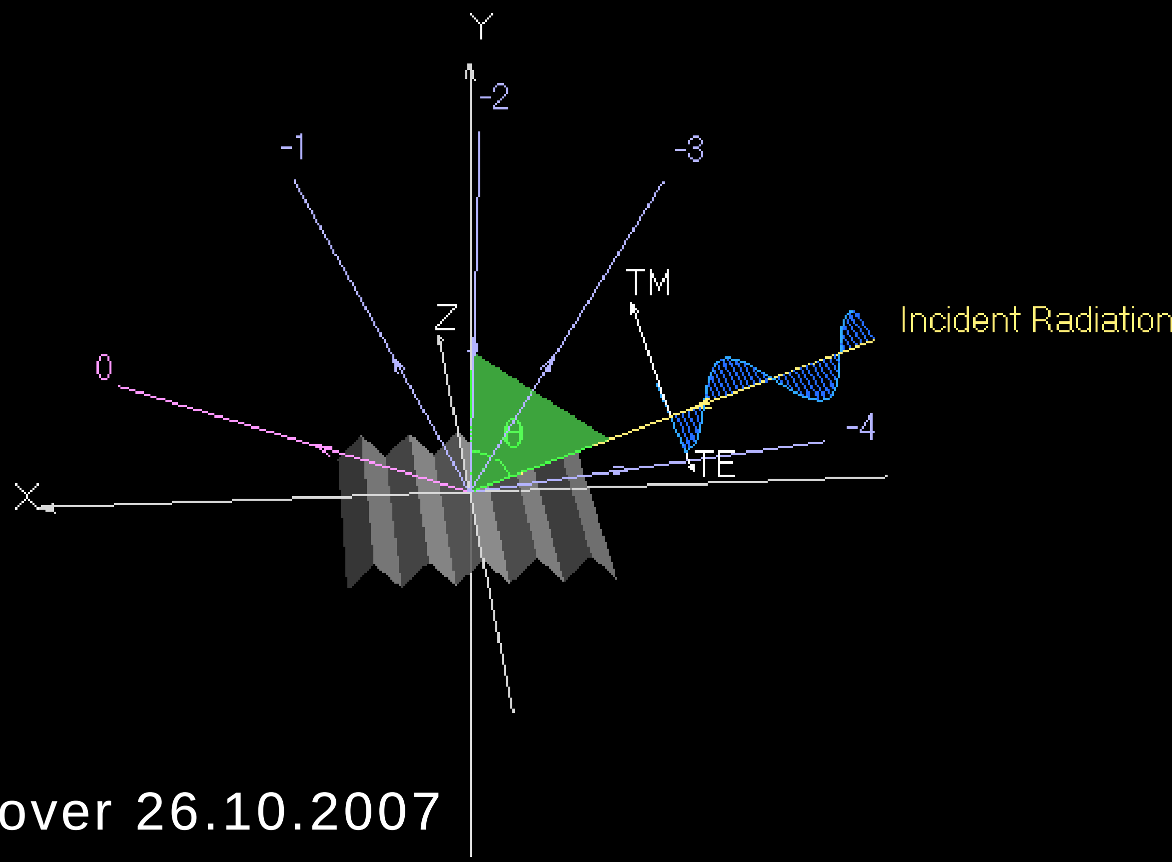
Phase and alignment noise in grating interferometers



