High-f Corrections to Ground-Based Searches for Long-Lived GW Signals

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Abstract

Most gravitational-wave (GW) searches with ground-based interferometers have been conducted in the long-wavelength (LW) approximation, where the variation of GW phase within the interferometer during the measurement is neglected. Here we assess the impact of using the LW approximation on two of the standard ground-based searches: a stochastic GW background and a periodic GW signal emitted e.g., by an pulsar.

Long-wavelength antenna pattern functions

The response of an interferometer to a plane polarized GW in the LW approximation can be written in terms of the antenna pattern functions:

$$F_{+,\times}(\hat{n}) = \frac{1}{2}(\hat{u}\otimes\hat{u} - \hat{v}\otimes\hat{v}) : e_{+,\times}(\hat{n})\,, \qquad (1)$$

where $\hat{n}$ is the direction to the GW source, $\hat{u}$, $\hat{v}$ are unit vectors pointing along the arms of the interferometer, and $e_{+,\times}(\hat{n})$ are the GW polarisation tensors. Figure 1 is a plot of the LW antenna pattern functions for $\hat{u} = \hat{x}$, $\hat{v} = \hat{y}$.

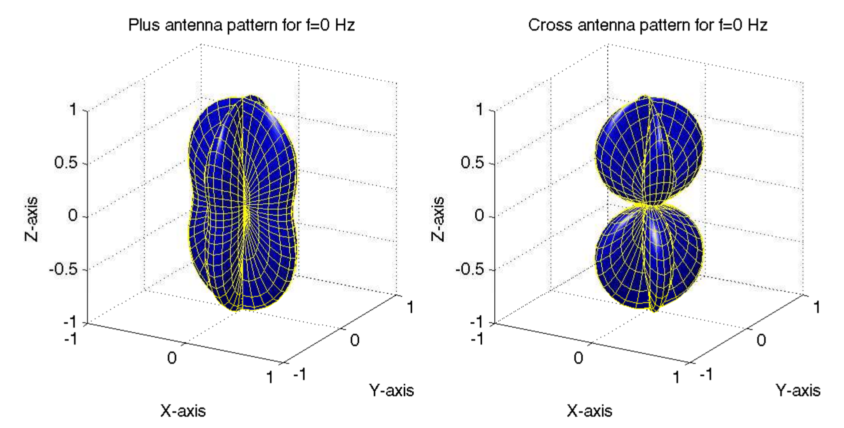

Figure 1: *LW antenna pattern functions $F_{+,\times}(\hat{n})$.*

Exact expressions including FP response

Taking into account the finite size of the detectors leads to frequency-dependent antenna patterns, as shown for example in early calculations for LISA [1] and LIGO [2]. The frequency-dependent antenna patterns are given by

$$H_{+,\times}(f,\hat{n}) = \frac{1}{2}(\mathcal{T}(f,\hat{u}\cdot\hat{n})\hat{u}\otimes\hat{u} - \mathcal{T}(f,\hat{v}\cdot\hat{n})\hat{v}\otimes\hat{v}) : e_{+,\times}(\hat{n})\,,$$

where

$$\mathcal{T}(f,\hat{u}\cdot\hat{n}) = \frac{e^{-i2\pi fT}}{2}\left[e^{i\pi fT_+}\mathrm{sinc}\,(\pi fT_-) + e^{-i\pi fT_-}\mathrm{sinc}\,(\pi fT_+)\right]$$

with $T \equiv L/c$ and $T_\pm \equiv T(1 \pm \hat{u}\cdot\hat{n})$. If we also include the fact that LIGO is a Fabry-Perot (FP) cavity, the antenna pattern functions become

$$G_{+,\times}(f,\hat{n}) = H_{+,\times}(f,\hat{n})H_{\rm FP}(f)\,, \qquad (2)$$

where

$$H_{\rm FP}(f) = e^{i2\pi fT}\frac{\sin(i2\pi f_0 T)}{\sin[i2\pi f_0 T(1+if/f_0)]}\,, \qquad f_0 \approx 86\ \mathrm{Hz}\,.$$

Figure 2 is a plot of the antenna pattern functions at the free-spectral range (FSR), where $L = \lambda/2$ (corresponding to $f = 37.5$ kHz).

