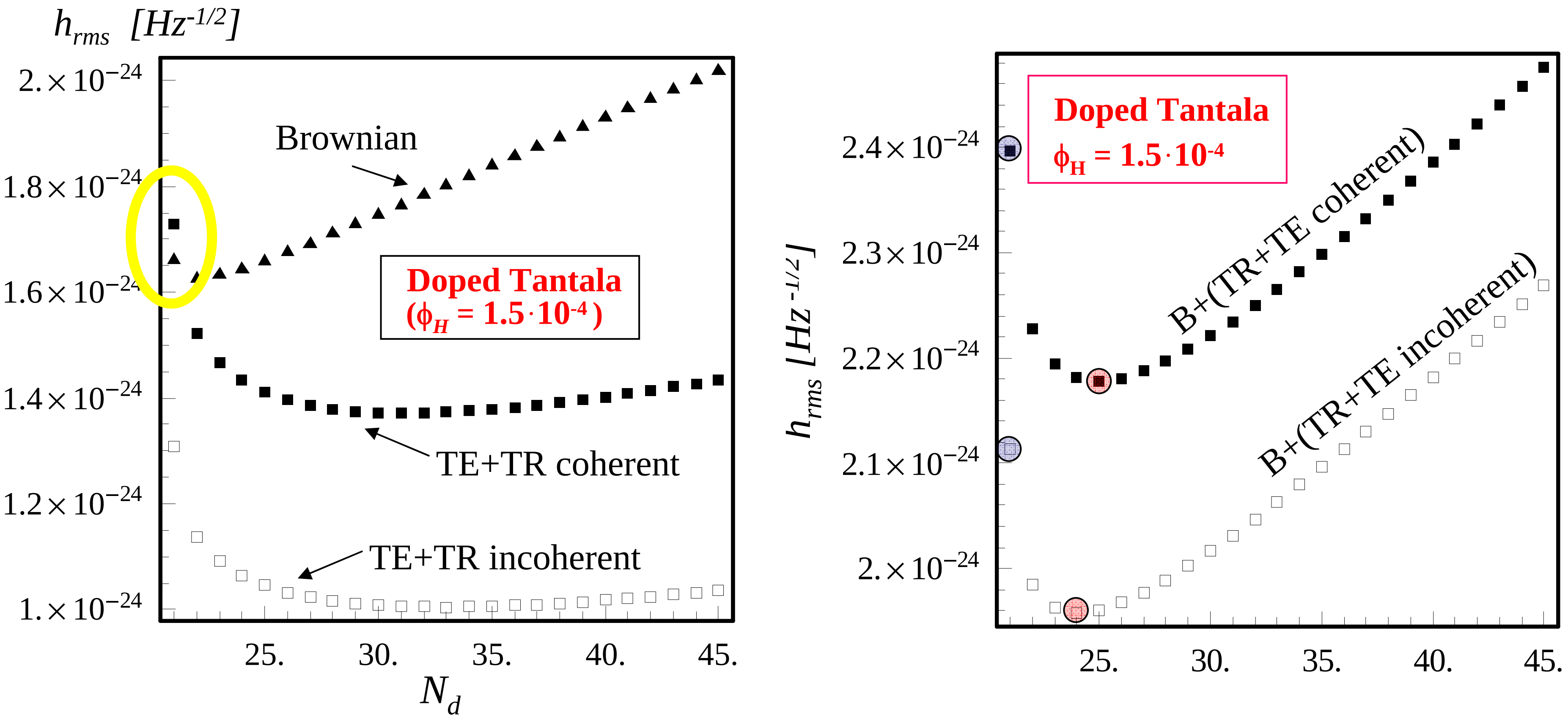
visibility volume,
event rate
(isotropic source distrib.)

$$\left.\right\} \propto PSD_{floor}^{-3/2}$$

⇨ **A 13% reduction in PSD_{floor} boosts the event rate by 23%, etc.**

1ppm OPTIMIZED COATING : DOPED TANTALA

$\tau = 0.9727\text{ppm}, \; f = 100\text{Hz}$



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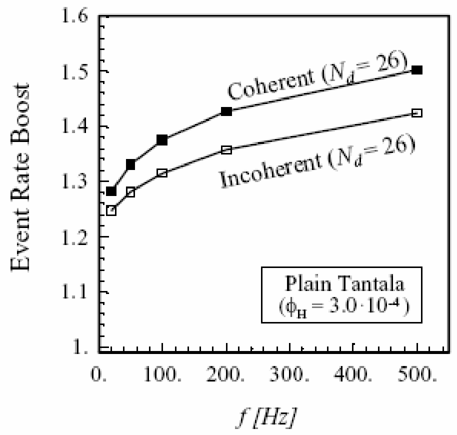
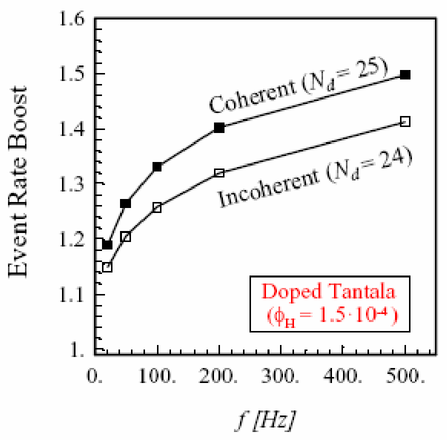
EVENT RATE BOOST (1ppm designs)

