LIGO G080029-00-R

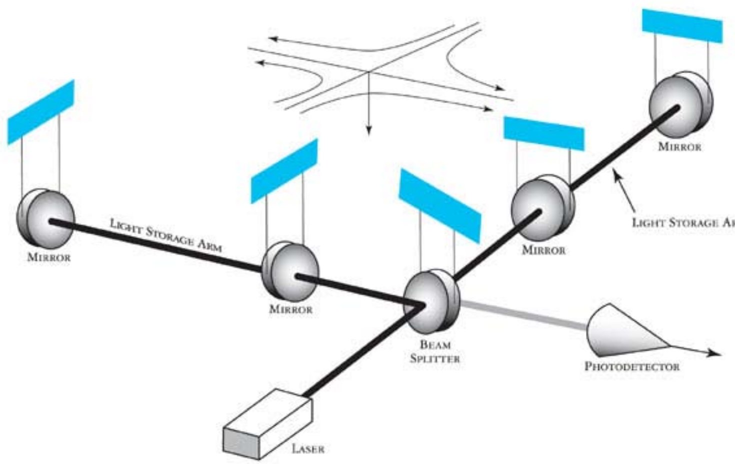
Researches on non-standard optics for advanced Gravitational Waves interferometers.

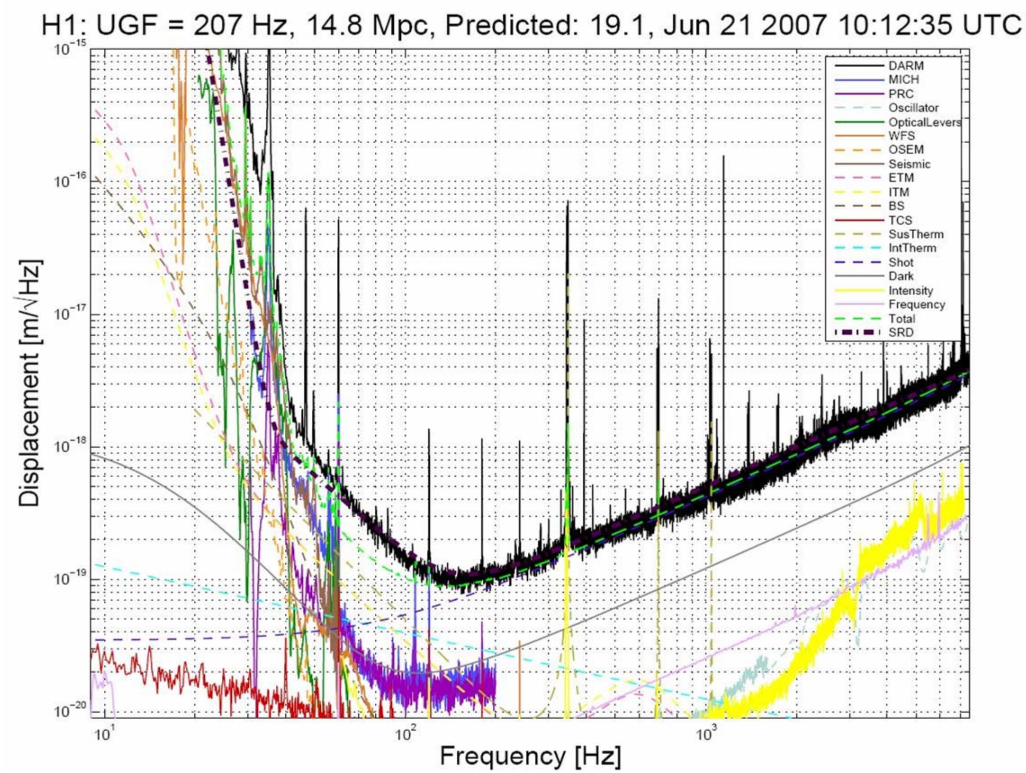
Ph.D. Thesis defence, Phisics, Pisa University, Italy

Juri Agresti

PhD thesis available at http://www.ligo.caltech.edu/docs/P/P080010-00.pdf

Current LIGO detectors





Science run **S5** just ended

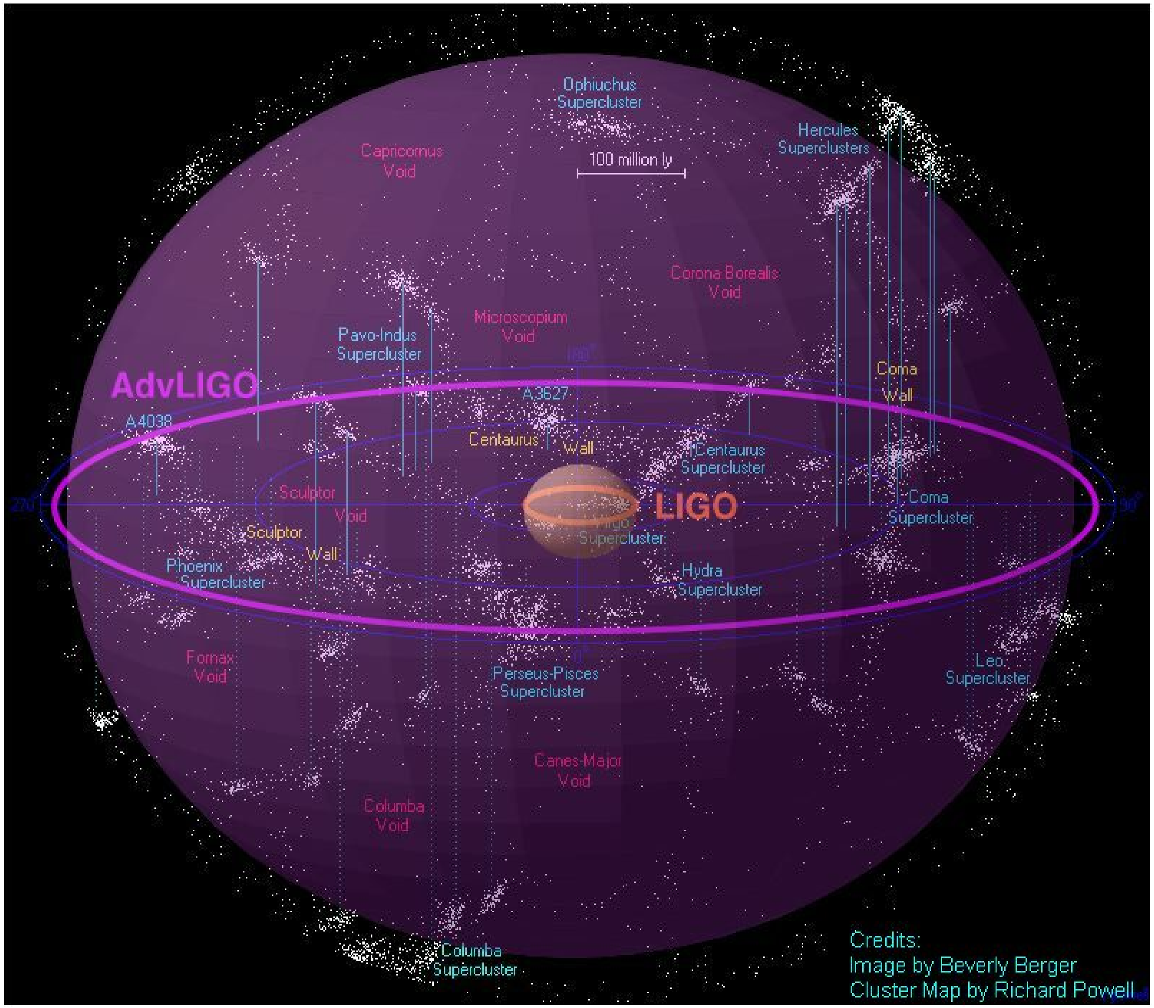
NS/NS range (S/N>8) ≈ 15 Mpc (4 Km)