Andreas Freise

QND Meeting, Hannover 26.10.2007

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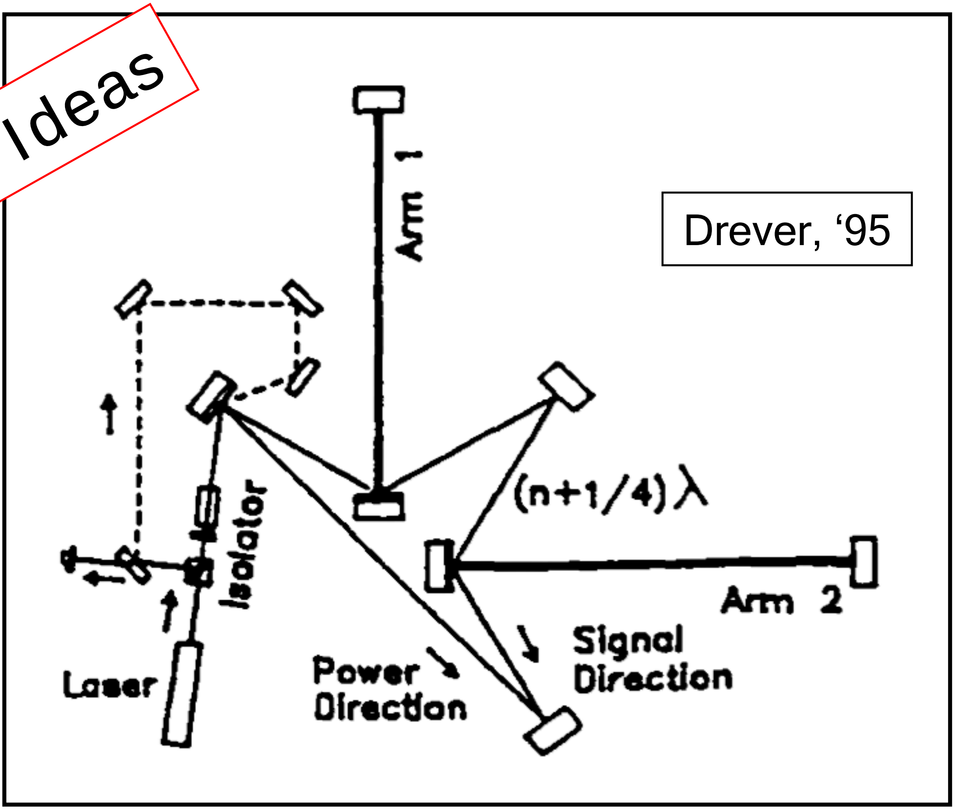
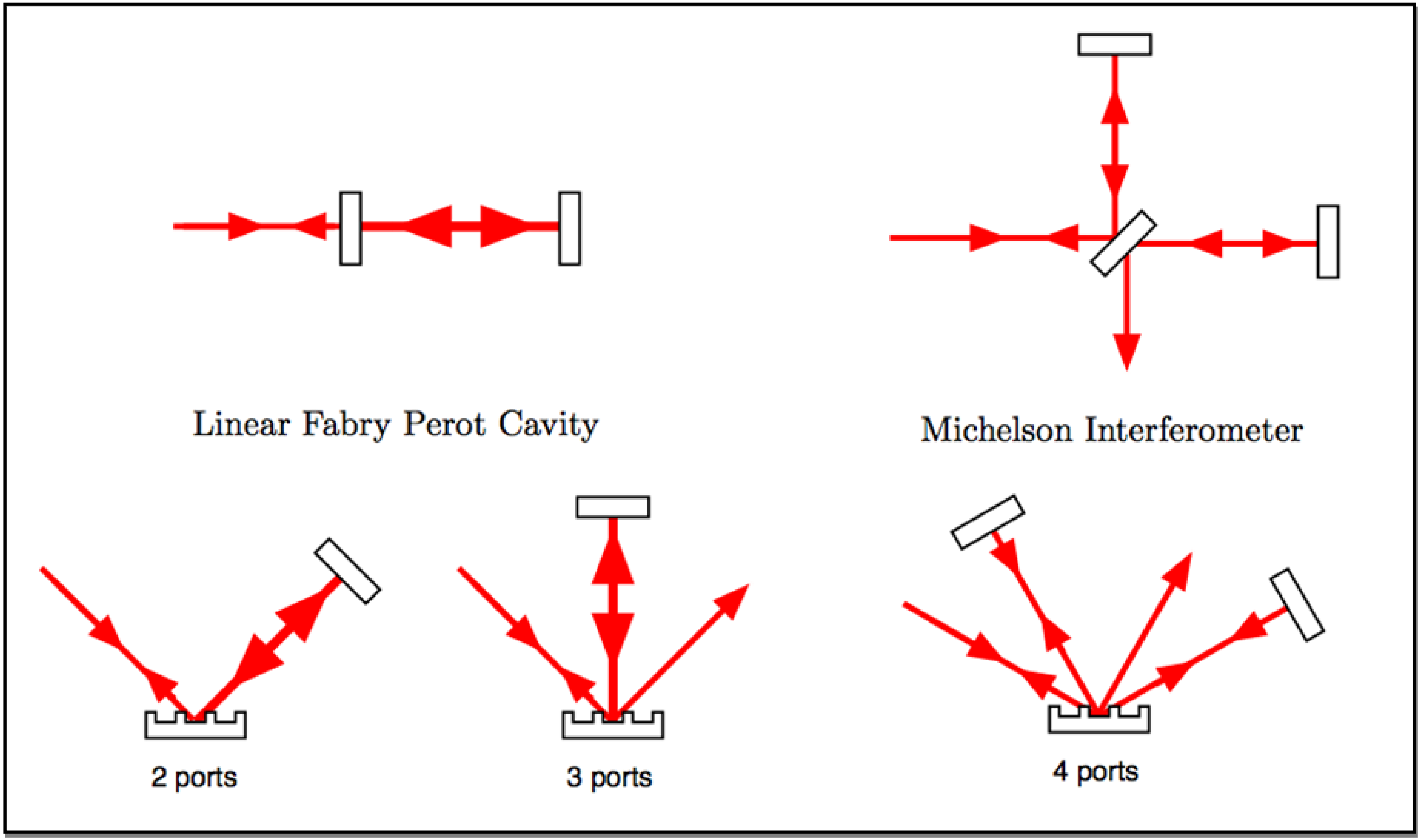
All-Reflective Interferometry

