Entanglement between test masses

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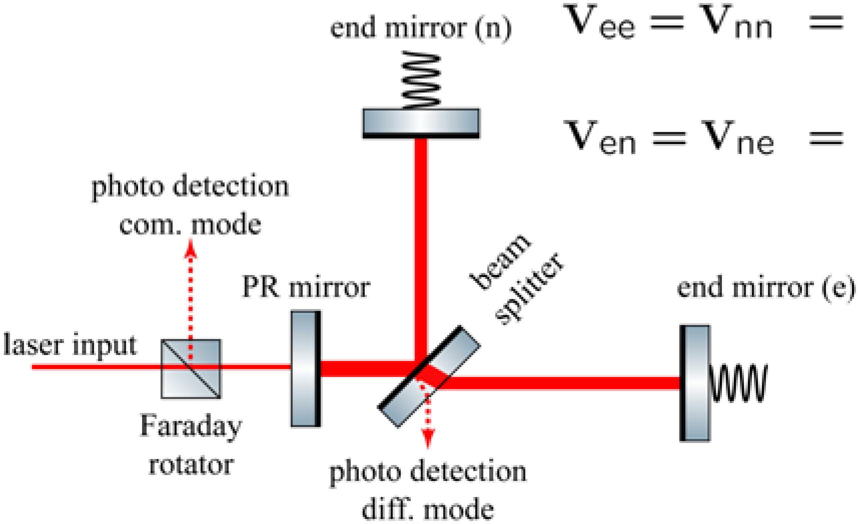
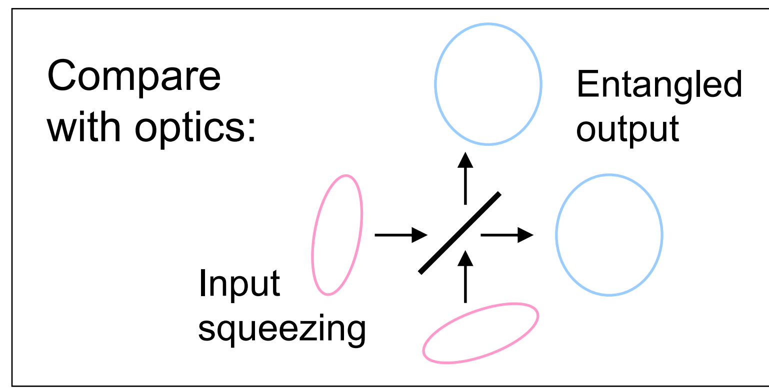
Entanglement between end mirrors

Total system: $\mathbf{V} = \begin{pmatrix} \mathbf{V}_{\text{ee}} & \mathbf{V}_{\text{en}} \\ \mathbf{V}_{\text{ne}} & \mathbf{V}_{\text{nn}} \end{pmatrix}$

$$\mathbf{V}_{\text{ee}} = \mathbf{V}_{\text{nn}} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \left[\begin{pmatrix} V^{\text{c}}_{xx} & V^{\text{c}}_{xp} \\ V^{\text{c}}_{xp} & V^{\text{c}}_{pp} \end{pmatrix} + \begin{pmatrix} V^{\text{d}}_{xx} & V^{\text{d}}_{xp} \\ V^{\text{d}}_{xp} & V^{\text{d}}_{pp} \end{pmatrix} \right] \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{V}_{\text{en}} = \mathbf{V}_{\text{ne}} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} \left[\begin{pmatrix} V^{\text{c}}_{xx} & V^{\text{c}}_{xp} \\ V^{\text{c}}_{xp} & V^{\text{c}}_{pp} \end{pmatrix} - \begin{pmatrix} V^{\text{d}}_{xx} & V^{\text{d}}_{xp} \\ V^{\text{d}}_{xp} & V^{\text{d}}_{pp} \end{pmatrix} \right] \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix}$$





