Random template banks and sensitivity gain through non-optimal parameter space covering

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LIGO-G070871-00-Z

Overview

1. GW parameter space searches
2. Introduction to the metric
3. Lattice coverings
4. A randomly placed template bank
5. Interpretation of the results
6. Implications

GW parameter space searches

- Matched filtering searches require prior knowledge of the signal waveform.
  - Continuous : sky position, frequency derivatives, binary parameters etc.
  - Inspiral : masses, spins, sky position for LISA (eg. EMRI, IMRI).
- Many current (and future) GW parameter space searches
  - are computationally bound.
  - have complicated, high dimensional, spaces.
- We therefore need
  - efficient parameter space template coverings.
  - simple, effectual, template placement strategies.

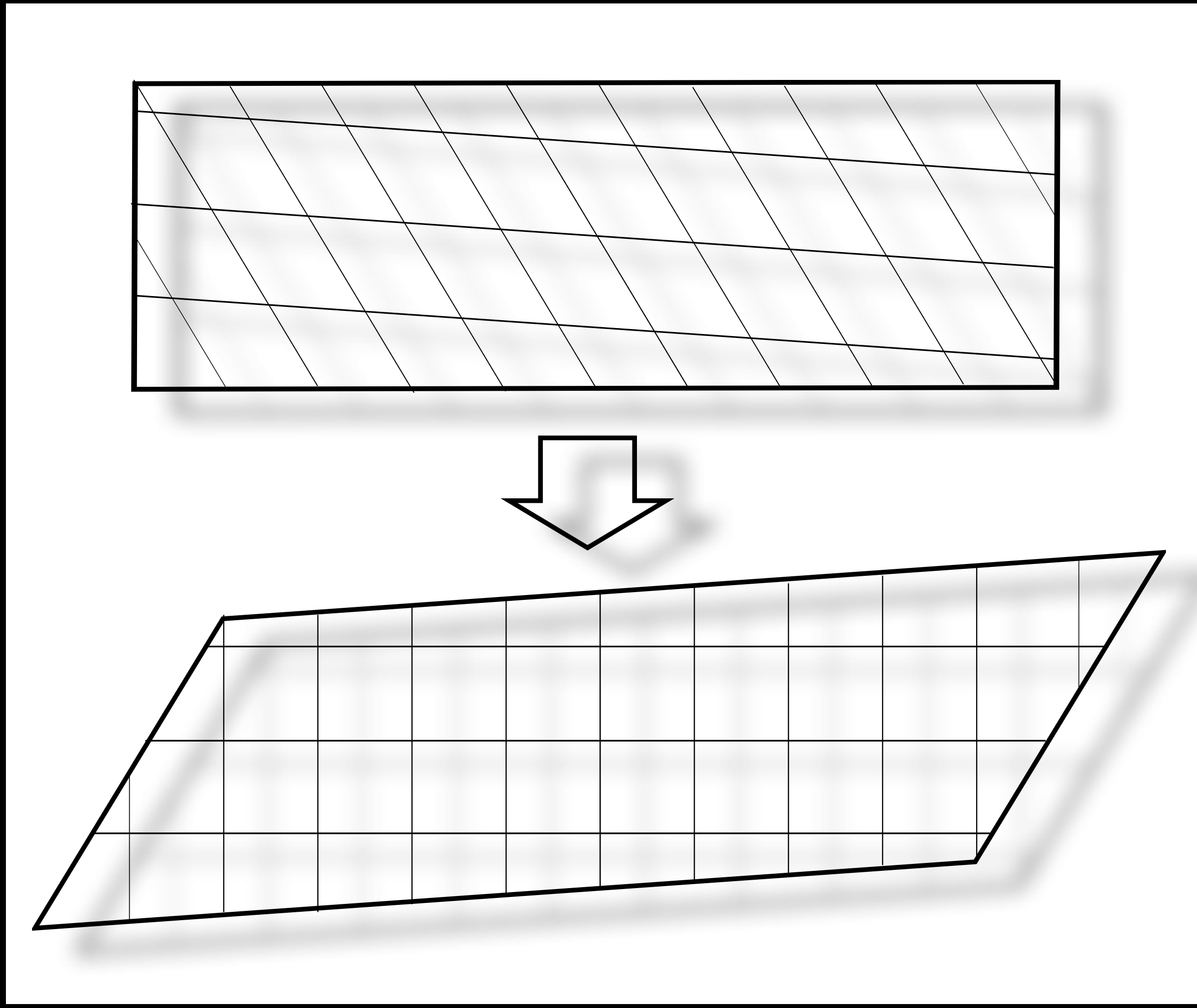
Introduction to the metric

- Construct a measure of distance in the parameter space equivalent to signal-template overlap.

