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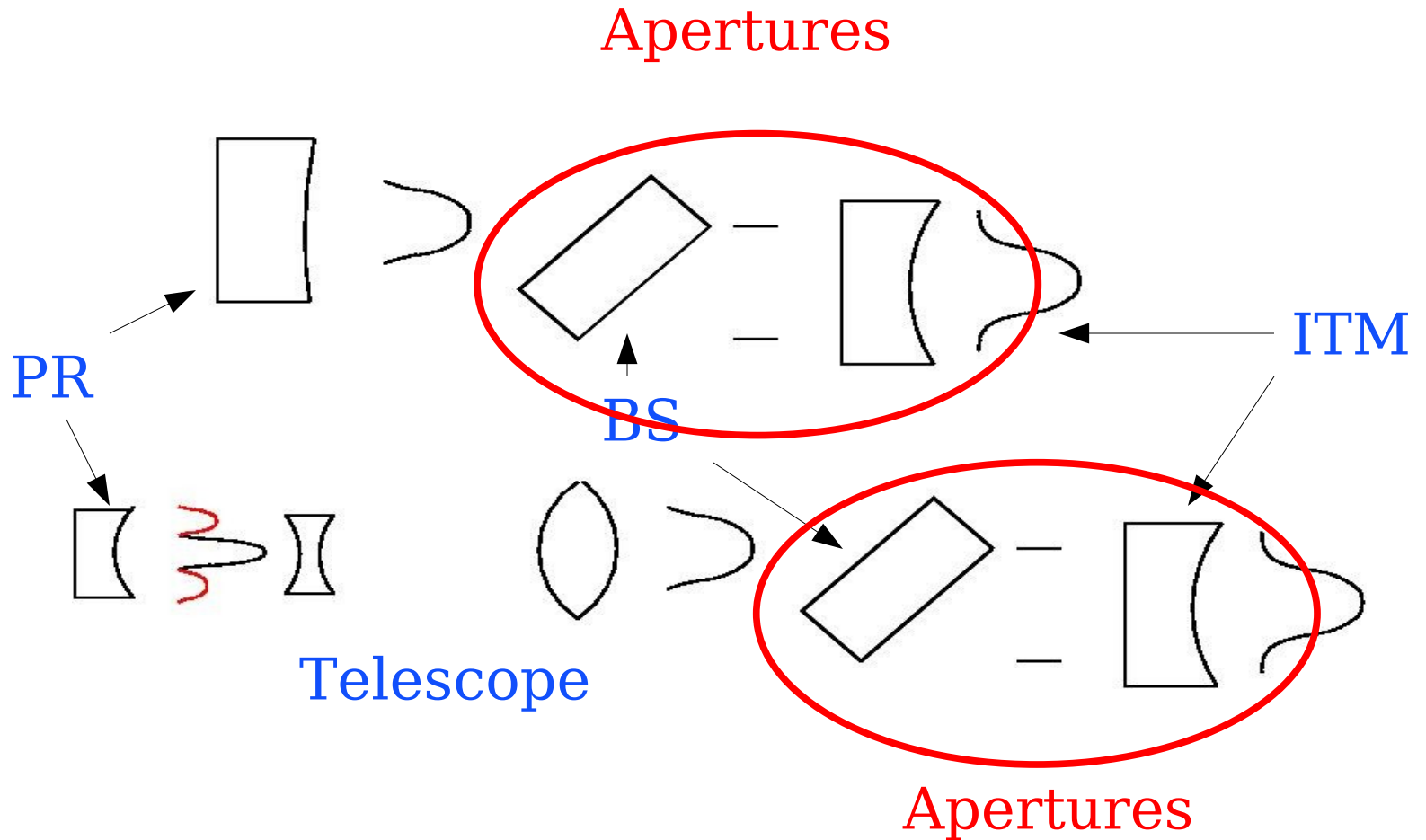
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# **Stable recycling cavities for Advanced LIGO**

**Guido Mueller**



- **Diffraction (clipping)**
- **Parametric Instabilities**
- **ASC**





## Explanation:

### ▪ Telescope-type discussion:

- » **Flat top beams create a ring pattern in diffraction limited telescopes**
  - Point spread function
- » **Apertured Gaussian will also create ring pattern**

### ▪ Modal picture (just to get an idea)

- » **Aperture might be described by additional very high order modes**
  - one with maxima at the edges to destructively interfere with 00-mode
  - several others to suppress the additional fringes in the non-apertured area
- » **HOMs will be out of phase with carrier after one roundtrip**  
--> **interferes now constructively with 00 mode and create real intensity at edges which will be cut off by aperture**



**Stable Recycling Cavity increases diffraction losses**

**Hiro's FFT results (work in progress):**

**Signal sideband**

- **6cm: 230ppm x recycling gain > 1%**
- **5.5cm: 65ppm x recycling gain < 1%**

**+ Carrier scatter in PR-cavity of same order**

**Assumes mode matched recycling cavities**



Mode mismatch between Recycling cavities and arm cavities:

- Modal model:

$$\text{LG}_{00}^{\text{In}} = \text{LG}_{00}^{\text{Cav}} + (\Delta w/w + i\Delta z/z_R) \text{LG}_{10}^{\text{Cav}}$$

$\Delta w, \Delta z$ : measure for mismatch

will be reflected at the cavity

$$\text{LG}_{00}^{\text{Out}} = \text{LG}_{00}^{\text{Cav}} - (\Delta w/w + i\Delta z/z_R) \text{LG}_{10}^{\text{Cav}}$$