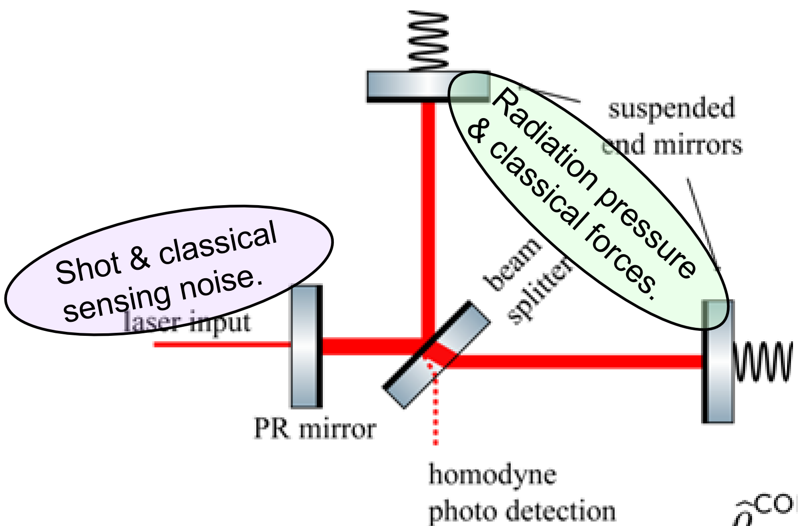
Logarithmic negativity [Vidal, Werner] a quantitative measure of entanglement:

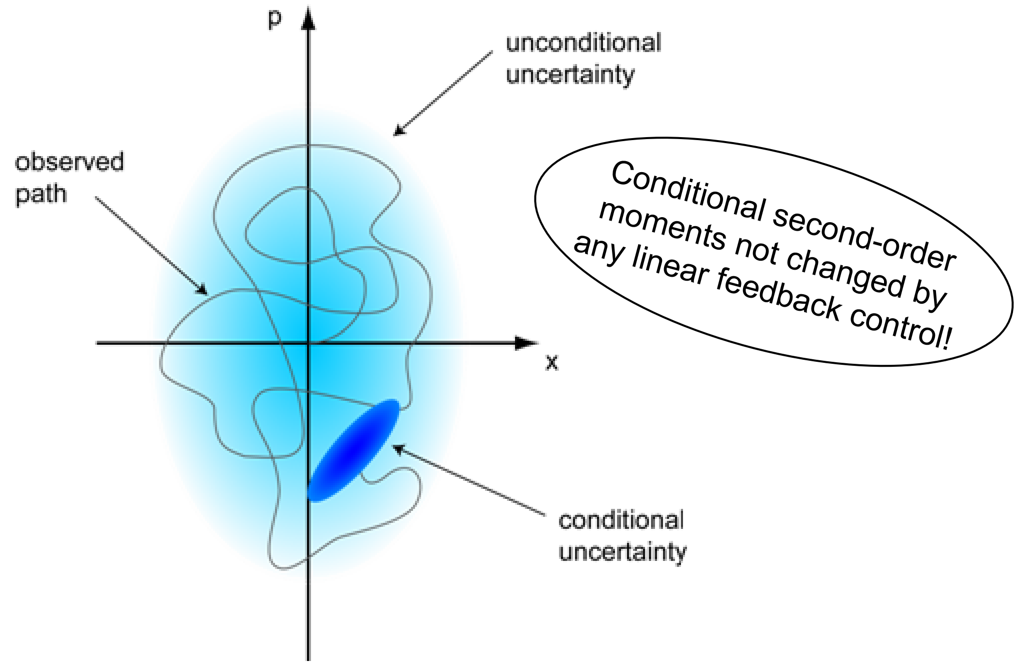
$$E_{\mathcal{N}} = \max \left[0, -\log_2 \left[2/\hbar \sqrt{(\Sigma - \sqrt{\Sigma^2 - 4 \det V})/2} \right] \right]$$

$$\Sigma = \det V_{\text{nn}} + \det V_{\text{ee}} - 2 \det V_{\text{ne}}$$

Conditioning on continuous measurement

Each (common and differential) mode:





$$\hat{\rho}^{\mathrm{cond}}(t) = \mathcal{P}_{[\hat{y}(t')=y(t'),t'<t]}\ \hat{\rho}(t)\ \mathcal{P}_{[\hat{y}(t')=y(t'),t'<t]}$$

Time domain:	Stochastic Master Equation
Frequency domain:	Wiener Filtering ✔

Prepare state by collecting data and finding optimal filter function. Need verification stage?

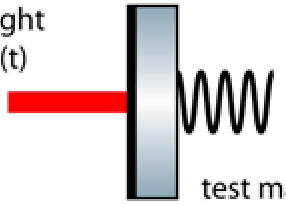
→ Stefan's talk.