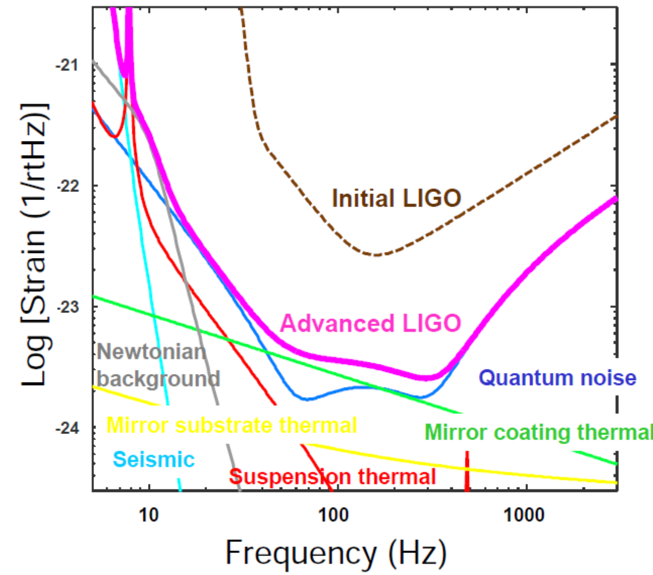
Event rate estimate for NS/NS inspirals

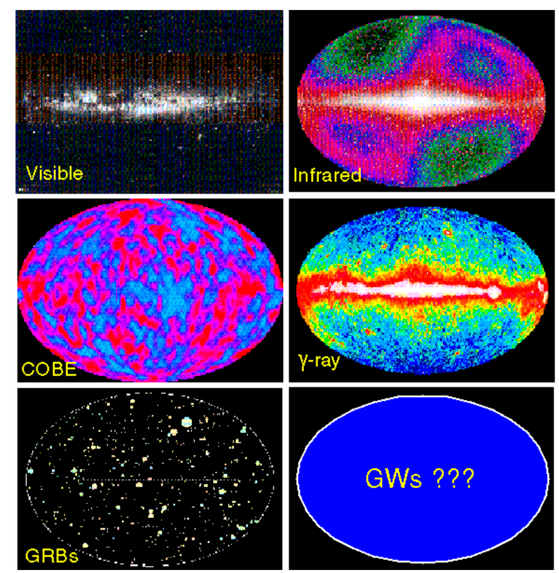
Current LIGO: 0.01 event/year

Advanced LIGO for GW astronomy

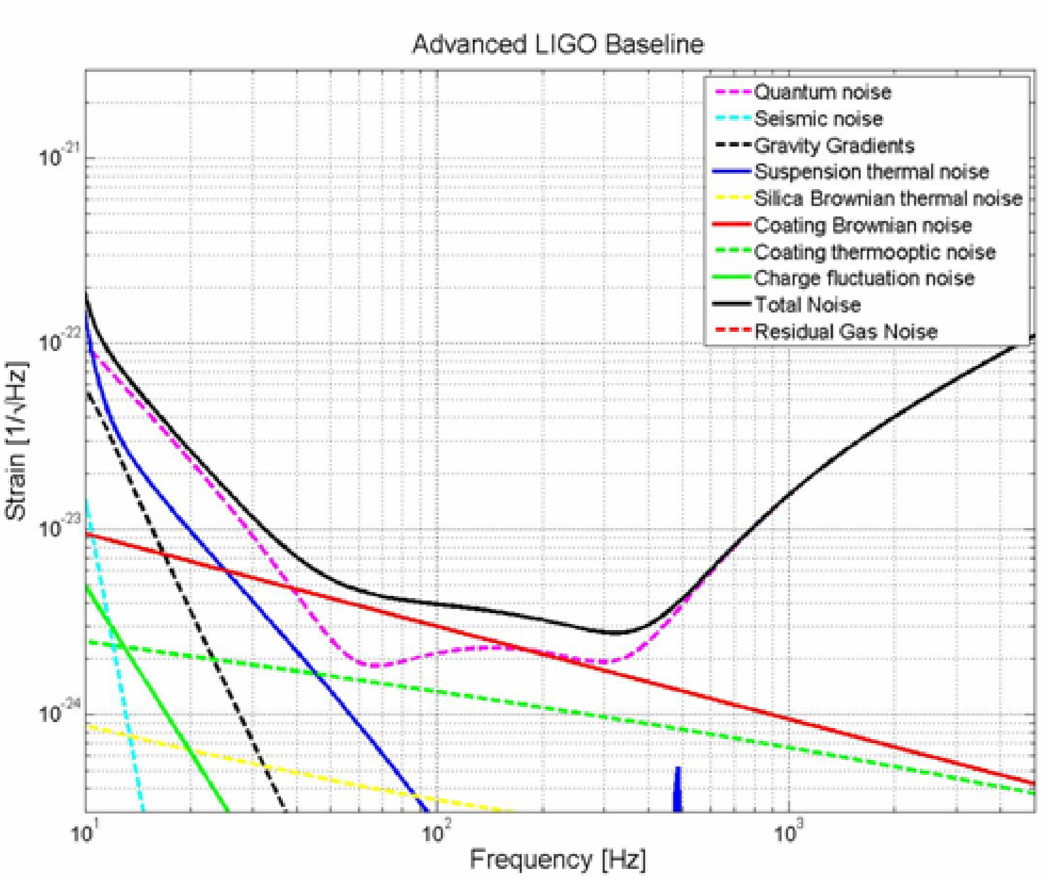
10x farther => 1000x more events

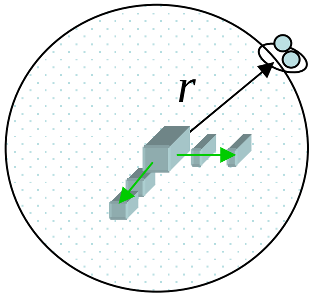






Beyond Ad-LIGO baseline design?





$$h_{min} \propto PSD_{floor}^{1/2}$$

$$h \propto r^{-1}$$

$$r_{max} \propto PSD_{floor}^{-1/2}$$

visibility volume

event rate

(isotropic source distrib.)

$$\left.\begin{matrix}\text{visibility volume} \\ \text{event rate}\end{matrix}\right\} \propto PSD_{floor}^{-3/2}$$

A 20% reduction in PSD_{floor} boosts the event rate by 30%, etc.

Coating Thermal Noises Limiting noise source in Adv LIGO

Multiple noise sources:

- **Brownian – Due to all forms of intrinsic dissipations within a material (impurities, dislocations of atoms, etc..)**

- **Thermo-optic Equilibrium fluctuations of the temperature of the test mass coatings cause fluctuations in physical parameters of the coating. Coupling parameters:**

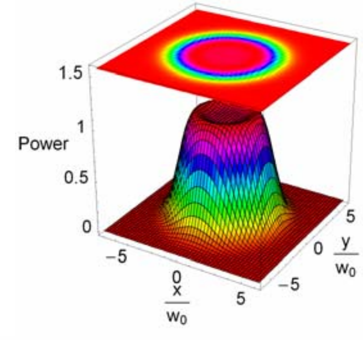
  - *dL/dT* **- thermoelastic**
  - *dn/dT* **- thermorefractive**

Take over by:

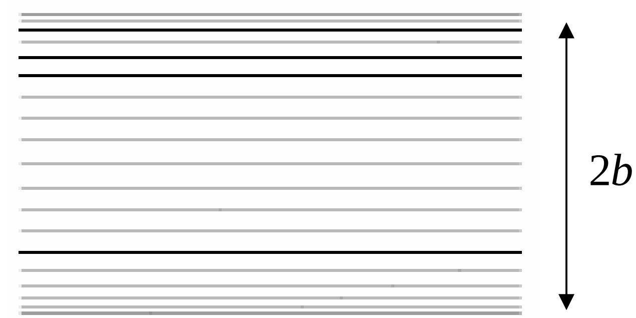
- Cryogenic temperature
- **Beam shape**
- **Coating geometry**
- Materials engineering (dopants)

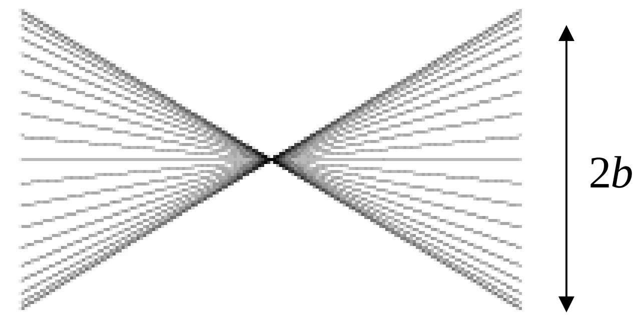
Mesa Beam from nearly flat to nearly concentric

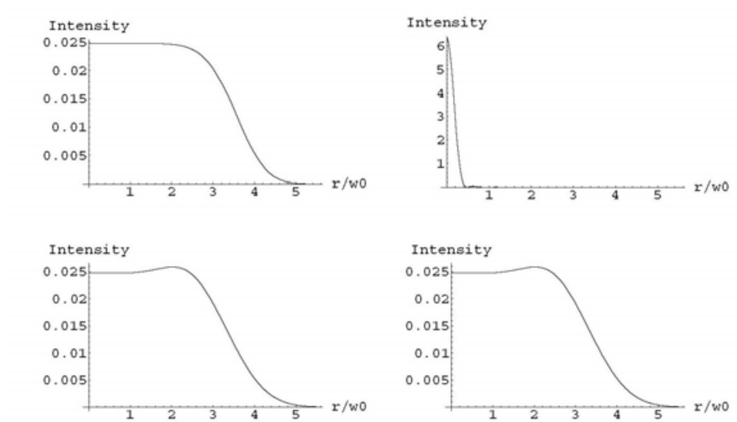
$$u_{FM} = \frac{1}{w_0^2\, b \sqrt{\pi^3}\, \Upsilon(b, w_0)} \int_{D(b)} e^{-\frac{(x-x_0)^2+(y-y_0)^2}{w_0{}^2}} dx_0 dy_0 \qquad w_0 = \sqrt{\frac{L}{k}}$$

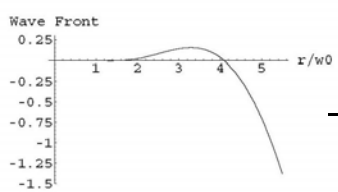


$$u_{CM} = \frac{1}{\sqrt{\pi\, \Upsilon(b, w_0)}} \frac{e^{-\frac{r^2}{w_0^2}}}{r} J_1\left(\frac{2\, r\, b}{w_0^2}\right)$$









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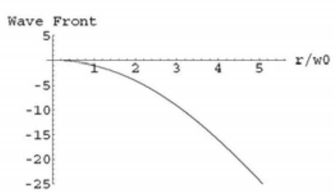


$$= k\,\frac{r^2}{L}$$

→ **Duality relation**

Duality relation between non spherical cavities:

Integral equation for cavity modes

$$\gamma u(\vec{r}) = \int_{Mirror\ Surface} K(\vec{r},\vec{r}')\, u(\vec{r}')\, d\vec{r}'$$

$K(\vec{r},\vec{r}')$ Propagator from surface to surface

$u(\vec{r})$ Field distribution over mirror surface

γ Eigenvalue

Nearly flat cavity

$$K_{flat}(\vec{r},\vec{r}') = \frac{ik}{2L\pi} Exp\left[-ikL + ikh(r) - \frac{ik}{2L}\left|\vec{r}-\vec{r}'\right|^2 + ikh(r')\right]$$

$h(r)$

Mirror profile of the nearly flat cavity

Equivalent nearly concentric cavity

$$K_{conc}(\vec{r},\vec{r}') = \frac{ik}{2L\pi} Exp\left[-ikL - ikh(r) + \frac{ik}{2L}\left|\vec{r}+\vec{r}'\right|^2 - ikh(r')\right]$$

$\frac{r^2}{L} - h(r)$

Mirror profile of the equivalent nearly concentric cavity configuration.

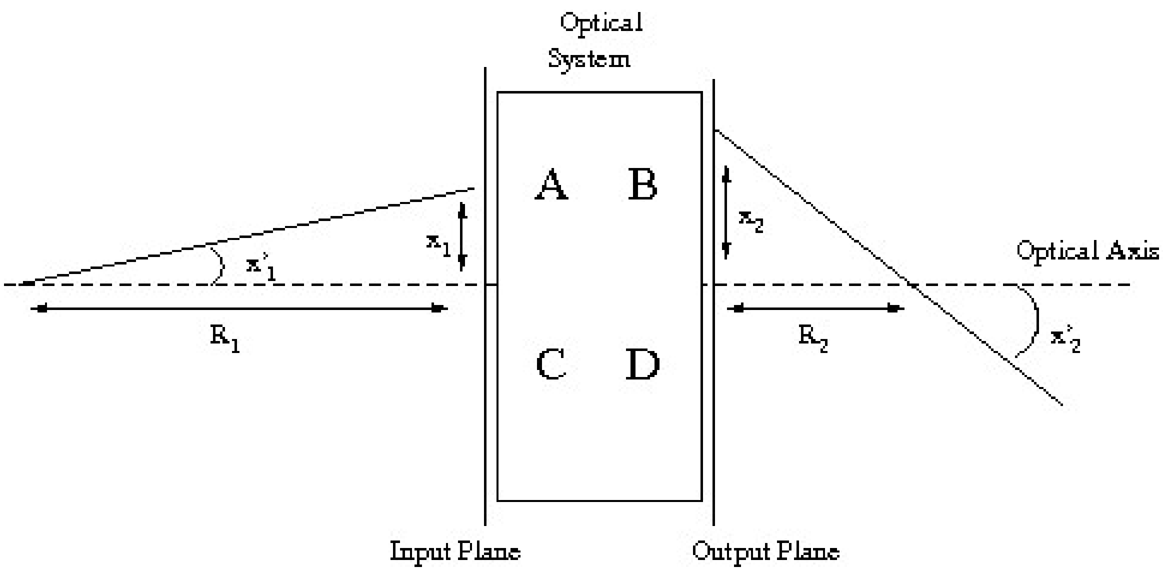
- The intensity distributions on the mirrors for the modes of the two equivalent resonators are the same. $\left|u_{lm}\right|^2$

- Unique mapping between the eigenvalues of the nearly concentric and nearly flat cavity for all orders. $e^{ikL}\gamma_{lm}^{conc} = (-1)^{m+1} e^{-ikL}\left(\gamma_{lm}^{flat}\right)^*$

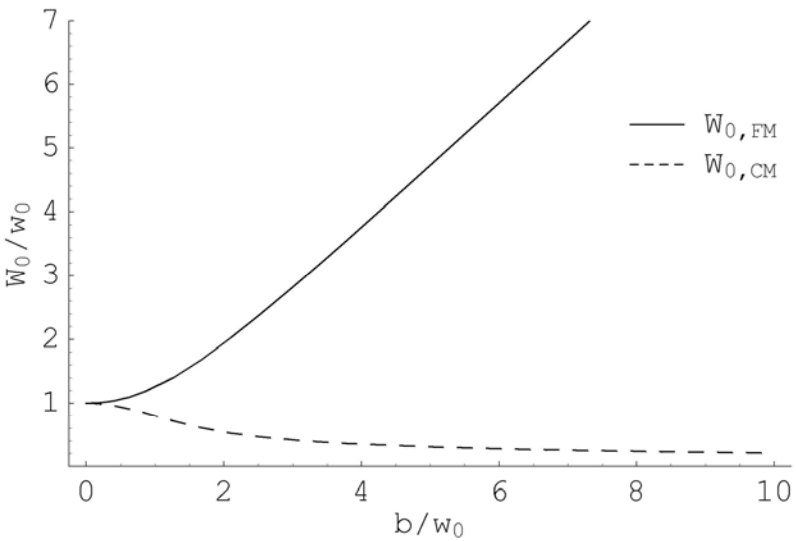
- The two cavities have the same diffraction loss per bounce. $1-\left|\gamma_{lm}\right|^2$