(Total noise budget included)

$\tau = 0.9727\,ppm$	ER boost @100Hz
Plain Tantala, QWL	1
Plain Tantala, OPT	1.38
Doped Tantala, QWL	1.54
Doped Tantala, OPT	2.05



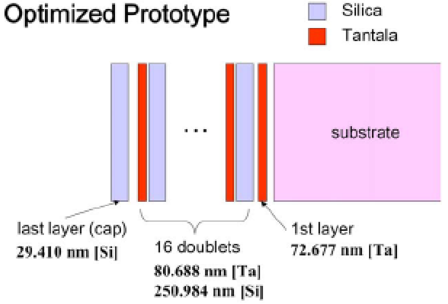


TNI PROTOTYPE ODISSEY

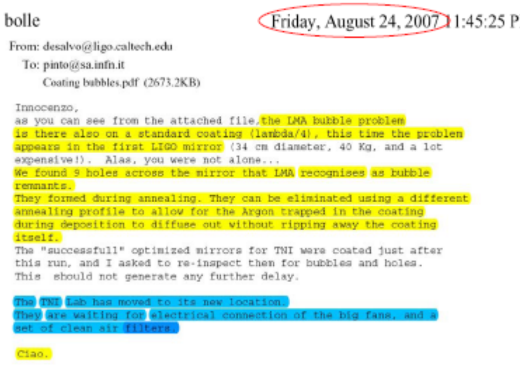
Coating	Substrate
$n_{Ta} = 2.035 - i\,1.8 \cdot 10^{-7}$	$n_S = 1.46543 - i\,4 \cdot 10^{-8}$
$n_{Si} = 1.46543 - i\,4 \cdot 10^{-8}$	$Y_S = 727 \cdot 10^8$
$Y_{Ta} = 140.0 \cdot 10^9$	$\sigma_S = 0.167$
$Y_{Si} = 727.0 \cdot 10^8$	$\phi_S = 0.0$
$\sigma_{Ta} = 0.23$	
$\sigma_{Si} = 0.167$	$\lambda = 1064.0$ nm
$\phi_{Ta} = 3.0 \cdot 10^{-4}$	
$\phi_{Si} = 5.0 \cdot 10^{-5}$	

$$\tau_p = 278.421\ ppm \quad \text{(OPT \& QWL)}$$

$$\frac{PSD_{OPT}}{PSD_{QWL}} = 0.843$$



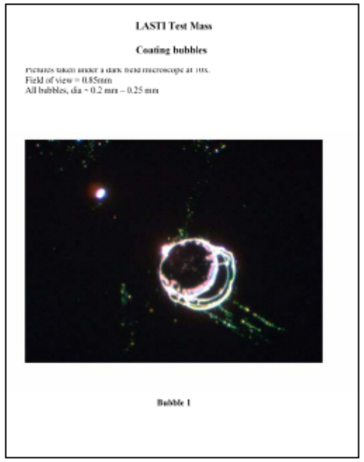
COAT – 300ppm optimized mirror, final design, november 2006



bolle Friday, August 24, 2007 11:45:25 PM

From: desalvo@ligo.caltech.edu
To: pinto@sa.infn.it
Coating bubbles.pdf (2673.2KB)

```
Innocenzo,
as you can see from the attached file,the LMA bubble problem
is there also on a standard coating (lambda/4), this time the problem
appears in the first LIGO mirror (34 cm diameter, 40 Kg, and a lot
expensive!).  Alas, you were not alone...
We found 9 holes across the mirror that LMA recognises as bubble
remnants.
They formed during annealing. They can be eliminated using a different
annealing profile to allow for the Argon trapped in the coating
during deposition to diffuse out without ripping away the coating
itself.
The "successfull" optimized mirrors for TNI were coated just after
this run, and I asked to re-inspect then for bubbles and holes.
This  should not generate any further delay.

The TNI Lab has moved to its new location.
They are waiting for electrical connection of the big fans, and a
set of clean air filters.

Ciao.
```