Early Ideas



Main optics do not use transmissive components

Grating Interferometers



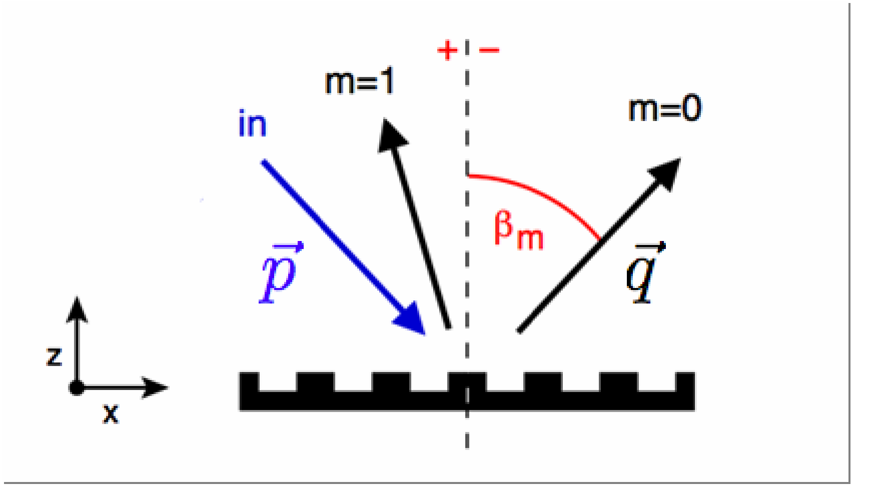
Phase and Alignment Noise

- **Motivation**: compute coupling coefficients related to the phase and alignment of the diffracted beams for the use in numerical simulations
- How does the motion of the grating (or the incoming beam) affect the direction of the outgoing beam?
- How does the motion of the grating affect the phase of the outgoing beam
- **Idea**: use only the geometry of the optical layout and ideal gratings (and compare to ideal transmissive optics)

Grating Interferometers

The grating equation:

$$\sin\alpha + \sin\beta_m = m\lambda/d$$



... in 3D:

$$\vec{q} \times \vec{e}_z - \vec{p} \times \vec{e}_z = \frac{m\lambda}{d}\vec{e}_y$$

