An Evidence Based Approach to Inspiral Followups

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Aim of this work

The goal of this work is to find a way to calculate the probability that an inspiral candidate signal is really a gravitational wave.

We use model selection within the Bayesian framework to compute the odds ratio between competing models.

For the purpose of development of the algorithm, we used Newtonian waveforms and synthetic Gaussian noise, but it should be readily extensible to post-Newtonian analysis and the use of real data.

Bayesian Hypothesis Testing

We want to know the probability ratio of an inspiral signal (H_S) being present in the data {d} to that of the data being just noise (H_N).

$$\frac{P(H_S|d,I)}{P(H_N|d,I)} = \frac{P(H_S|I)}{P(H_N|I)}\frac{P(d|H_S,I)}{P(d|H_N,I)}$$

Prior odds   "Bayes Factor"

To calculate marginal likelihood or "evidence" of signal model, marginalise the PDF over all model parameters.

$$P(d|H_S,I) = \int_{\Theta} p(d|\underline{\theta},H_S,I)p(\underline{\theta}|H_S,I)d\underline{\theta}$$

This is the same approach used in Clark et al 2007 & Searle et al 2007.

Computing Evidence

As a proof of concept, perform this analysis on Newtonian waveform in stationary synthetic Gaussian noise.

Integrate over 4 parameters using a probabilistic algorithm.

$$P(d|H_S, I) = \int_{\mathbf{\Theta}} p(d|\underline{\theta}, H_S, I) p(\underline{\theta}|H_S, I) d\underline{\theta}$$

$$P(d|H_S, I) = \int p(A, \mathcal{M}, t_c, \phi_c|H_S, I) p(d|A, \mathcal{M}, t_c, \phi_c, H_S, I) dA d\mathcal{M} dt_c d\phi_c$$

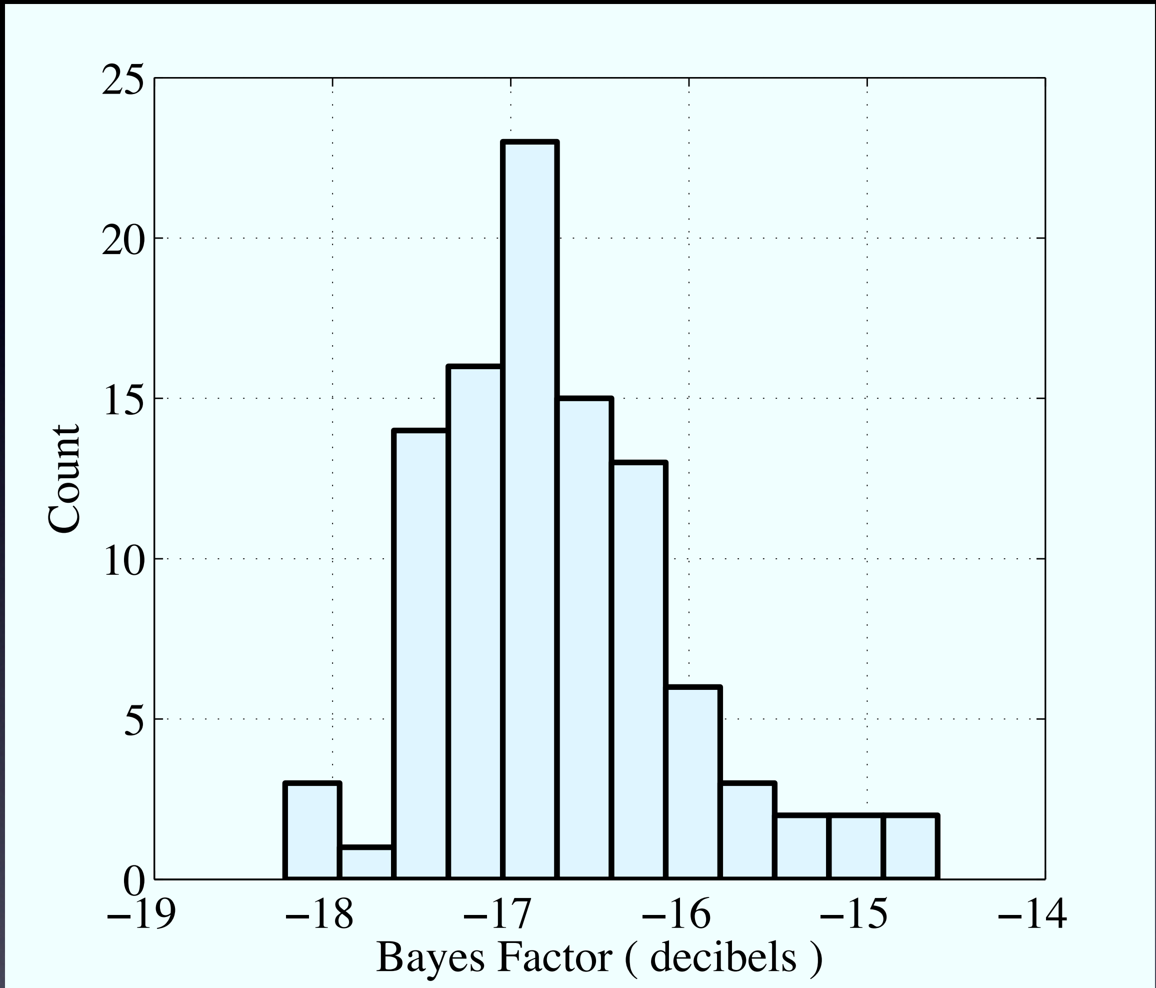
Using flat priors and a Gaussian model of the noise in the frequency domain, likelihood function is