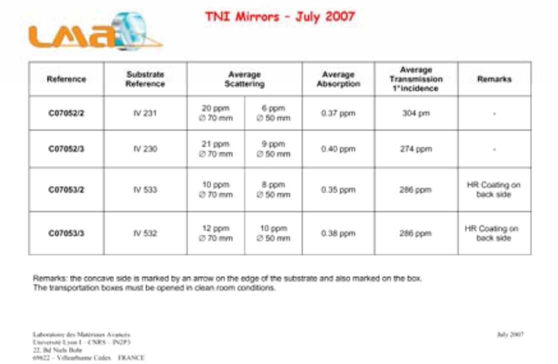


LASTI Test Mass

Coating bubbles

Pictures taken under a dark field microscope at 10x.
Field of view = 0.85mm
All bubbles, dia ~ 0.2 mm – 0.25 mm

Bubble 1



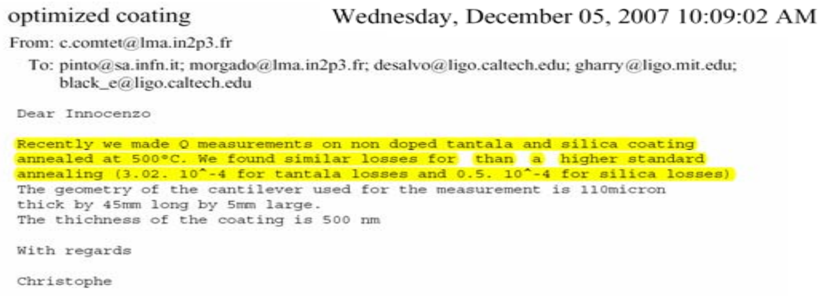
TNI Mirrors - July 2007

Reference	Substrate Reference	Average Scattering		Average Absorption	Average Transmission 1°incidence	Remarks
C07052/2	IV 231	20 ppm ∅ 70 mm	6 ppm ∅ 50 mm	0.37 ppm	304 pm	-
C07052/3	IV 230	21 ppm ∅ 70 mm	9 ppm ∅ 50 mm	0.40 ppm	274 ppm	-
C07053/2	IV 533	10 ppm ∅ 70 mm	8 ppm ∅ 50 mm	0.35 ppm	286 ppm	HR Coating on back side
C07053/3	IV 532	12 ppm ∅ 70 mm	10 ppm ∅ 50 mm	0.38 ppm	286 ppm	HR Coating on back side

Remarks: the concave side is marked by an arrow on the edge of the substrate and also marked on the box.
The transportation boxes must be opened in clean room conditions.

Laboratoire des Matériaux Avancés
Université Lyon I - CNRS - IN2P3
22, Bd Niels Bohr
69622 – Villeurbanne Cedex FRANCE

July 2007



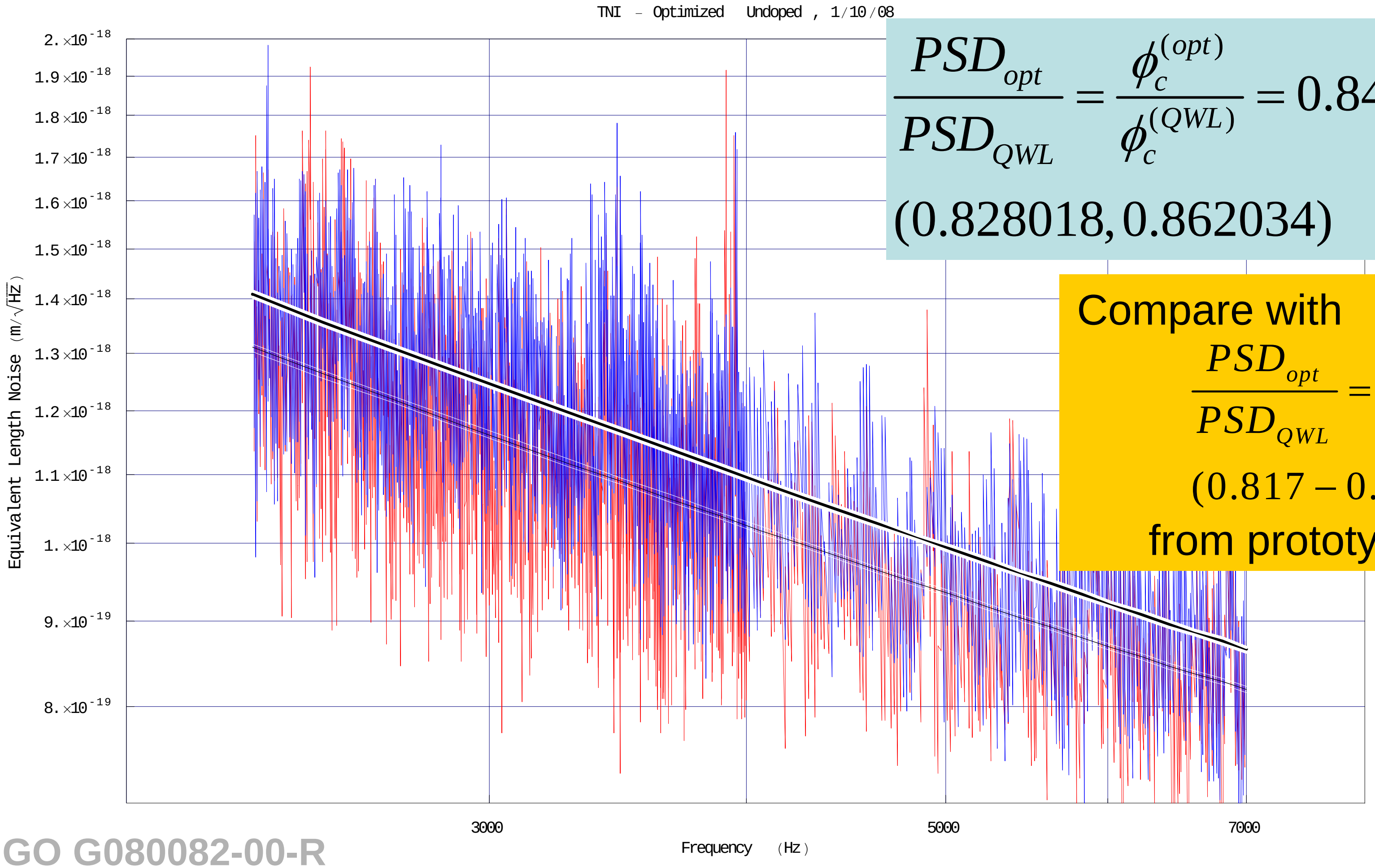
optimized coating Wednesday, December 05, 2007 10:09:02 AM