Note: Grating normal along z, grating grooves along y

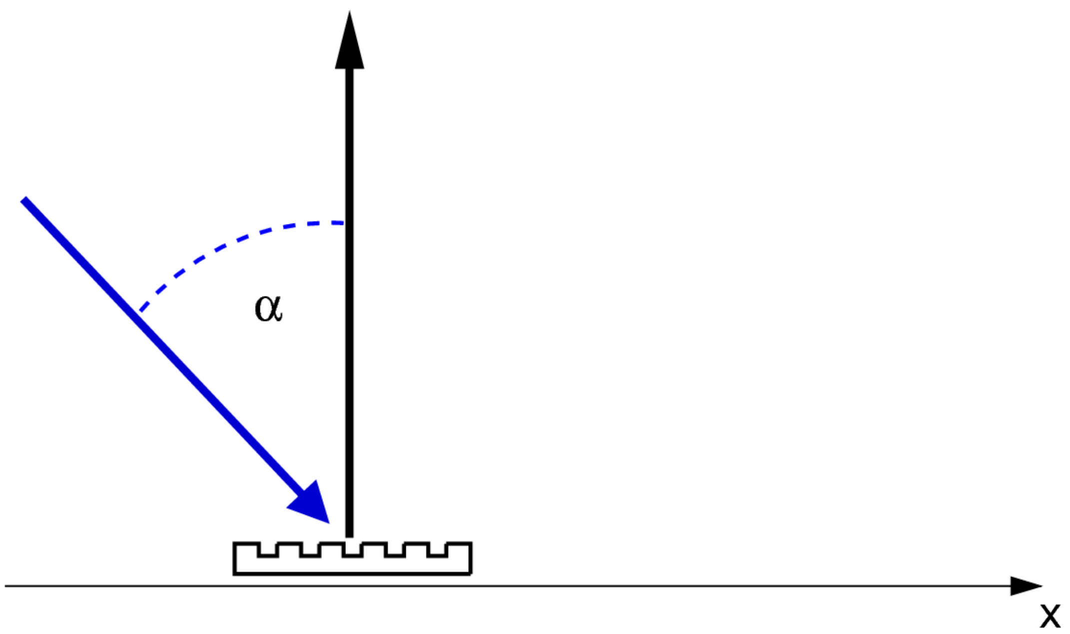
Alignment Effects

- Rotation around x- and y-axis has the same effects like with standard mirrors
- Rotation around z-axis (roll) is different from standard optics: causes approximately the same effect as tilt (rotation around x-axis), same type and amount of beam rotation
- Rotation of the grating has no effect on the phase of the light