$$p(d|H_S, I) = \int_{\mathbf{\Theta}} \prod_{n=1}^{N} [2\pi \mathcal{R}(\sigma_n) \mathcal{I}(\sigma_n)]^{-1} \exp\left(- \frac{\left|\tilde{h}(f_n; \underline{\theta}) - \tilde{d}_n\right|^2}{2\left|\tilde{\sigma}_n\right|^2} \right) d\underline{\theta}$$

Can be extended to any parametric model!

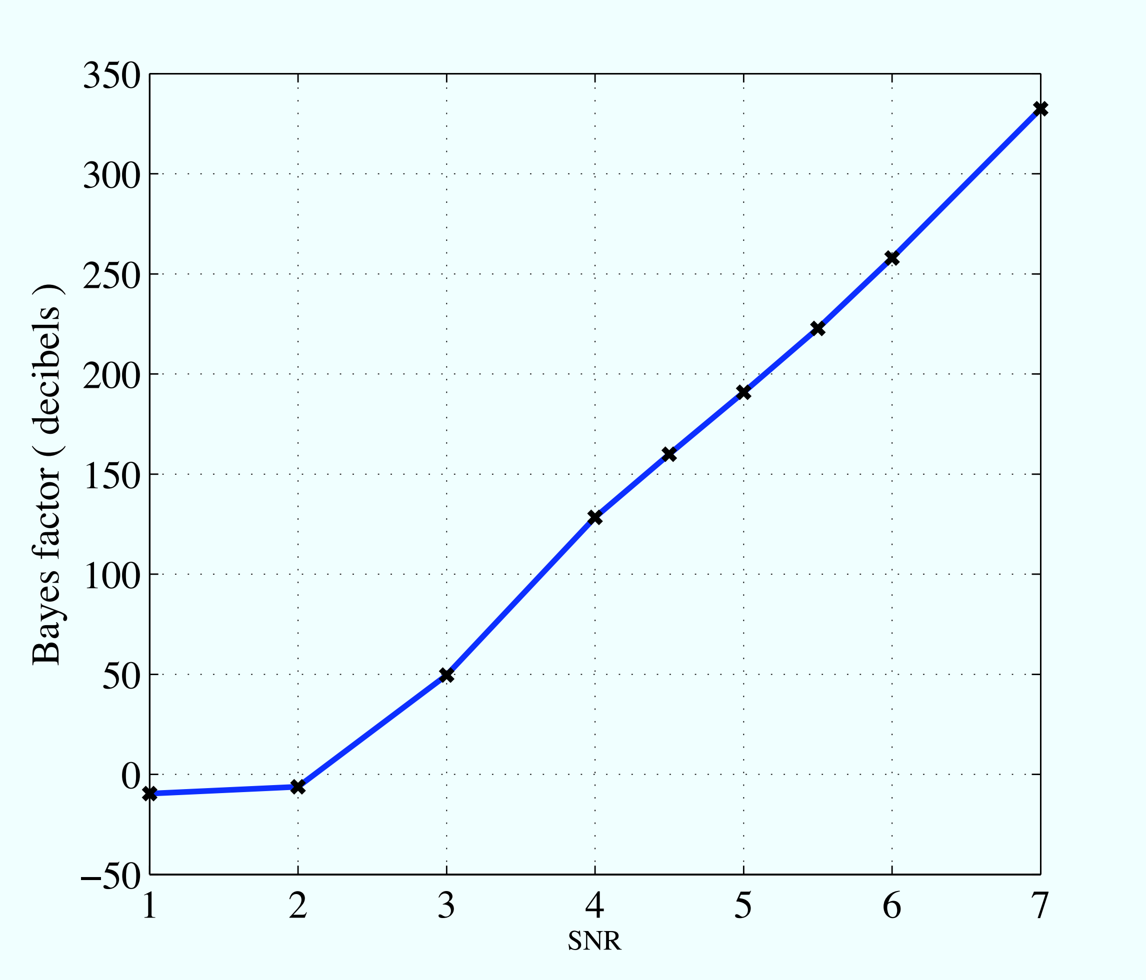
Results - Gaussian noise only

- Produce 100 different realisations of Gaussian stationary noise with no signal.
- The algorithm returns a Bayes Factor < 1. This indicates that the noise model is preferred.
- However with these inconclusive results the final odds ratio will be dominated by the prior.
- Code runs in <2 hours for 100s data at 512 Hz Nyquist frequency.



