A χ^2 veto for Continuous Wave Searches

12th Gravitational Wave Data Analysis Workshop

December 2007

L. Sancho de la J.

A.M. Sintes

(LIGO-G070826-00-Z)

Universitat de les Illes Balears

Outline

- Motivation
- Hough Transform
- The χ^2 veto
- Results

Motivation

Motivation

Hough Transform

The χ^2 test

Results

- Type of source → Continuous Sources
  - Very small amplitude ($h_0 \sim 10^{-26}$)
  - Long integration time needed to build up enough SNR
  - Relative motion of the detector with respect to the source (amplitude and frequency modulated)
  - System evolves during the observational period
- Type of search → All-sky search
  - Computational cost increases rapidly with total observation time.
- Type of Methods →
  - Coherent Methods *(Matched filtering)*
  - Semi-Coherent Methods *(Stack Slide, Power Flux, Hough)*
  - → Hierarchical Methods

Reduce the number of candidates to be followed up → Improve the sensitivity keeping the computational cost.

UIB

LSC

The Hough Transform

- Robust pattern detection technique.

- We use the Hough Transform to find the pattern produced by the **Doppler modulation** (due to the relative motion of the detector with respect to the source) and **spin-down** of a GW signal in the time – frequency plane of our data:

$$f(t) - \hat{f}(t) = \hat{f}(t)\,\frac{\vec{v}(t)\cdot\vec{n}}{c}$$

Detector RF — SSB

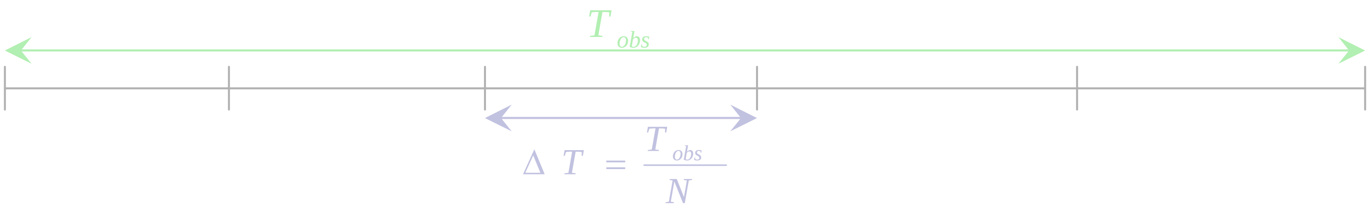
$$\hat{f}(t) = \hat{f}_0 + \dot{\hat{f}}(t - t_0) + \ldots$$

- For isolated NS the expected pattern depends on the parameters: $\{\alpha,\ \delta,\ f,\ \dot{f},\ldots\}$

The Hough Transform

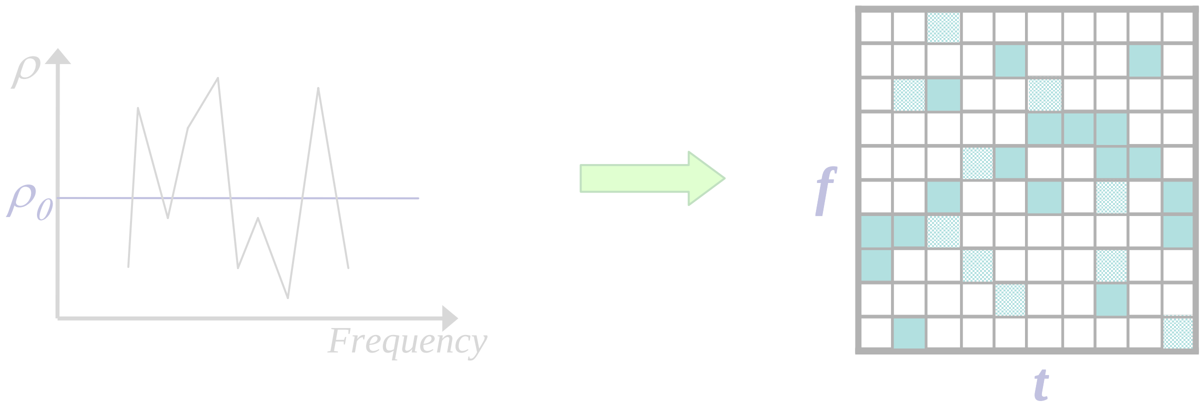
- Procedure:

① Break up data (*x(t) vs t*) into segments.