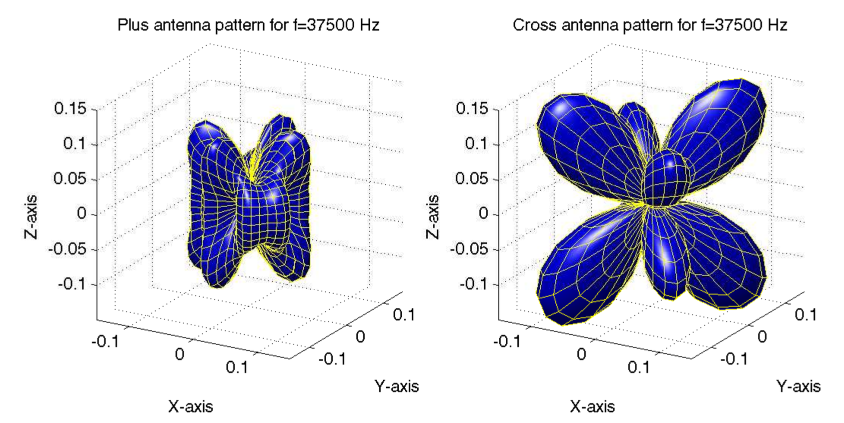

Figure 2: *Antenna pattern functions at FSR: $f = 37.5$ kHz.*

Corrections to long-wavelength approximation

If we consider frequencies where $2\pi fT \ll 1$, then

$$G_{+,\times}(f,\hat{n}) \approx F_{+,\times}(\hat{n})H_{\rm pole}(f)\,, \qquad H_{\rm pole}(f) = \frac{1}{1+if/f_0}\,. \qquad (3)$$

Equation (3) defines the LW approximation to the antenna pattern functions, including the FP response. The corrections to the LW approximation are given by the difference $G_{+,\times} - F_{+,\times}H_{\rm pole}$. The bottom two panels of Fig. 3 show the magnitude of the correction terms at $f = 1024$ Hz, where $2\pi fT = 0.086$ for LIGO. The top two panels show that, relative to Fig. 1, the magnitude of the LW antenna pattern functions are down by a factor of 0.08, due to the value of $H_{\rm pole}$ at $f = 1024$ Hz. Thus, the maximum value of the correction terms are $\sim 1\%$ of the maximum LW values. This is $\sim 10\times$ smaller than the nominal $\sim 10\%$ value, due to the angular dependence of the correction terms. The effect of the frequency corrections on searches for GWs from periodic sources was recently analyzed by Baskaran and Grishchuk [3] in the linear approximation to Eq. (2), based on the electro-magnetic analogy introduced in [4].

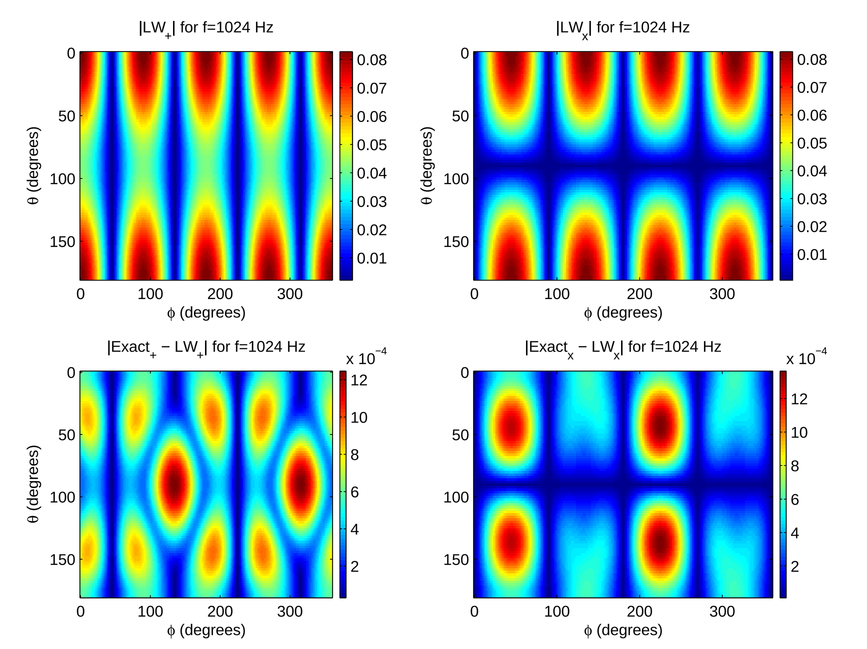

Figure 3: *The LW antenna pattern functions and the magnitude of the correction terms at $f = 1024$ Hz.*

Effect on stochastic searches

The antenna pattern function influences the response of a stochastic background search through the *overlap reduction function*:

$$\gamma_{12}(f) = \frac{5}{8\pi}\iint d^2\Omega_{\hat{n}} \sum_{A=+,\times} G_1^{A*}(f,\hat{n})\,G_2^{A}(f,\hat{n})\,e^{-\frac{i2\pi f\hat{n}\cdot(\vec{r}_1-\vec{r}_2)}{c}}\,.$$

