The Search for High Frequency Gravitational Waves (using the LIGO interferometers)

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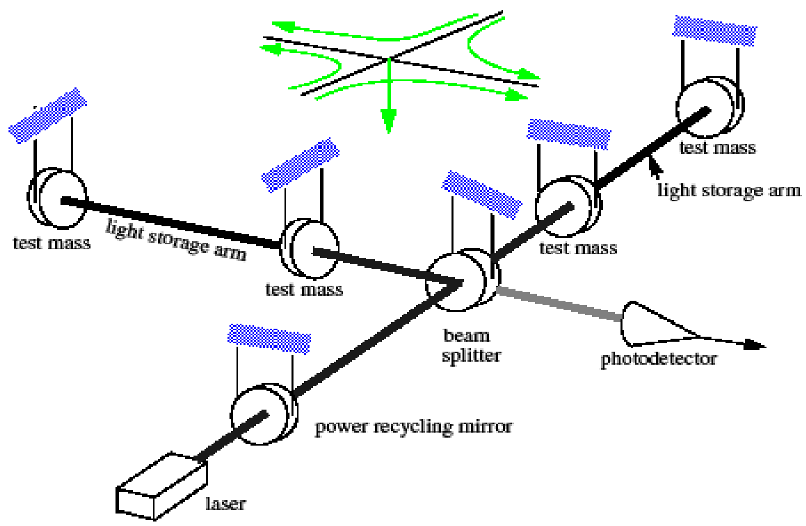
Interferometers as G.W. detectors



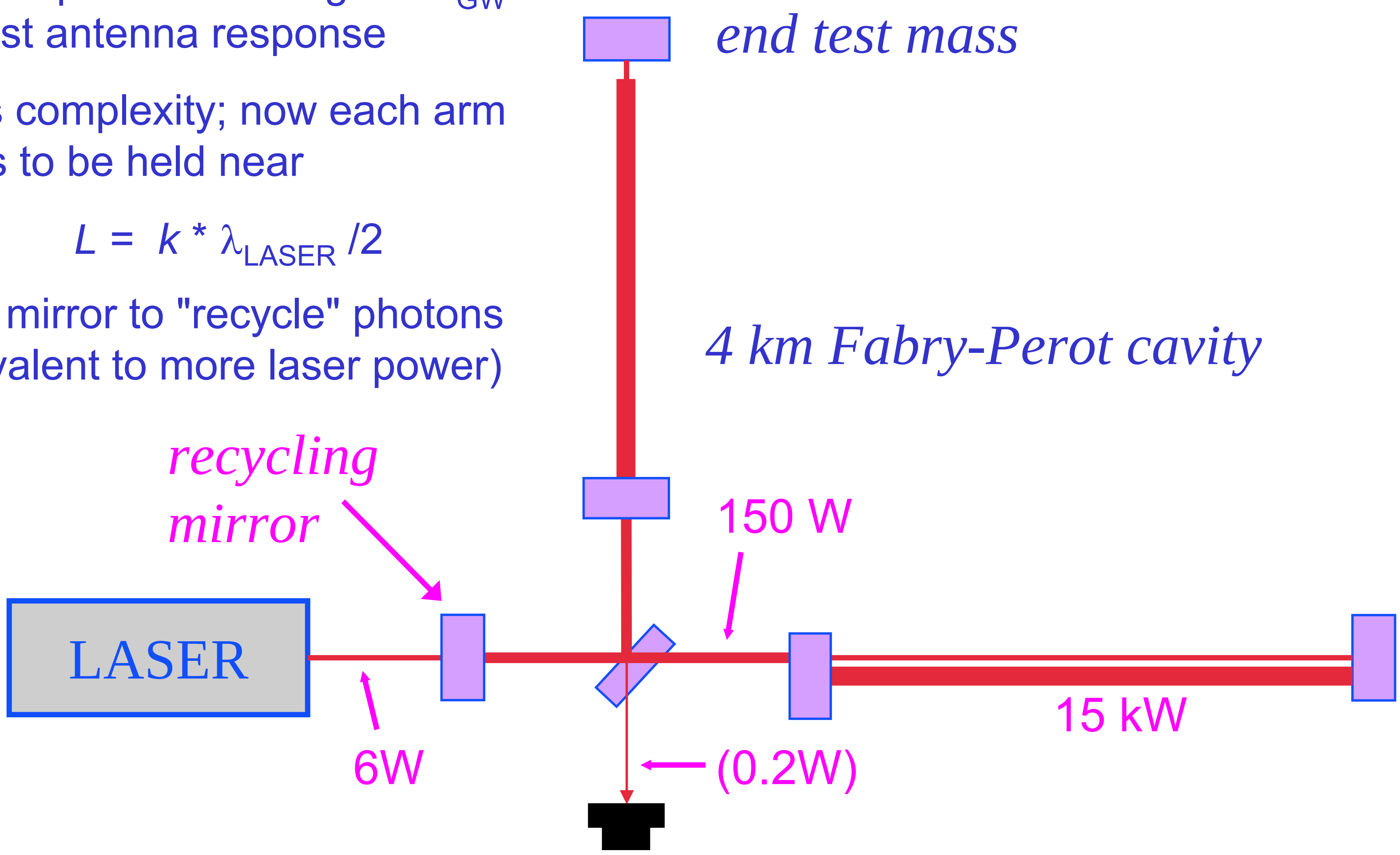
1. Measure phase shift of light propagating in the arms
2. Orthogonal arms give rise to differential signal
3. Fabry-Perot cavities in arms increase the effective arm length $L_{eff} \sim 130\ L$ with $L = 4$ km
4. $h = \Delta L/L = (L_x - L_y)/L$ g.w. strain

Recycled Michelson Interferometer with Fabry-Perot arms

- Want "optical" arm length ~ λ_{GW} /4 for best antenna response
- Adds complexity; now each arm needs to be held near

$$L = k * \lambda_{LASER} /2$$

- Add mirror to "recycle" photons (equivalent to more laser power)