$$\mu = \mathrm{d}s^2 = g_{ij}\mathrm{d}x_i\mathrm{d}x_j \qquad g_{ij} = \langle \partial_i\phi\partial_j\phi\rangle - \langle\partial\phi_i\rangle\langle\partial\phi_j\rangle$$

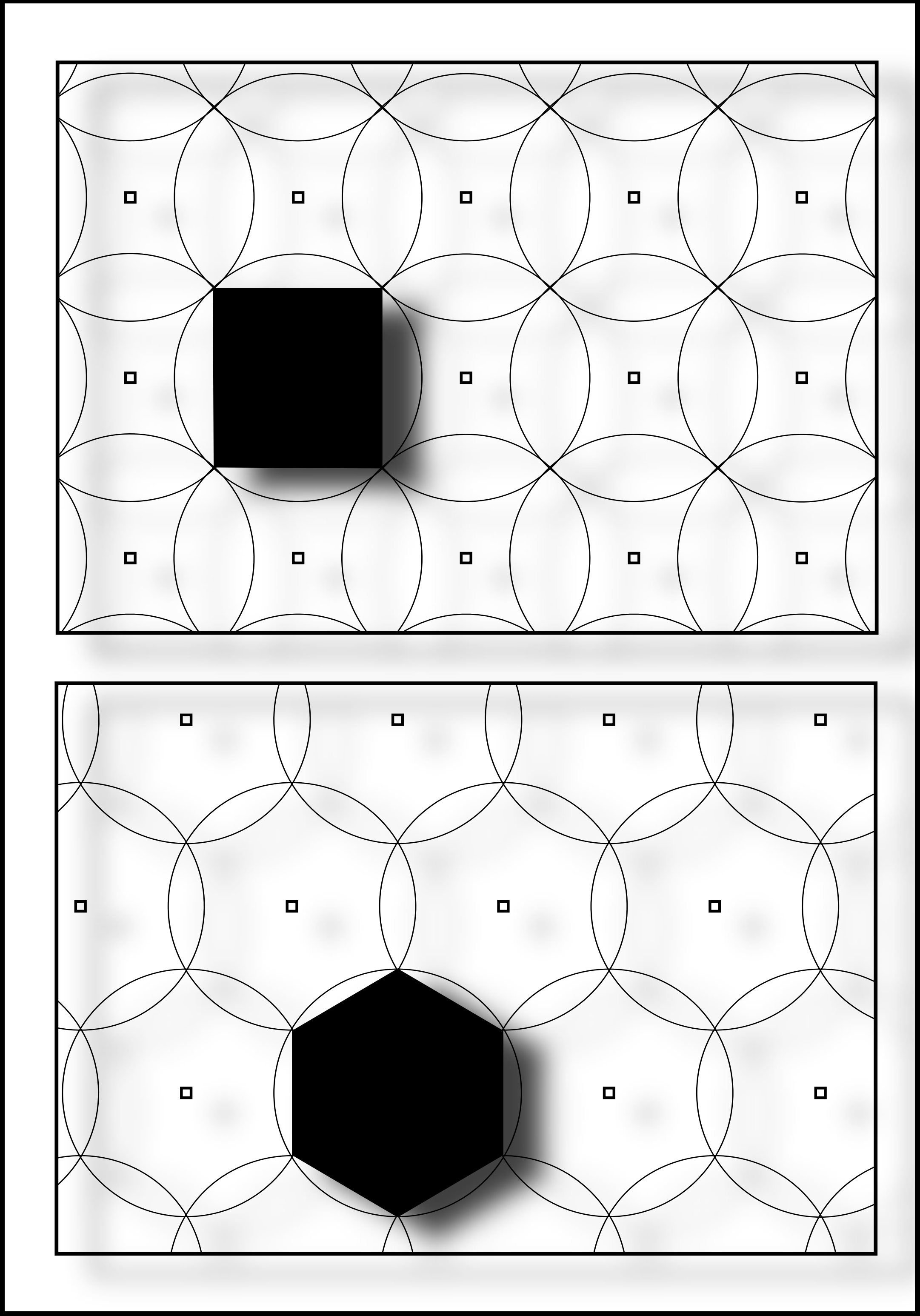
- Use the eigenvectors and eigenvalues to define a set of local "unit" basis "directions" in the parameter space.

- Templates can then be placed using the diagonalised and normalised basis as an underlying guide.



Lattice coverings

- In the new basis, in general the space is *locally* Cartesian however we will consider globally *flat* spaces.
- The problem becomes the standard mathematical "covering" problem.
- The *simplest* n-dimensional lattice is a cubic lattice Z^n (sub-optimal ie. high n-sphere overlap).
- The "*best known*" class of lattice (for n<24) is known as the A_n^* lattice.