Pending on sign of  $\Delta w$  and  $\Delta z$  either

**the In or the Out mode will be larger than the cavity mode**

**--> More diffraction losses**

Details require FFT code (Hiro) but

**stable recycling cavities will require careful mode matching**



- **Diffraction (clipping)**
- **Parametric Instabilities**
- **ASC**



## Parametric Instabilities

- Low order modes driven by the

- » 00-mode inside arm cavities

- » mechanical resonances of the mirrors substrates

Could build-up as PIs inside the arm cavities if

- » the optical losses are smaller than the opto-mechanical gain

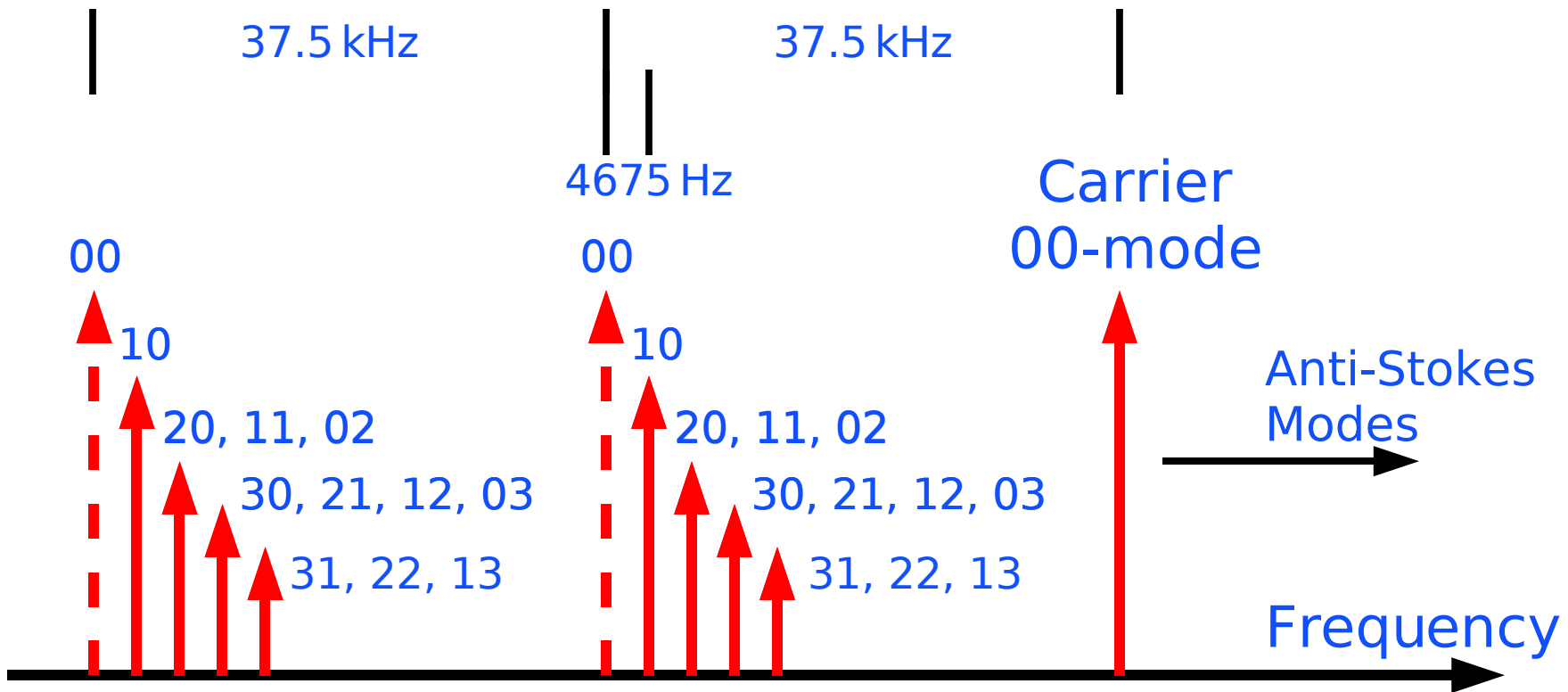
→ Don't want to do signal recycling on the PI modes

Ideal: Resonant sideband extraction for the PI mode





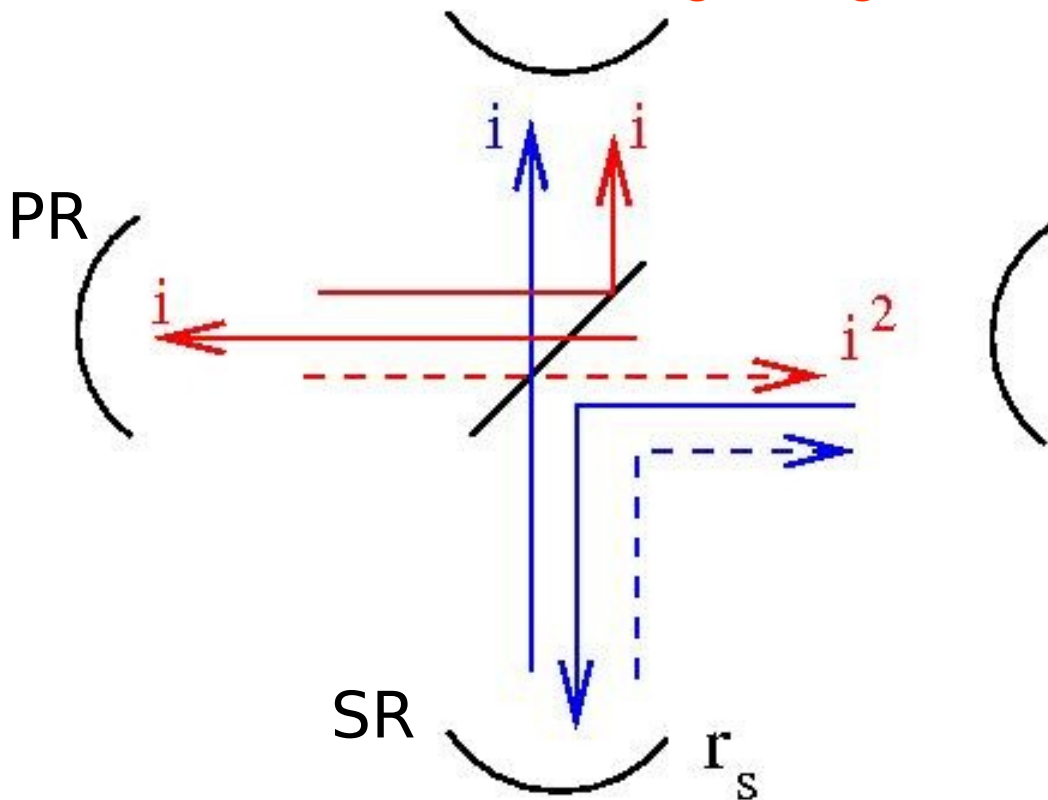
## PI-Spectrum:



My understanding: PI limited to Hermite-Gauss modes up to  $n$  and  $m < 5$   
 Higher order modes have Diffraction losses  $>$  ITM transmission



- Start discussion with following configuration:
  - » symmetric BS:  $t_{BS} = r_{BS}$ , no Gouy phases in recycling arms
  - » PR mirror is power recycling the carrier
  - » SR mirror is extracting the signal mode (RSE configuration)

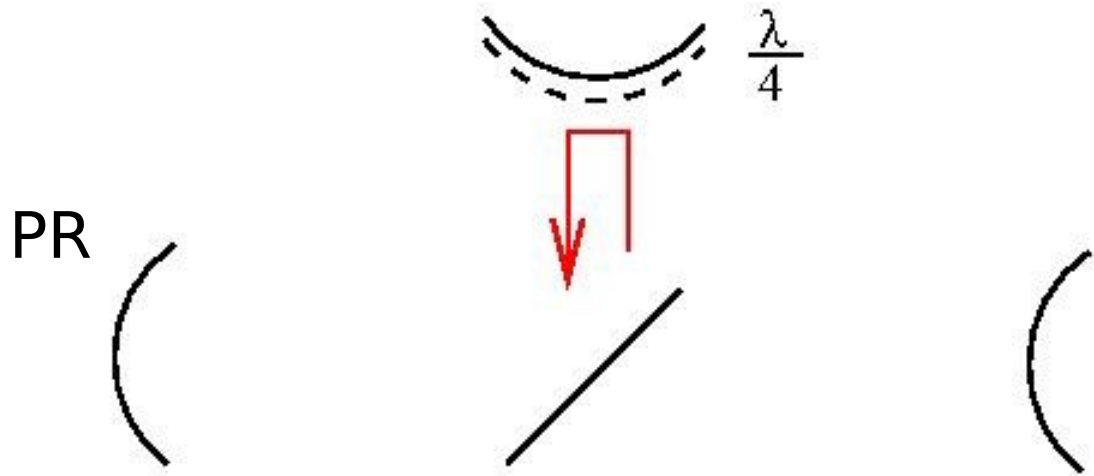


Recycling arm  
 Michelson IFO  
 would be bright!

☞ PI-mode travels  
 mainly to 2<sup>nd</sup> Cavity



2<sup>nd</sup> Cavity moved by  $\lambda/4$  compared to  
1<sup>st</sup> cavity to have MI dark (compensates the  $(it)^2$  in BS)



- 2<sup>nd</sup> Cavity off-resonant
  - ☞ Phase shift:  $\pi$
- 2<sup>nd</sup> Cavity on-resonance
  - ☞ Phase shift:  $2\pi$

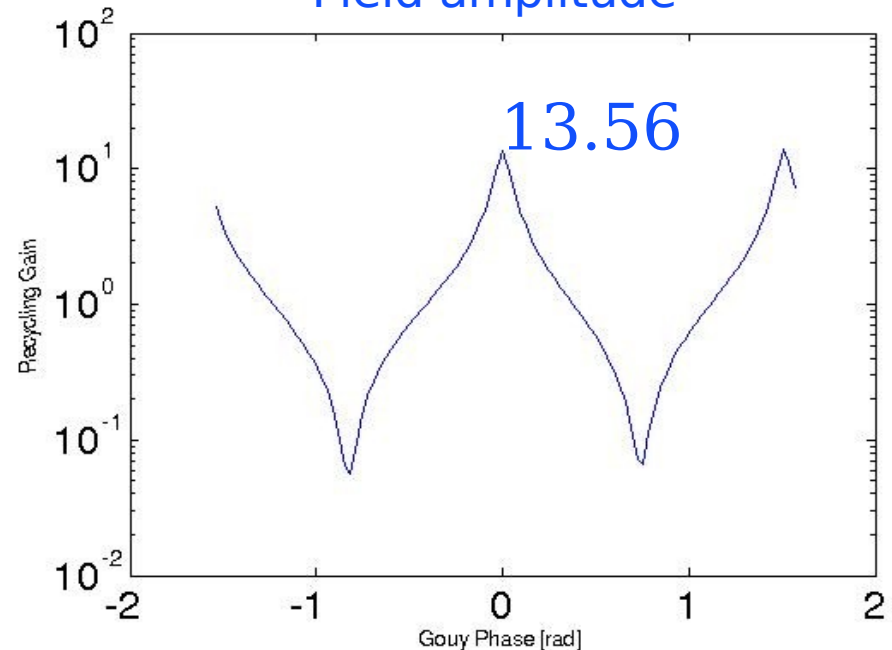


## 1<sup>st</sup> Case:

- 2<sup>nd</sup> Cavity resonant with PI mode (unlikely):  
Light reflects back into short recycling cavity MI  
Picks up another 90deg phase shift