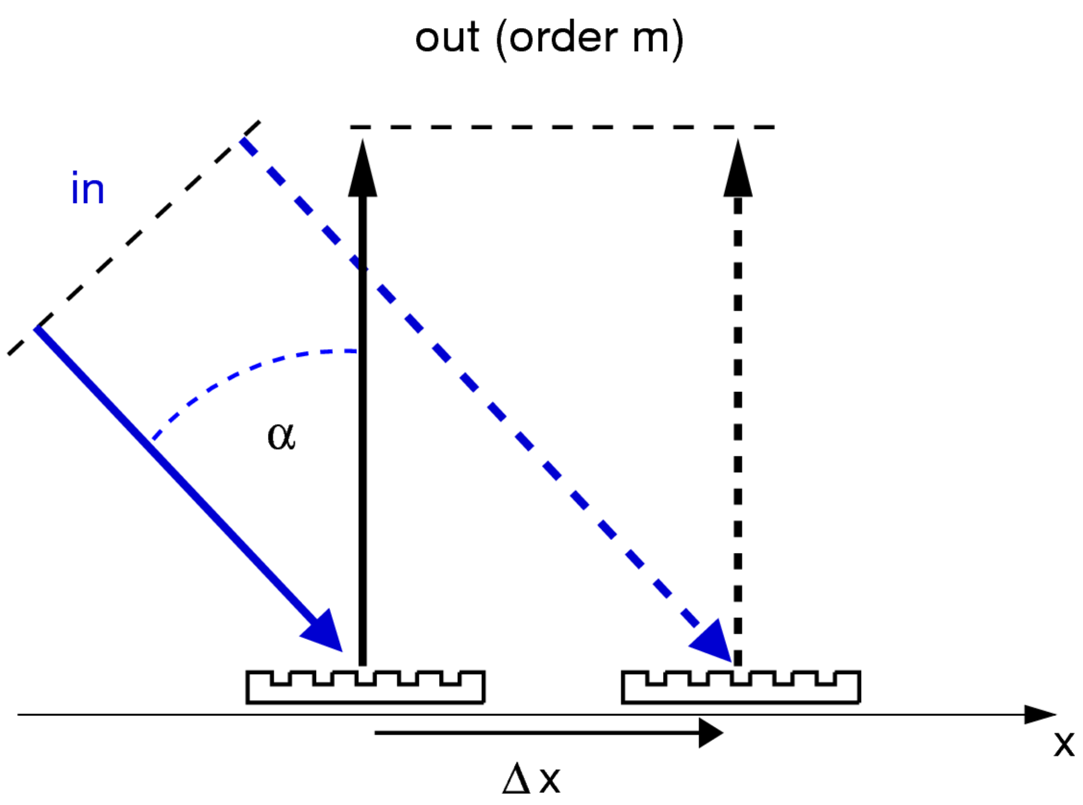
Translation Effects

- Translation of the grating or of the beam has no non-obvious effects on the alignment of the outgoing beam
- In the following we will describe the effects of translation on the phase of the diffracted beam