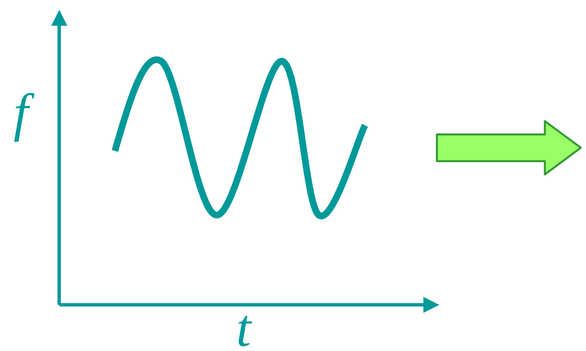
② Take the FT of each segment and calculate the corresponding normalized power in each case (ρ_k).

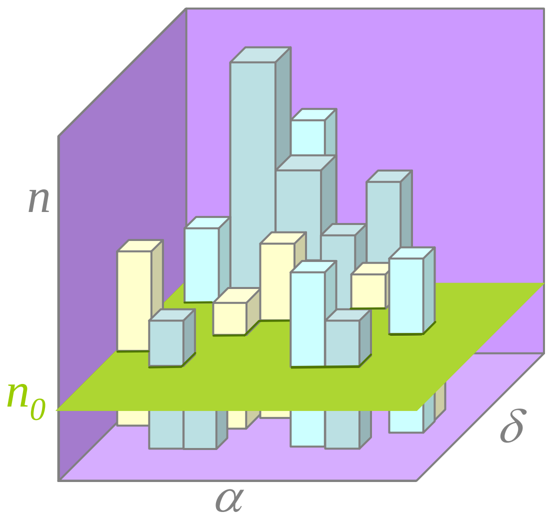
③ Select just those that are over a certain threshold ρ_{th}.

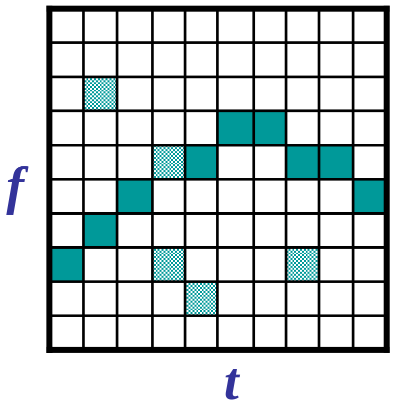


The Hough Transform

- Procedure:







Hough Transform Statistics

- The probability for any pixel on the *time - frequency* plane of being selected is:

$$p = \begin{cases} \text{Signal absent} & q = e^{\rho_{th}} \\ \text{Signal present} & \eta = e^{\rho_{th}} \left\{ 1 + \frac{\rho_{th}}{2}\lambda_k + O\left(\lambda_k^2\right) \right\} \end{cases}$$

SNR for a single SFT $\lambda_k = \frac{4\left|\tilde{h}(f_k)\right|^2}{T_{coh}\, S_n(f_k)}$

- After performing the Hough Transform N SFTs, the probability that the pixel $\{\alpha,\ \delta,\ f,\ \dot{f},\ldots\}$ has a number count n is given by

Without Weights

Number Count $n = \sum_{i=1}^{N} n_i$

$$p(n) = \binom{N}{n} p^n (1-p)^{N-n}$$

	Signal absent	Signal present
Mean	$\langle n \rangle = N\,q$	$\langle n \rangle = N\eta$
Variance	$\sigma^2 = N\,q\,(1-q)$	$\sigma^2 = N\eta(1-\eta)$

With Weights

Number Count $n = \sum_{i=1}^{N} \omega_i n_i$ $\sum_{i=1}^{N} \omega_i = N$

$$p(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(n-\langle n \rangle)^2}{2\sigma^2}}$$

	Signal absent	Signal present
Mean	$\langle n \rangle = N\,q$	$\langle n \rangle = qN + \frac{q\rho_{th}}{2}\sum_{i=1}^{N}\omega_i\lambda_i$
Variance	$\sigma^2 = \sum_{i=1}^{N}\omega_i^2\, q\,(1-q)$	$\sigma^2 = \sum_{i=1}^{N}\omega_i^2 \eta_i (1-\eta_i)$

Need of the χ^2 discriminator

- We define the **significance** of a number count as

$$s = \frac{n - \langle n \rangle}{\sigma}$$

($\langle n \rangle$ and σ are the expected mean and variance for pure noise)

- The Hough significance will be large if the data stream contains the desired signal, but it can also be driven to large values by spurious noise.
- We would like to discriminate which of those could actually be from a real signal.
- It is important to reduce the number of candidates in a Hierarchical search → improvement in sensitivity for a given finite computational power.