- 👉 Round trip phase shift is 180deg 👉 signal recycling (very bad)
- » Add now identical Gouy phases to recycling cavities
  - recycling cavity MI stays bright
  - move from signal recycling to signal extraction

relative recycling gain:  
Field amplitude



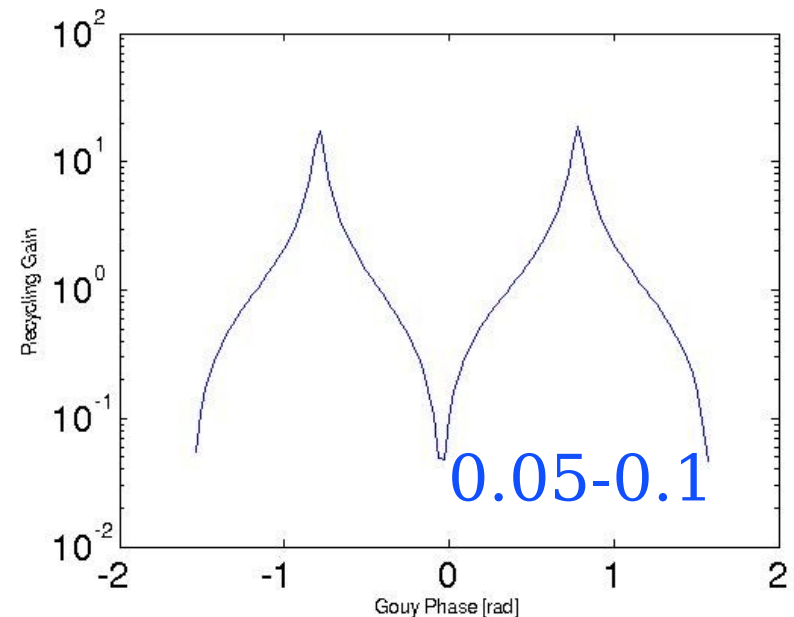


## 2<sup>nd</sup> Case:

- 2<sup>nd</sup> Cavity non-resonant with PI mode (likely):
  - Picks up 180deg phase shift at 2<sup>nd</sup> Cavity
  - Light reflects back into short recycling cavity MI
  - Picks up another 90deg phase shift

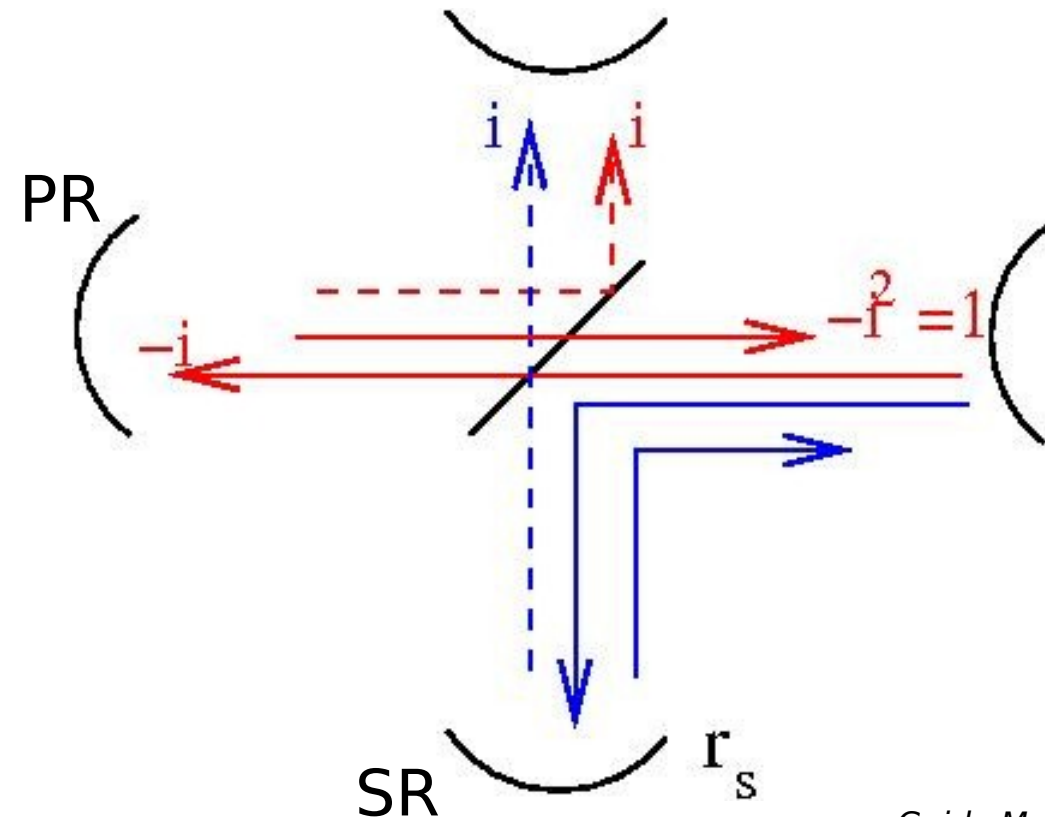
- ☞ Round trip phase shift is 360deg ☞ signal extraction (very good)
- » Add now identical Gouy phases to recycling cavities
  - recycling cavity MI stays bright
  - move from signal extraction to signal recycling

relative recycling gain:  
Field amplitude





- Optimized for alignment sensing (detect the 10-mode):
  - » PR recycling cavity would have  $\Psi_G \sim \pi/2$  (to extract the 10-mode)
  - » SR mirror is extracting signal mode (RSE configuration)



Recycling arm  
Michelson IFO  
is dark for odd modes!

☞ 2<sup>nd</sup> Cavity doesn't  
matter anymore

Note:  
Nothing has changed  
for even modes!