Wiener Filtering

$$\hat{x}(t) = \underbrace{\int_{-\infty}^{t} dt' K_x(t-t')\,\hat{y}(t')}_{\text{known}} + \underbrace{\hat{R}_x(t)}_{\text{unknown}} \quad \text{with} \quad \langle \hat{R}_x(t)\,\hat{y}(t')\rangle_{\text{sym}} = 0$$

$$[\hat{y}(t), \hat{y}(t')] = 0$$

$$[\hat{x}(t), \hat{y}(t')] = 0 \;\; \forall t > t'$$



Conditional moments of Gaussian state:

$$\tilde{x}(t) = \int_{-\infty}^{t} dt' K_x(t-t')\, y(t') \qquad \tilde{p}(t) = \int_{-\infty}^{t} dt' K_p(t-t')\, y(t')$$

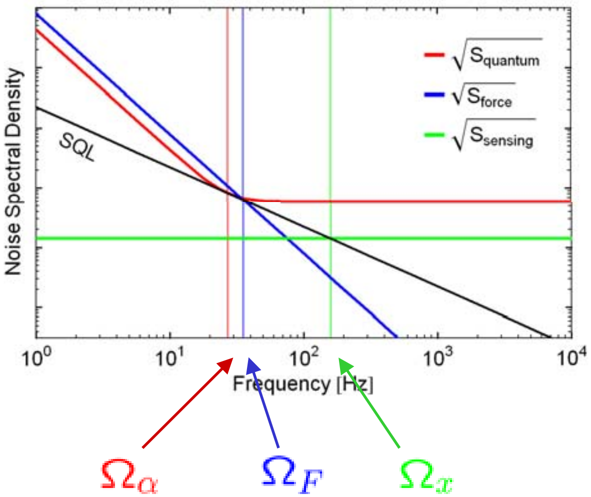
$$V_{xx} = \langle \hat{R}_x^2 \rangle = \int_0^\infty \frac{d\Omega}{2\pi} \left(S_x - \left[\frac{S_{xy}}{s_y^*}\right]_+ \left[\frac{S_{xy}}{s_y^*}\right]_+^* \right)$$

$$V_{pp} = \langle \hat{R}_p^2 \rangle = \int_0^\infty \frac{d\Omega}{2\pi} \left(S_p - \left[\frac{S_{py}}{s_y^*}\right]_+ \left[\frac{S_{py}}{s_y^*}\right]_+^* \right)$$

$$V_{xp} = \langle \hat{R}_x \hat{R}_p \rangle_{\text{sym}} = \int_0^\infty \frac{d\Omega}{2\pi}\, \Re \left\{ S_{xp} - \left[\frac{S_{xy}}{s_y^*}\right]_+ \left[\frac{S_{py}}{s_y^*}\right]_+^* \right\}$$

Steady state!

Conditional test-mass state



$\phi = 0$

$\omega_m,\ \gamma_m \ll \Omega_\alpha$

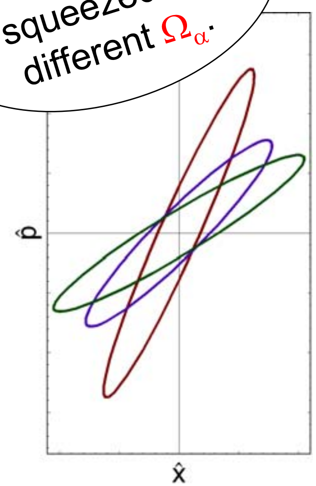
$$V_{xx} = \frac{\hbar}{\sqrt{2} m \Omega_\alpha}\left[\left(1+\frac{2{\Omega_F}^2}{{\Omega_\alpha}^2}\right)\left(1+\frac{2{\Omega_\alpha}^2}{{\Omega_x}^2}\right)^3\right]^{1/4} \ll \frac{\hbar}{2m\omega_m}$$

$$V_{pp} = \frac{\hbar m \Omega_\alpha}{\sqrt{2}}\left[\left(1+\frac{2{\Omega_F}^2}{{\Omega_\alpha}^2}\right)^3\left(1+\frac{2{\Omega_\alpha}^2}{{\Omega_x}^2}\right)\right]^{1/4} \gg \frac{\hbar m \omega_m}{2}$$

$$V_{xp} = \frac{\hbar}{2}\left[\left(1+\frac{2{\Omega_F}^2}{{\Omega_\alpha}^2}\right)\left(1+\frac{2{\Omega_\alpha}^2}{{\Omega_x}^2}\right)\right]^{1/2} > 0$$

Ω_x / Ω_F > 2 → non-zero frequency band in which classical noise is <u>completely</u> below the SQL!