From: c.comtet@lma.in2p3.fr
To: pinto@sa.infn.it; morgado@lma.in2p3.fr; desalvo@ligo.caltech.edu; gharry@ligo.mit.edu; black_e@ligo.caltech.edu

```
Dear Innocenzo

Recently we made Q measurements on non doped tantala and silica coating
annealed at 500°C. We found similar losses for  than  a  higher standard
annealing (3.02. 10^-4 for tantala losses and 0.5. 10^-4 for silica losses)
The geometry of the cantilever used for the measurement is 110micron
thick by 45mm long by 5mm large.
The thichness of the coating is 500 nm

With regards

Christophe
```

TNI PROTYPES TESTED

[see Akira's talk, this meeting]



$$\frac{PSD_{opt}}{PSD_{QWL}} = \frac{\phi_c^{(opt)}}{\phi_c^{(QWL)}} = 0.844869$$

$$(0.828018, 0.862034)$$

Compare with

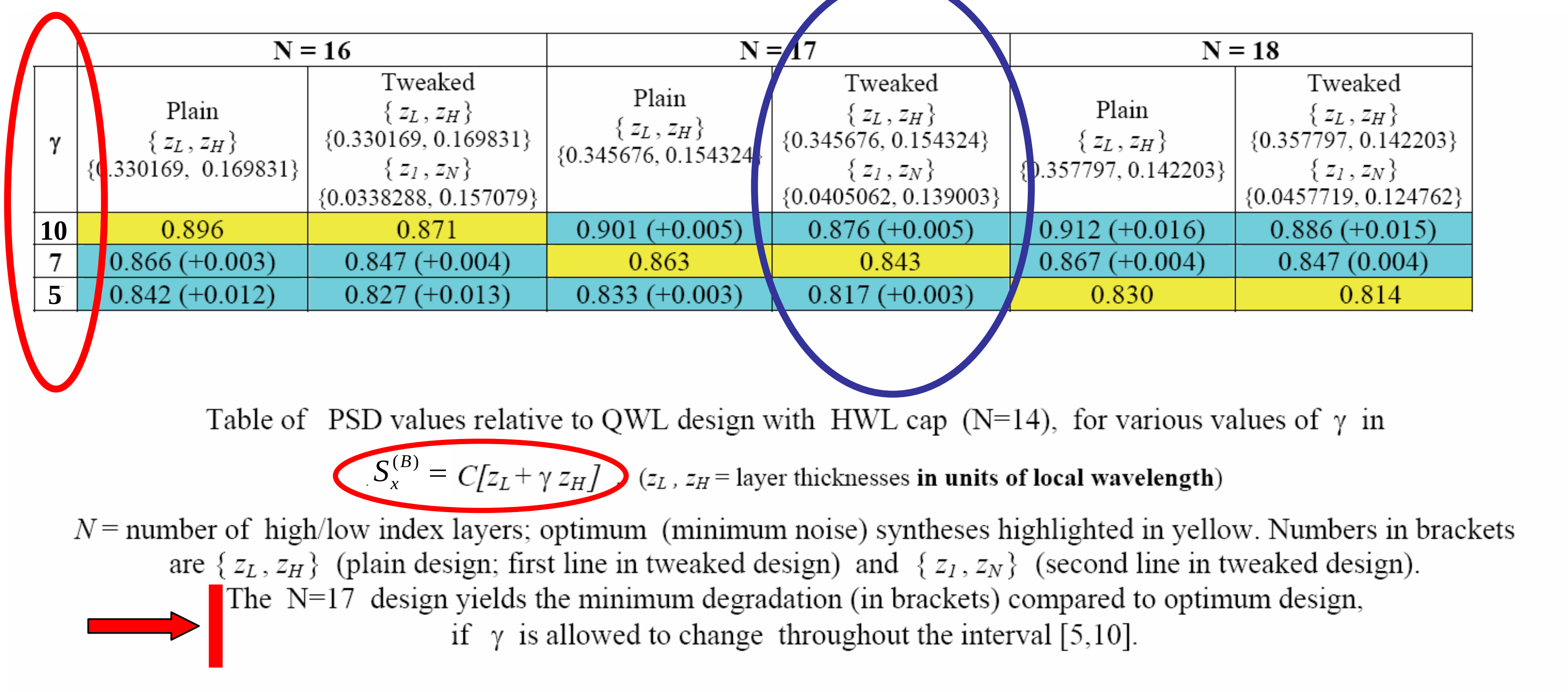
$$\frac{PSD_{opt}}{PSD_{QWL}} = 0.843$$

$$(0.817 - 0.876)$$

from prototype model

ROBUSTNESS - I