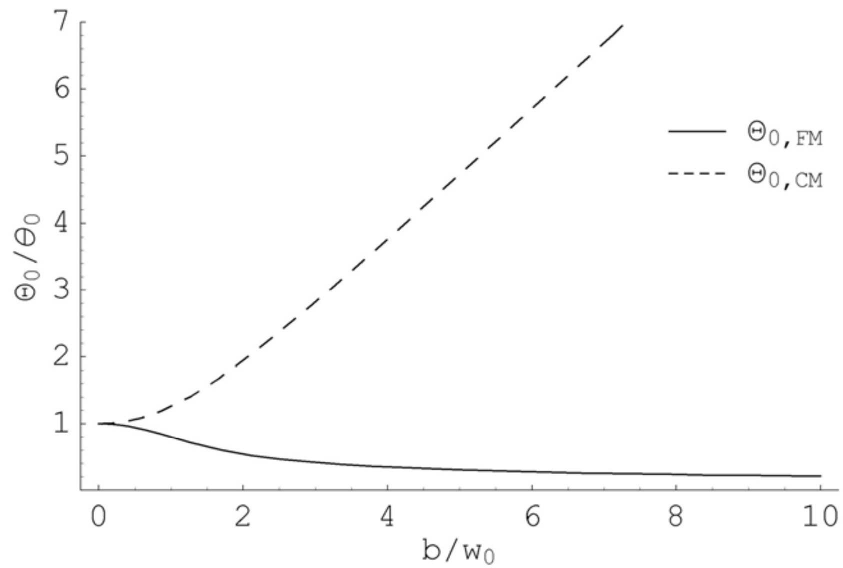
Analytical investigations of mesa beams

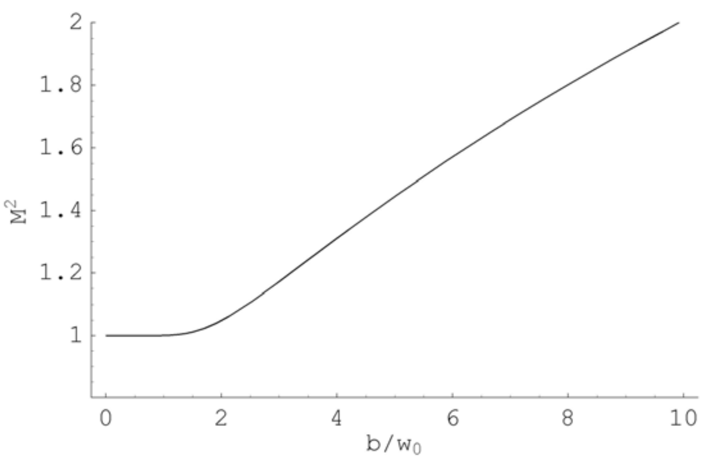
- Calculation of the generalized W, Θ_0, R
- Calculation of the beam propagation factor (invariant) $M^2 = \frac{\pi}{\lambda} W_0 \Theta_0$
- Misalignment sensitivity (invariant) $\left|\eta_m\right|^2 \approx 1 - M^4 \left(\frac{\alpha^2}{\Theta_0^2} + \frac{\delta^2}{W_0^2} \right)$



Nearly flat and nearly concentric mesa beam optical parametrs





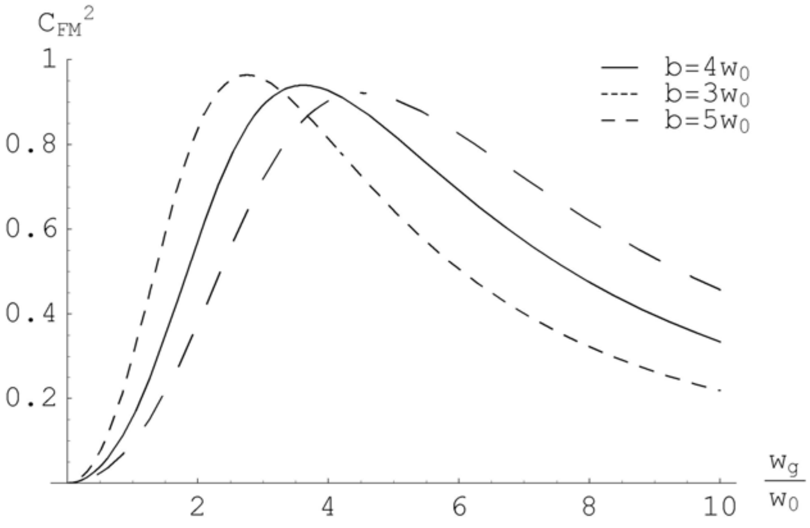


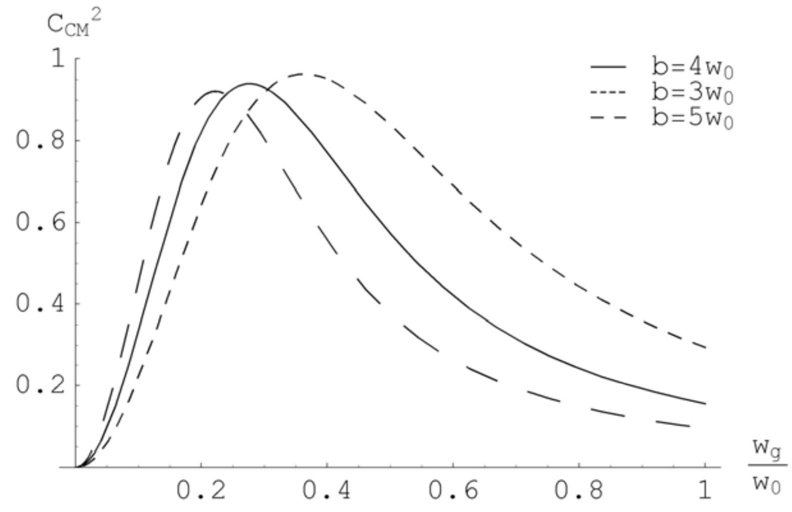
Ad-LIGO fiducial Gaussian

$$\frac{W_0^G}{w_0} = \sqrt[4]{\frac{1+g}{1-g}} = \left(\frac{\Theta_0^G}{\theta_0}\right)^{-1} \rightarrow \begin{cases} 2.26 & \text{FG} \\ 0.44 & \text{CG} \end{cases}$$

It is not worst than Gaussian beam...

Gaussian beam – mesa beam power coupling





94% of the power of a Gaussian beam can feed into the mesa beam

Development of simulation programs for optical cavities with arbitrary mirror shape.

Optical cavity's eigenmodes

$$\gamma\, u(\vec{r}) = \int_{\substack{Mirror\\Surface}} K(\vec{r},\vec{r}')\, u(\vec{r}')\, d\vec{r}'$$

Fredholm integral

Finite Element Method approach

Reduce the integral equation to a 2-D matrix eigenvalue problem

Cylindrical simmetry

$$u(r,\varphi) = R(r)e^{-im\varphi}$$

1-D radial equation for each *m*

$$\gamma\, R(x_i) = \sum_j^N K(x_i, x_j)\, x_j w_j R(x_j)$$

N x N matrix

Gaussian quadrature

• greater accuracy with fewer points.

No simmetry

Map a 4-D problem to 2-D

Grid nodes on the mirror *N x N*

M cells

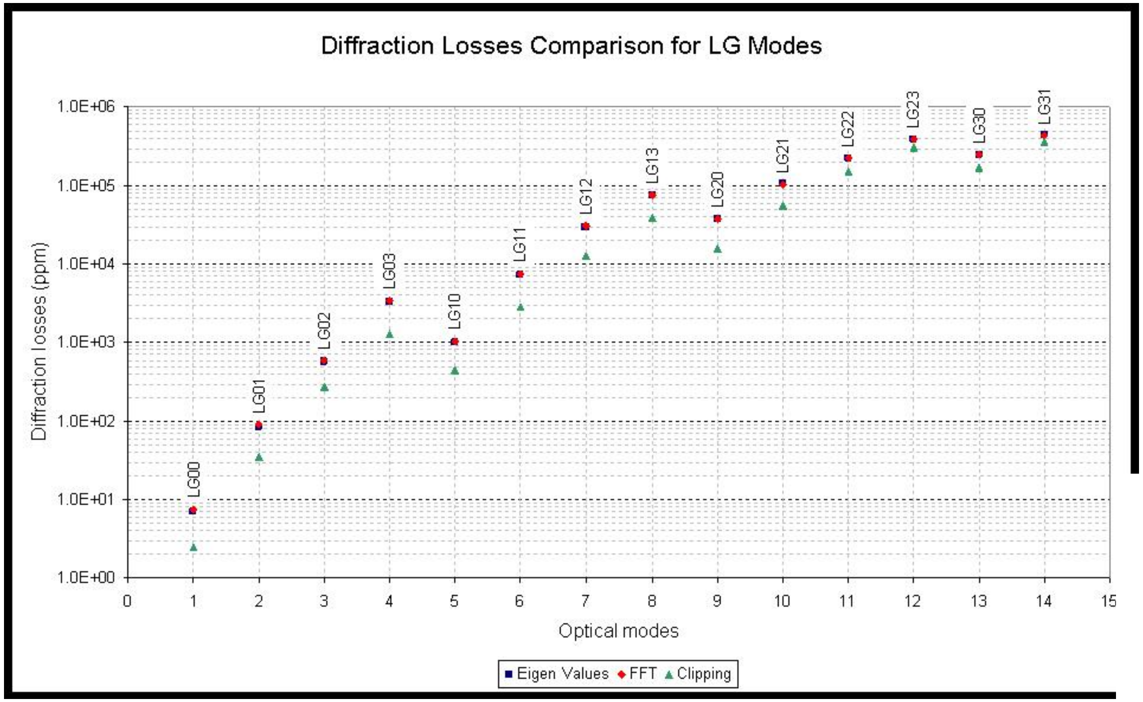
Propagator between cells

M x M matrix

Drawback $Dim[K] \approx N^4$

Validation and performance

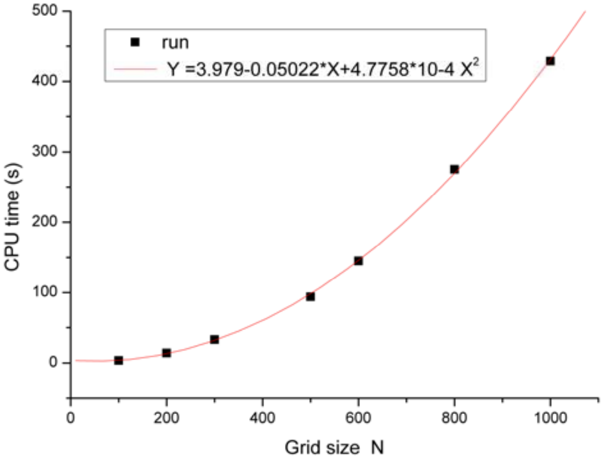
Calculation of the diffraction losses of Ad-LIGO for parametric instability analysis (Internal oscillations of mirror beat against light)