Lattice normalised thickness

- We can define a measure of covering efficiency, referred to as the *normalised thickness*.

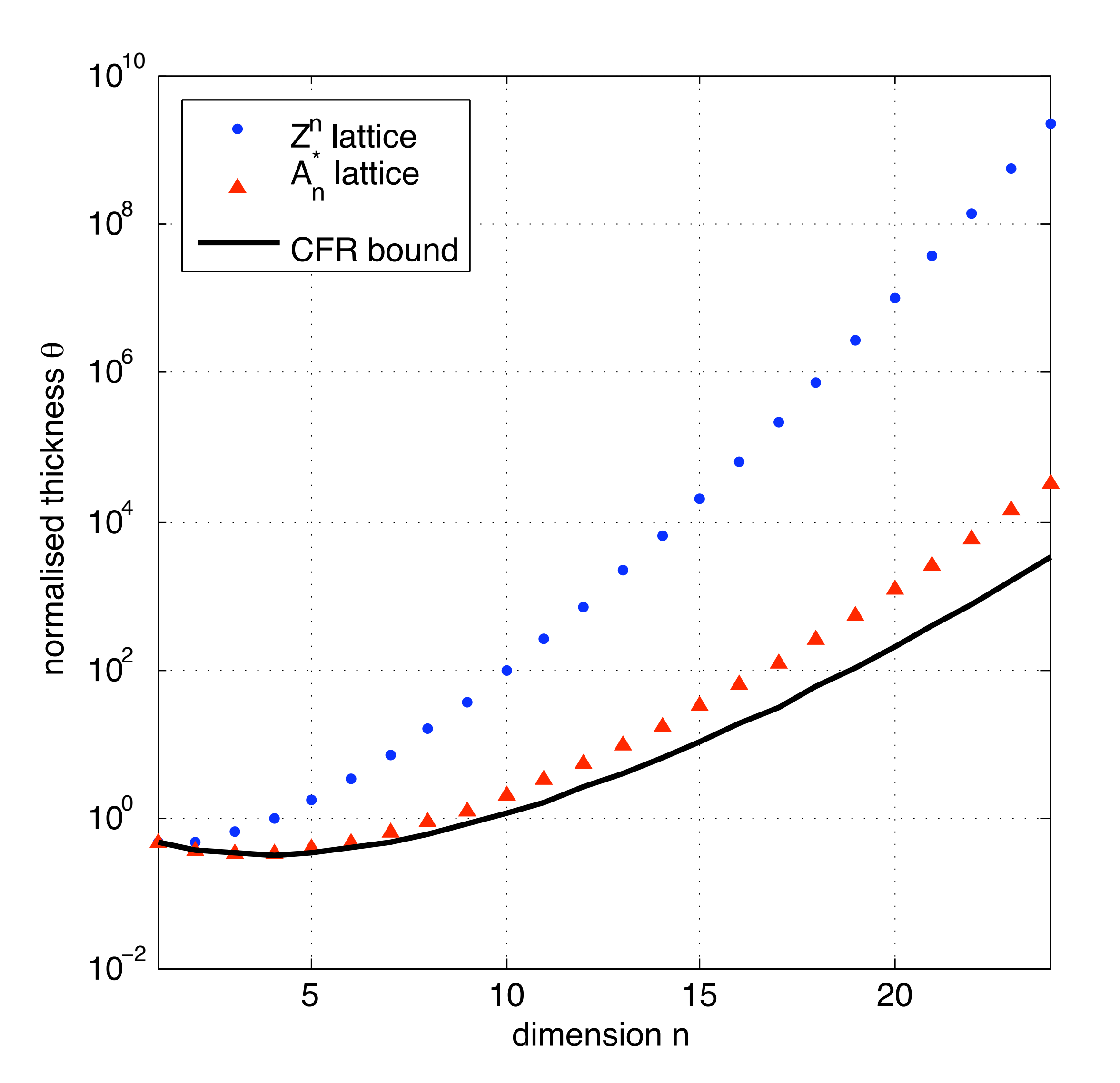
$$\theta = \frac{R^n}{\sqrt{||g_{ij}||}}$$

θ is proportional to the density of templates.