Differently squeezed for different Ω_α.



Homodyne detection angle

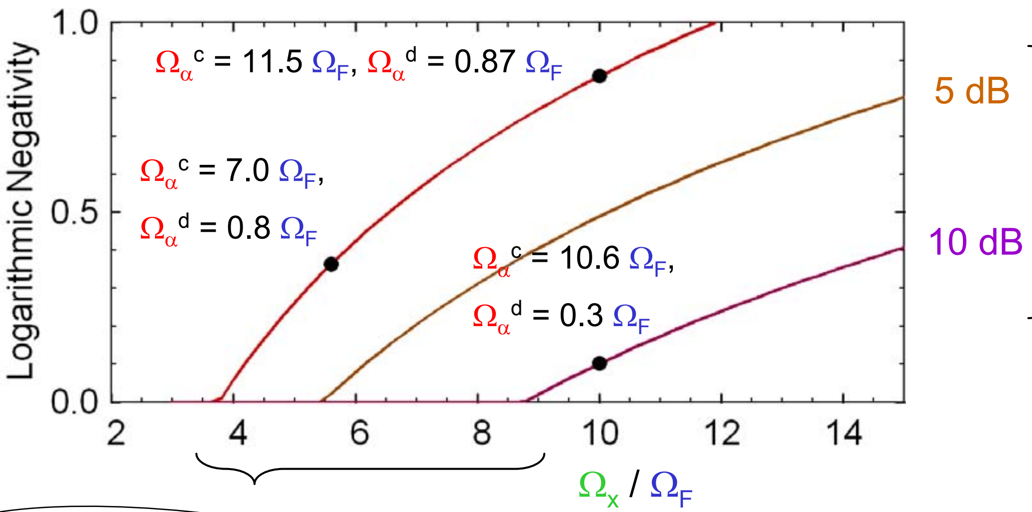
$$V_{xx}V_{pp} - V_{xp}^2 = \frac{\hbar^2}{4}\left[\left(1+\frac{2{\Omega_F}^2}{{\Omega_\alpha}^2}\right)\left(1+\frac{2{\Omega_\alpha}^2}{{\Omega_x}^2}\right) + \frac{2{\Omega_F}^2}{{\Omega_\alpha}^2}\tan^2\phi\right]$$

$$\geq \frac{\hbar^2}{4}\left(1+\frac{2\Omega_F}{\Omega_x}\right)^2 \geq \frac{\hbar^2}{4}$$

Equality for: $\phi = 0$, $\Omega_\alpha = \sqrt{\Omega_x \Omega_F}$

Constrains on conditional mirror entanglement

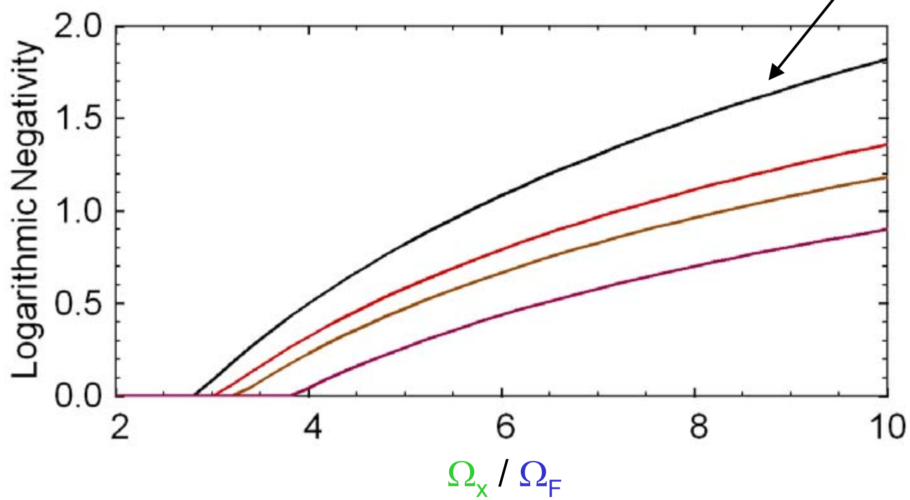
[arXiv:quant-ph/0702258]



Laser noise level.