The observable phase shift

G.W. propagating along the z-axis, in the TT gauge

$$h_{\mu\nu}(t,z) = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \exp[i\Omega(t + z/c)]$$

where $g_{\mu\nu} = \eta_{\mu\nu} - h_{\mu\nu}$

Let the detector be at $z = 0$, and $T = L/c$. The mirrors are **free**

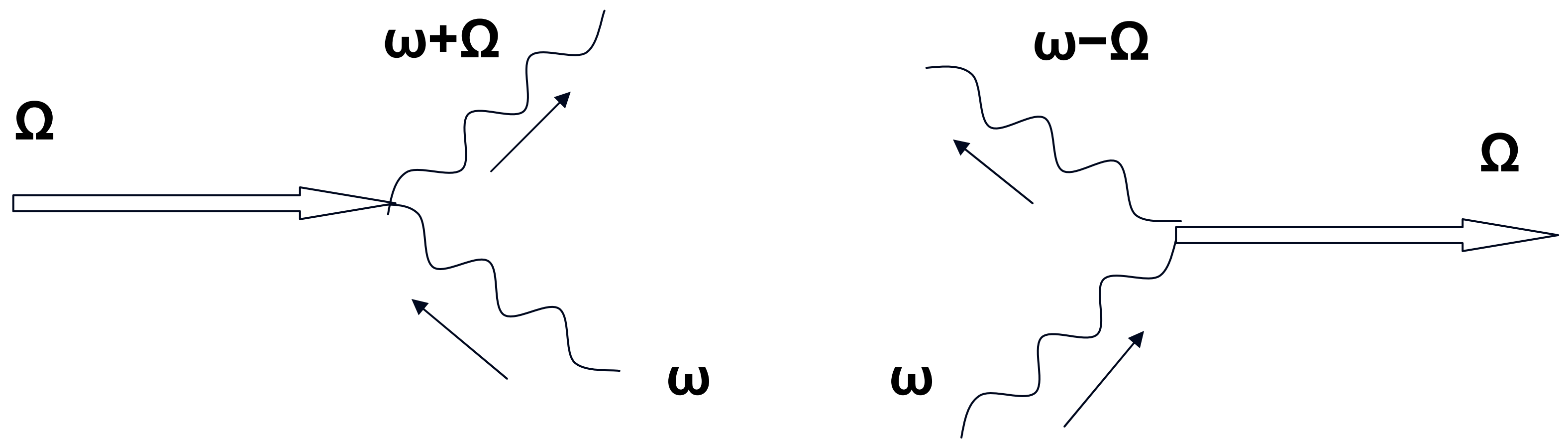
$$\Delta\varphi(t) = (2\pi/\lambda)\, \Delta x(t) = \omega_c T\, h(t)\, [\sin(\Omega T)/\Omega T]\, \exp(-i\Omega T)$$

In the TT frame : mirror coordinates do not change

In local frame : mirror positions change

When $\Omega T = \pi$, $\Delta\varphi = 0$ (but only for normal incidence)

Another view of the gravitational coupling



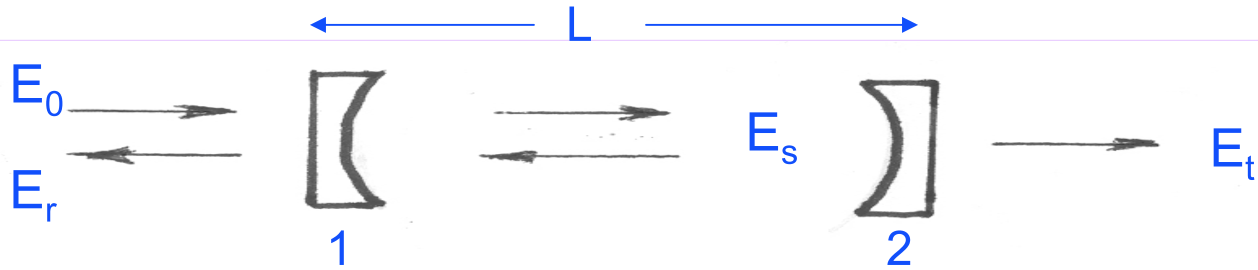
Absorption of graviton by laser field

Stimulated emission of a graviton

The G.W field is classical: the occupation number is ~ 10^{35}
Absorption and emission have equal probabilities. There is no energy exchange with the G.W. field; only a phase shift of the carrier. Since $\Delta\varphi = \alpha\, h \cos(\Omega t)$ the field has **sidebands**

$$A_\omega(t) = e^{i(\omega t+\Delta\varphi)} \approx e^{i\omega t} + \tfrac{1}{2}\, i\, \alpha\, e^{i(\omega+\Omega)t} + \tfrac{1}{2}\, i\, \alpha\, e^{i(\omega-\Omega)t}$$

The Fabry-Perot cavity (1897)



Amplitude reflectivity	r_1 , r_2	
transmittance	t_1 , t_2	$r^2 + t^2 + A = 1$
Absorption (power)	A_1, A_2	

The fields depend on the phase accumulated in a round trip

$$\varphi = \omega_c(2L/c) = 2\pi\,(2L/\lambda_c)$$

$$E_r = E_0\,[\,r_1 - r_2\,(r_1^{\,2} + t_1^{\,2})\,e^{-i\varphi}\,] / [\,1 - r_1 r_2\,e^{-i\varphi}\,]$$

$$E_s = E_0\,[\,i t_1\,] / [\,1 - r_1 r_2\,e^{-i\varphi}\,]$$

$$E_t = E_0\,[\,-\,t_1\,t_2\,e^{-i\varphi/2}\,] / [\,1 - r_1 r_2\,e^{-i\varphi}\,]$$

For LIGO $r_1 = 0.985$, $r_2 \sim 1$ thus (when $\varphi = 0$) **$P_s \sim 130\,P_0$**

Mode spectrum of the F-P cavity

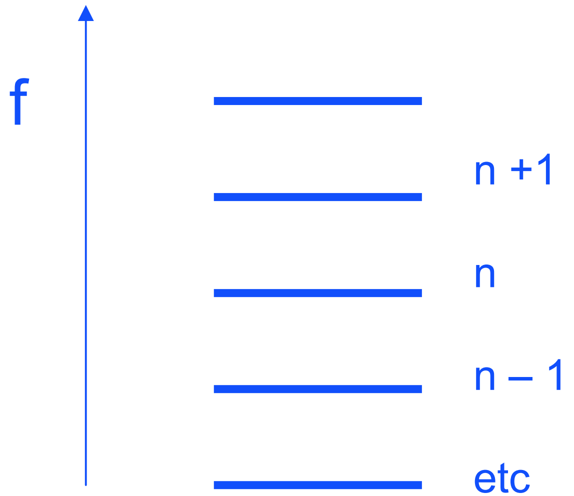
Q (Quality factor) = 2π[Stored energy / Energy lost per cycle]