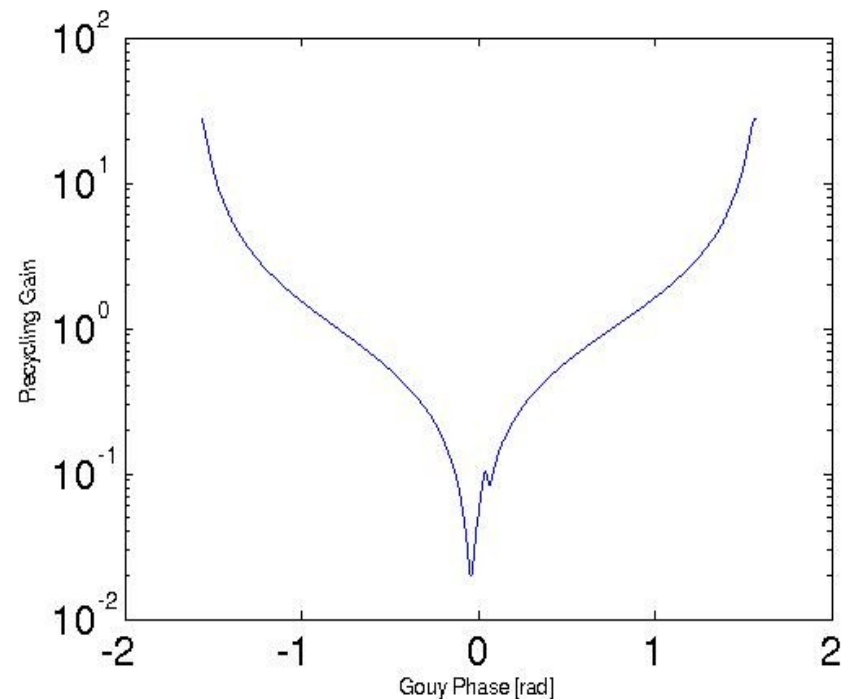


Only new case: (Odd modes only, max gain higher because they see PR/SR only once per roundtrip)

- ☞ **Round trip phase shift is 360deg** ☞ **signal extraction (very good)**
- » **Add now identical Gouy phases to recycling cavities**
  - recycling cavity MI stays dark
  - move from signal extraction to signal recycling

(Structure in center caused by SR-detuning)

relative recycling gain:  
Field amplitude

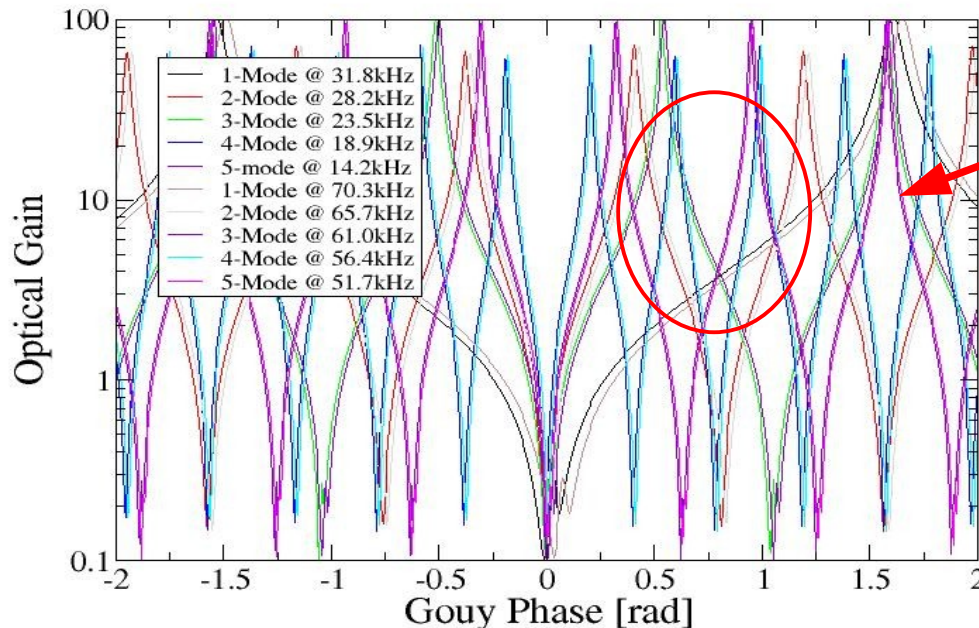




Optical gain (compared to no recycling) as a function of  $\Psi_g$  (Power recycling:  $\pi/2 + \Psi_g$ )

Assumes arm cavities are NOT identical and PI mode from one cavity is not entering second cavity.