θ is therefore proportional to the required number of search templates.

$$\theta_{Z^n} = \frac{n^{\frac{n}{2}}}{2^n}$$

$$\theta_{A_{n*}} = \sqrt{n+1}\left[\frac{n(n+2)}{12(n+1)}\right]^{\frac{n}{2}}$$



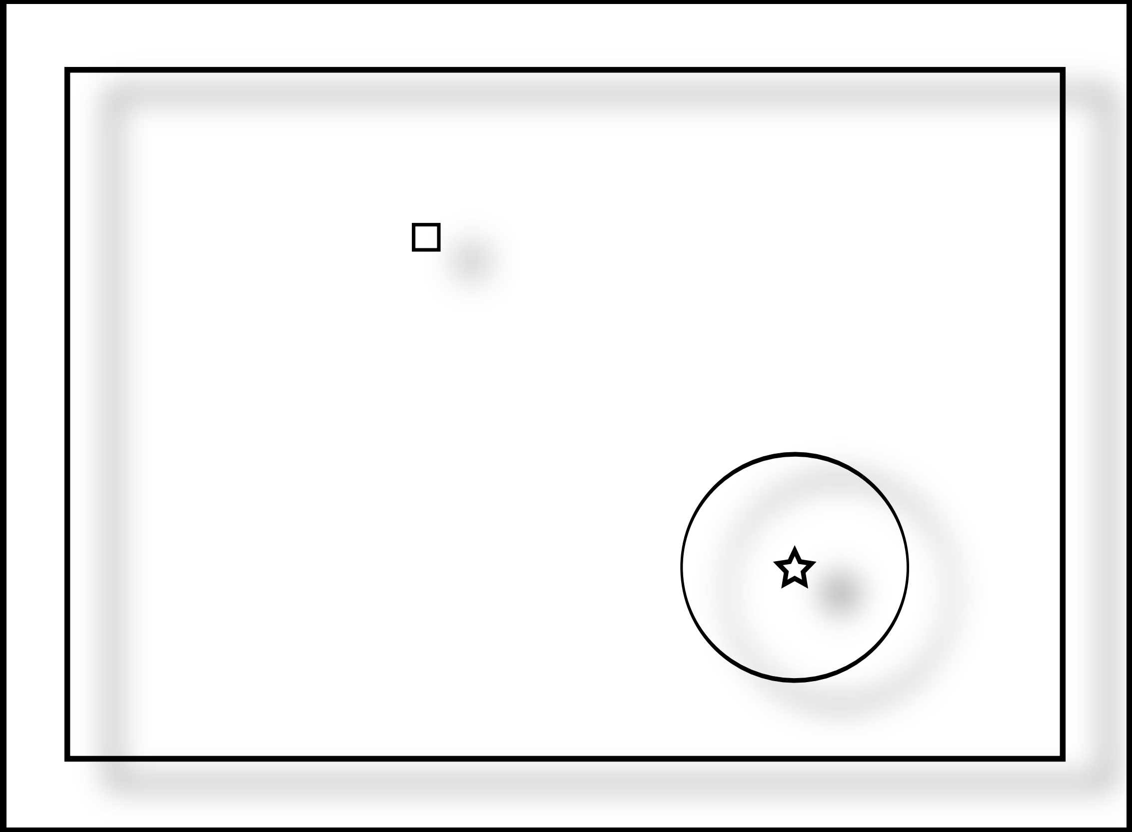
Random template placement

- How efficient is a randomly placed template bank ?
- Assuming a large Euclidean space of volume V_S,

1. We place a *single* random template.
2. We place N random templates.

The probability that the randomly located template lies outside a spherical region of maximum mismatch μ centered on the signal is