A typical overlap reduction function is shown in Fig. 4

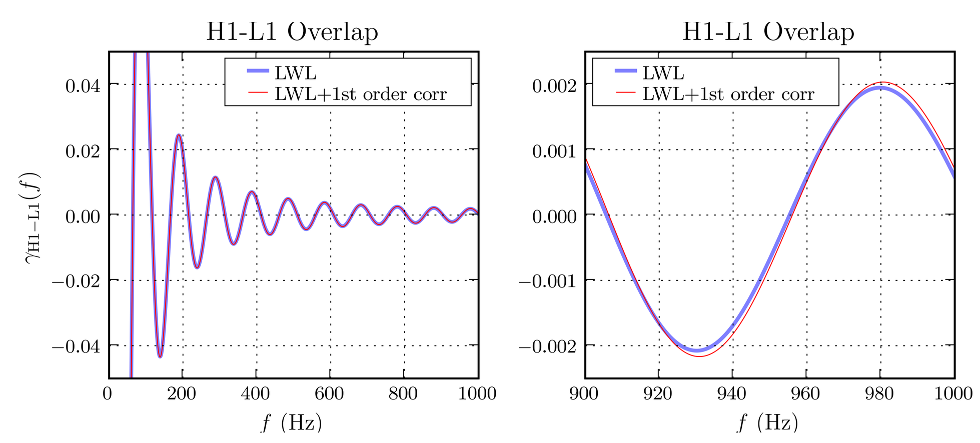

Figure 4: *LHO-LLO overlap reduction function. The standard long-wavelength limit (LWL) form used in most ground-based analyses is shown, along with a version including 1st-order corrections.*

Using the explicit form of $G_{+,\times}$ given in Eq. (2), it is possible to derive analytic expressions not only for the LW limit, but also for the 1st-order correction [5]. The one stochastic search so far influenced by kHz frequencies was the LLO-ALLEGRO search [6]. Table 1 illustrates the corrections relevant at $f \approx 915$ Hz due to the finite length of the LLO arms.

	$\gamma^{\rm LWL}(f)$	$\delta\gamma(f)$	$\delta\gamma(f)/\gamma^{\rm LWL}(f)$
XARM	0.95333	0.00298	0.00313
YARM	-0.89466	-0.00167	0.00187
NULL	0.03181	-0.00061	-0.01914

Table 1: *Impact of 1st-order corrections on LLO-ALLEGRO search. The corrections are less than 1%, except for the null orientation. The upper limits in [6] are not affected to the stated precision by these corrections.*

Previous LLO-LHO correlation work has concentrated on frequencies $f \lesssim 300$ Hz, but inclusion of Virgo in S5 has added interest in frequencies around 1 kHz. A typical measure of the impact of 1st-order corrections is the fractional change in the stastical one-sigma error bar σ:

$$\frac{\delta\sigma}{\sigma^{\rm LWL}} = -\left(\int_{f_{\rm min}}^{f_{\rm max}} df\,\frac{\delta\gamma(f)\,\gamma^{\rm LWL}(f)}{P_1(f)\,P_2(f)}\right)\Bigg/\left(\int_{f_{\rm min}}^{f_{\rm max}} df\,\frac{[\gamma^{\rm LWL}(f)]^2}{P_1(f)\,P_2(f)}\right)$$

This contribution is less than 1% at frequencies considered in previous searches, and for LLO-Virgo and LHO-Virgo pairs at about 1 kHz, but for LHO-LLO near 1 kHz, it turns out to become significant, as shown in Table. 2.

	H1-L1	H2-L1	H1-V1
$50-150$ Hz	-1.9×10^{-3}	1.9×10^{-4}	2.5×10^{-4}
$900-1000$ Hz	-4.2×10^{-2}	8.9×10^{-4}	1.1×10^{-3}

Table 2: *Impact of 1st-order corrections on error bars for pairs of interferometers. The numbers in the table are $\delta\sigma/\sigma^{\rm LWL}$, calculated assuming a white stochastic backgrounds across the band shown, using the nominal design sensitivities of the instruments. The 4% correction for L1-H1 means that kHz searches of LLO-LHO pairs should include 1st-order corrections and not just the LWL overlap reduction function.*