100 Points Gaussian

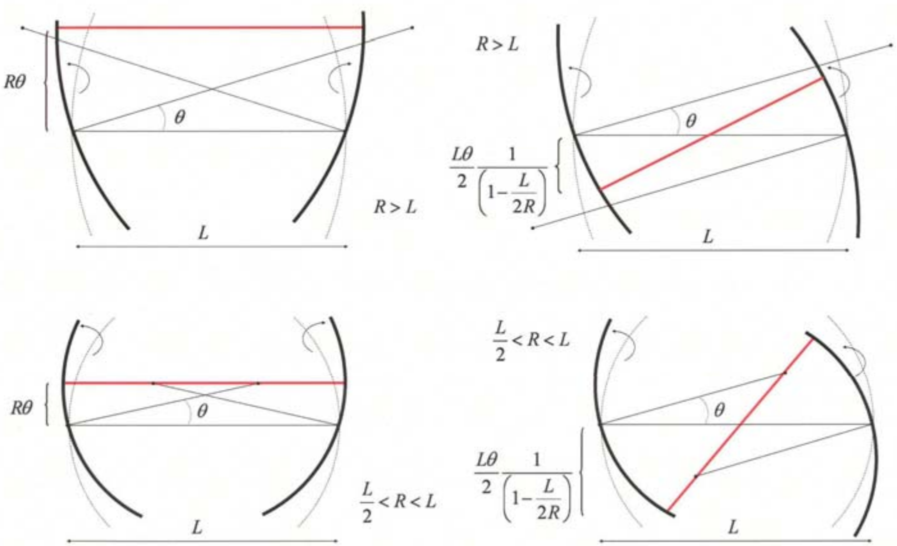
3.6 s for 10 eigenmodes

FFT about 1 hr per mode



•<u>Control of optical-mechanical instabilities</u>:

Nearly concentric cavities are more stable than nearly flat cavities for misalignment coupled to radiation pressure.



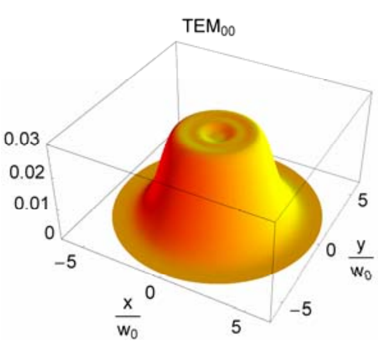
For non-spherical mirrors (MH) we use perturbative approach with FEM modes

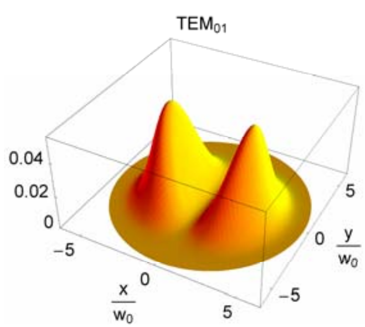
$$T = \frac{2P}{c}\alpha\,\theta$$

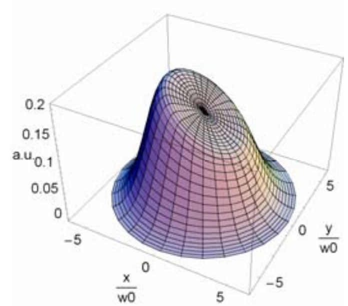
Using the duality relations

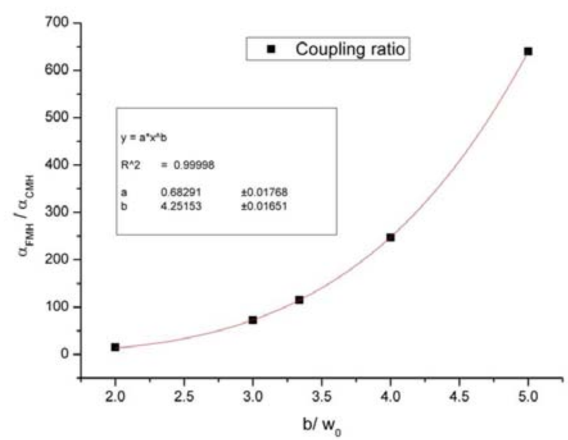
$$\frac{\alpha^{MB}_{conc}}{\alpha^{MB}_{flat}} \approx \frac{1}{247}$$

$$u_{tilt} = u_{00} + c\,u_{01}$$









FFT simulation tool for the mesa beam cavity prototype

Using paraxial approximation, FFT codes can simulate the propagation of actual TEM patterns on optical cavities

A Mathematica FFT routine has been dedicated to simulate our cavity beam behavior with rescpect to mirror imperfections and misalignment.