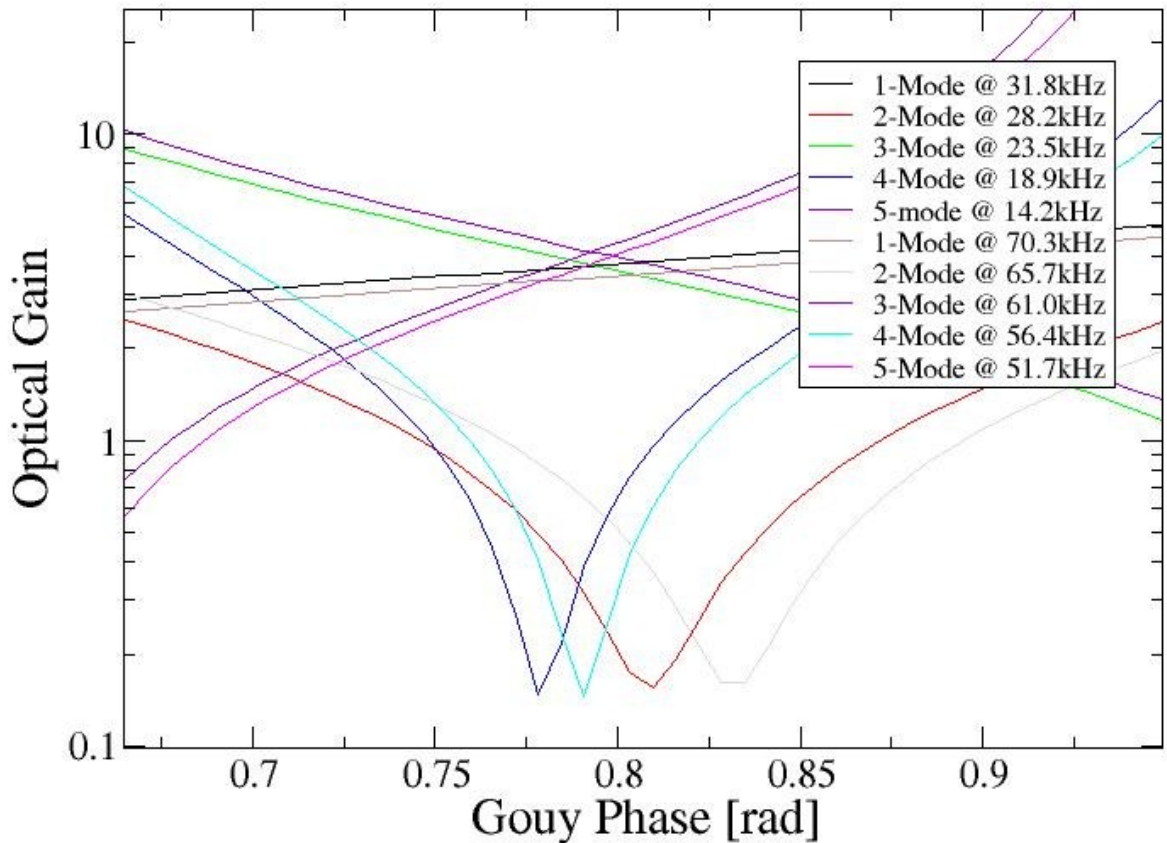
Parametric gain for modes with mode number  $m+n < 6$



Window of opportunity:  
 $\sim \pi/4$

Warning:  
All these peaks  
will move for high  
SR detunings!





Window of opportunity:  
 $\sim \pi/4$

still good at 0.7  
 main problem are the 30, 21, 12, 03 modes.



- Like to have the Gouy phases in PR and SR cavities equal or different by  $\pi/2$ 
  - » **Allows to track the spatial modes better**
- **Prefer non-identical arm cavities**
  - » **Transversal mode spacing changes by 30Hz/m difference in ITM ROC**
    - 10m difference should have no significant impact on mode matching
  - » **Having second cavity on or near resonance adds a large and very sensitive phase shift to HOM -> changes resonance condition fast**
    - or could this be used to tweak optical gain of PI?
- **All this needs to be confirmed for**
  - » **arm cavities with 5.5cm beam size on ITM**
  - » **new reflectivities ( $T_{ITM}=1\%$ ,  $T_{PR}=3.6\%$ ,  $T_{SR}=11.8\%$ )**



- **Diffraction (clipping)**
- **Parametric Instabilities**
- **ASC**



- **Based on new Length Sensing Scheme (T06xxxx)**
  - »  $T_{ITM} = 1\%$       $T_{PR} = 3.6\%$       $T_{SR} = 11.8\%$
  - » **Schnupp asymmetry: 5cm**
  - »  $L_{PR} = 55.815\text{m}$       $L_{SR} = 57.410\text{m}$
  - »  $f_1 = 9.4\text{MHz}$       $f_2 = 5f_1 = 47\text{MHz}$
  
- **3 Modes of operation**
  - » **Low Power Case:  $P_{in} = 4\text{W}$ ,  $\Delta L_{SR} : 177\text{nm}$  (Detuning)**
    - $\Gamma_1 = 0.14$       $\Gamma_2 = 0.14$      (0.2 inside MZ)
  - » **RSE-Case:  $P_{in} = 125\text{W}$ ,  $\Delta L_{SR} : 0\text{nm}$  (Detuning)**
    - $\Gamma_1 = 0.14$       $\Gamma_2 = 0.14$
  - » **NS/NS-Case:  $P_{in} = 125\text{W}$ ,  $\Delta L_{SR} : 29.5\text{nm}$  (Detuning)**
    - $\Gamma_1 = 0.14$       $\Gamma_2 = 0.14$



## Displacement:

### ■ BH/BH total

- » TM:  $8 \times 10^{-19}$  m/rtHz @ 10Hz,  $3 \times 10^{-20}$  m/rtHz @ 30Hz,  $1.4 \times 10^{-20}$  m/rtHz @ 50Hz
- » RM:  $3 \times 10^{-16}$  m/rtHz @ 10Hz

### ■ RSE

- » TM:  $8 \times 10^{-19}$  m/rtHz @ 10Hz,  $4 \times 10^{-20}$  m/rtHz @ 30Hz,  $2.3 \times 10^{-20}$  m/rtHz @ 50Hz
- » RM:  $3 \times 10^{-16}$  m/rtHz @ 10Hz

### ■ NS/NS

- » TM:  $8 \times 10^{-19}$  m/rtHz @ 10Hz,  $4 \times 10^{-20}$  m/rtHz @ 30Hz,  $2 \times 10^{-20}$  m/rtHz @ 50Hz
- » RM:  $3 \times 10^{-16}$  m/rtHz @ 10Hz