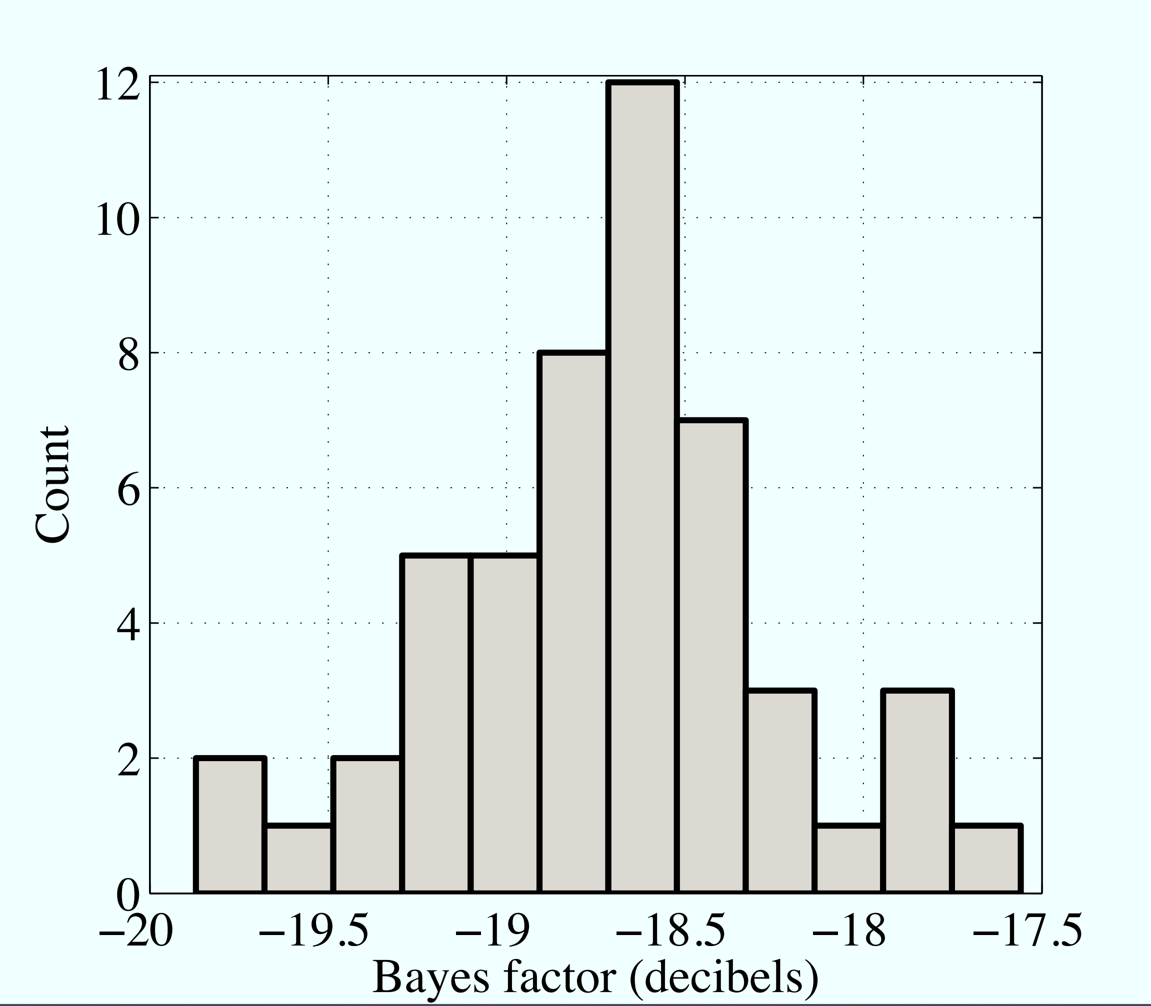
Gaussian + Signal, SNR 1 to 7

- Signals with varying SNR injected into 13800 data points of synthetic Gaussian noise.
- Algorithm clearly detects signals, and gives large Bayes factors for significant SNRs.
- The prior odds ratio then determines whether this is significant or not.