(TNI prototype)



γ	N = 16		N = 17		N = 18	
	Plain $\{z_L, z_H\}$ {0.330169, 0.169831}	Tweaked $\{z_L, z_H\}$ {0.330169, 0.169831} $\{z_1, z_N\}$ {0.0338288, 0.157079}	Plain $\{z_L, z_H\}$ {0.345676, 0.154324}	Tweaked $\{z_L, z_H\}$ {0.345676, 0.154324} $\{z_1, z_N\}$ {0.0405062, 0.139003}	Plain $\{z_L, z_H\}$ {0.357797, 0.142203}	Tweaked $\{z_L, z_H\}$ {0.357797, 0.142203} $\{z_1, z_N\}$ {0.0457719, 0.124762}
10	0.896	0.871	0.901 (+0.005)	0.876 (+0.005)	0.912 (+0.016)	0.886 (+0.015)
7	0.866 (+0.003)	0.847 (+0.004)	0.863	0.843	0.867 (+0.004)	0.847 (0.004)
5	0.842 (+0.012)	0.827 (+0.013)	0.833 (+0.003)	0.817 (+0.003)	0.830	0.814

Table of PSD values relative to QWL design with HWL cap (N=14), for various values of γ in

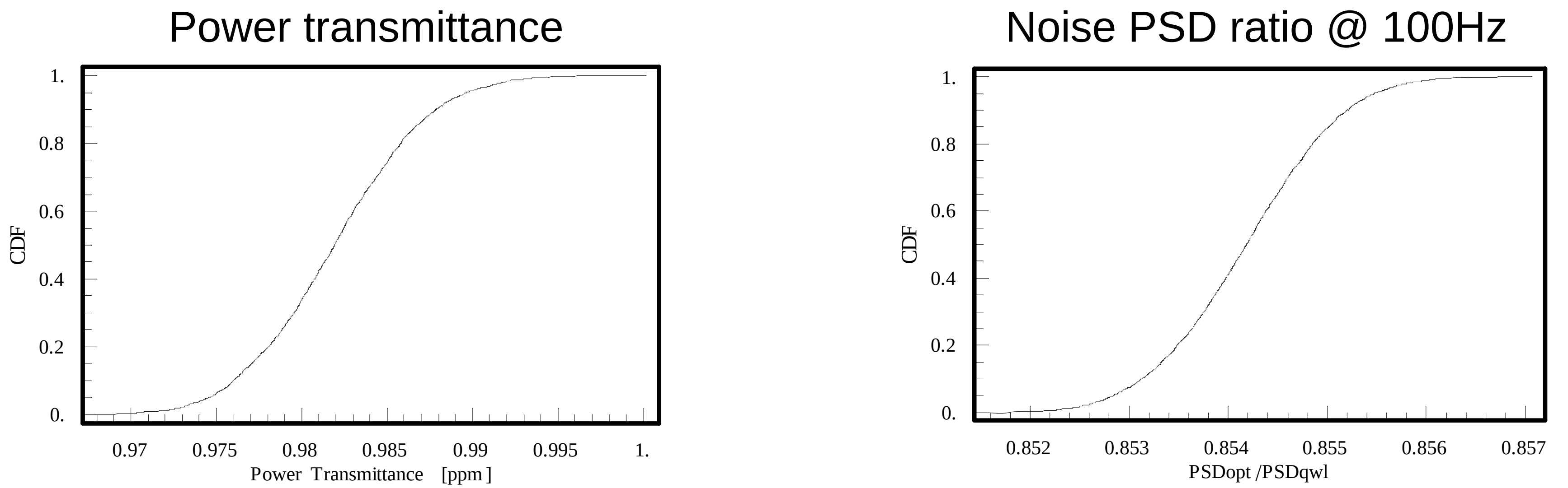
$S_x^{(B)} = C[z_L + \gamma z_H]$ (z_L , z_H = layer thicknesses **in units of local wavelength**)