- Angular stability requirements scales with centering with respect to angular actuator axes of rotation
  - assume  $\Delta l = 100\mu\text{m}$
- BH/BH total
  - » TM:  $8 \times 10^{-15} \text{ rad/rtHz}$  @ 10Hz,  $3 \times 10^{-16} \text{ rad/rtHz}$  @ 30Hz,  $1.4 \times 10^{-16} \text{ rad/rtHz}$  @ 50Hz
  - » RM:  $3 \times 10^{-12} \text{ rad/rtHz}$  @ 10Hz (independent of beam size!)
- RSE
  - » TM:  $8 \times 10^{-15} \text{ rad/rtHz}$  @ 10Hz,  $4 \times 10^{-16} \text{ rad/rtHz}$  @ 30Hz,  $2.3 \times 10^{-16} \text{ rad/rtHz}$  @ 50Hz
  - » RM:  $3 \times 10^{-12} \text{ rad/rtHz}$  @ 10Hz (independent of beam size!)
- NS/NS
  - » TM:  $8 \times 10^{-15} \text{ rad/rtHz}$  @ 10Hz,  $4 \times 10^{-16} \text{ rad/rtHz}$  @ 30Hz,  $2 \times 10^{-16} \text{ rad/rtHz}$  @ 50Hz
  - » RM:  $3 \times 10^{-12} \text{ rad/rtHz}$  @ 10Hz (independent of beam size!)

No safety margin! Independent of beam size!



- RMS-Stability (T070999-00-I, Rana, Peter):
  - » Test mass angles:  $10^{-9}$ rad rms
  - » BS and RC-mirrors:  $10^{-8}$ rad rms
    - Driven by Beam Jitter coupling
      - scales with  $1/\text{beamsize}$  = as do ASC signals
      - ➔ Invariant to beam size
    - What other effects (especially some which don't scale)?



- Sensor has to meet the following requirements
  - » RMS:
    - Has to provide an error signal for all mirrors which is clearly below the rms requirement. Need to check:
      - Shot noise limit of WFS
      - Saturation
  - » In Band (only test masses, ASC BW on RC < Adv.LIGO band):
    - Has to meet requirements up to  $\sim 3 \times \text{UGF}$  of the ASC servo loops
      - at  $3 \times \text{UGF}$  we can roll the gain down faster than  $1/f^2$  ( $\sim$ LIGO sensitivity slope)
  - » UGF has to be above Sidles-Sigg instability
    - Low Power (BH/BH) Case:  $P_{\text{in}} = 4\text{W} \rightarrow \text{UGF} \sim 3\text{Hz}$  (above pendulum freq.)
      - Sensor needs to meet requirements up to  $\sim 10\text{Hz}$  (?)
    - RSE and NS/NS case:  $P_{\text{in}} = 125\text{W} \rightarrow \text{UGF} \sim 10\text{Hz}$ 
      - Sensor needs to meet requirements up to  $\sim 30\text{Hz}$





- Sensor has to meet the following requirements
  - » RMS:
    - Has to provide an error signal for all mirrors which is clearly below the rms requirement. Need to check:
      - Shot noise limit of WFS
      - Saturation

All this assumes that the suspension system keeps the mirrors quiet enough in the Adv. LIGO Band!

pendulum freq.)

- Sensor needs to meet requirements up to  $\sim 10\text{Hz}$  (?)
- RSE and NS/NS case:  $P_{\text{in}} = 125\text{W} \rightarrow \text{UGF} \sim 10\text{Hz}$ 
  - Sensor needs to meet requirements up to  $\sim 30\text{Hz}$



- **Based on Valera's code**

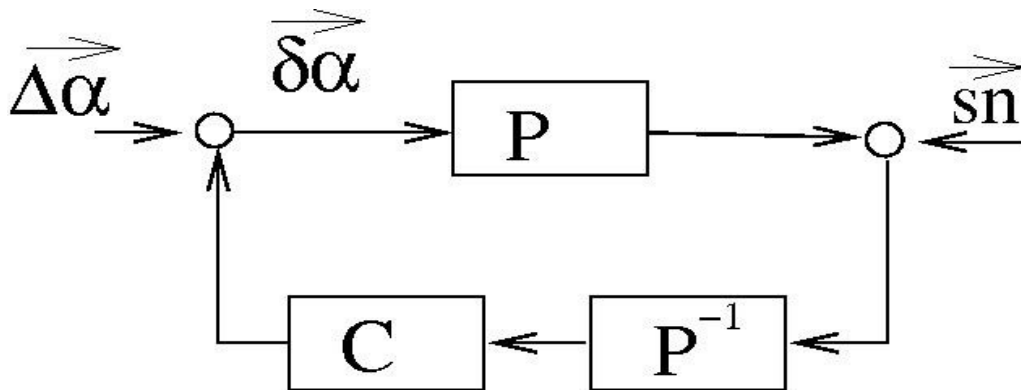
- » **all signals still scaled with 6cm beam size**

- results for stable recycling cavity mirrors need to be multiplied by  $\sim 30$  (beam size ratio on mirrors)

- » **Shot noise not correct**

- Still some factor 2 type uncertainty

- have the theory but didn't implement it yet



$$\vec{\delta\alpha} = \frac{\vec{\Delta\alpha} + CP^{-1}\vec{sn}}{1+C}$$



$\Delta\alpha$  : Fluctuations w/o feedback

In-band:

- Should be lower than requirements

Below band:

- Sidles-Sigg ( $\sim 4\text{Hz}$  at 125W, Test masses only)
- Suspension Eigenfrequencies (pendulum frequency)

Loop Gain (Test masses)

- About 2-3 at Sidles Sigg instability
- Roll-off with  $1/f$  to  $3\times\text{UGF}$ 
  - UGF @ 125W:  $\sim 10\text{Hz}$ 
    - $1/f$  slope until  $\sim 30\text{Hz}$  (Most difficult part)
- Above that with  $1/f^{2+}$ , faster than Adv. LIGO sensitivity

Loop Gain (RM, TM at low power (?))