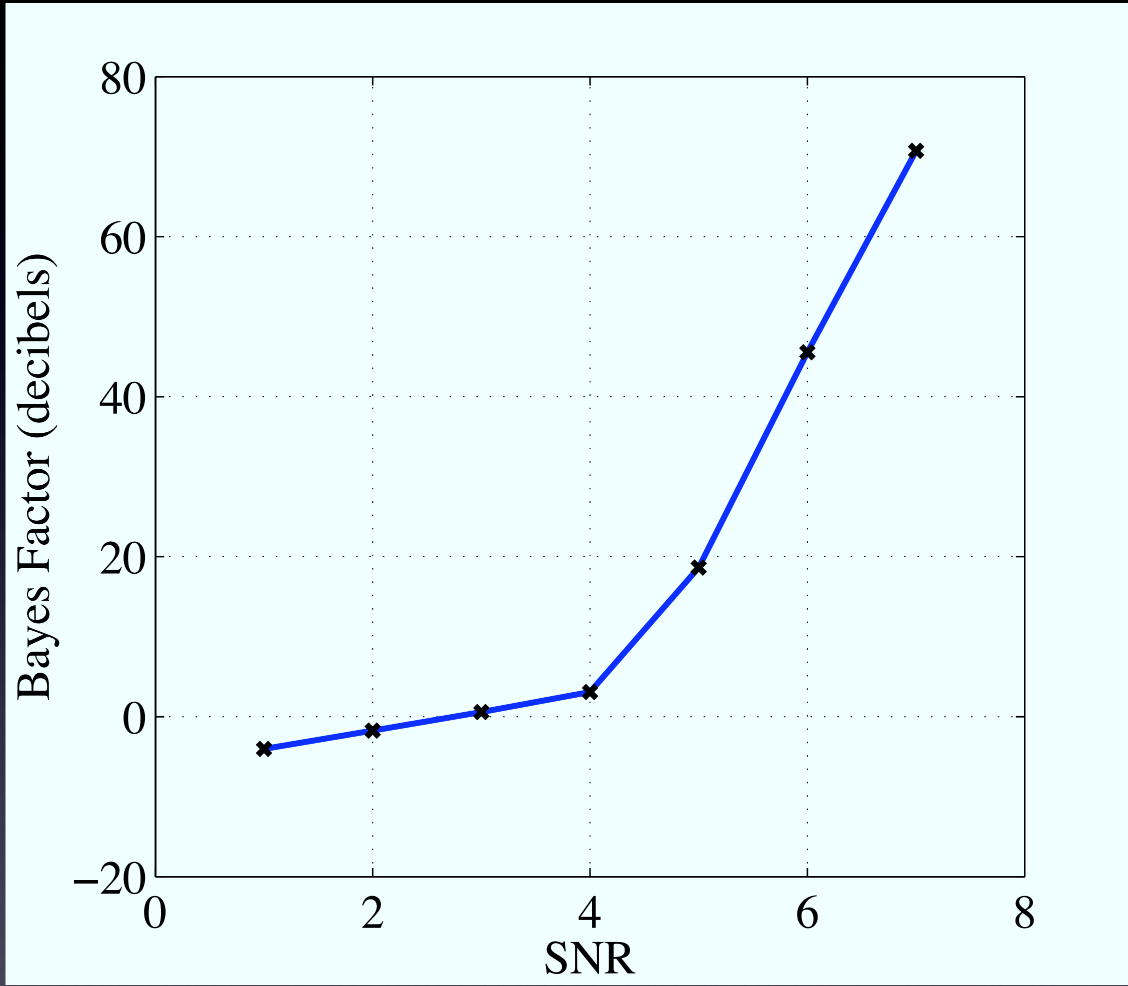
Gaussian + Poissonian Noise Only

- Using 100 Gaussian data sets, now inject points with amplitude 100 and Poisson time-domain PDF Pois(0.1).
- The algorithm does not present any Bayes factor >1 for the signal hypothesis on the data.
- Robust against random glitches in the data.
- How does this affect detection of signals?