$Q = 2\pi(\tau/T) = \omega\tau$ τ is the decay time of the stored energy

$\Delta f/f = \Delta\omega/\omega = 1/Q$ Δf is the FWHM of the optical line

The **Finesse** $F = Q/(2L/\lambda) = \pi / (1 - R - A)$

$\Delta f = (c/2L)/F = f_{fsr}/F$ $\mathbf{f_{fsr} = c/2L}$ is the free spectral range

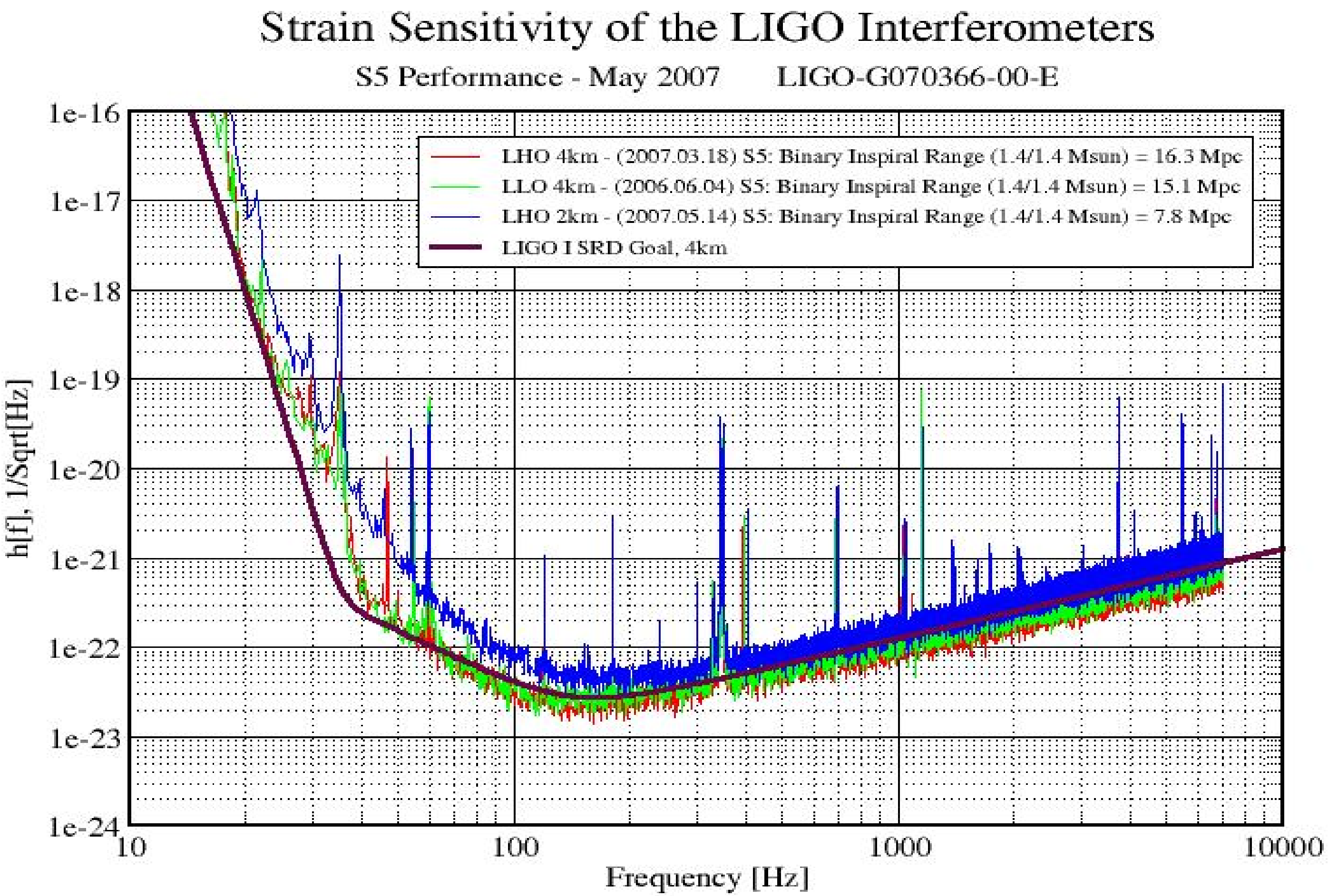


The modes are equally spaced and differ by the fsr frequency. Normally only one mode is populated; excitation to other modes is possible.

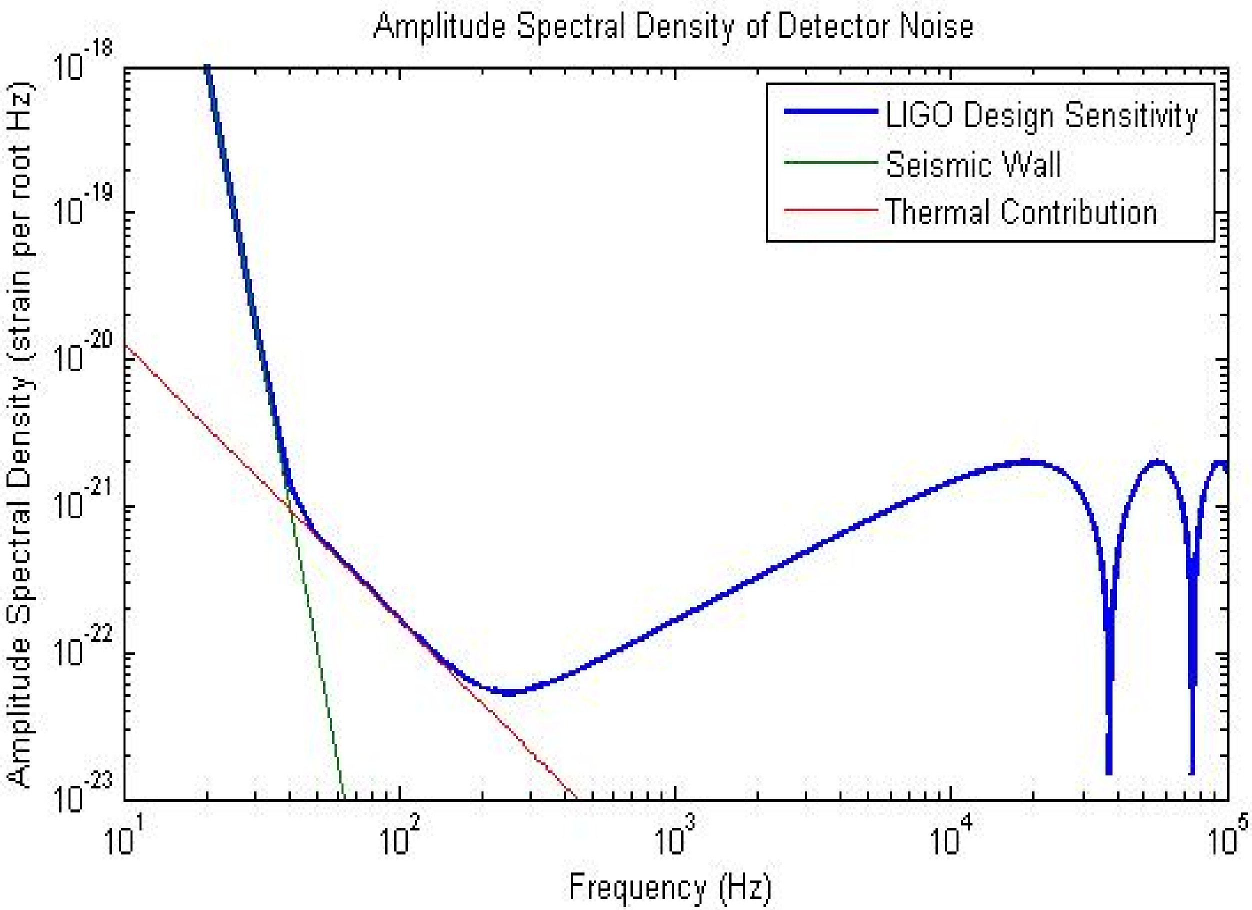
For LIGO $\mathbf{n = 2L/\lambda = 8\times10^{9}}$