N = number of high/low index layers; optimum (minimum noise) syntheses highlighted in yellow. Numbers in brackets are $\{z_L, z_H\}$ (plain design; first line in tweaked design) and $\{z_1, z_N\}$ (second line in tweaked design).

The N=17 design yields the minimum degradation (in brackets) compared to optimum design, if γ is allowed to change throughout the interval [5,10].

ROBUSTNESS - II

Random uniform layer-thickness errors, $|\delta\ell| \leq 1nm$, 1ppm prototype.

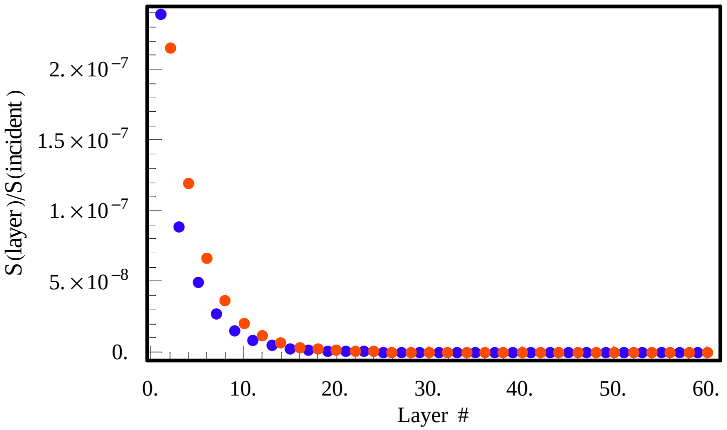


(10^4 realizations)

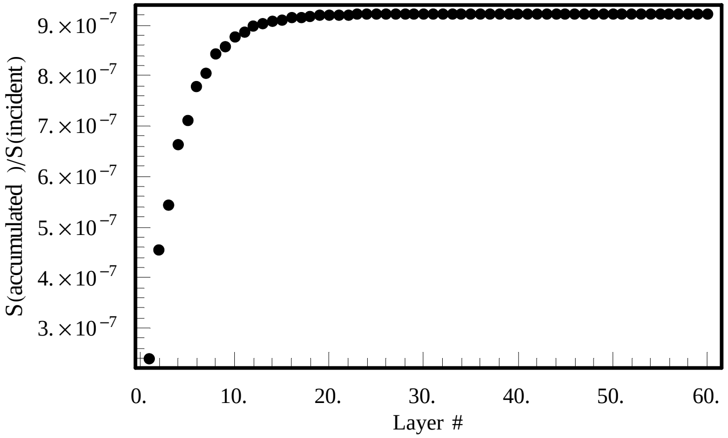
OPTICAL LOSSES

(1ppm prototype).