- Use the Hough Statistics information to veto the disturbances:

Hypothesis: Data = random Gaussian Noise + Signal

↓

Construct a χ^2 test to validate this hypothesis

The χ^2 test for the Weighted Hough Transform

1) Divide the SFTs into p non-overlapping blocks of data

						TOTAL
# SFTs	N_1	N_2	N_3	...	N_p	N
Number count	n_1	n_2	n_3	...	n_p	n
Sum weights	$\approx \frac{N}{p}$	$\approx \frac{N}{p}$	$\approx \frac{N}{p}$		$\approx \frac{N}{p}$	N

2) Analyze them separately

3) Construct a χ^2 statistic looking along the different blocks to see if the Hough number count accumulates in a way that is consistent with our hypothesis.

$$\langle n \rangle = \sum_{i=1}^{N} w_i \eta_i = \sum_{j=1}^{p} \langle n_j \rangle$$

$$\sigma^2 = \sum_{i=1}^{N} w_i^2 \eta_i (1-\eta_i)$$

$$\langle n_j \rangle = \sum_{i \in I_j} w_i \eta_i$$

$$\sigma_{n_j}^2 = \sum_{i \in I_j} w_i^2 \eta_i (1-\eta_i)$$

$$\Delta n_j \equiv n_j - n \frac{\sum_{i \in I_j} w_i \eta_i}{\sum_{i=1}^{N} w_i \eta_i} \qquad \begin{cases} \langle \Delta n_j \rangle = 0 \\ \sum_{j=1}^{p} \Delta n_j = 0 \end{cases}$$

$$\chi^2 = \chi^2(n_1, \ldots, n_p) = \sum_{j=1}^{p} \frac{(\Delta n_j)^2}{\sigma_{n_j}^2}$$

$$\chi^2 = \sum_{j=1}^{p} \frac{\left(n_j - n\left(\sum_{i \in I_j} w_i \eta_i\right) / \left(\sum_{i=1}^{N} w_i \eta_i\right)\right)^2}{\sum_{i \in I_j} w_i^2 \eta_i (1-\eta_i)}$$

$$\eta^* = \frac{n}{N}$$

$$\bar{\chi}^2 = \sum_{j=1}^{p} \frac{\left(n_j - n\left(\sum_{i \in I_j} w_i\right)/N\right)^2}{\eta^*(1-\eta^*) \sum_{i \in I_j} w_i^2}$$

- If Signal present: *small* χ^2
- If due to spurious noise: *big* χ^2

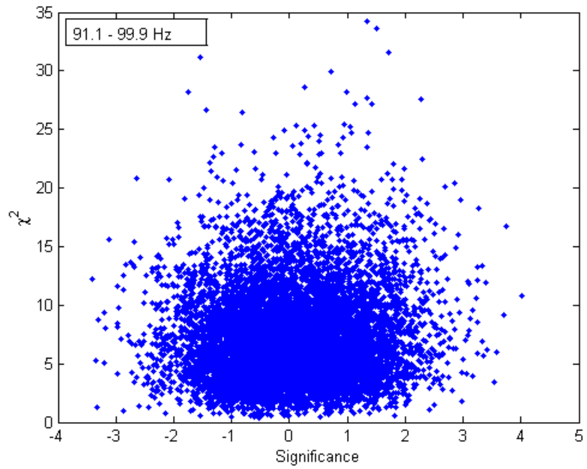
The χ^2 test: $\chi^2 - significance$ plane characterization