Optimal: Detect both modes at $\phi \approx -\phi/2 \rightarrow$ then independent from laser noise!

Ω_x / Ω_F - threshold larger than 2 for entanglement.



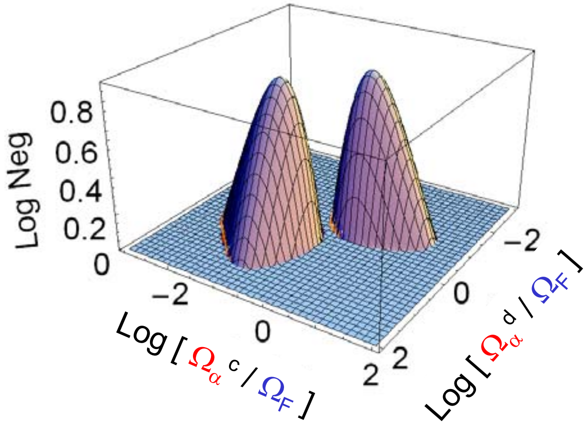
Common mode detection at $\phi \approx -\pi/2$.

Detection close to amplitude quadrature ($\phi \approx -\pi/2$) $\rightarrow$ modes more squeezed – but at same time even more anti-squeezed!

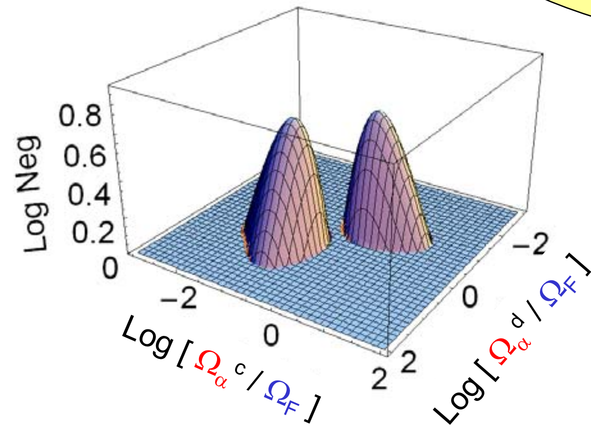
Conditional mirror entanglement

No laser noise!