Propagation in free space is a multiplication in the Fourier domain

Mirror profiles become phase multiplication pixel by pixel in the real space

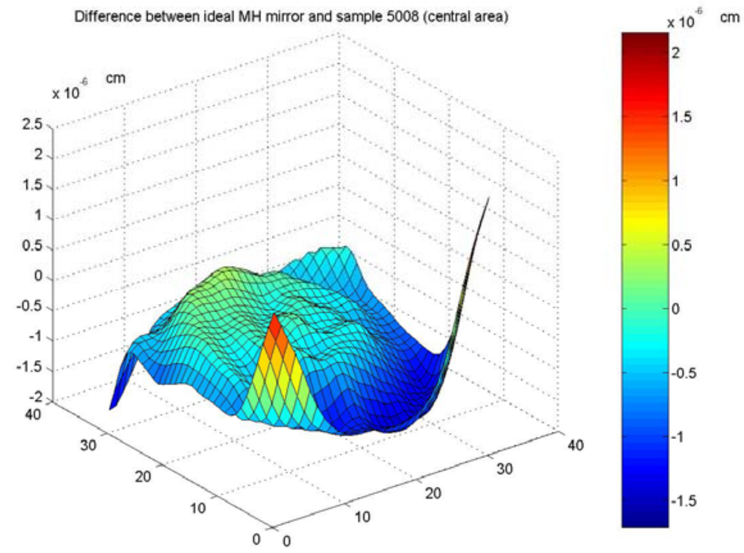
Aliasing

$$\Delta h_{neighboring} < \frac{\lambda}{2}$$

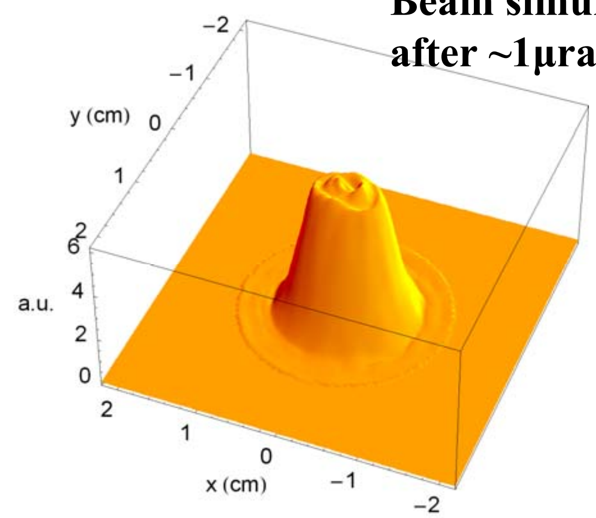
Large N and W+ zero-padding

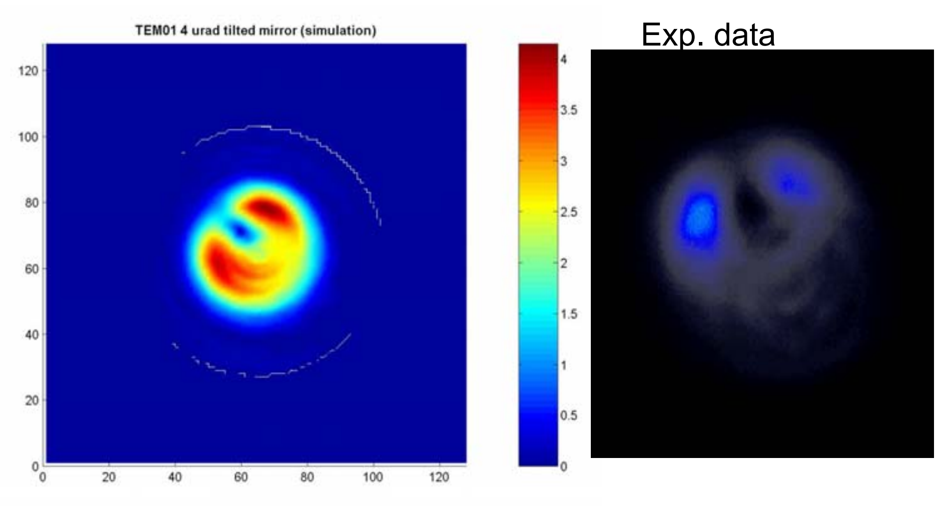
FFT simulations

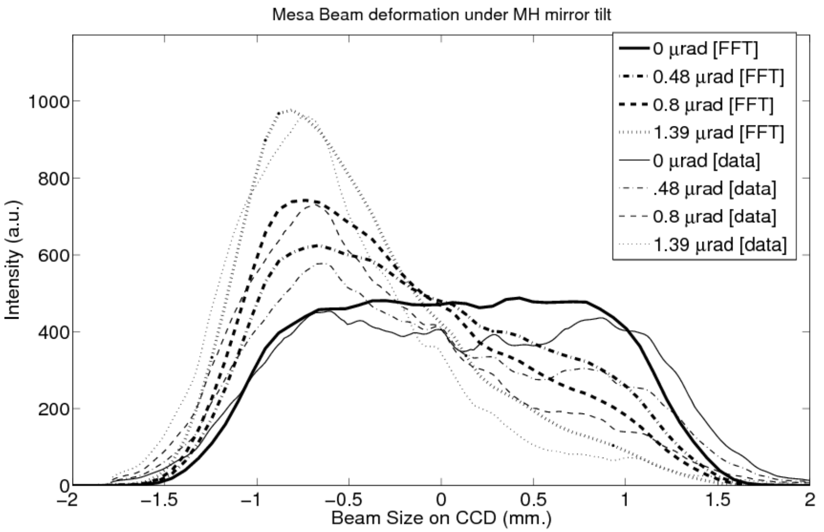
Deviation from ideal mirror profile



Beam simulated after ~1µrad tilt

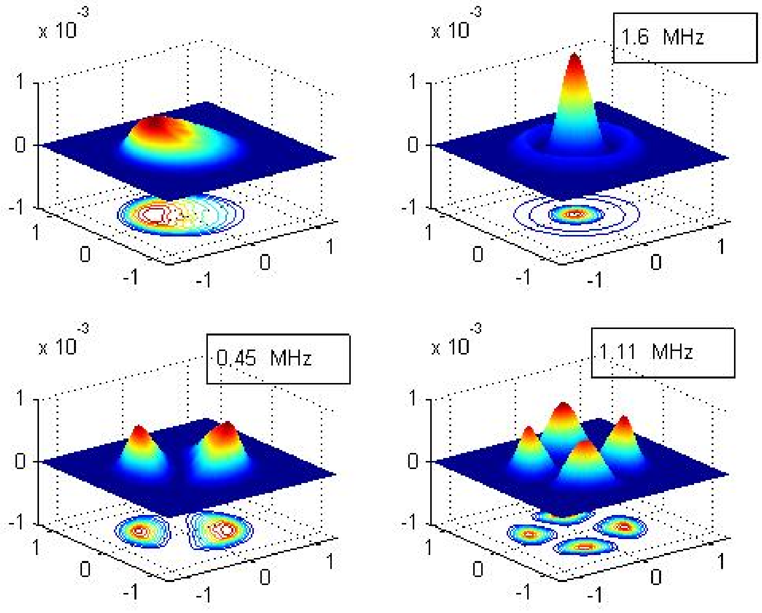




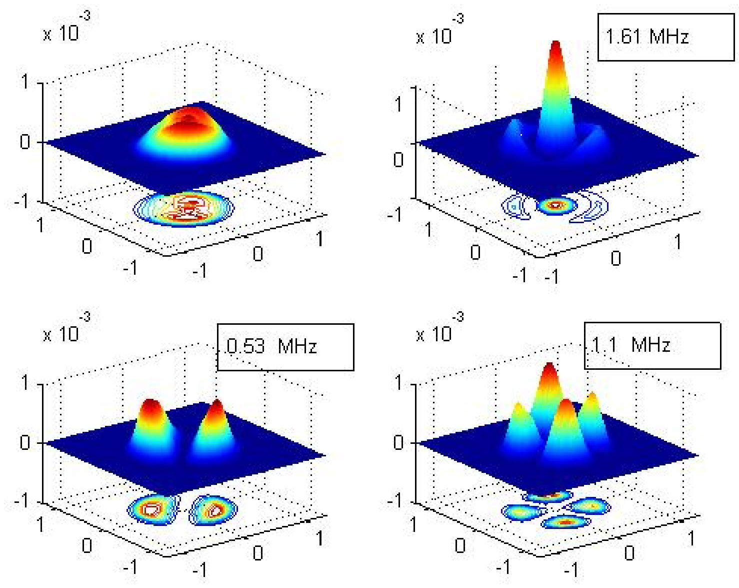


Eigenmodes simulations

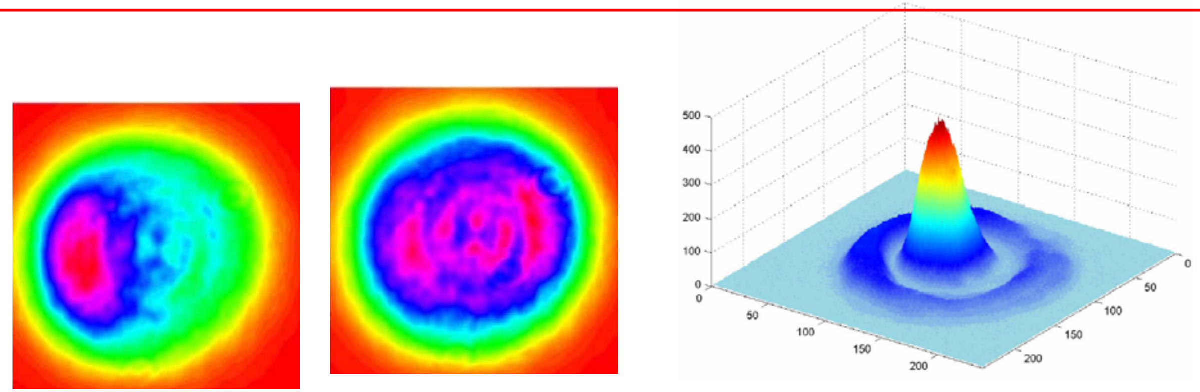
1 µrad tilt +Flat input



Input real saddle shape



The modelled effects are seen in experimental data



Thermal noise for finite sized mirrors:

1. **Precise comparative estimation of the various thermal noise contributions for finite test masses (design optimization).**
2. **Noise suppression using Mesa beam and other beam geometry**

Levin's approach to Fluctation Dissipation Theorem

$$S_X(\omega) = \frac{8\,k_B\,T}{\omega^2}\frac{W_{diss}}{F_0^2}$$

W_{diss} **Is the energy dissipated by the mirror in responce to the oscillating pressure**

$$P(\vec{r},t) = F_0\, f(\vec{r})\cos(\omega\, t)$$

Fluctuation dissipation theorem

The dissipative properties of the dynamical system are directly related to the **equilibrium** fluctuations.

The responce of a system in thermodynamic equilibrium to small external perturbation (thermal bath) is the same as its responce to spontaneous fluctuations.

Generalized coordinate X

Driving external force F

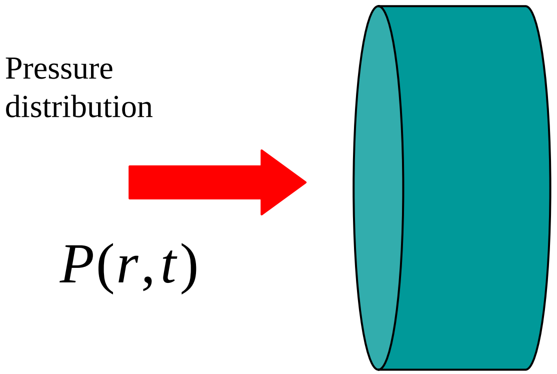
$$S_X(\omega) = \frac{4 k_B T}{\omega^2} \mathrm{Re}[Y(\omega)]$$

$$\frac{1}{Y(\omega)} = Z(\omega) = \frac{F(\omega)}{\dot{X}(\omega)}$$

Levin method $$S_X(\omega) = \frac{8 k_B T}{\omega^2} \langle W_{diss}(\omega) \rangle$$

The calculation of the dissipated energy is simpler than the admittance

Assumptions in our analysis



BHV+LT (accurate) approximate analyical solution of elasicity equations for a cylindrical test mass

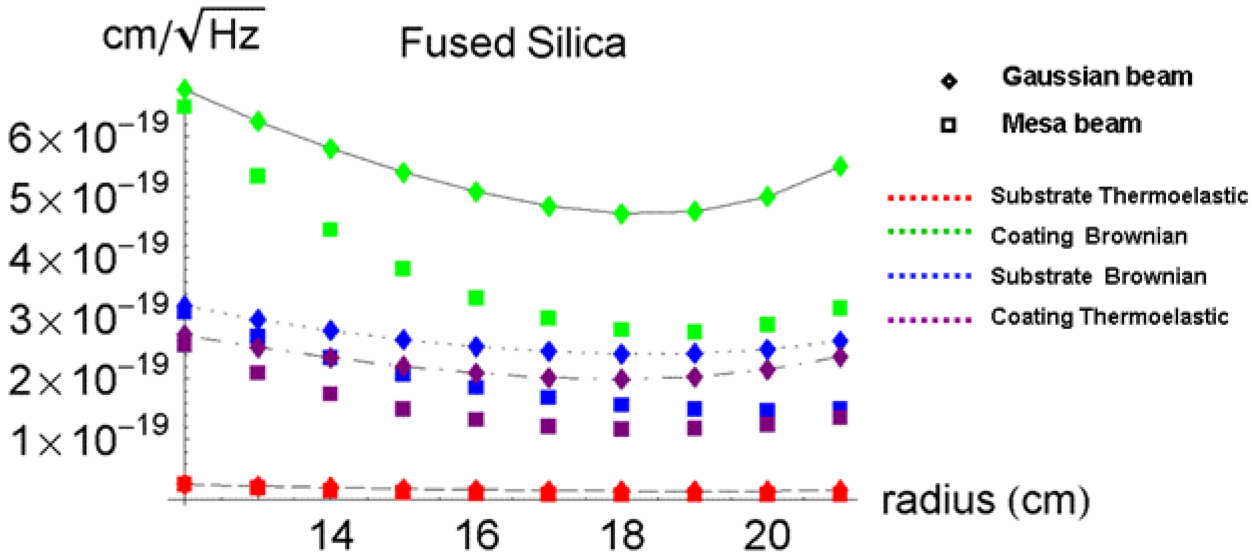
Quasistatic approximation for the oscillations of stress and strain induced by P. $\tau_{sound} << \tau_{GW}$

Adiabatic approximation for the substrate thermoelastic problem (negligible heat flow during elastic deformation). $r_{heat} << r_{beam}$

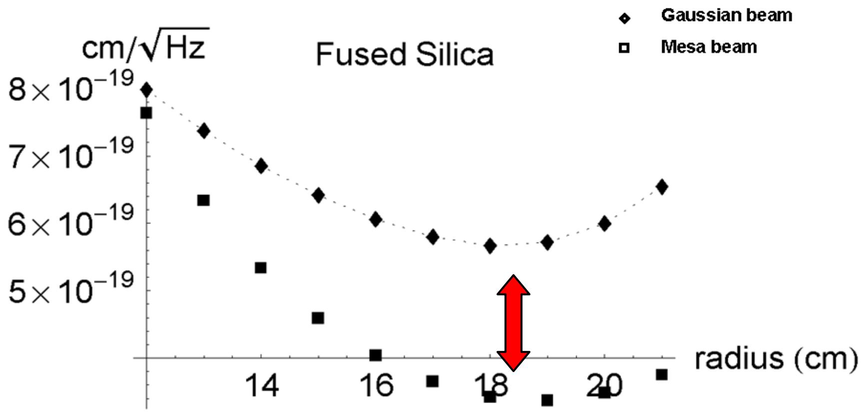
Brakes down for coating thermoelastic problem → **Perturbative approach**

Coating is an isotropic and homogeneous thin film

- **Fixed total mirror mass = 40 Kg.**
- **The beam radius is dynamically adjusted to maintain a fixed diffraction loss = 1ppm (clipping approximation).**
- **The mirror thickness is also dynamically adjusted as a function of the mirror radius in order to maintain the total 40 Kg mass fixed.**
- **Calculation at the frequency 100 Hz**