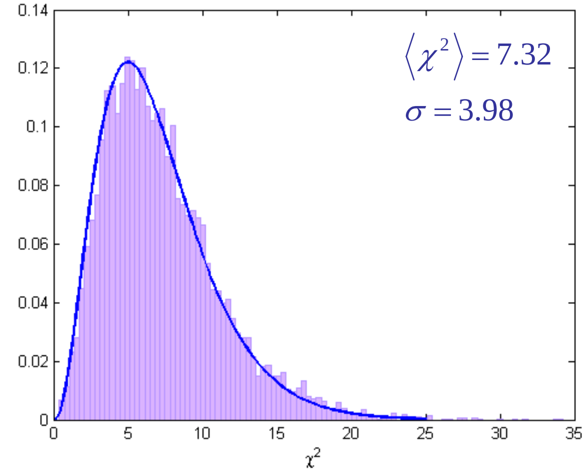
Gaussian Noise

$$\langle \chi^2 \rangle = p - 1$$

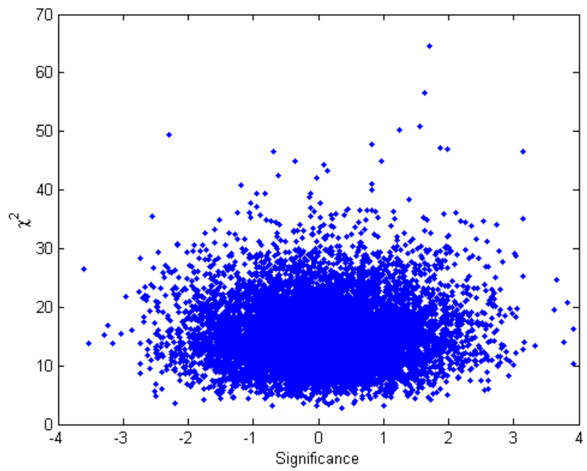
$$\sigma = \sqrt{2p}$$

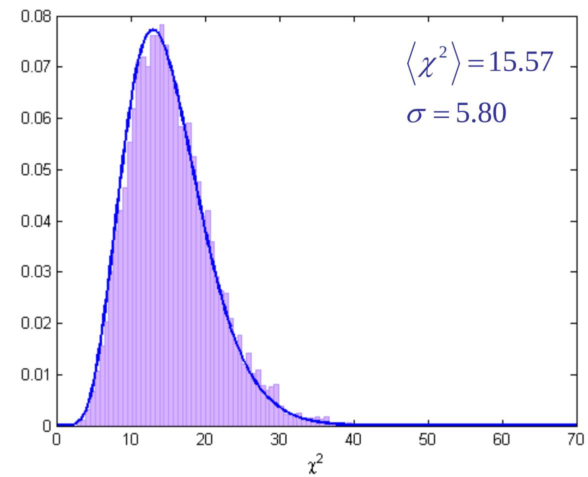
p = 8





p = 16



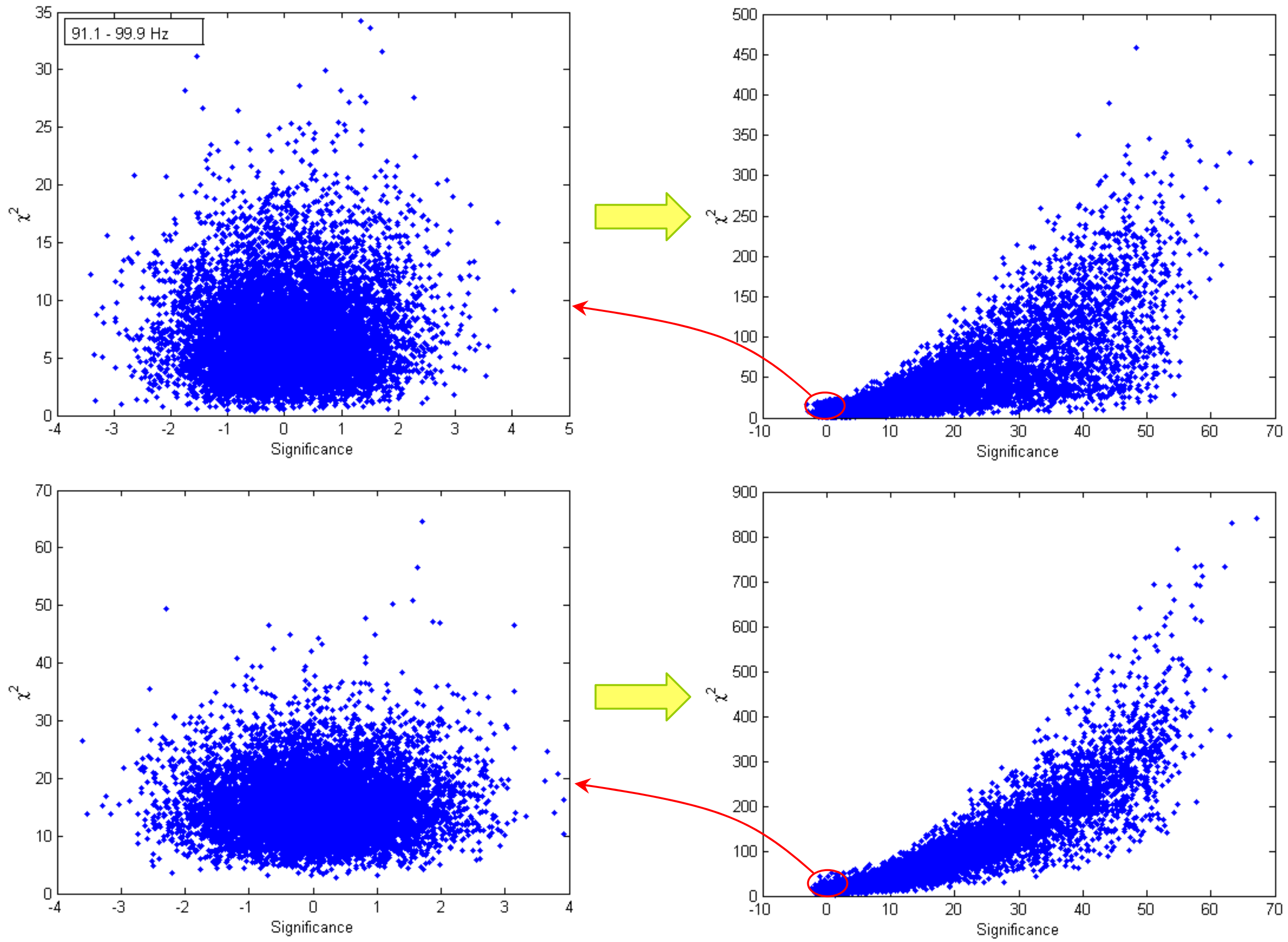


The χ^2 test: $\chi^2 - significance$ plane characterization

Software Injected Signals

p = 8

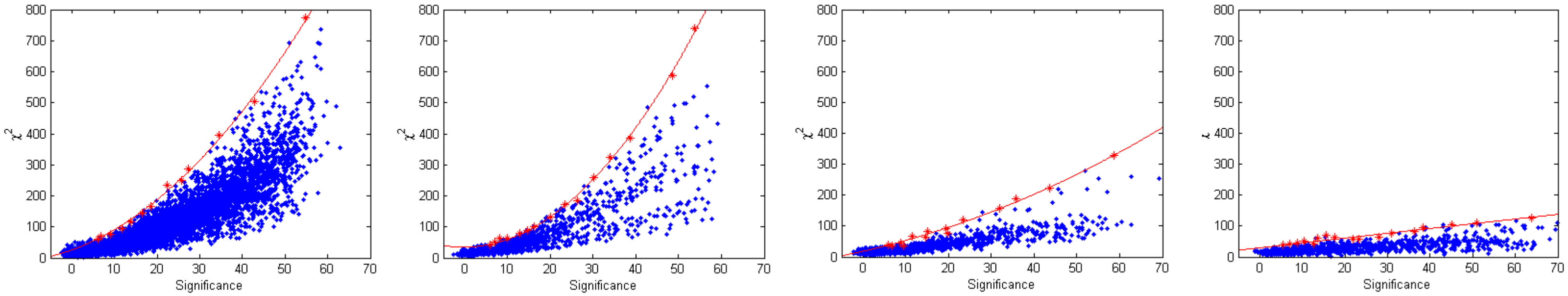
p = 16



The χ^2 test: $\chi^2 - significance$ plane characterization

Software Injected Signals

91.1 – 99.9 Hz | 101.1 – 101.9 Hz | 252.1 – 252.9 Hz | 420.1 – 420.9 Hz



- 1 month of LIGO data.
- Example for p = 16.
- 22 small 0.8 Hz bands between 50 and 1000 Hz were analyzed.
- In each 'quiet' band we do 1000 Monte Carlo injections for different h_0 values covering uniformly all the sky, *f-band*, *spindown* $\in [-1 \cdot 10^{-9} - 0]$ Hz s^{-1}, and pulsar orientations (9000 MC 91-100 Hz)
- Find the best fit in the selected bands $\rightarrow$ fitting coefficients should be frequency dependent.