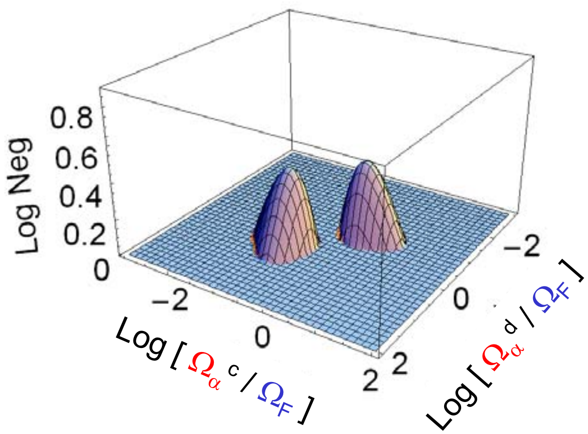
$\Omega_x / \Omega_F = 10$



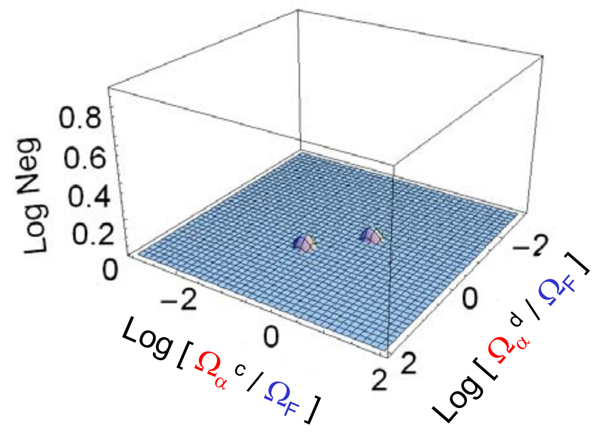
$\Omega_x / \Omega_F = 8$



$\Omega_x / \Omega_F = 6$



$\Omega_x / \Omega_F = 4$



Survival of conditional mirror entanglement

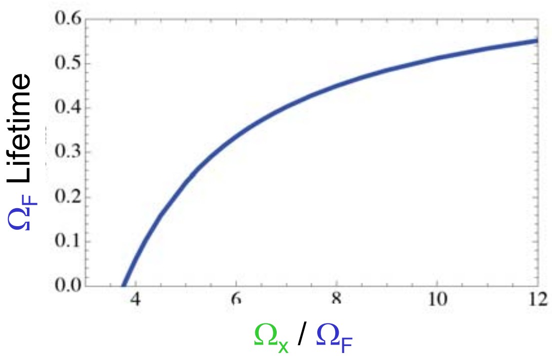
Second-order moments at $t = \tau$ conditioned on measurement at $t < 0$:

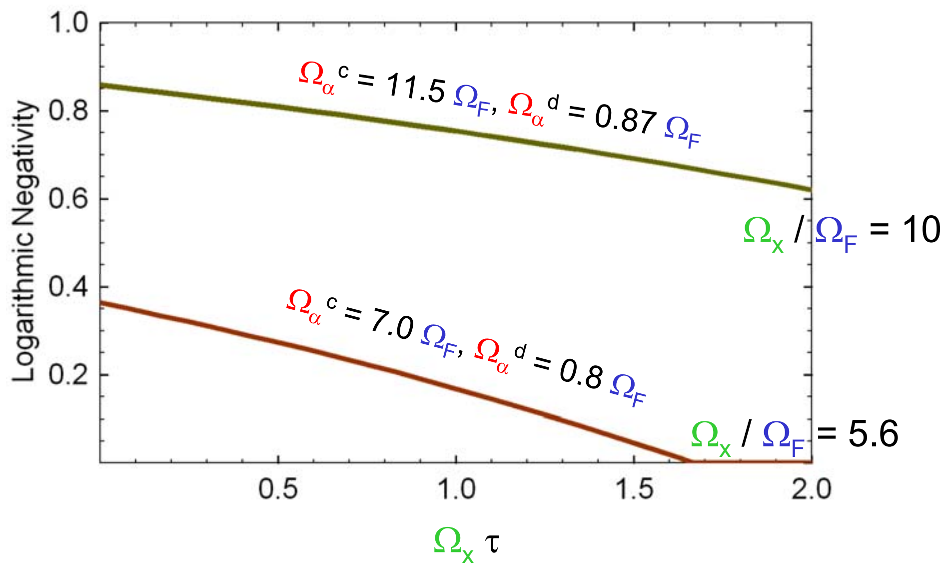
$$
\begin{aligned}
V_{xx}^{\text{rot}}(\tau) &= V_{xx}(0) + \frac{\hbar}{2m}\,\Omega_\alpha{}^2\left(1 + \frac{2\Omega_F{}^2}{\Omega_\alpha{}^2}\right)\frac{\tau^3}{3} \\
V_{pp}^{\text{rot}}(\tau) &= V_{pp}(0) + \frac{\hbar m}{2}\,\Omega_\alpha{}^2\left(1 + \frac{2\Omega_F{}^2}{\Omega_\alpha{}^2}\right)\tau \\
V_{xp}^{\text{rot}}(\tau) &= V_{xp}(0) - \frac{\hbar}{2}\,\Omega_\alpha{}^2\left(1 + \frac{2\Omega_F{}^2}{\Omega_\alpha{}^2}\right)\frac{\tau^2}{2}
\end{aligned}
$$

In “rotating frame”.

No laser noise!

Erase, if laser turned off at t = 0.