Optical Power Loss

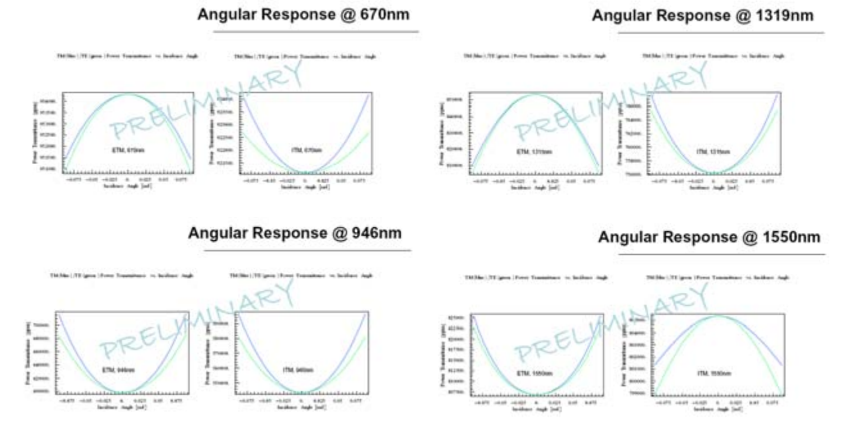


Accumulated Optical Power Loss

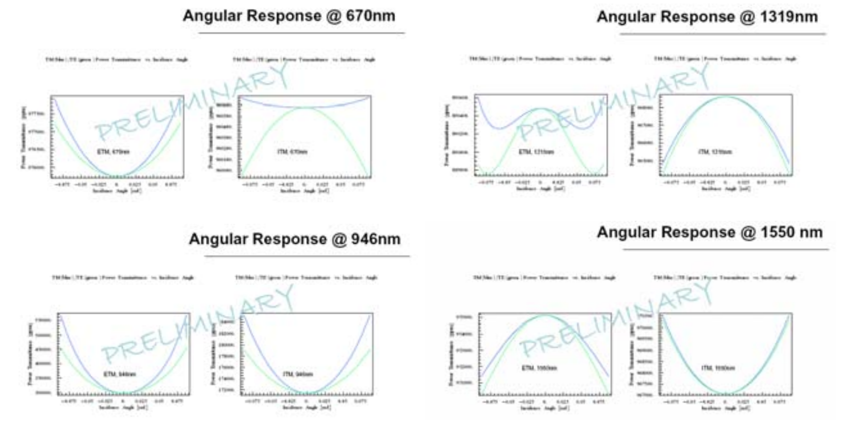


FURTHER USABLE REFLECTANCE WINDOWs

optimized



QWL



Power reflectance at various wavelengths. QWL vs optimized design.

λ [nm]		670		946		1319		1550	
		QWL	OPT	QWL	OPT	QWL	OPT	QWL	OPT
ETM	$R_{TE,TM}$ (0 deg)	0.024	0.046	0.56	0.40	0.11	0.15	0.045	0.19
	R_{TE} (±5 deg)	0.023	0.049	0.60	0.31	0.11	0.19	0.050	0.18
	R_{TM} (±5 deg)	0.022	0.048	0.61	0.28	0.10	0.19	0.048	0.17
ITM	$R_{TE,TM}$ (0 deg)	0.039	0.079	0.83	0.46	0.031	0.24	0.033	0.19
	R_{TE} (±5 deg)	0.040	0.077	0.82	0.42	0.036	0.23	0.030	0.20
	R_{TM} (±5 deg)	0.039	0.076	0.81	0.40	0.035	0.23	0.029	0.20

Nominal power transmittances @ 1064nm, normal incidence:
6.324 ppm (ETM), 14172 ppm (ITM)

- •No need to constrain noise - optimization in order to get useful power reflectance at other (laser) wavelength(s) .
- •Optimized design no worse than QWL in terms of power - reflectance at other (laser) wavelengths (worse but still OK @ 946nm).
- •No critical dependence of result on material parameters.

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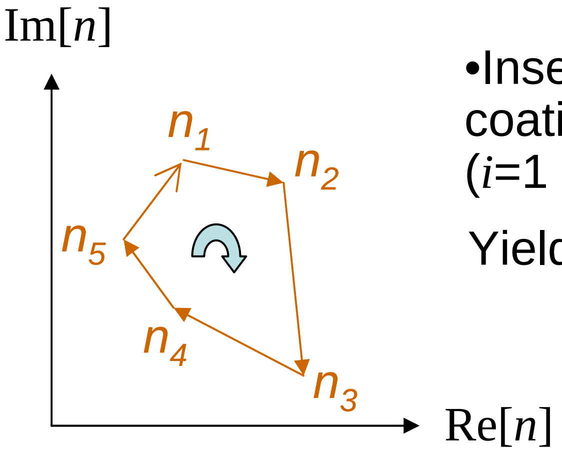
M-ary COATINGS

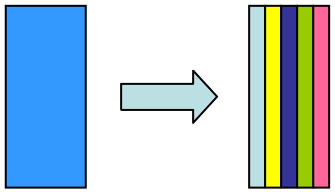


- Insertion of a (different) dielectric layer in a stacked doublet coating boosts reflectance provided $\mathrm{Im}[(n_{i+1}-n_i)/(n_i-n_{i-1})]<0$ (i=1 is top layer) [J. I. Larruquert, Opt. Comm. 206 (2002) 259]

Yields simple material selection rule (“turn clockwise”).

May result into reduced coating noise
Material downselection **TBD.**

- Use materials with *different* properties (contrart, losses) in topmost / bottom layers to reduce coating thickness without increasing optical loss [P.G. Verly, Appl.Opt. 37 (1998) 7327]

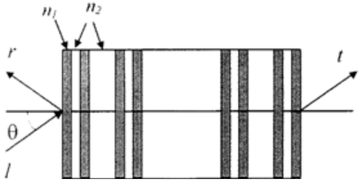


- Stack of *sub-wavelength* layers “equivalent” to *homogeneous* medium [E. Tuncer, Physical Review B 71 (2005) 012101]. *E,n,*ϕ *etc.* obtained from appropriate *mixture* formulas. Way to engineering *new* materials ?