Effect on searches for periodic GWs

The heterodyned GW signal for an isolated pulsar at a fixed position in the sky is given by [7]:

$$y(t) = \frac{1}{4}G_+(t;f,\hat{n})h_0(1+\cos^2\iota)e^{i\phi_0} - \frac{i}{2}G_\times(t;f,\hat{n})h_0\cos\iota\,e^{i\phi_0}$$

where h_0 is the amplitude, ι the inclination angle, and ϕ_0 the initial phase of the incident GW. The (slowly-varying) time-dependence in the antenna pattern functions comes from the Earth's sidereal rotational motion. To assess the effect of using the LW approximation on a directed pulsar search, we calculate the *match*:

$$\mathrm{match} = \frac{\frac{1}{2}\int dt\,(y_1 y_2^* + y_1^* y_2)}{\sqrt{\int dt\,|y_1|^2}\sqrt{\int dt\,|y_2|^2}}$$

between the signals $y_1(t)$, $y_2(t)$ defined by Eqs. (2) and (3), respectively. The time integration is over 1 sidereal day. $(1-\mathrm{match})$ is the reduction in SNR that results from using the LW antenna pattern functions instead of the exact ones. Figure 5 shows the match versus declination for fixed GW frequency (2048 Hz) for the simple case where ψ, ι, and ϕ_0 are assumed to be known with values $\psi = 0$, $\iota = 0$, and $\phi_0 = 0$. This corresponds to a *circularly-polarised* GW. The detector arms $\hat{u}$, $\hat{v}$ were taken to be that of LHO. Note that maximum reduction in SNR is much less than 1%, so a negligible effect for LIGO.

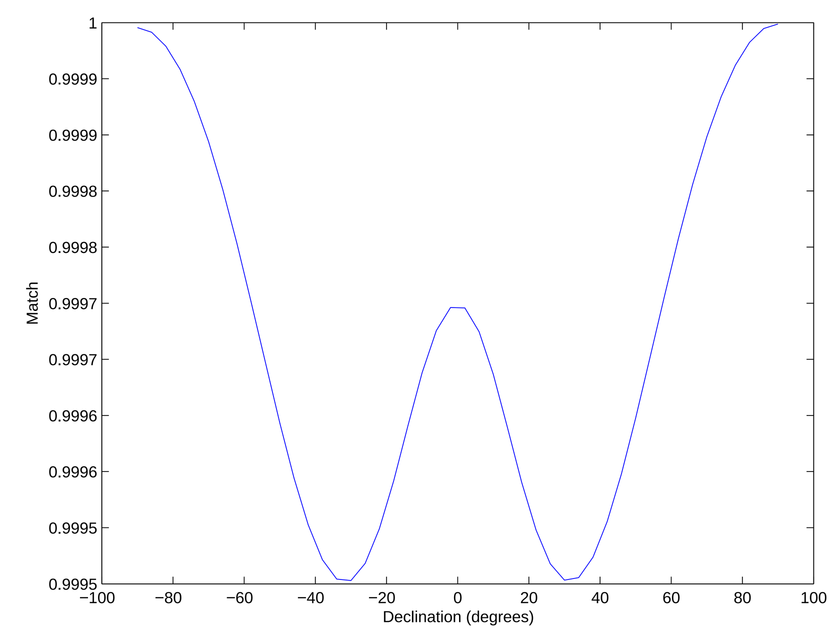

Figure 5: *Fraction of maximum available SNR (i.e., match) as a function of source declination for a fixed GW frequency $f = 2048$ Hz.*

Searches for GWs at the FSR

The presence of FP cavities in the interferometer arms gives us another frequency band to search for GWs. For the 4-km LIGO interferometers this is approximately a 200-Hz peak centered at the FSR (37.5 kHz). Enhanced sensitivity of the detectors (only a factor of 5-8 less than at DC) motivated installation of high-sampling rate (262-kHz) digitizers at both LIGO sites to produce Fast AS-Q data for searches of GW signals at the FSR. Efforts to analyze the data from the Fast AS-Q channel during S4 and S5 are underway at the University of Rochester (stochastic searches) [8] and LHO (bursts) [9, 10].

Summary

For frequencies of order a couple of kHz or below, the effect of corrections to the LW approximation are negligible for searches performed so far, being $\sim 1\%$ or smaller. However, 1st-order corrections should be included in LLO-LHO analyses around $f \sim 1$ kHz, and exact expressions such as (2) should be used at much higher frequencies, like those at the FSR.

References

[1] R. Schilling, *CQG* 14, 1513 (1997)
[2] D. Sigg, LIGO-T970101-B
[3] D. Baskaran and L.P. Grishchuk, *CQG* 21, 4041, (2004)
[4] L.P. Grishchuk, Soviet Physics, Uspekhi 20, 319 (1977)
[5] J.T. Whelan, LIGO-T070172-00-Z
[6] LSC, *PRD* **76**, 022001 (2007)
[7] R.J. Dupuis and G. Woan, *PRD*, **72**, 102002, (2005)
[8] C. Forrest et al., LIGO-T070228-00-Z
[9] R. Savage et al., LIGO-G060667
[10] J. Parker, LIGO-T070037-00-W