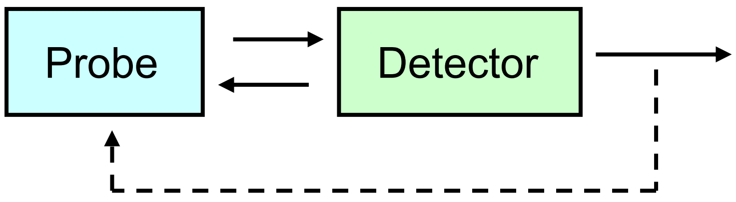




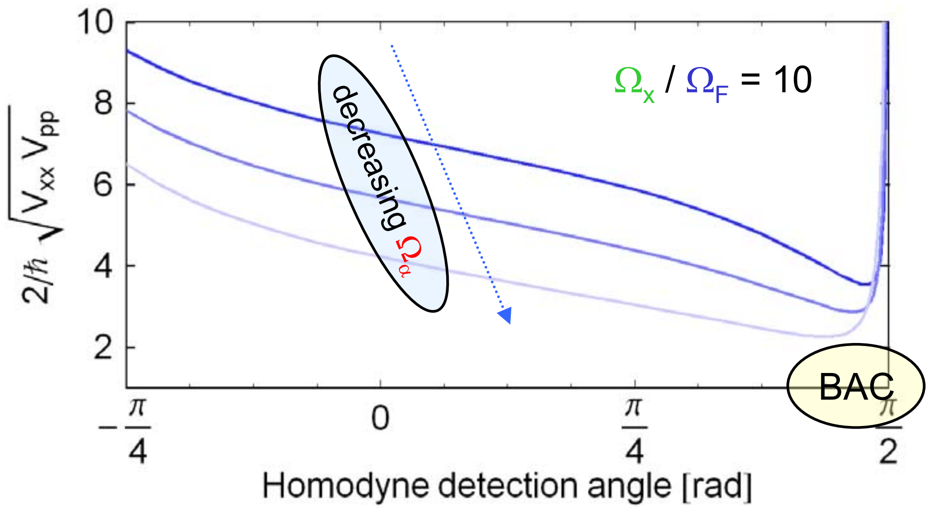
Controlled test-mass state

With optimal controller in terms of conditional second-order moments:

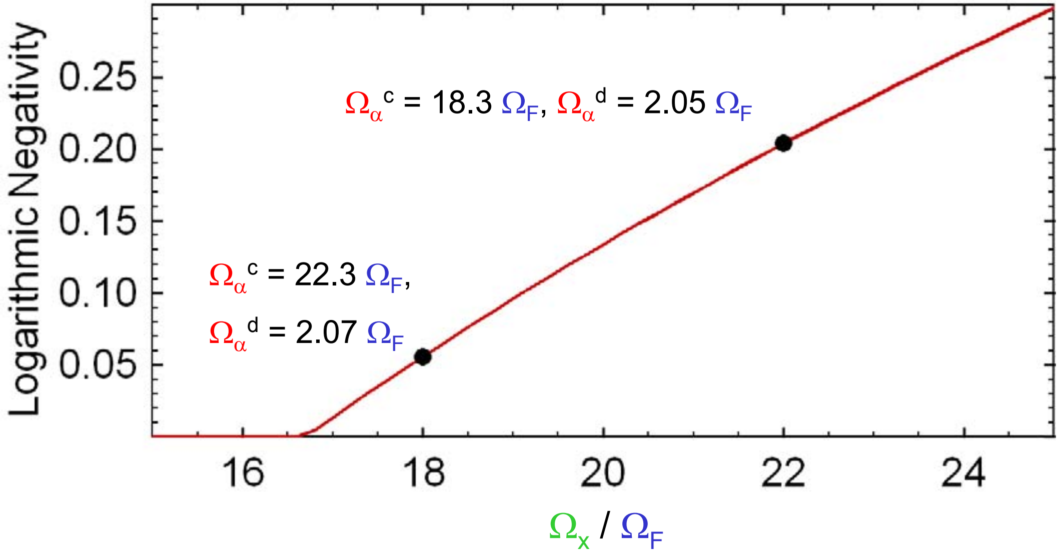
$$V_{xx}^{\mathrm{ctrl}} = V_{xx} + \sqrt{\frac{V_{xx}}{V_{pp}}}\, V_{xp}$$
$$V_{pp}^{\mathrm{ctrl}} = V_{pp} + \sqrt{\frac{V_{pp}}{V_{xx}}}\, V_{xp}$$
$$V_{xp}^{\mathrm{ctrl}} = 0$$



- Controlled state always more mixed than conditional state.
- Test masses fixed → can do simultaneous verification.
- Weak measurement optimal for low noise - but phase transition.
- Less mixed at quadrature close to amplitude → back-action-compensating (BAC).