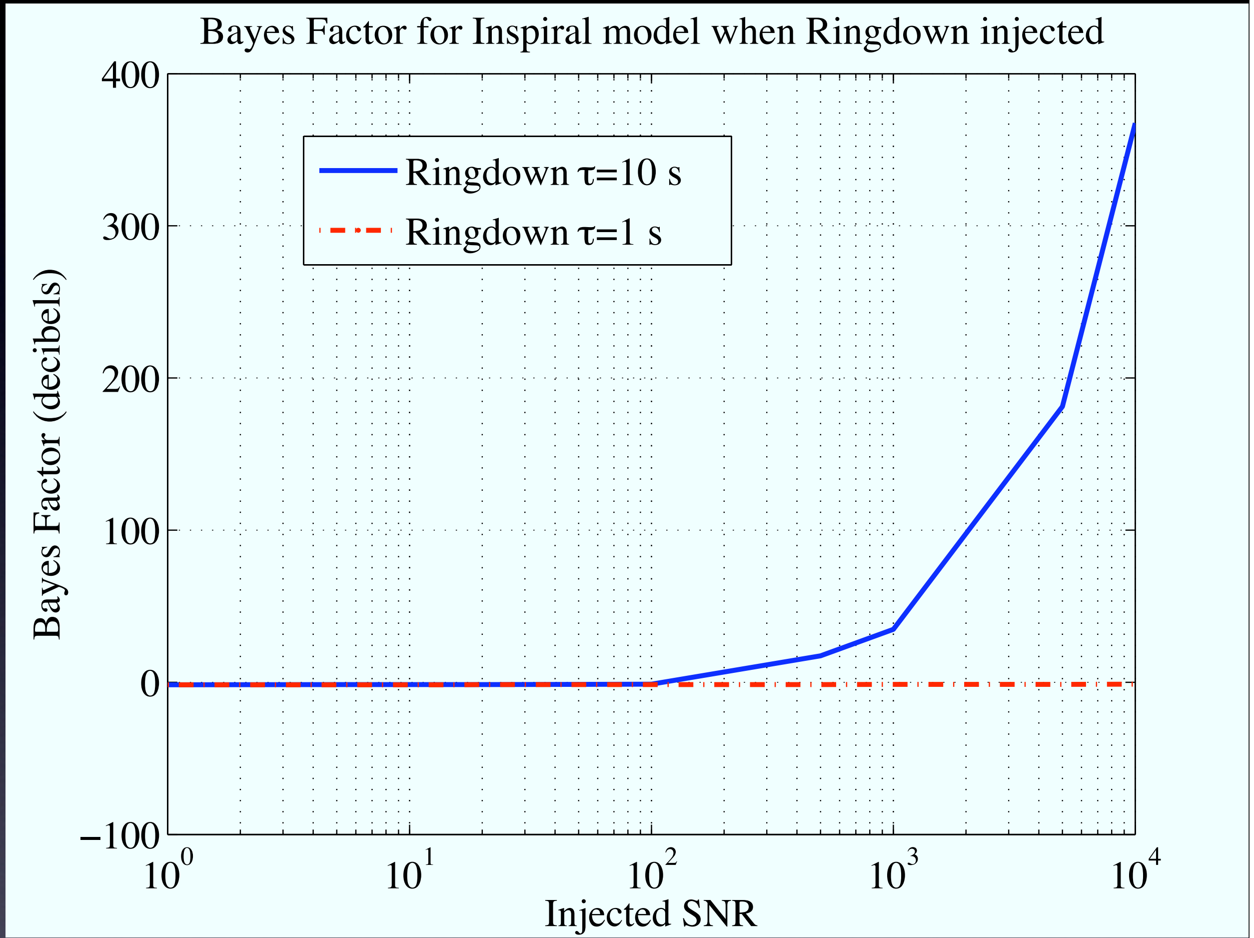
Injections in Gaussian + Poissonian noise

- Signals were injected into Poissonian + Gaussian noise at increasing SNR.
- Bayes Factor still favours the signal model at SNR > 4, but compared to the pure Gaussian case sensitivity is reduced.
- However, the signal model is not triggered by the Poisson noise, so should not cause false alarms.



Gaussian noise + instrumental ringdown

- Non-astrophysical ringdown signals with different decay times (1s and 10s) are injected into the data at increasing SNRs, to mimic the effects of noise.
- Above SNR=100, does the 10s ringdown start to trigger the inspiral model, whereas the 1s injection does not affect the results.
- Only very loud ringdowns will trigger inspiral model, by which point neither model accurately reflects the data.
- Introducing a ringdown model would allow a natural way to distinguish this type of signal.



Conclusions

- Bayesian hypothesis testing provides a clear and conceptually straightforward way of determining the odds of a candidate signal being real.
- Ideally suited to being used in the follow up of stage of an analysis.
- Calculation of the full Bayesian evidence can be done efficiently.
- Can be expanded to use other signal models.

Ongoing work

- Add Post-Newtonian waveforms from LAL.
- Analyse large amounts of IFO data to fully understand the response of algorithm to the types of noise encountered. Find suitable prior odds ratio from background studies?
- Integrate with Bayesian follow-up framework for
  - Multi-IFO coherent analysis
  - Automatic use of any inspiral waveform
  - Integration into the inspiral pipeline