$\mathbf{f_{fsr} = 37.520\ kHz}$ $\mathbf{Q \sim 10^{14}}$

Frequency dependence of the interferometer response

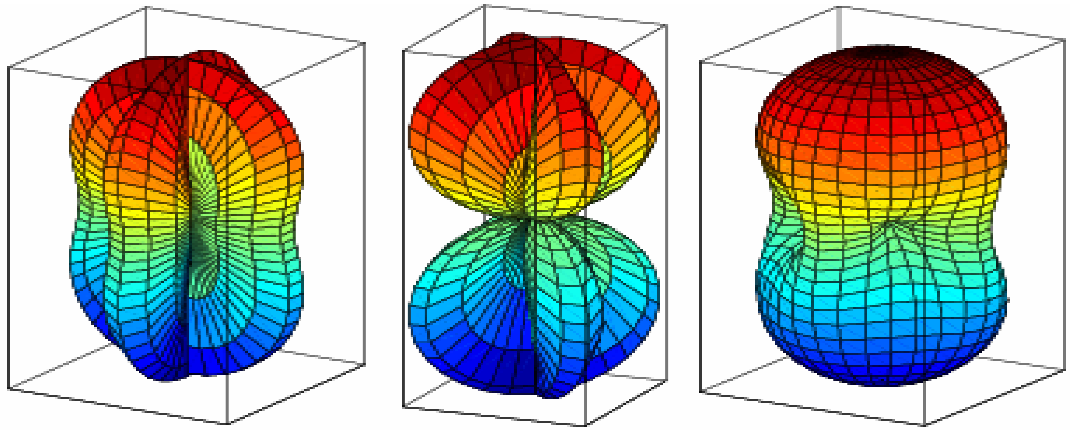


Calculated response to 100 kHz



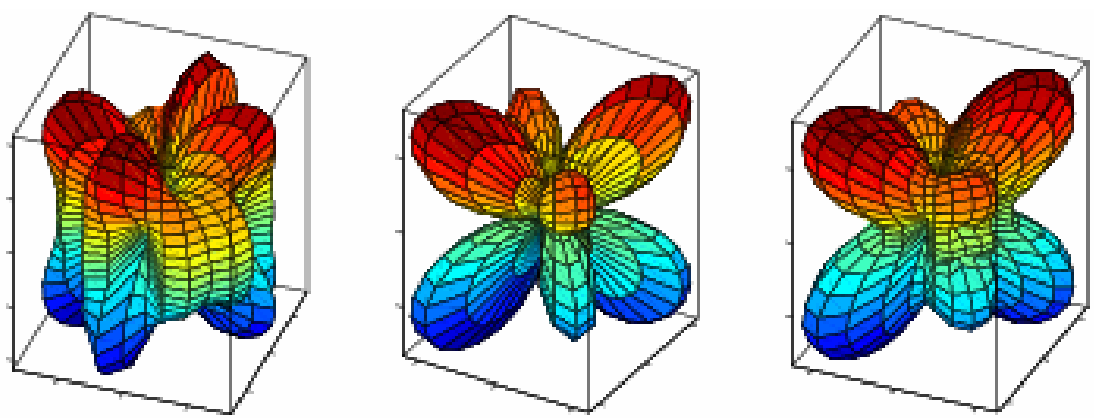
Response as a function of direction of incidence & polarization of the G.W.

Low frequencies



h_+ polarization **$h_\times$ polarization** **rms polarization**

fsr ~ 37.5 kHz



Data acquisition and analysis

At low frequency we use the ERROR signal. At high frequency the antisymmetric port signal AS_Q.

The light intensity is sampled at f_s = 16,384 Hz (limited readout at f_s = 262,144 Hz); $\mathbf{f_{max} = f_s/2}$

Fourier transform h(t) to the frequency domain

$$h(f) = S(f)^{1/2} = \{ (1/T) \, |\int h(t) \, e^{-i2\pi ft} \, dt|^2 \}^{1/2}$$

The spectrum must be calibrated,

$$h(f)_{calibrated} = R(f) \, h(f) \qquad \text{strain}/\sqrt{\text{Hz}}$$

LIGO data is analyzed for: (1) **Bursts,** (2) **Inspiraling binaries,** (3) **cw signals,** (4) **Stochastic background**

The stochastic background

Can be either of Cosmological or Astrophysical origin.

h(t) has zero mean, is isotropic, and unpolarized.