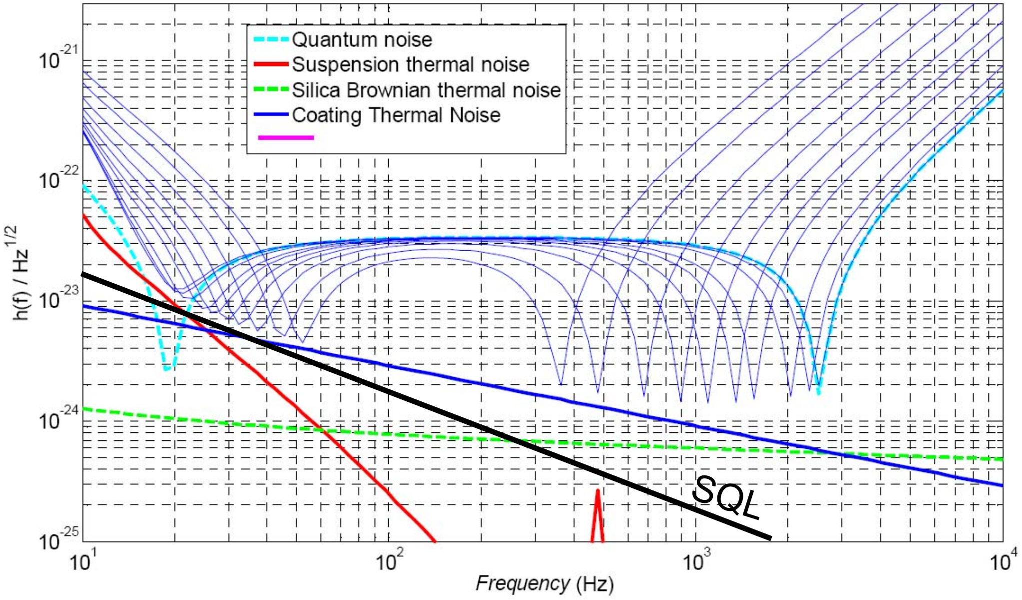
Constrains on controlled mirror entanglement



- Much higher demand on classical noise SQL-beating, i.e. higher Ω_x / Ω_F.
- Optimal detection at ϕ » $\pi / 3$.
- More power needed: here even $\Omega_\alpha^d > \Omega_F$.

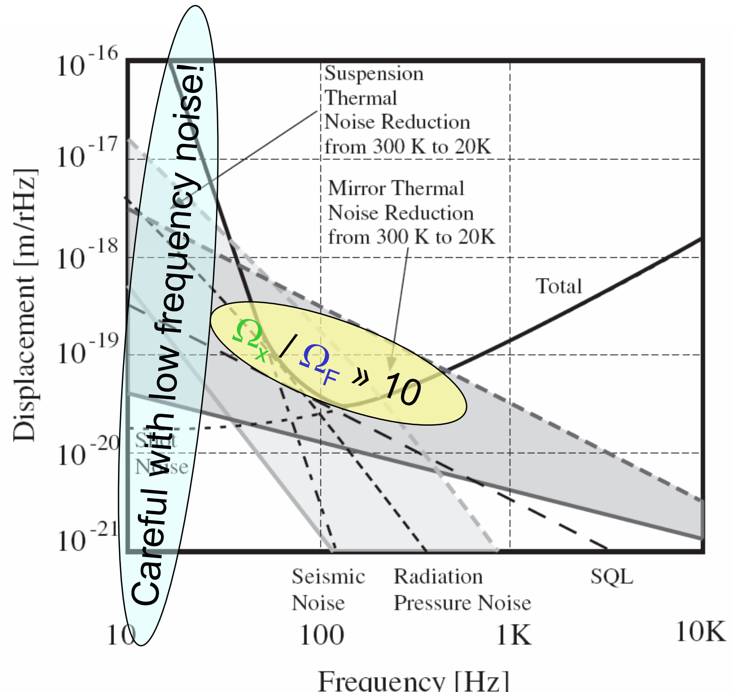
Sub-SQL classical noise

Noise budget advanced LIGO:



[Report LIGO-060056-07-M]

Noise budget CLIO:



[Miyoki et al., Class. Quantum Grav. 21, S1173 (2004)]

Here Ω_x » Ω_F » 30 Hz → ratio far too small!!!

Hannover prototype

- 10 m prototype with 100 g to 1 kg end mirrors.
- Low mechanical loss (silica) in end mirrors and thin coating/ or just gratings.
- Cool mirrors down to 20 K?
- Inject squeezed input!
- High power and frequency stabilized laser system…
- Implement double readout or entangle differential motion of a pair of coherently operated interferometers.
- → Need rigorous study of noise model.