$$P_{1,n} = 1 - \frac{V_n R^n}{V_S}$$



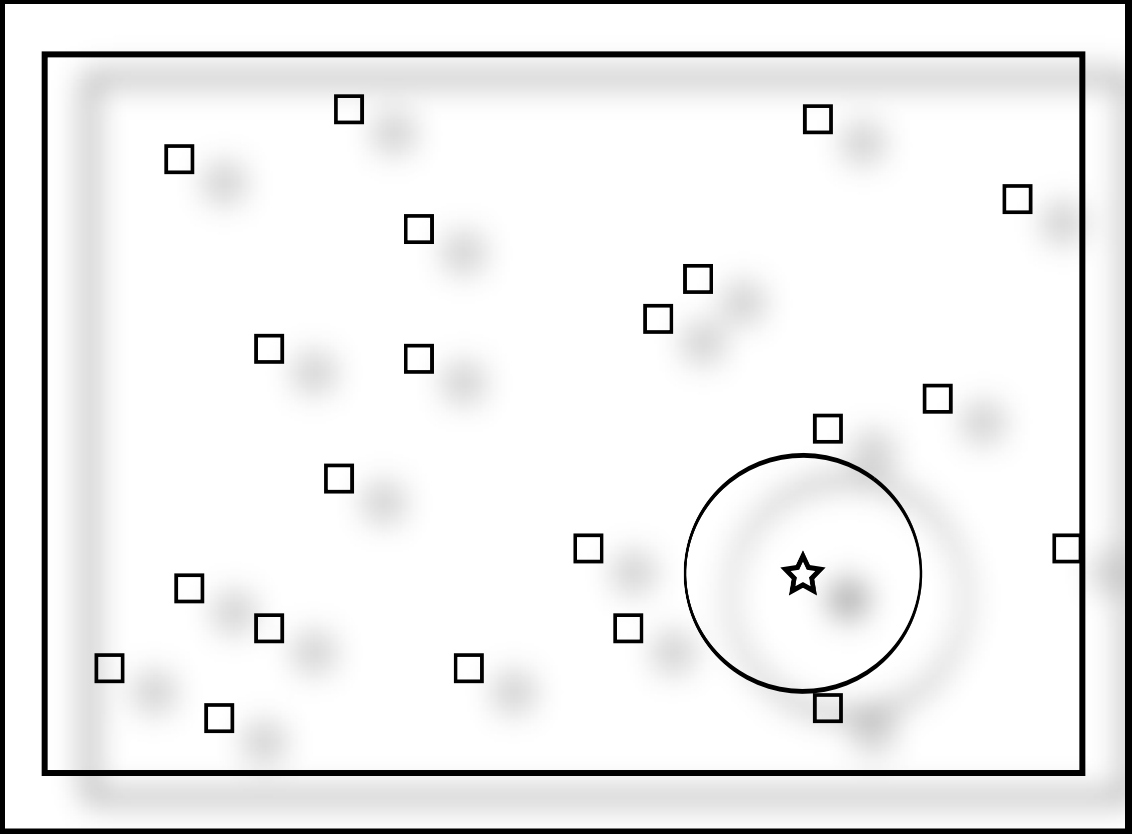
Random template placement

- How efficient is a randomly placed template bank ?
- Assuming a large Euclidean space of volume V_S,

1. We place a single random template.
2. We place N random templates.

The probability that all randomly located templates lie outside the spherical region of maximum mismatch μ centered on the signal is

$$P_{N,n} = \left[1 - \frac{V_n R^n}{V_S}\right]^N$$



Random template placement

- It follows that the probability of achieving a mismatch of μ or less using N randomly placed templates, assuming a single signal, is

$$P'_{N,n} = 1 - \left[1 - \frac{V_n R^n}{V_S}\right]^N$$

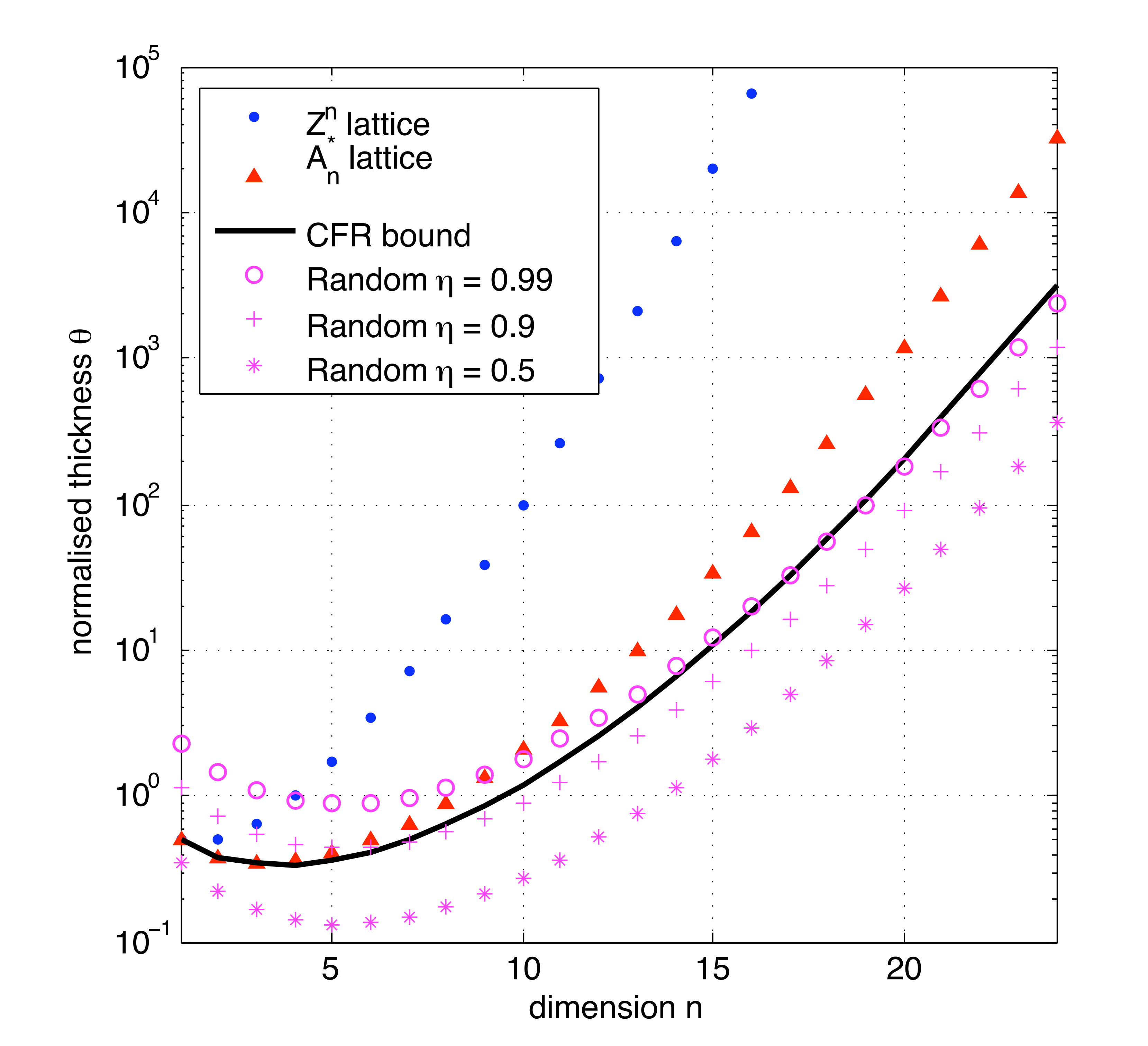
- For $V_n R^n < V_S$ we always have $P'_{N,n} < 1$ and so *we enforce that only a fraction* η *of the space is covered*, ie. solve the following for N