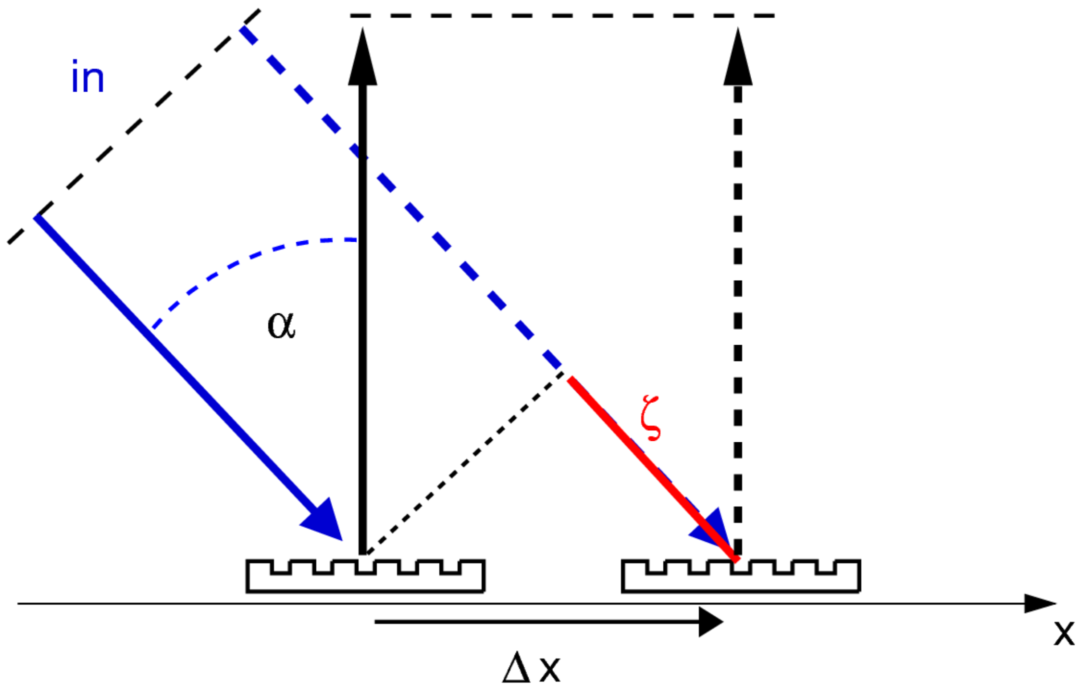
Translation Phase Noise



Translation Phase Noise



Translation Phase Noise



$$\xi_{\Delta x} = -\Delta x \, \frac{m\lambda}{d}$$

Translation Phase Noise

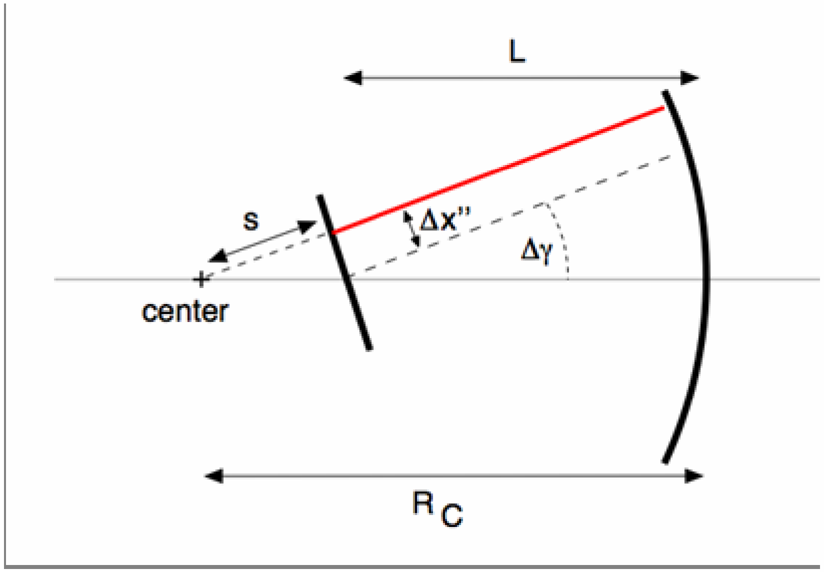
$$\xi_{\Delta x} = -\Delta x \boxed{\frac{m\lambda}{d}}$$

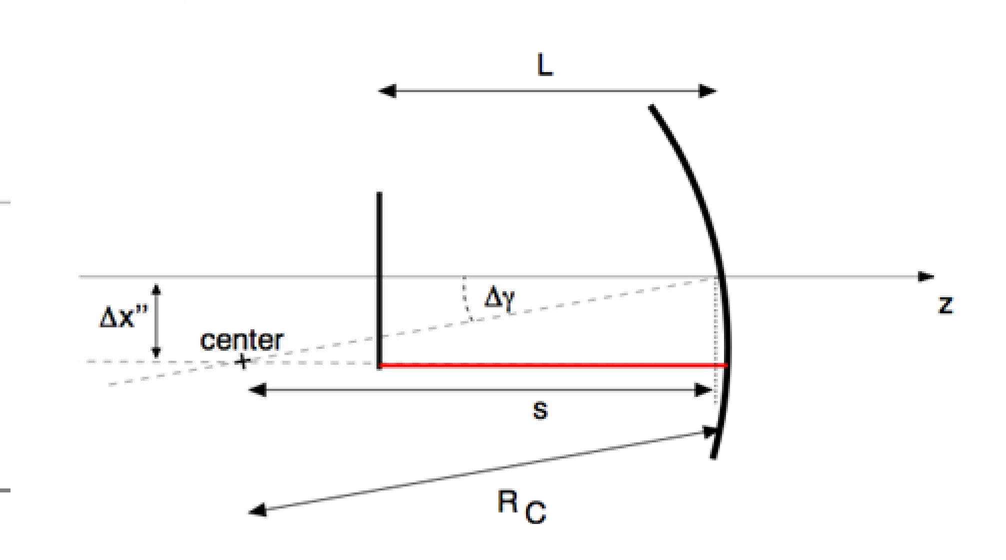
→ ≈ 1 for gratings with few diffraction orders

Optical phase change is proportional to beam translation!