- Pendulum frequencies ( $\sim 1\text{Hz}$ )
  - UGF: 3Hz, slope of  $1/f$  until 10Hz, then  $1/f^{2+}$



- **Low Power Case:  $P_{in} = 4W$** 
  - » **DC-Power (TEM00 in mW, divide by 10 for pick off):**

Port	CR	SBU1	SBL1	SBU2	SBL2	DC
Dark	10.4	0.1	0.0	0.2	0.2	10.9
Refl	1.6	19.6	19.7	19.5	19.6	80.0

- » **RF-Power (TEM00 in mW, divide by 10 for pick off):**

Port	f1	f2	2xf1	2xf2	f2-f1	f2+f1
Dark	0.9	2.6	0.1	0.4	0.3	0.2
Refl	11.0	8.1	39.3	39.1	64.9	78.1



- **Low Power Case:  $P_{in} = 4W$  (Current status)**

- **Loop Gain**

- » **3Hz = 1 (UGF)**
- » **10Hz = 0.3**
- » **30Hz = 0.03**

Requirements (frad/rtHz):

- 10Hz: 8 (TM)
- 30Hz: 0.3 (TM)
- 10Hz: 3000 (RM)

frad/rtHz	Port	Sensor	3Hz	10Hz	30Hz
PR	BP f1	7.8	3	1.8	0.23
<b>SR</b>	<b>DP f2</b>	<b>6110</b>	<b>3055</b>	<b>1410</b>	<b>178</b>
DITM	DP f2	2.7	1.3	0.6	0.08
CITM	BP f2	0.1	0.05	0.02	0.003
DETM	DP f2	7.7	3.8	1.8	0.22
CETM	BP f1	0.8	0.39	0.18	0.02

Note that PR and SR angles x30 for stable rec.



- **RSE Case:  $P_{in} = 125W$** 
  - » **DC-Power (TEM00 in mW, divide by 10 for pick off):**

Port	CR	SBU1	SBL1	SBU2	SBL2	DC
Dark	82	0.2	0.2	543.9	543.9	1170.5
Refl	12.5	615.7	615.7	76.9	76.9	1397.8

- » **RF-Power (TEM00 in mW, divide by 10 for pick off):**

Port	f1	f2	2xf1	2xf2	f2-f1	f2+f1
Dark	0.8	416.8	0.5	1088	3.6	44.3
Refl	173	61.9	1231	154	846	871



- **RSE Case:  $P_{in} = 125W$  (Current status)**
  - **Loop Gain (RM)**
    - » **3Hz = 3 (1)**
    - » **10Hz = 1 (0.3)**
    - » **30Hz = 0.3 (0.03)**
- Requirements (frad/rtHz):
- 10Hz: 8 (TM)
  - 30Hz: 0.4 (TM)
  - 10Hz: 3000 (RM)

frad/rtHz	Port	Sensor	3Hz	10Hz	30Hz
PR	BP f1	2.1	1.1	0.5	0.06
SR	DP f2	71	35	16	2.1
DITM	DP f2	1	0.8	0.5	0.24
CITM	BP f1	0.8	0.6	0.4	0.18
DETM	DP f2	1.4	1	0.7	0.31
CETM	BP f1	0.5	0.4	0.3	0.12

Note that PR and SR angles x30 for stable rec.



- **NS/NS Case:  $P_{in} = 125W$**

- » **DC-Power (TEM00 in mW, divide by 10 for pick off):**

Port	CR	SBu1	SBl1	SBu2	SBl2	DC
Dark	85	0.3	0.2	140	99.7	325
Refl	12.9	615	615	477	51.9	2240

- » **RF-Power (TEM00 in mW, divide by 10 for pick off):**

Port	f1	f2	2xf1	2xf2	f2-f1	f2+f1
Dark	0.9	190	0.5	236	5.9	9.6
Refl	176	1222	1231	995	1893	1818





- **NS/NS Case:  $P_{in} = 125W$  (Current status)**
  - **Loop Gain (RM)**
    - » **3Hz = 3 (1)**
    - » **10Hz = 1 (0.3)**
    - » **30Hz = 0.3 (0.03)**
- Requirements (frad/rtHz):
- 10Hz: 8 (TM)
  - 30Hz: 0.4 (TM)
  - 10Hz: 3000 (RM)

frad/rtHz	Port	Sensor	3Hz	10Hz	30Hz
PR	BP f1	0.8	0.4	0.2	0.03
SR	DP f2	105	52	24	3
DITM	DP f2	0.3	0.26	0.17	0.08
CITM	BP f1	0.9	0.7	0.47	0.22
DETM	DP f2	0.1	0.09	0.06	0.03
CETM	BP f1	1.6	1.2	0.8	0.37

Note that PR and SR angles x30 for stable rec.



- **Further necessary improvements:**
  - » **Need to check if expected fluctuations  $\Delta\alpha$  of recycling mirrors is really low enough**
  - » **Try to improve WFS using:**
    - **Double demod signals**
    - **Pick off signals from recycling cavities**
  - » **Fix shot noise**
  - » **Check for different Gouy phases**
  - » **Try to understand why the signals are what they are**
  - » **...**



PR:  $\Psi_g = \pi/2 + 0.7$  SR:  $\Psi_g = 0.7$

- **ASC**

- » **Nearly there. Fairly optimistic**

- **Beam Jitter**

- » **Coupling increases by  $\sim 1.5$  compared to marginally stable cavity**

- **Signal loss**

- » **No low order HOM on resonance.**

- **Parametric Instabilities**

- » **Additional optical gain for all except the 3-modes at or below 3.**

- 3-Modes have gain of  $\sim 8$  (compared to gain in non-recycled cavities).

- » **Prefer non-identical cavities (ROC mismatch of order 5m)**

- » **Need to be checked for new transmissivities**