Results (on 1 month of LIGO data)

Motivation

Hough Transform

The χ^2 test

Results

- Loudest significance in every *0.25Hz* band obtained with Hough:

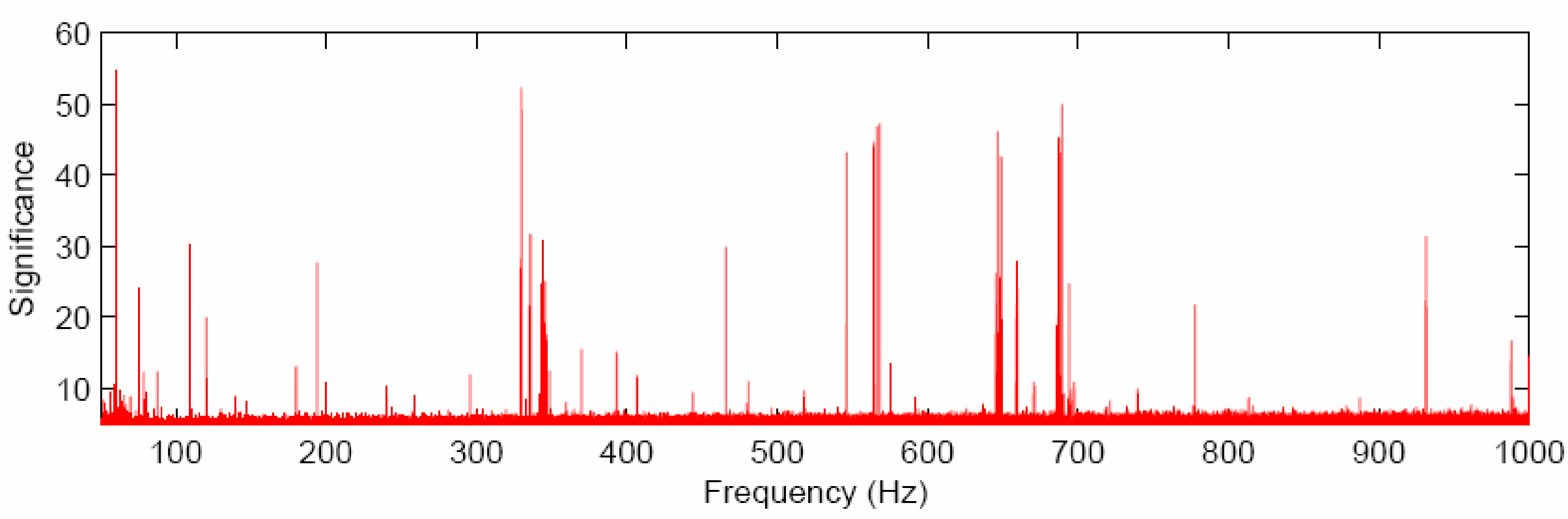
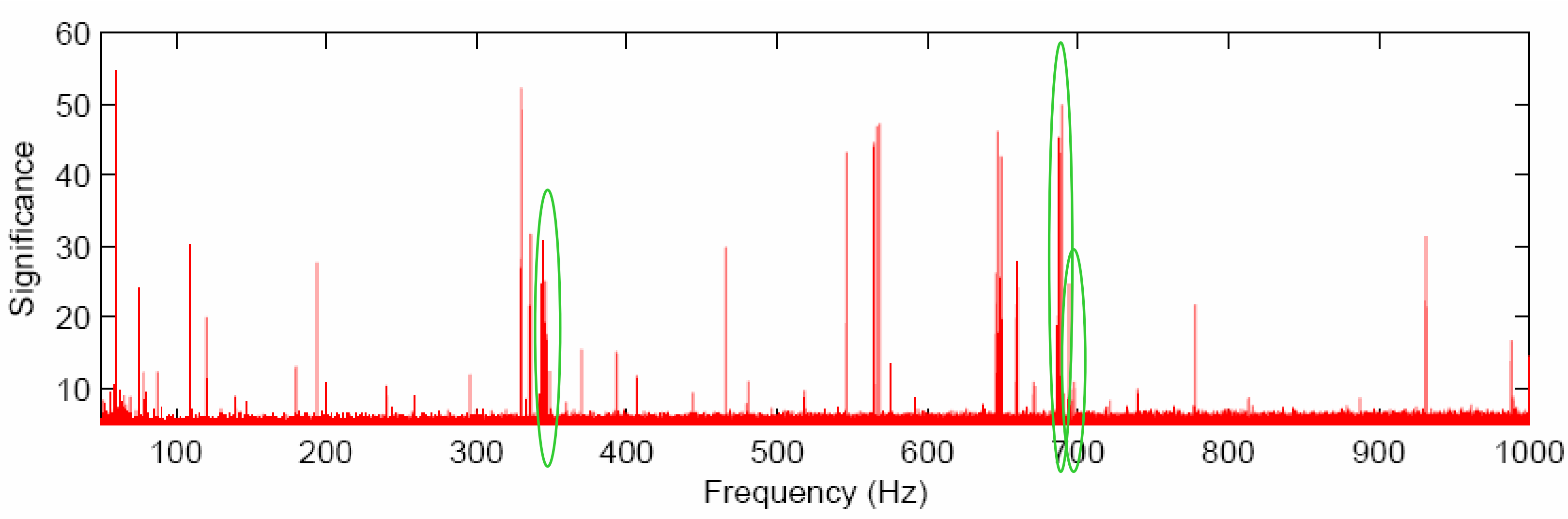