Effect is proportional to m, especially it is zero for the zeroth order.

Cavity Alignment



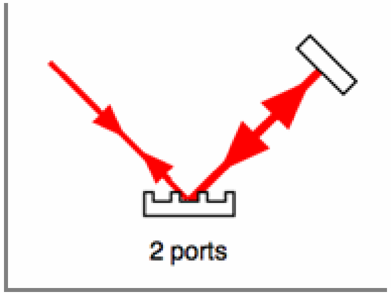


$$\Delta L \approx R_c \Delta\gamma_1 \Delta\gamma_2$$ → quadratic

$$x'' \approx (R_c - L)\Delta\gamma_2 + R_c \Delta\gamma_1$$ → linear

Alignment Limits

For a linear cavity (Virgo like) with a 2-port grating and typical values we can compute alignment limits:



$$\Delta L = 2 R_c \, \gamma_1 \, \gamma_2$$ yields:

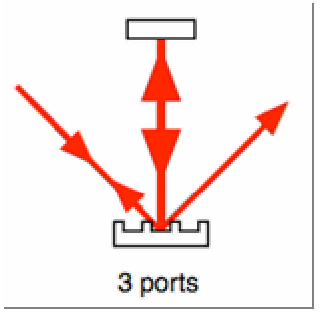
$$\gamma_2 < 2 \cdot 10^{-16} \, \frac{\mathrm{rad}}{\sqrt{\mathrm{Hz}}} \left(\frac{h}{10^{-23}/\sqrt{\mathrm{Hz}}} \right) \left(\frac{L}{3\,\mathrm{km}} \right) \left(\frac{3.5\,\mathrm{km}}{R_c} \right) \left(\frac{10\,\mathrm{nrad}}{\gamma_1} \right)$$

$$\Delta x = \frac{R_C}{\cos(\alpha)} \, \gamma_2$$ yields:

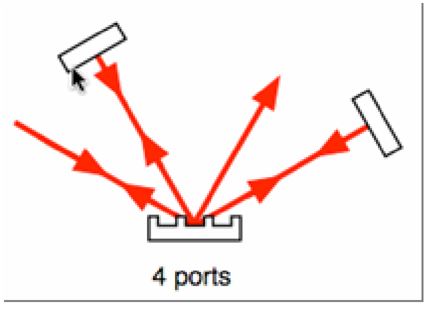
$$\gamma_2 < 7 \cdot 10^{-24} \, \frac{\mathrm{rad}}{\sqrt{\mathrm{Hz}}} \left(\frac{h}{10^{-23}/\sqrt{\mathrm{Hz}}} \right) \left(\frac{L}{3\,\mathrm{km}} \right) \left(\frac{\cos(\alpha)}{\cos(30^\circ)} \right) \left(\frac{3.5\,\mathrm{km}}{R_C} \right) \left(\frac{d}{\lambda} \right)$$

Other Layouts (3-port, 4-port)

- Other diffractive layouts (using the same Virgo like parameters)
  - 3-port input coupler (cavity mode reflected at normal incidence):
    10^{-21} rad/sqrt(Hz)