FS	$\sqrt{S_X^{GB} / S_X^{MB}}$
CB	1.7
CT	1.7
SB	1.55
ST	1.92



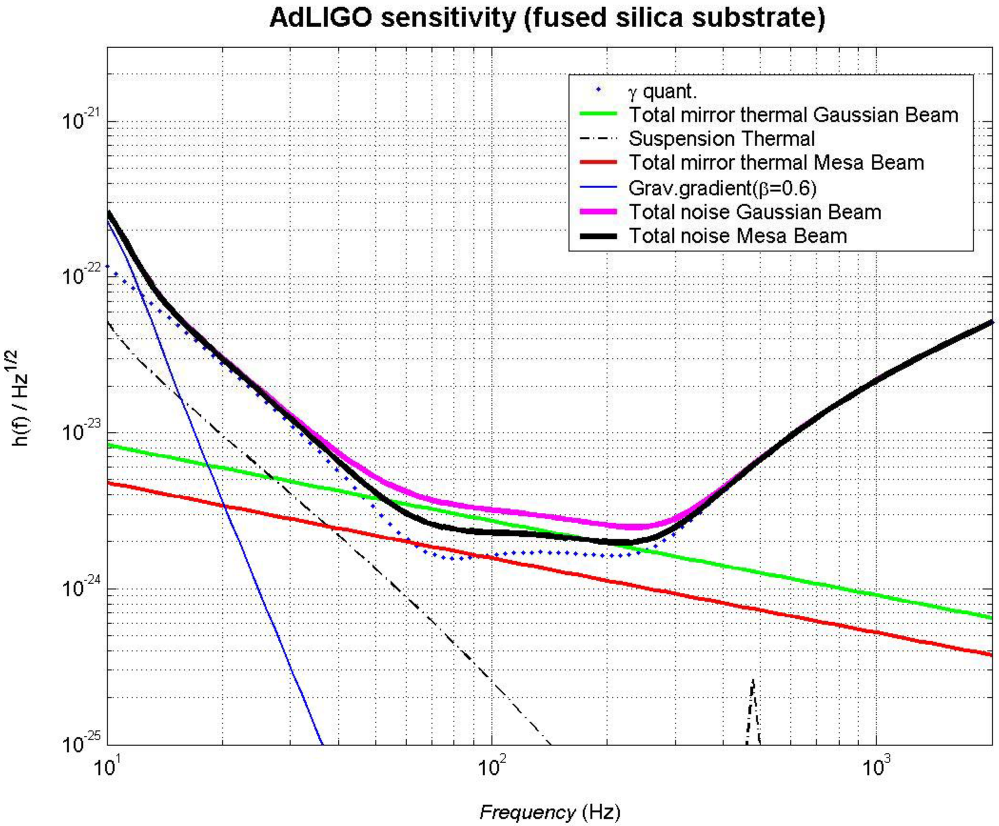
$2a/H \approx 2-2.4$

Gain factor ≈ 1.7

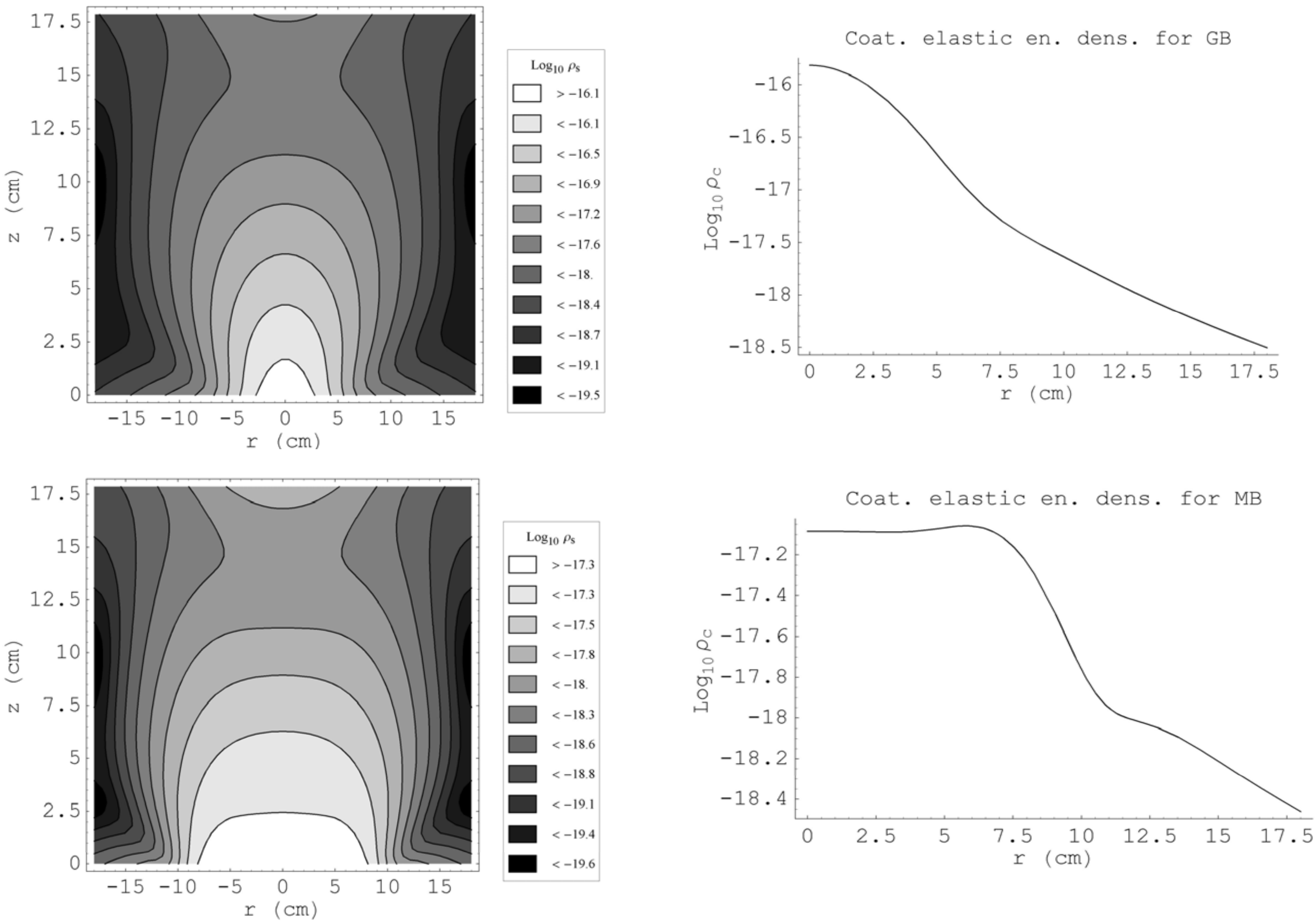
Thermo-refractive noise

$$\sqrt{\frac{S_X^{GB}}{S_X^{FT}}(f=100Hz)} \approx \sqrt{3}$$

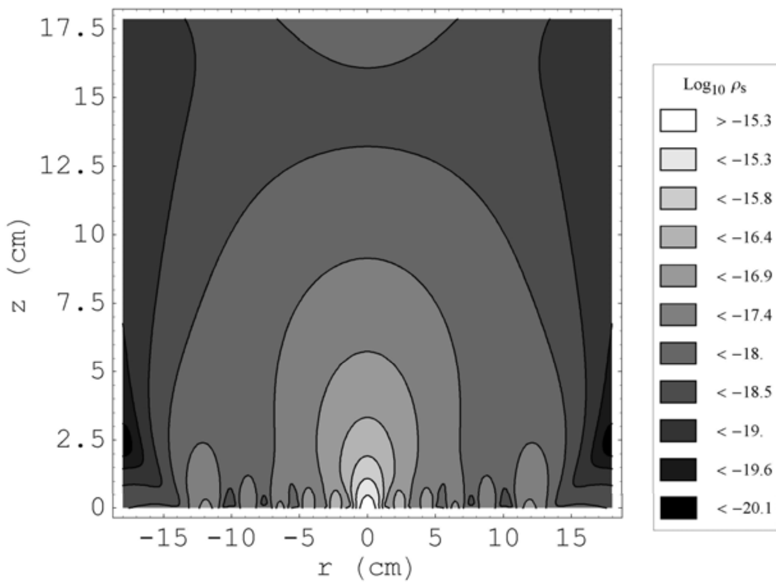
	GB	MB
NS-NS range	**177** Mpc	**228** Mpc

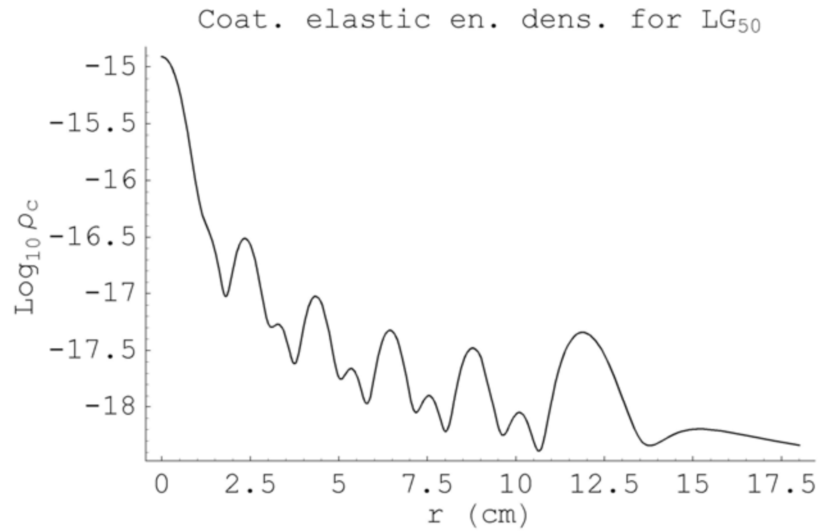


Consideration on High order LG modes for thermal noise reduction

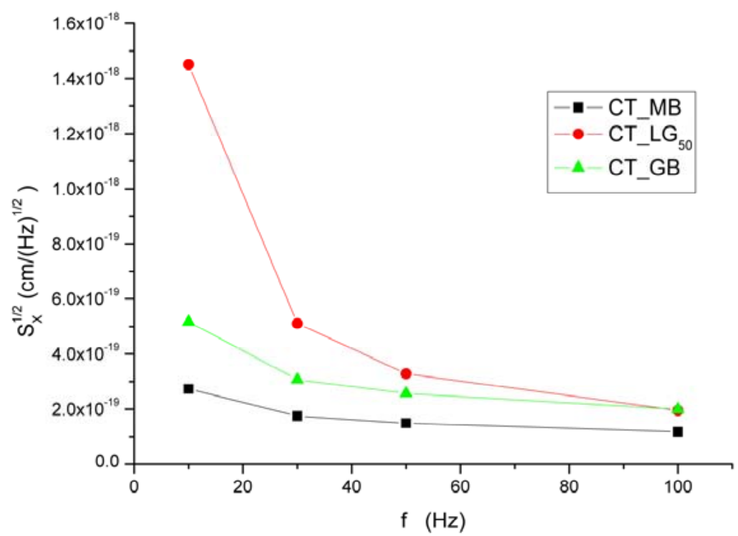


Rapidly variation of the elastic fields near the coating produces high elastic energy stored in the coating and hiher thermal gradients





$$r_t = \sqrt{\frac{\kappa}{\rho C \omega}}$$



Noise	GB	MB	LG_{50}
CB	$4.72 \cdot 10^{-19}$	$2.8 \cdot 10^{-19}$	$3.2 \cdot 10^{-19}$
CT	$1.99 \cdot 10^{-19}$	$1.17 \cdot 10^{-19}$	$1.95 \cdot 10^{-19}$

Noise calculation for GW sensitivity at 100 Hz. Units $cm/\sqrt{Hz}$

Optimized coating project: coll. Sannio Univ., TNI, LMA

- **Experiments suggest**
  - **Ta_2O_5 is the dominant source of dissipation in current SiO_2/Ta_2O_5 coatings**
- **Research ongoing to:**
  - **'optimise' coating designs by minimising volume of Ta_2O_5 present in the coatings**

•Current coating design: stacked doublets of *quarter-wavelength* (QWL) SiO_2 - Ta_2O_5 layers.

•Yields *largest reflectance* among all stacked-doublet designs for any *fixed* no. of layers (or equivalently, *smallest* no. of layers at any *fixed reflectance*).

•Does *not* yield the mimimum noise for a prescribed reflectivity, hence *not optimal.*

Position noise PSD

Boltzmann constant

Absolute temperature

Poisson ratio of substrate

$$S_x(f) = \frac{2k_B T}{\pi^{3/2} f} \frac{(1-\sigma^2)}{wY} \phi_{eff}$$

Beam half-width

Young modulus of substrate

Effective loss - angle of mirror (complicated function of layers' thicknesses, loss-angles, Young moduli & Poisson coeff.s)

$$\begin{aligned} \phi_{\rm eff}^{\rm coat} &= \frac{d_{\rm coat}}{\sqrt{\pi} w} \frac{1}{Y_\perp} \left\{ \left[\frac{Y}{1-\sigma^2} - \frac{2\sigma_\perp^2 Y Y_\parallel}{Y_\perp (1-\sigma^2)(1-\sigma_\parallel)} \right] \phi_\perp + \right. \\ &+ \frac{Y_\parallel \sigma_\perp (1-2\sigma)}{(1-\sigma_\parallel)(1-\sigma)} (\phi_\parallel - \phi_\perp) + \\ &+ \left. \frac{Y_\parallel Y_\perp (1+\sigma)(1-2\sigma)^2}{Y(1-\sigma_\parallel^2)(1-\sigma)} \phi_\parallel \right\}, \end{aligned}$$

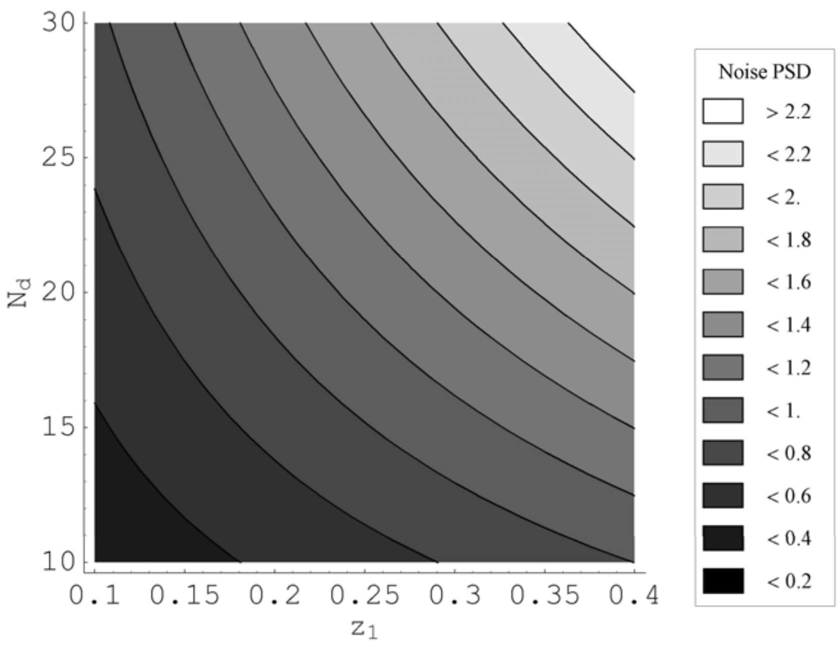
Developed the best model for coating effective parameters.