It is characterized by a spectral density H(f)

The energy density in a G.W. is

$$\rho_G = \frac{c^2}{32\pi G} \langle \dot{h}_{ab}(t,\vec{x}) \dot{h}^{ab}(t,\vec{x}) \rangle$$

The normalized energy density per log frequency interval

$$\Omega(f) = \frac{1}{\rho_c} \frac{d\rho_G}{d\ln f} = f \frac{1}{\rho_c} \frac{d\rho_G}{df} = \frac{10\pi^2}{3H_0^2} f^3 < h_1^*(f) h_2(f) >$$

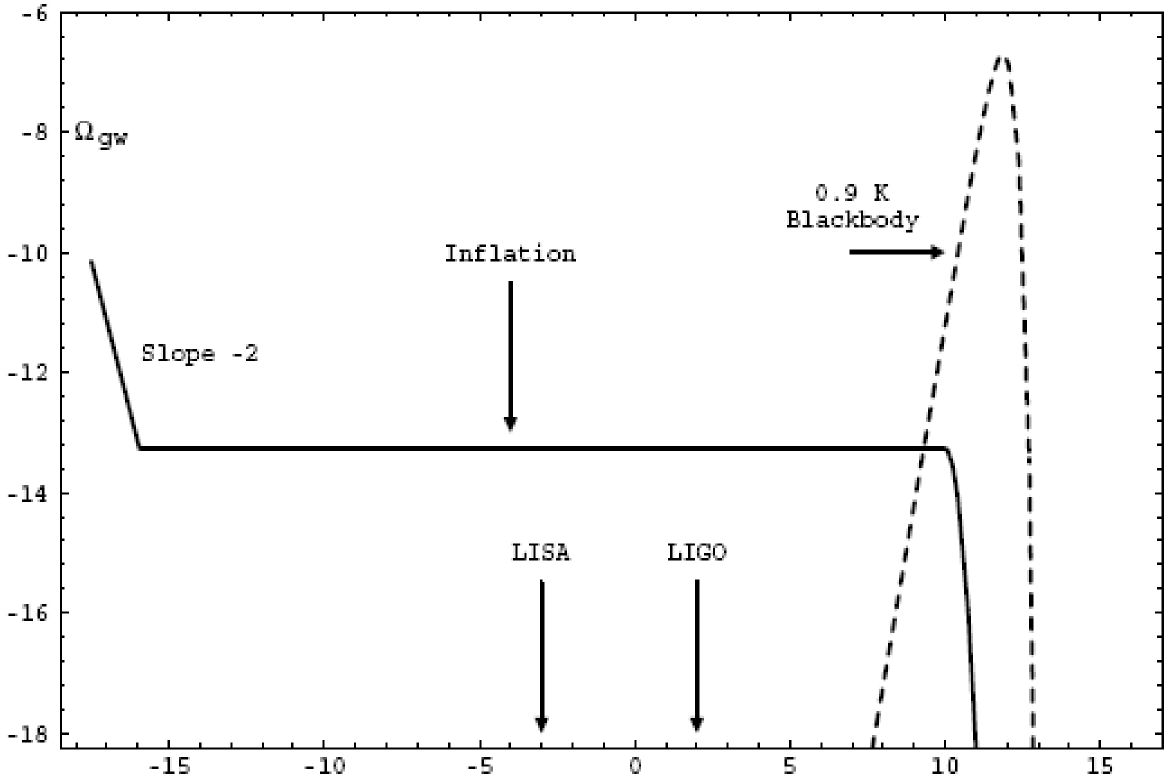
BigBang nucleosynthesis limits the integral of Ω(f)

$$\int_0^\infty \Omega(f) d(\ln f) = \Omega$$

The observable strain h(f) **falls off as $1/f^{3/2}$** for fixed Ω(f)

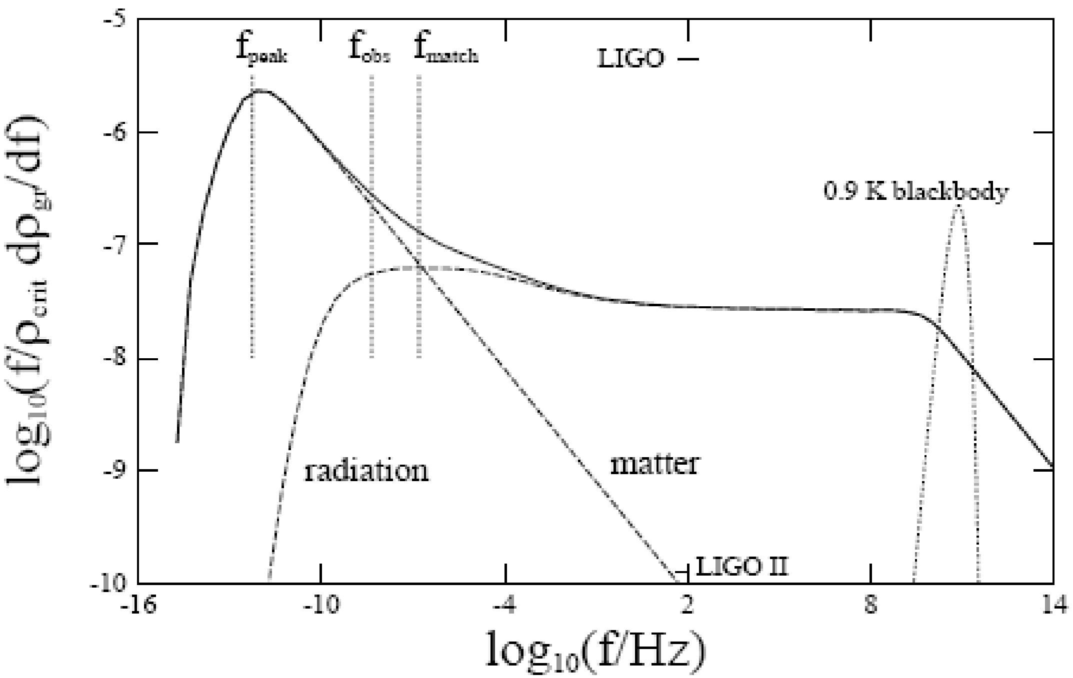
Stochastic power spectrum from inflation

log(Ω(f))



$\log_{10}$ (f / Hz)

Stochastic power spectrum from cosmic strings



Cross-correlation of two detectors

The level of the stochastic signal is expected to be **below the noise** level. This in contrast to the discovery of the 3^0K microwave radiation by Penzias and Wilson where the signal exceeded the noise.