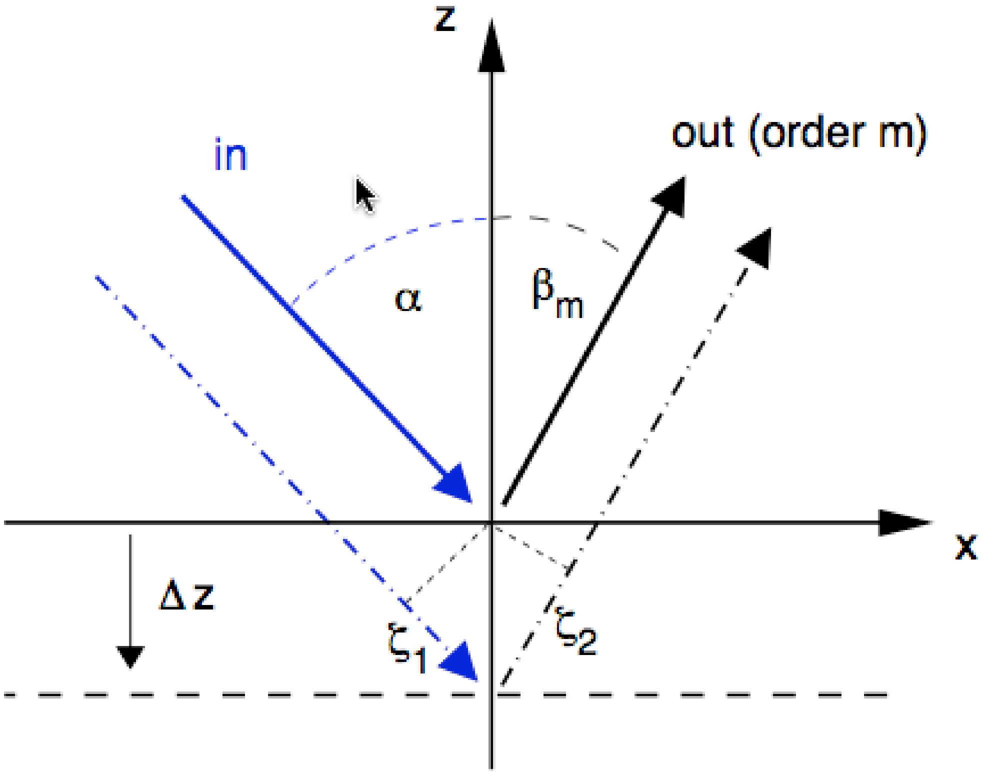


  - Diffractive beam splitter:
    10^{-21} rad/sqrt(Hz)



Compensating Phase Noise?

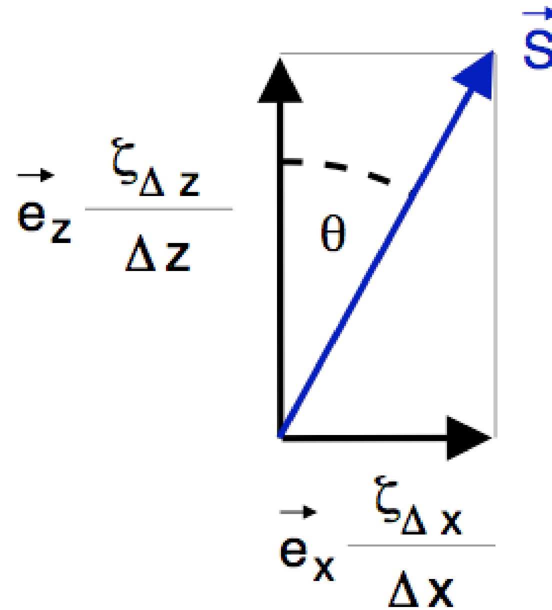
Change of optical phase due to translation along z:



Finding a Special Eigensystem

- Special eigenvector in the direction of the bi-section between incoming and diffracted beam:

$$\Theta = \tan\left(\frac{\alpha + \beta_m}{2}\right)$$



- Grating motion orthogonal to this vector causes **no** phase noise

M. C. Rushford, W. A. Molander, J. D. Nissen, I. Jovanovic, J. A. Britten, and C. P. Barty. Diffraction grating eigenvector for translational and rotational motion. *Optics Letters*, 31:155–157, January 2006.

Summary

- Gratings introduce much more stringent alignment requirements
- Possible scenarios:
  - High-power high-frequency detectors
  - Alignment insensitive topologies?
- Grating roll has to be controlled and isolated as well as the other alignment degrees of freedom
- The sensitive/in-sensitive axes of motion are not parallel to the beams or the grating normal, suspension design should take this into account

Freise et al, LIGO-P070094-00-Z, to be published in NJP

... end.