Fig.29 of *"B. Abbott et al., All-sky search for periodic gravitational waves in LIGO S4 data, 2007* (arXiv:0708.3818)"

- Veto ~ 92% of the frequency bins with significance greater than 7

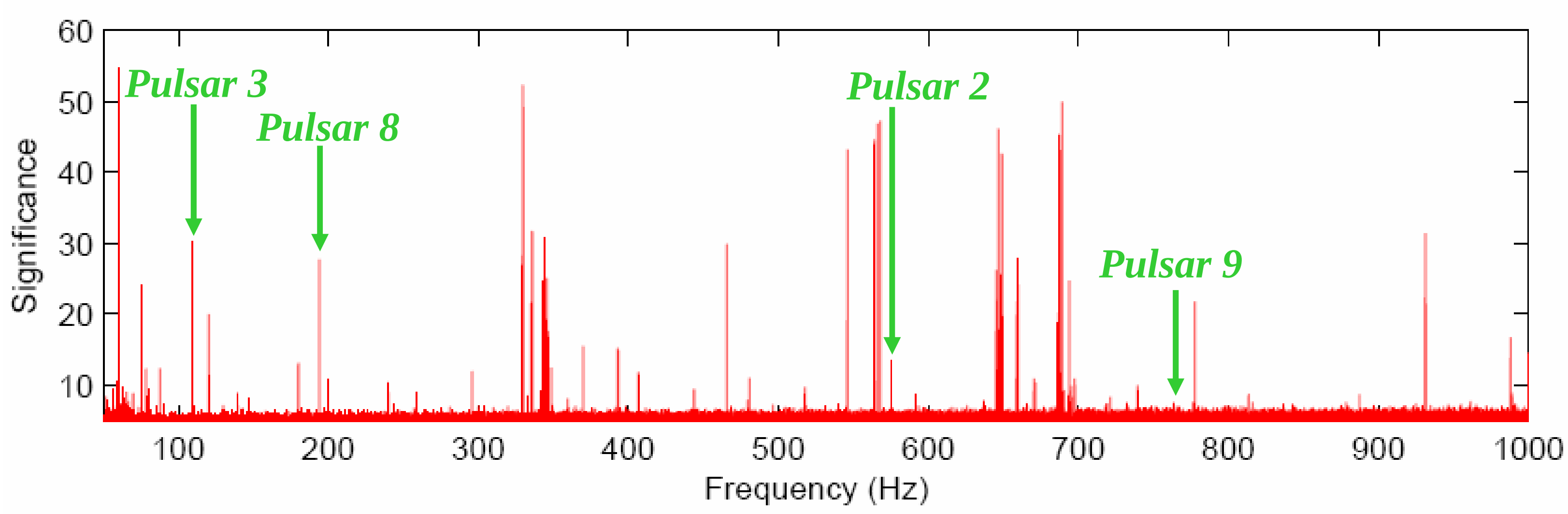
Instrumental Disturbances

- VIOLIN MODES:



Hardware Injected Signals

- PULSARS: (injected 50% of the time → look like disturbances!)



Conclusions and future work

Motivation

Hough Transform

The χ^2 test

Results

- We have developed a χ^2 veto for the Hough Transform in the CW search and we have characterized it in the presence of a signal and in the presence just of noise (Gaussian noise and also instrumental perturbations).

- We have proven the efficiency of this veto using 1month of LIGO data.

- Under development : This χ^2 veto is being implemented for an all-sky search using the LIGO S5 data.