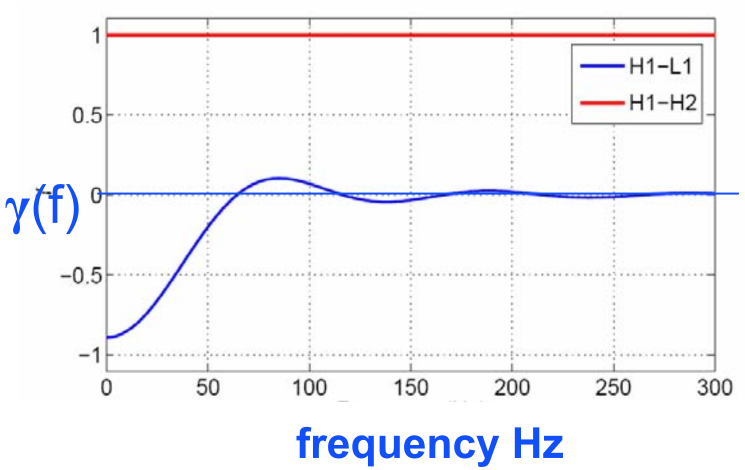
To extract the signal **correlate** two co-located detectors

$$h_1(t) = n_1(t) + s(t) \qquad h_2(t) = n_2(t) + s(t)$$

In the frequency domain

$$\langle h_1^*(f)\, h_2(f)\rangle = \langle |s(f)|^2\rangle + \{\text{terms} \rightarrow 0 \text{ for long averaging time}\}$$

If the detectors are at different sites there is a direction dependent time delay in the arrival of the signals. This modifies the cross-correlation by the **overlap reduction factor** γ**(f)**



Upper Limits from LIGO (AAS meeting 2008)

Preliminary result using data through Jan. 2007

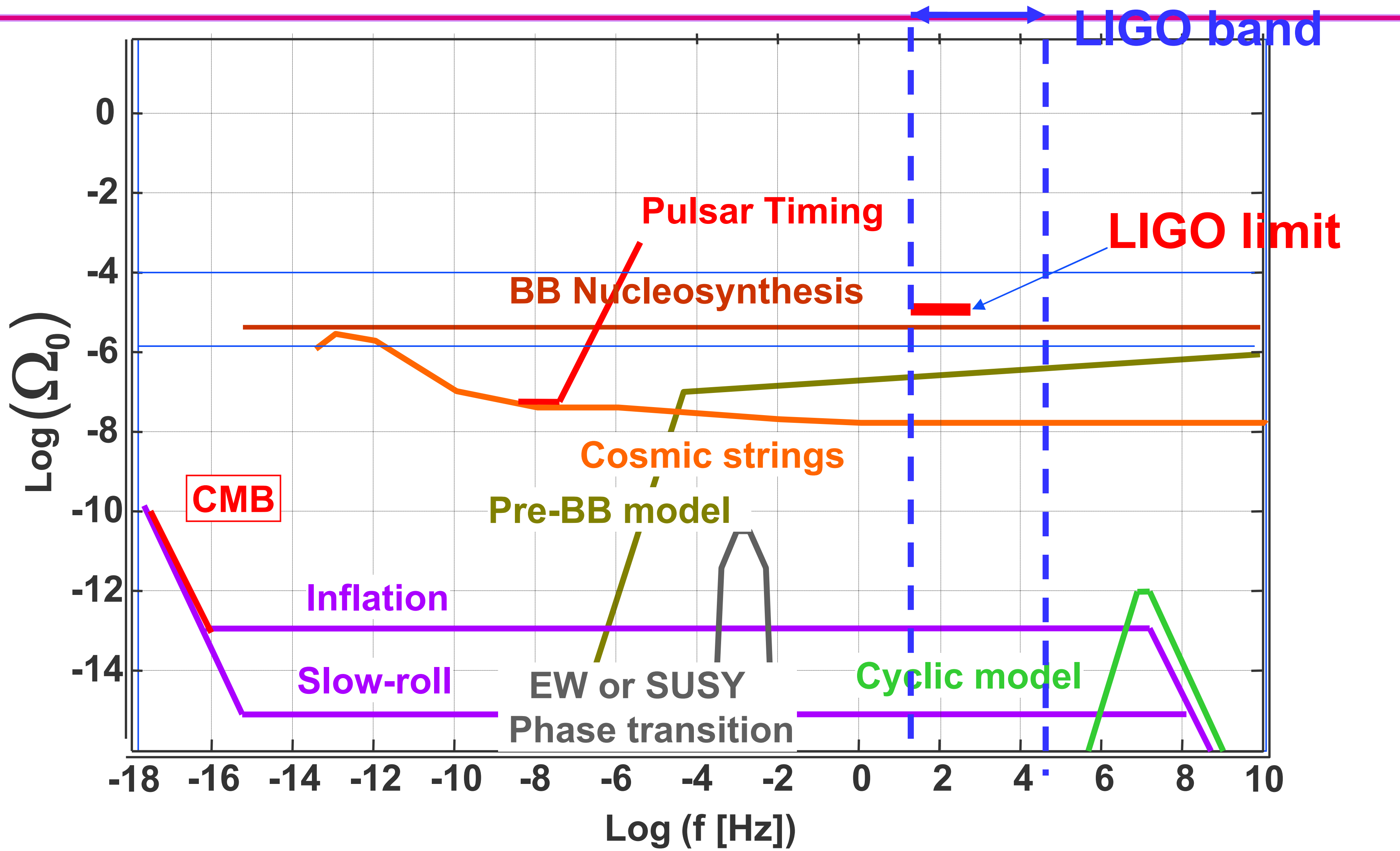
Hanford – Livingston correlation

$50 < f < 140$ Hz

$$\Omega(f) < (1.0 \pm 5.2)\times 10^{-6}$$

$$\Omega_{GW}(40 < f < 175 \text{ Hz}) < 9 \times 10^{-6}$$ 90% confidence

Predictions and limits on the stochastic background



Earth Tides

Origin: Moon and Sun (half as strong)

Mainly diurnal and twice-daily frequencies

Amplitude $\delta g/g \sim 4\times10^{-8}$

Increase in the radius of the Earth's surface causes extension in the horizontal plane ("Love" numbers)