$$P'_{N,n} = \eta$$

- This gives an *effective* normalised thickness of

$$\theta_{\mathrm{r}}(\eta) = \ln\left(\frac{1}{1-\eta}\right)\frac{1}{V_n}$$

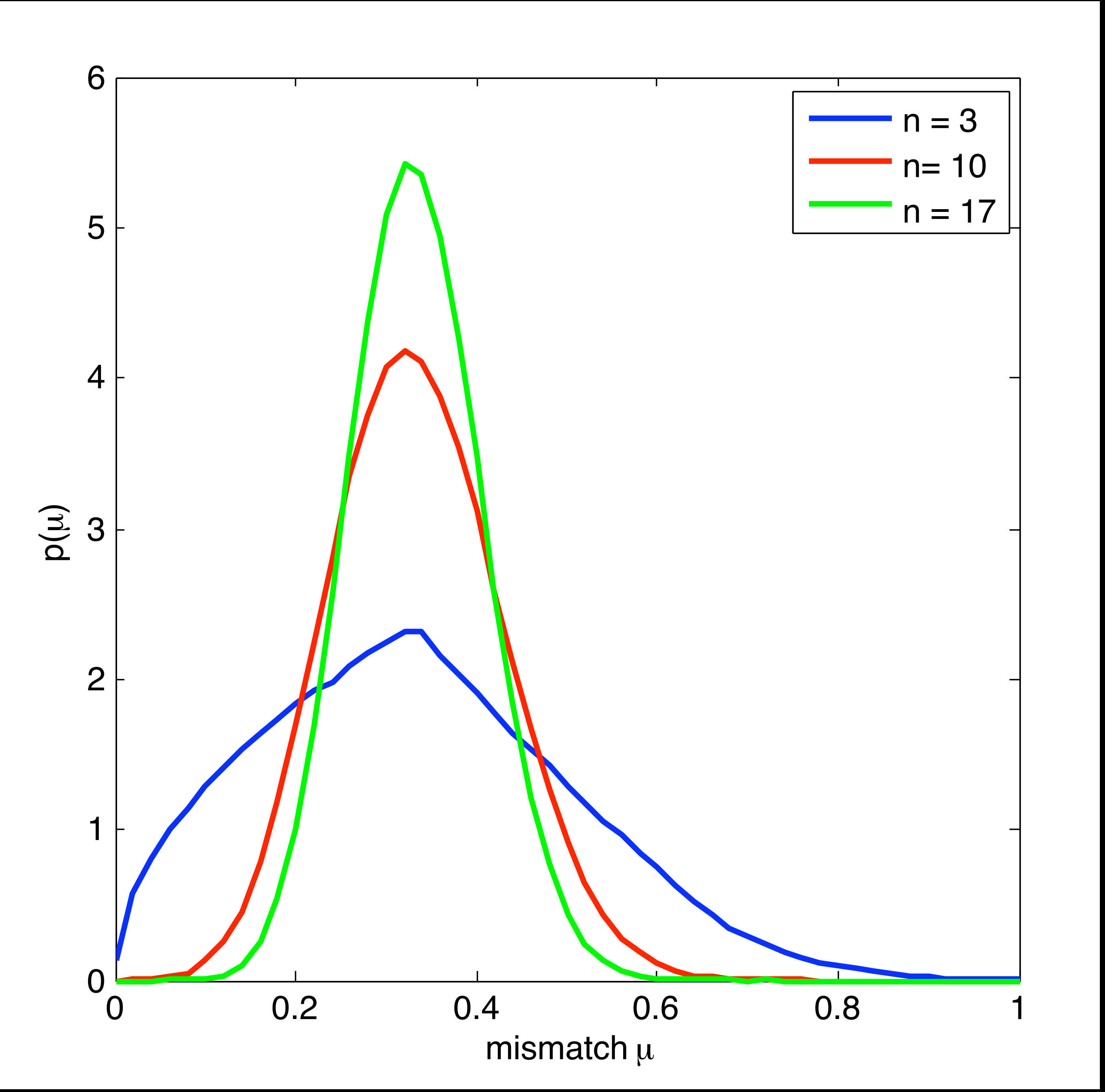
Effective normalised thickness



Interpretation