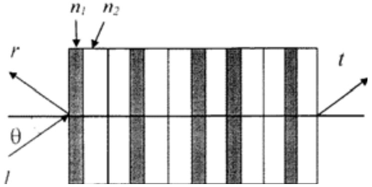
BAND-GAP ENGINEERED COATINGS

“Photonic Band Gap Engineering” [Yablonovitch, Opt. Lett.23 (1998) 1648]

(pre)fractal
Cantor 3-adic



substitutional
Fibonacci
A,B → AB,A

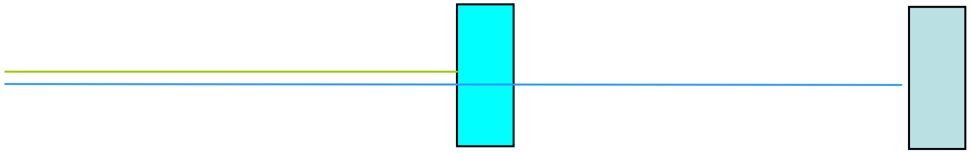


Nearly *omni-directional (and wideband) reflection* obtainable [D. Lusk and F. Placido, Thin Solid Films, 392 (2005) 226];

⇨ Expected to *mitigate misalignment instability* in (otherwise stable) spherical / confocal mesa - beam (or hyperboloidal beam) cavities w. coated mirrors

Perfect transmission bands also obtainable [R.W. Peng et al., Appl. Phys. Lett. 80 (2002) 3063]

⇨ May permit building *two (almost independent) interferometers in a single beam pipe)…*



CONCLUSIONS

- Coating thickness optimization *almost mandatory* to minimize coating noise when using doped Tantala, yielding a substantial boost (> 30%@ 100Hz) in the expected event rate, as compared to QWL design.
- Optimal design *almost the same for both* the incoherent and the coherent thermooptic noise formula.
- Optimal design is *robust* against thickness deposition errors, and can be made *judiciously tolerant* w.r.t. uncertainties in the relevant material parameters.
- Among all proposed coating noise reduction techniques (new materials, cryogenic mirrors, flat-top beams) thickness optimization is the *cheapest reliable option*

ONGOING & FUTURE WORK

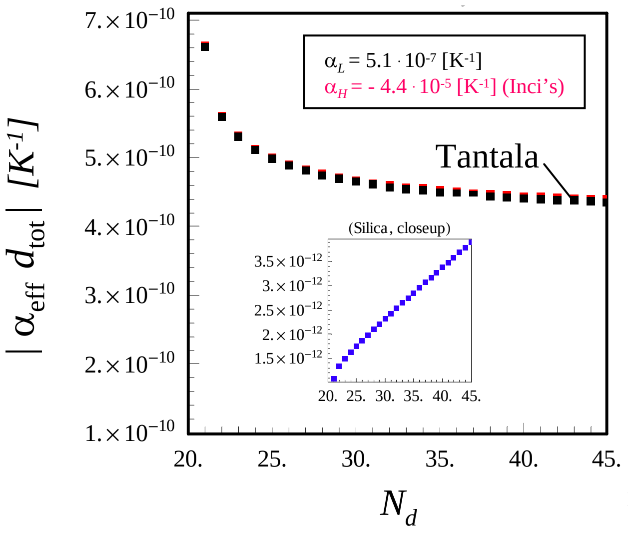
- Design of optimized doped-Tantala mirrors for TNI.
  - Sensitivity study w.r.t. to uncertainties in material params.
  - Identification of most tolerant quasi-optimal design **Funding requested to INFN**.

- M-ary and sub-wavelength coatings optimization study started.

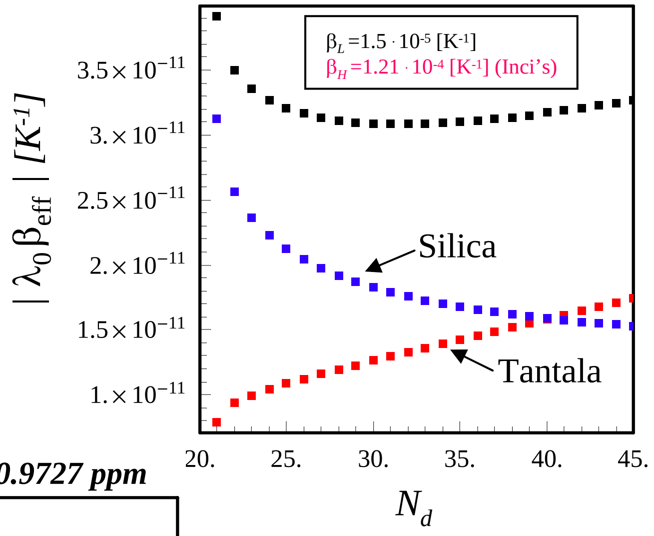
- Band-gap engineered optimised coatings study restarted.

INCI’S NUMBERS

[Naci-Inci, J. Phys. D37 (2004) 3151.]



(TE Coefficient)



(TR Coefficient)