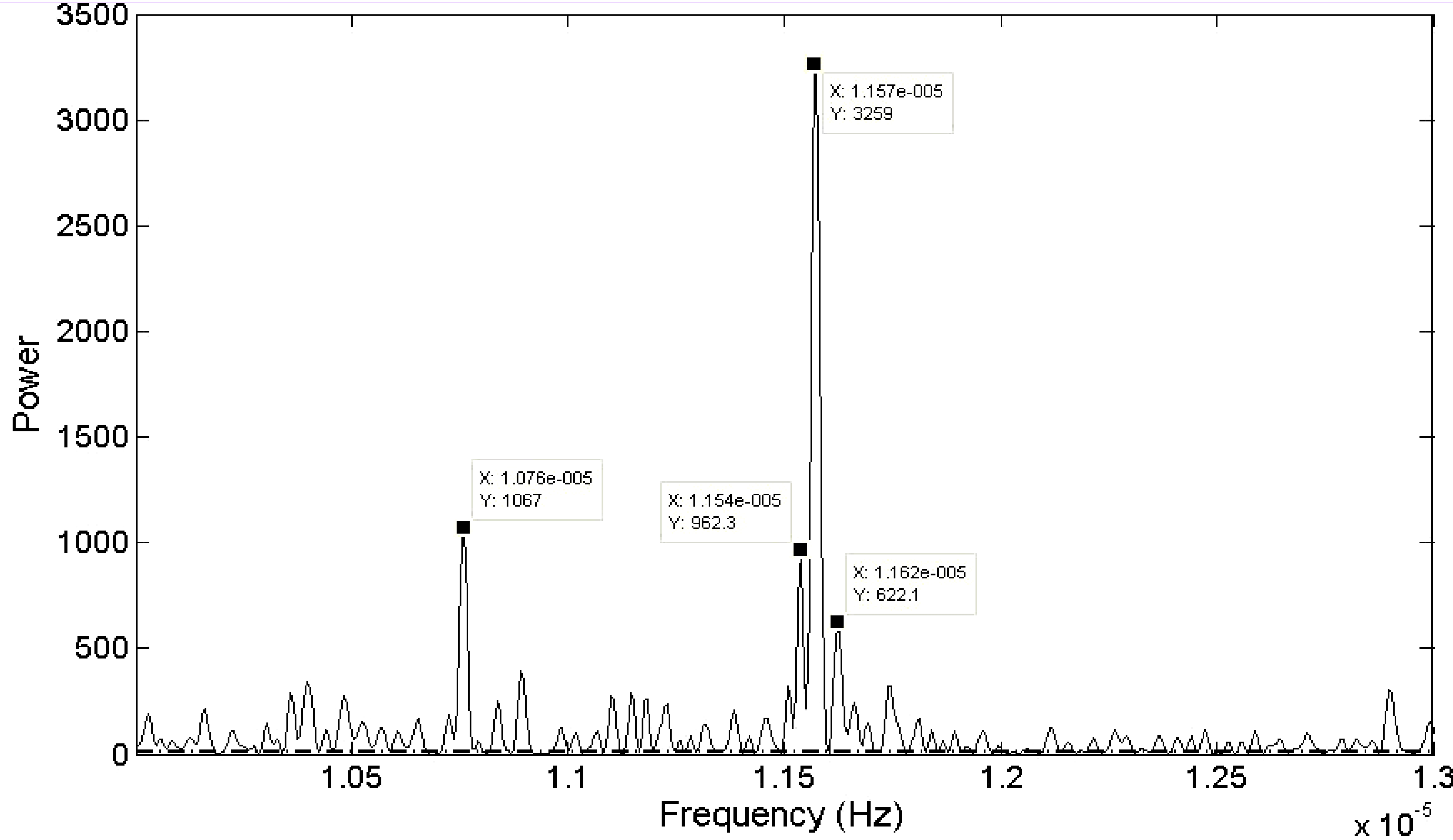
For the LIGO 4 km arms the extensions are ~ 100 μm and give rise to both common and differential arm changes.

To correct this effect the **tidal servo** changes the carrier frequency (common mode) and a piezo drive moves the entire end mirror assemblies (differential mode).

Frequencies of the principal Earth Tides

Wave	Frequency (Hz)	L= Lunar, S = Solar
Long Period component		
Ss_a	6.338×10^{-8}	S decl.
Diurnal components		
O_1	1.07585×10^{-5}	L principal
P_1	1.15424	S principal
S_1	1.15741	S elliptic
${}^{m}K_1, {}^{s}K_1$	1.16058	L,S decl.
Semi-diurnal components		
M_2	2.23643×10^{-5}	L principal
S_2	2.31481	S principal
${}^{m}K_2, {}^{s}K_2$	2.32115	L,S decl.

Tidal frequency spectrum in the diurnal region



The y-axis in a.u. is proportional to the modulation of the psd [(strain)^2/Hz]

Gravitational frequency shifts

The frequency of e.m. radiation is shifted by the gradient of a gravitational field. The IFO arm lengths must be adjusted.

For a point source of mass M at a distance R the "red shift" is

$$\Delta f/f = (GM/c^2R^2)\ \Delta R$$

For the Sun, and LIGO (ΔR = 4 km) $\Delta f/f = 2.6\times10^{-16}$ (= h)

However locally the tides introduce a tangential acceleration of only $\delta g \sim 10^{-7}$ g leading to

$$\Delta f/f = (\delta g/c^2)\times L \sim 4\times10^{-20}$$

This effect could be observable because δg is time dependent and in principle imposes a diurnal variation on the error signal. However the much larger effect from the physical extension of the arms at the same frequencies (and the compensation for such motion) introduce difficult systematic effects which, as yet, have not been resolved.

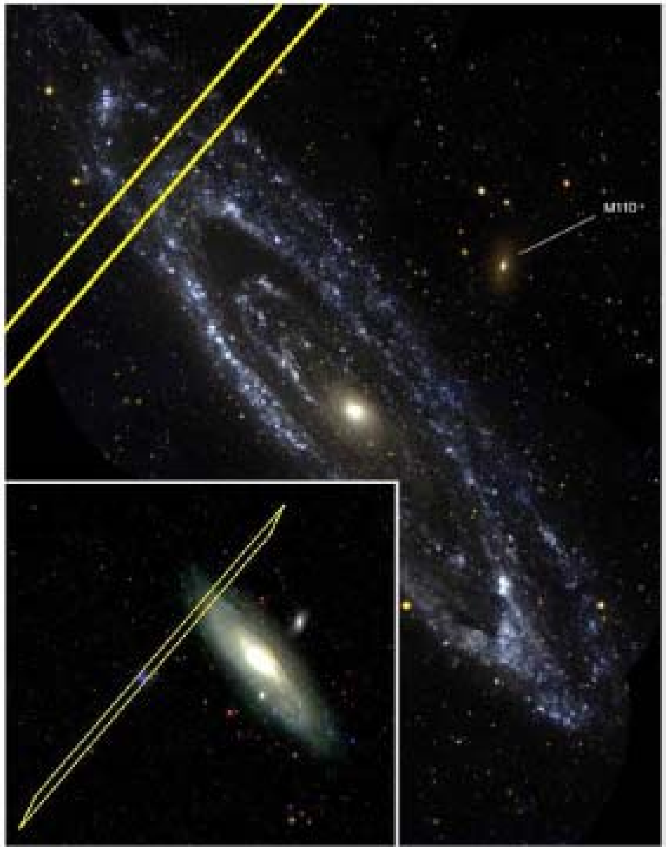
Search for a signal from GRB 070221

Intense, short duration ~0.15 s hard spectrum GRB detected by 5 space craft.
Located 1.1^{o} from the center of M31 (Andromeda Galaxy)