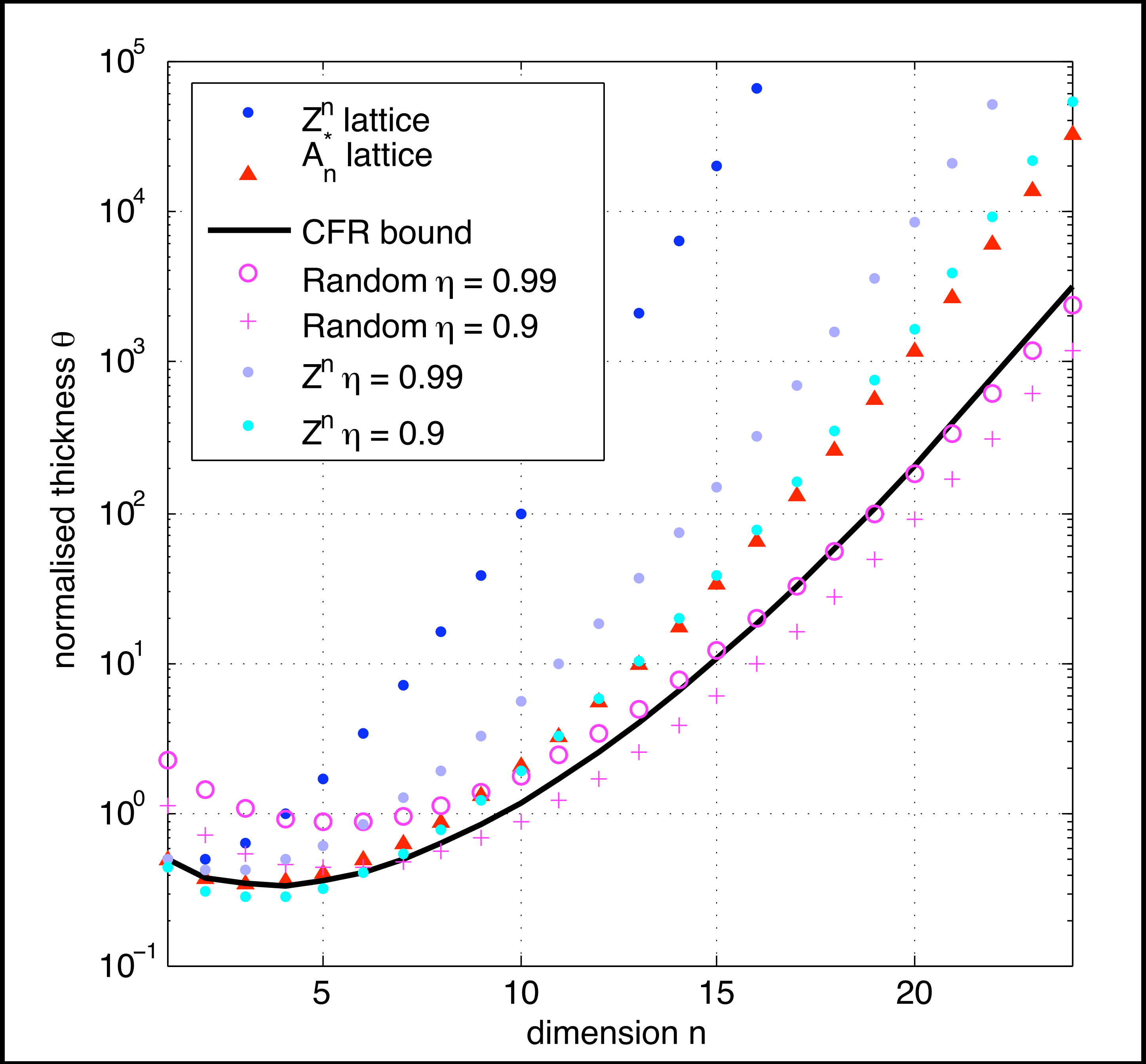
CUBIC LATTICE

- Lattices go to a *lot* of extra effort to cover the *last few %* of the space.
- The random template bank wins because it's lazy and doesn't try to get those last few %.
- So, shouldn't we have a "lazy" lattice ?