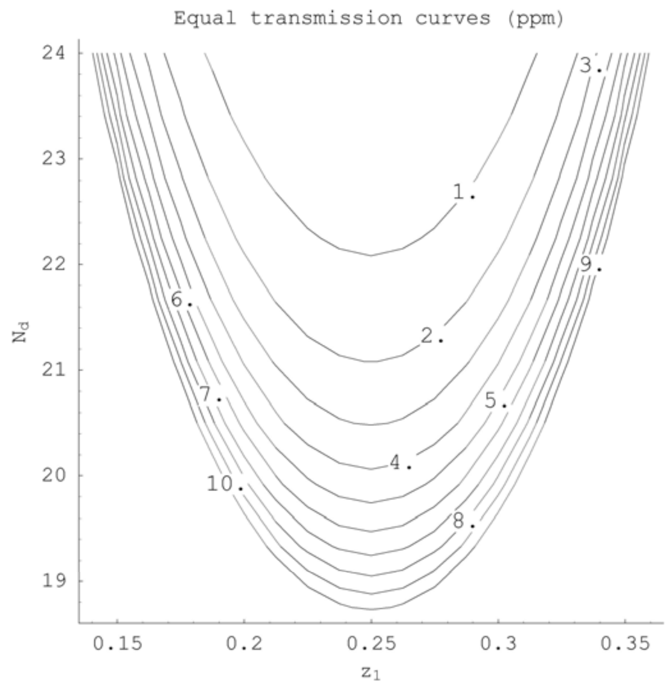
Minimize the thermal noise for a prescribed mirror transmissivity

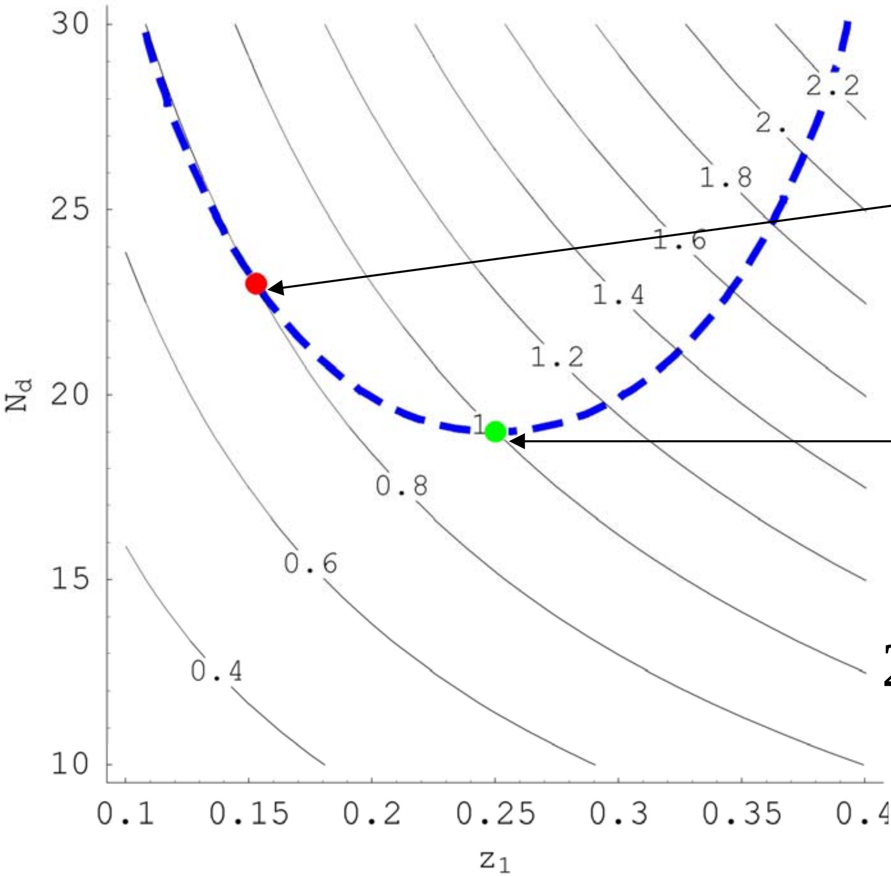
GA-engineered coatings for minimum noise at prescribed reflectivity show trend toward non-QWL stacked-doublet configurations.

$$z_1 + z_2 = \frac{1}{2}, \quad z_i = \frac{n_i l_i}{\lambda}$$

The parameters in the optimization are reduced to N_d , z_1





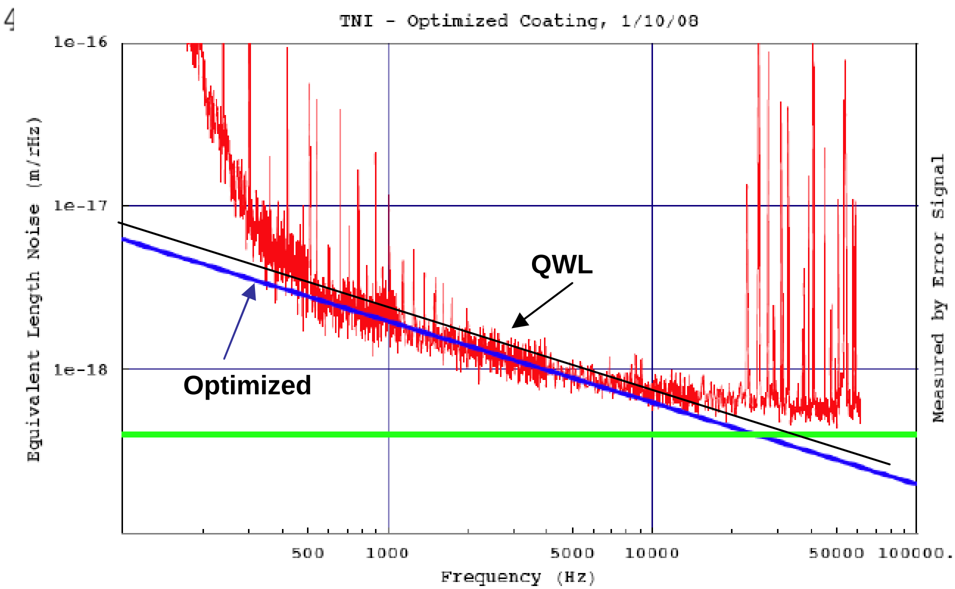


Optimized design

QWL design (8.3 ppm)

$$20\% \; in \; PSD \Rightarrow 30\% \; in \; \underline{rate} \; for \; GW$$

Measurement at TNI (preliminary)



Technicalities

Substrate Brownian noise

$$W_{diss} = 2\omega\phi_s \langle U \rangle$$

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\phi\phi} = \frac{u_r}{r}, \quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}, \quad \varepsilon_{rz} = \tfrac{1}{2}\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) \; ,$$

$$U = \int_{test\ mass} \tfrac{1}{2}\varepsilon_{ij}\sigma_{ij}\, dV$$

$$\sigma_{ii} = \lambda\varepsilon + 2\mu\varepsilon_{ii}, \quad \sigma_{rz} = 2\mu\varepsilon_{rz}, \quad \varepsilon = \varepsilon_{rr} + \varepsilon_{\phi\phi} + \varepsilon_{zz}$$

Substrate thermoelastic noise

$$W_{diss} = \left\langle \int_{test\ mass} \frac{\kappa}{T}\left(\vec{\nabla}\delta T\right)^2 dV \right\rangle$$

$$r_{beam} >> r_t \qquad r_t = \sqrt{\frac{\kappa}{\rho C \omega}}$$

$$\delta T = -\frac{\alpha Y T}{C\rho\left(1-2\sigma\right)}\varepsilon$$

Coating Brownian noise

$$W_{diss} = 2\omega\phi_c < U_c >$$

$$U_c \approx \delta U_c\, d$$

$$\delta U_c = \int_S \tfrac{1}{2}\varepsilon_{ij}^c \sigma_{ij}^c\, dS$$

Boundary condition

$$\varepsilon^c{}_{rr} = \varepsilon_{rr}(z=0) \qquad \varepsilon^c{}_{\phi\phi} = \varepsilon_{\phi\phi}(z=0) \qquad \sigma^c{}_{zz} = \sigma_{zz}(z=0)$$

$$\sigma^c{}_{ii} = \lambda_c \varepsilon^c + 2\mu_c \varepsilon^c{}_{ii}, \quad \sigma^c{}_{rz} = 2\mu_c \varepsilon^c{}_{rz}, \quad \varepsilon^c = \varepsilon^c{}_{rr} + \varepsilon^c{}_{\phi\phi} + \varepsilon^c{}_{zz}$$

$$\sigma_{rz}^c = 0$$

Coating thermoelastic noise

$$d \ll r_t \ll r_{beam}$$

$$\left(\frac{\partial}{\partial t} - K_\beta \frac{\partial^2}{\partial z^2}\right)\delta T_\beta = -\left(\frac{Y\alpha T}{(1-2\sigma)C\rho}\frac{\partial \varepsilon}{\partial t}\right)_\beta = -\mathrm{B}_\beta \qquad \beta = s, c$$

at the surface

$$(i\omega - K_\beta)\delta T_\beta = -i\omega \mathrm{B}_\beta$$

Boundary condition $\left.\frac{\partial \delta T_c}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial \delta T_s}{\partial z}\right|_{z=H} = 0, \quad \delta T_c = \left.\delta T_s\right|_{z=d}, \quad K_c \frac{\partial \delta T_c}{\partial z} = \left. K_s \frac{\partial \delta T_s}{\partial z}\right|_{z=d}$

$$W_{diss} = \left\langle \int_{V_s} \frac{\kappa_s}{T}\left(\frac{\partial \delta T_s}{\partial z}\right)^2 dV_s \right\rangle + \left\langle \int_{V_c} \frac{\kappa_c}{T}\left(\frac{\partial \delta T_c}{\partial z}\right)^2 dV_c \right\rangle$$