Distance to M31 is ~ 770 kpc

Typical GRB luminosities of
10^{48} - 10^{52} erg
place it at ~ 23 Mpc

H1 and H2 (but not L1) were in Science mode at that time

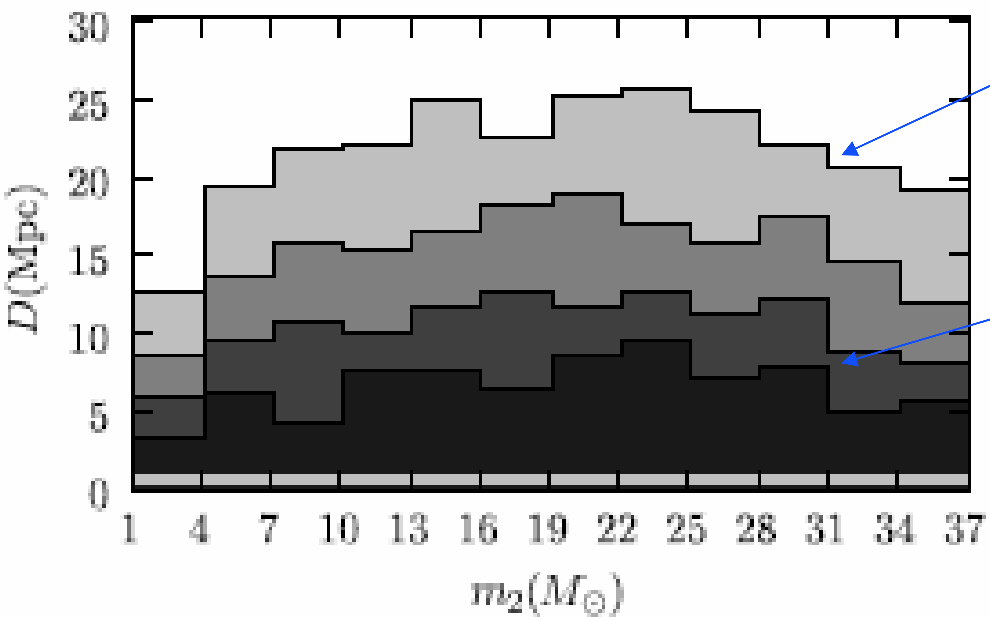


Search for compact binary inspirals at the GRB time

Mass range covered

$1\ M_o < m_1 < 3\ M_o$ $1\ M_o < m_2 < 40\ M_o$

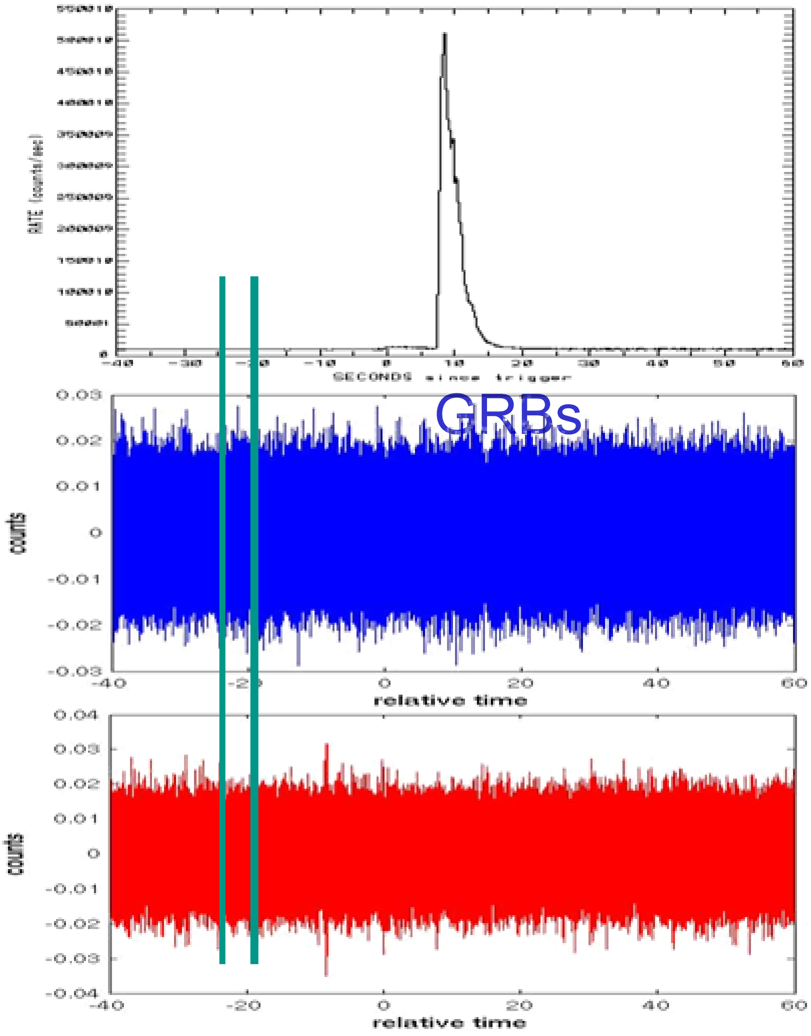
~7,000 templates in H_1 and ~ 5,400 templates in H_2



Probability 25% that progenitor was at a larger distance

90% probability

Search for bursts coincident with GRB time



Use triggers from **satellites** Swift, HETE-2, INTEGRAL, IPN, Konus-Wind

Cross correlate data between pairs of detectors around time of event

25 – 100 ms target signal duration

[-2,+1] min around GRB

Compare largest measured CC to background distribution of CCs (from neighboring times with no GRB signal).

For 100 ms interval at f ~150 Hz, if the source is at M31

$E_{GW} < 7.9 \times 10^{50}$ ergs

e.m. radiation ~10^{45} $(D/770\text{kpc})^2$ ergs

Concluding Remarks

Search for cw signal at the crab frequency

Search for a stochastic signal from Sco-X

Radiometer search for point sources in the sky

Difficulties with h.f. Gravitational Waves

Enhanced/Advanced LIGO