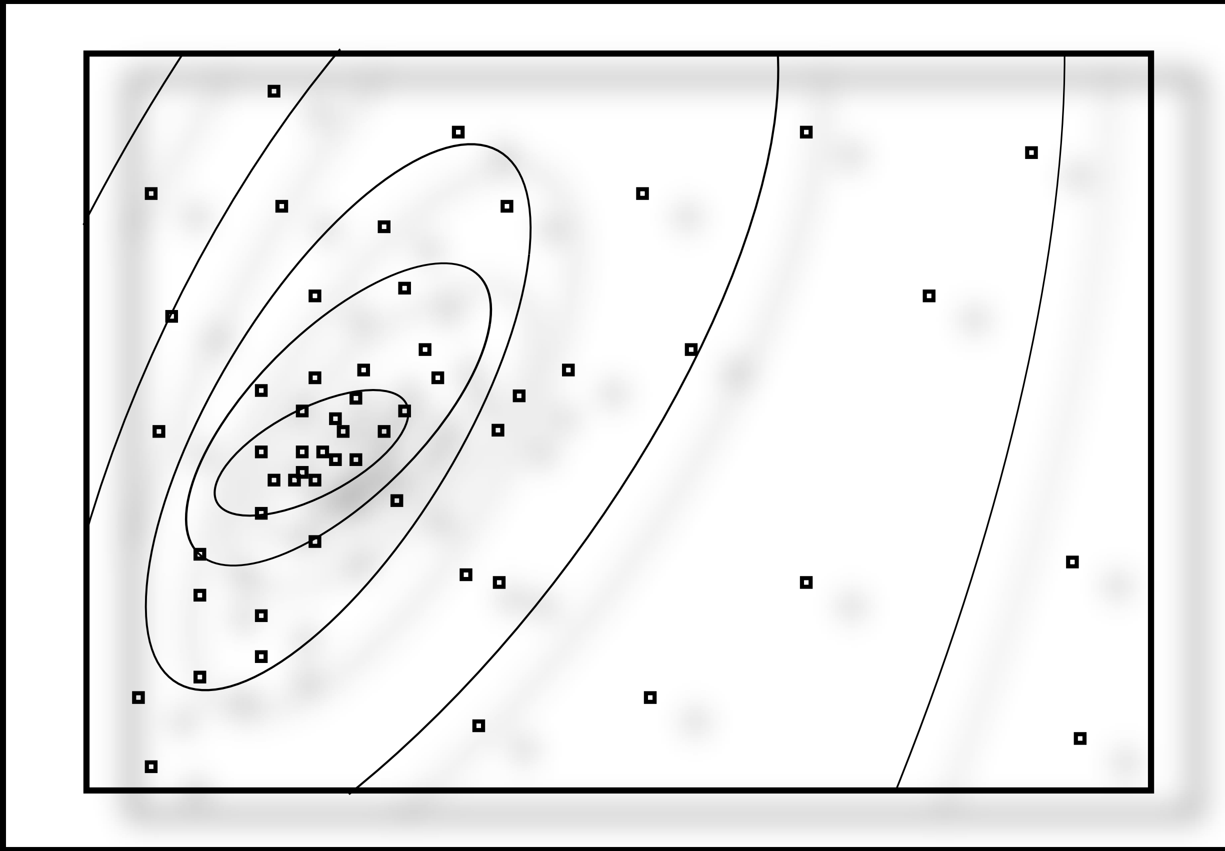
A fair comparison

- Let us now allow the same fraction of space to be under-covered using the cubic lattice.



A more realistic scenario

- In practice the only information required in order to place a controlled random template bank is the *determinant* of the metric.
- This allows us to compute the proper volume of the space V_S, (needed to compute the number of templates N).
- In *non-flat* space we propose the generation of a scalar template density function.
- Templates can then be placed randomly according to this density.



Implications

1. *If* we are prepared to disregard a predefined fraction of the parameter space then we can make significant computational gains using both

   a) random template banks.
   b) "lazy" lattices.

2. For non-flat space, if the determinant is known, placing a random template bank is *far* simpler than placing a lattice.

3. Simpler and quicker to implement than existing stochastic banks.

4. All of our present GW results have an associated statistical uncertainty. A random template bank simply adds to this uncertainty.