Coating Thermo-refractive noise estimation

$$\beta = \frac{dn}{dT}$$

- Infinite mirrors
- Perfect square beam

$$S_X(\omega) = \lambda^2 \beta_{eff}^2 \frac{4k_b T^2 K}{\rho C} \int_{-\infty}^{\infty} dq_z \int_0^{\infty} \frac{q_\perp dq_\perp}{(2\pi)^2} \frac{2q^2}{K^2 q^4 + \omega^2} \frac{1}{1 + q_\perp^2 d^2} \left|\tilde{g}(q_\perp)\right|^2$$

$$\tilde{g}(q_\perp) = 2\pi \int_0^{\infty} r\,dr\, f(r) J_0(q_\perp r)$$

$$\beta_{eff} = \frac{n_2^2 \beta_1 + n_1^2 \beta_2}{4(n_1^2 - n_2^2)}$$

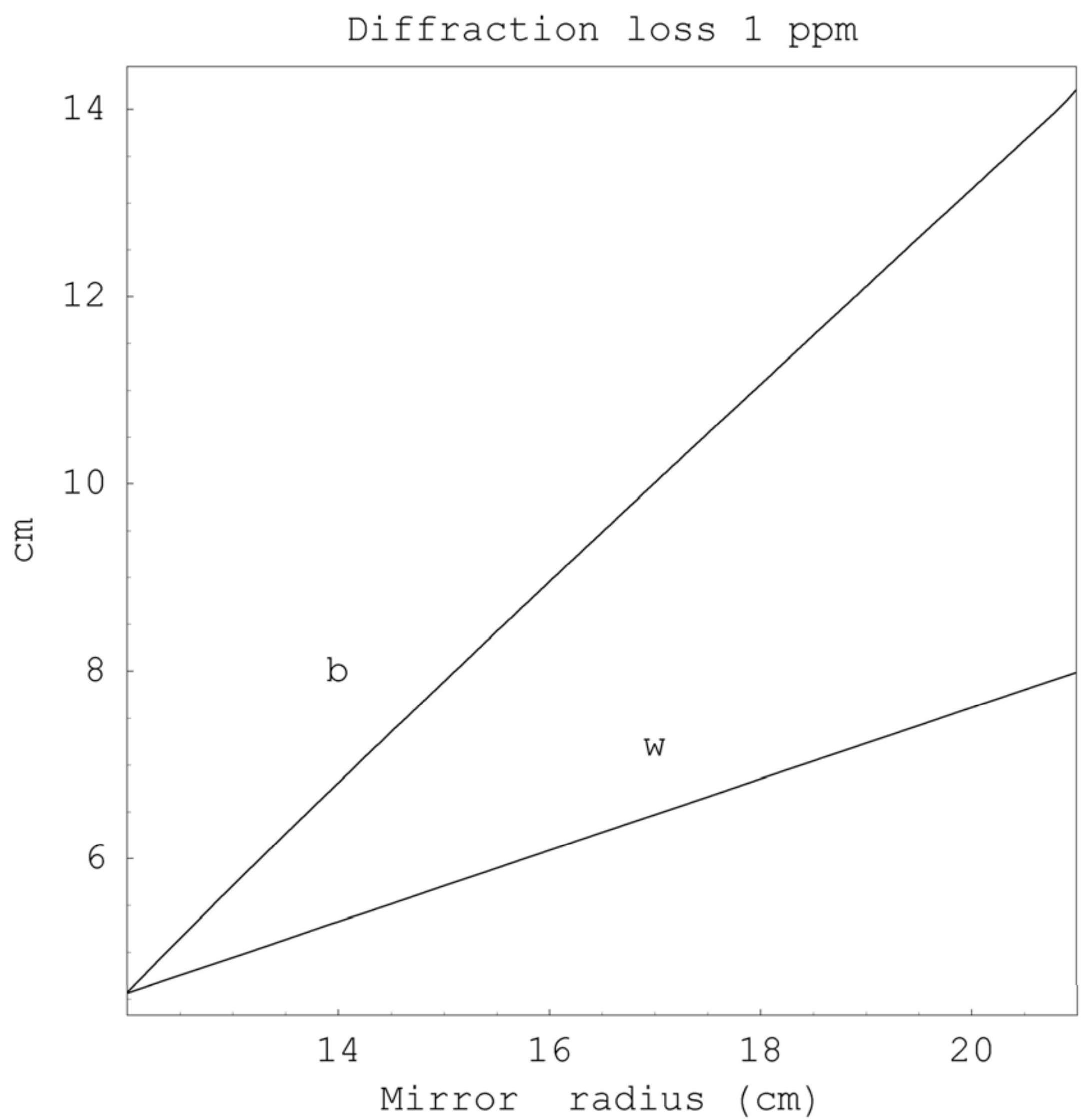
$$f_{FT}(r) = \frac{1}{\pi D^2} \quad for \quad r \le D, \quad 0 \quad for \quad r > D$$

$$\sqrt{\frac{S_X^{GB}}{S_X^{FT}}(f = 100Hz)} \approx \sqrt{3}$$

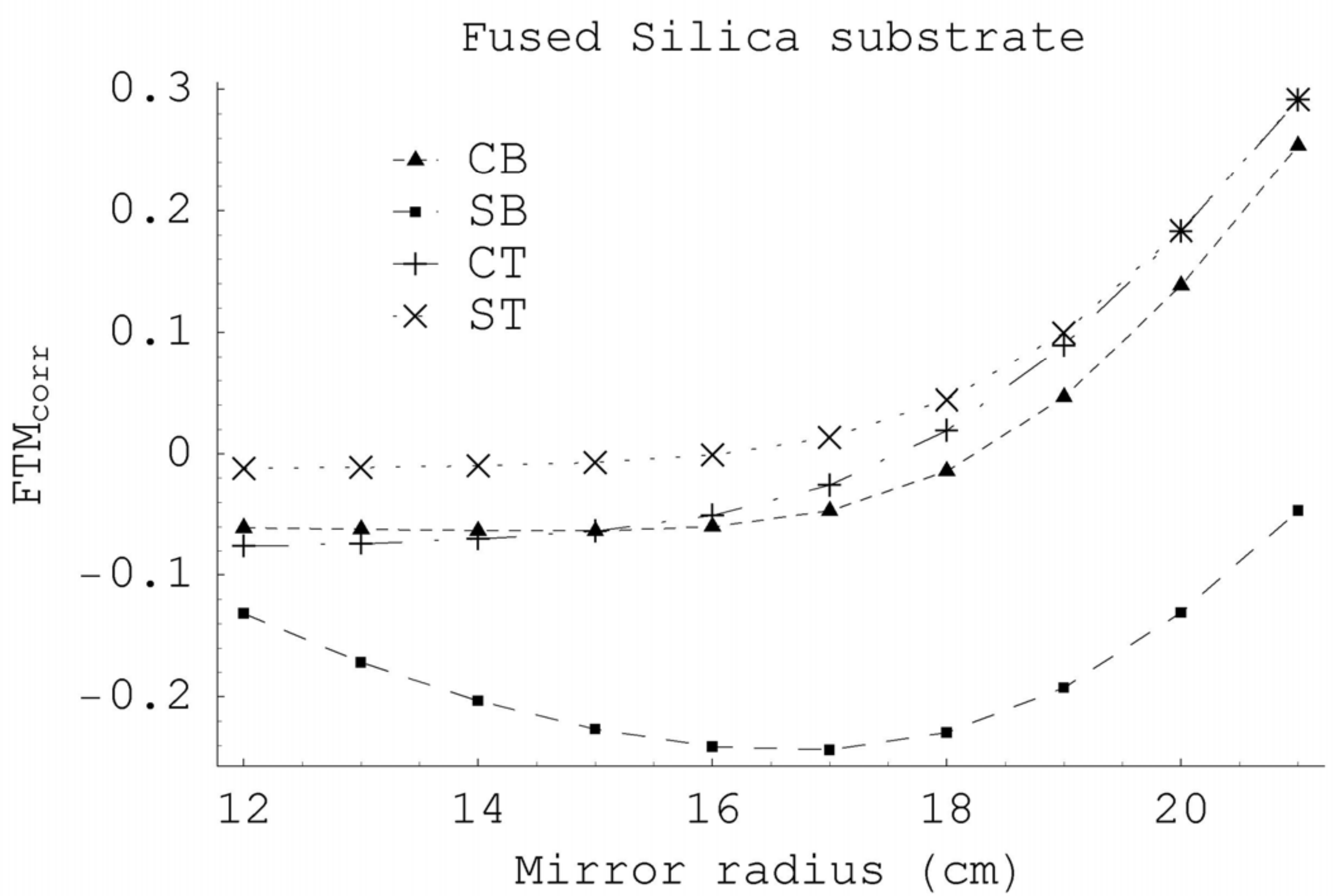
$D = 4w_0$ $\quad w_0 = 2.6\,cm$

$w = 6\,cm$

Gaussian and mesa beam parameters with diff. loss constraint



Mirror size effect in thermal noise evaluations (Gaussian beam)



Averaged elastic parameters

$$Y_1^* = Y_2^* = \frac{Y_1^2(1-\nu_2^2)\delta_1^2 + 2Y_1Y_2(1-\nu_1\nu_2)\delta_1\delta_2 + Y_2^2(1-\nu_1^2)\delta_2^2}{Y_1(1-\nu_2^2)\delta_1 + Y_2(1-\nu_1^2)\delta_2} \qquad (4.97)$$

$$Y_3^* = \frac{Y_1Y_2\left[Y_1(1-\nu_2)\delta_1 + Y_2(1-\nu_1)\delta_2\right]}{Y_2^2(1-\nu_1-2\nu_1^2)\delta_1\delta_2 + Y_2^2(1-\nu_2-2\nu_2^2)\delta_1\delta_2 + Y_1Y_2\left[(1-\nu_2)\delta_1^2 + 4\nu_1\nu_2\delta_1\delta_2 + (1-\nu_1)\delta_2^2\right]}$$

$$\nu_{12}^* = \frac{Y_1\nu_1(1-\nu_2^2)\delta_1 + Y_2\nu_2(1-\nu_1^2)\delta_2}{Y_1(1-\nu_2^2)\delta_1 + Y_2(1-\nu_1^2)\delta_2} \qquad (4.98)$$

$$\nu_{13}^* = \frac{Y_1Y_2\left[(1-\nu_2)\nu_1\delta_1 + (1-\nu_1)\nu_2\delta_2\right]}{Y_1^2(1-\nu_2-2\nu_2^2)\delta_1\delta_2 + Y_2^2(1-\nu_1-2\nu_1^2)\delta_1\delta_2 + Y_1Y_2\left[(1-\nu_2)\delta_1^2 + 4\nu_1\nu_2\delta_1\delta_2 + (1-\nu_1)\delta_2^2\right]}$$

$$G_\perp^* = \frac{Y_1Y_2}{2\left[Y_2(1+\nu_1)\delta_1 + Y_2(1+\nu_2)\delta_2\right]}$$

$$G_\parallel^* = \frac{Y_1\delta_1}{2(1+\nu_1)} + \frac{Y_2\delta_2}{2(1+\nu_2)}$$

Small Poisson ratios expansion

$$Y_\parallel = Y_1\delta_1 + Y_2\delta_2 + O(\nu^2)$$

$$Y_\perp = \frac{Y_1Y_2}{Y_2\delta_1 + Y_1\delta_2} + O(\nu^2)$$

$$\nu_\parallel = \frac{Y_1\nu_1\delta_1 + Y_2\nu_2\delta_2}{Y_1\delta_1 + Y_2\delta_2} + O(\nu^2)$$

$$\nu_\perp = \frac{Y_1Y_2\nu_1\delta_1 + Y_1Y_2\nu_2\delta_2}{(Y_1\delta_1 + Y_2\delta_2)(Y_2\delta_1 + Y_1\delta_2)} + O(\nu^2)$$