Electrostatic force noise and free-fall for LISA

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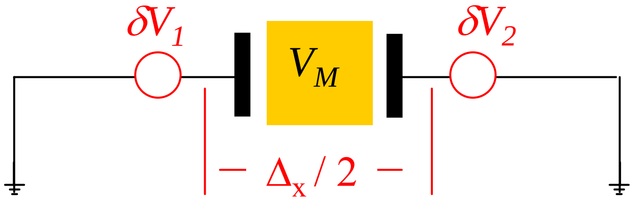
Charging issues in experimental gravity
MIT, July 26-27, 2007
G070571-00-R

Noise source: stray low frequency electrostatics



$$k = -\frac{\partial F}{\partial x} = -\frac{1}{2}\sum_i \frac{\partial^2 C_i}{\partial x^2}\left(V_i - V_{TM}\right)^2 \quad \left\{ \begin{array}{l} \propto Q^2 \\ \propto \left\langle \delta V^2 \right\rangle \end{array} \right.$$

Electrostatic stiffness

[For zero net DC bias imbalance]

$$F = \frac{Q}{C_{TOT}} \sum \frac{\partial C_i}{\partial x} \delta V_i \quad \left\{ \begin{array}{l} S_F^{1/2} = \dfrac{\sqrt{2e^2 \lambda_{EFF}}}{\omega C_T} \left| \dfrac{\partial C}{\partial x} \right| \Delta_x \\ S_F^{1/2} = \dfrac{\langle Q \rangle}{C_T} \left| \dfrac{\partial C}{\partial x} \right| S_{\Delta_x}^{1/2} \end{array} \right.$$

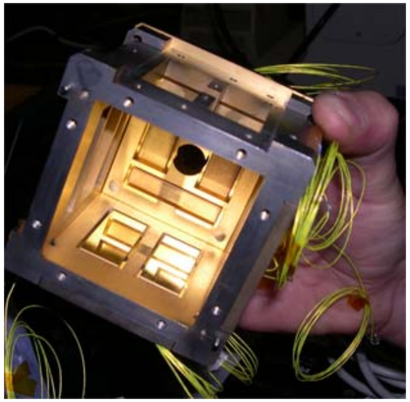
Random charge noise mixing with DC bias (Δ_x)

Noisy average “DC” bias ($S_{\Delta x}$) mixing with mean charge

$$S_F^{1/2} = \sqrt{\sum \left| \frac{\partial C_i}{\partial x} \right|^2 \delta V_i^{\,2} S_{\delta V_i}}$$

Noisy “DC” biases interacting with themselves

Measurements of electrostatic force noise with LISA GRS

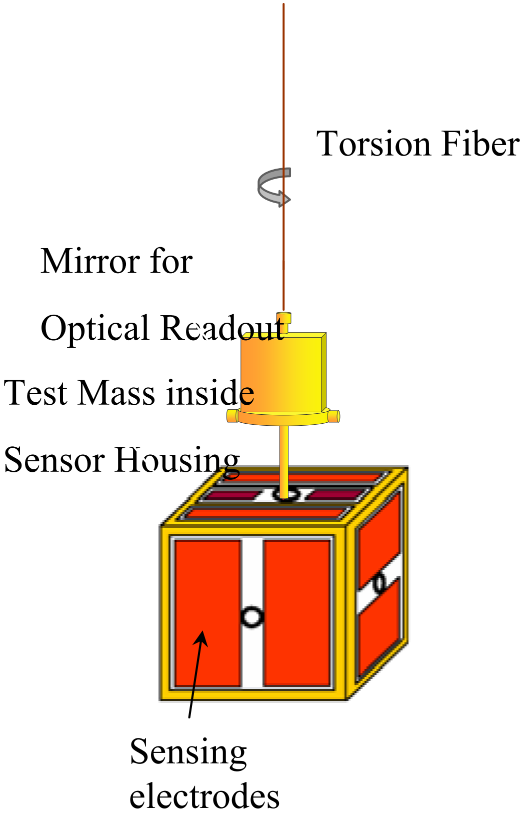


GRS capacitive sensor for LTP / LISA

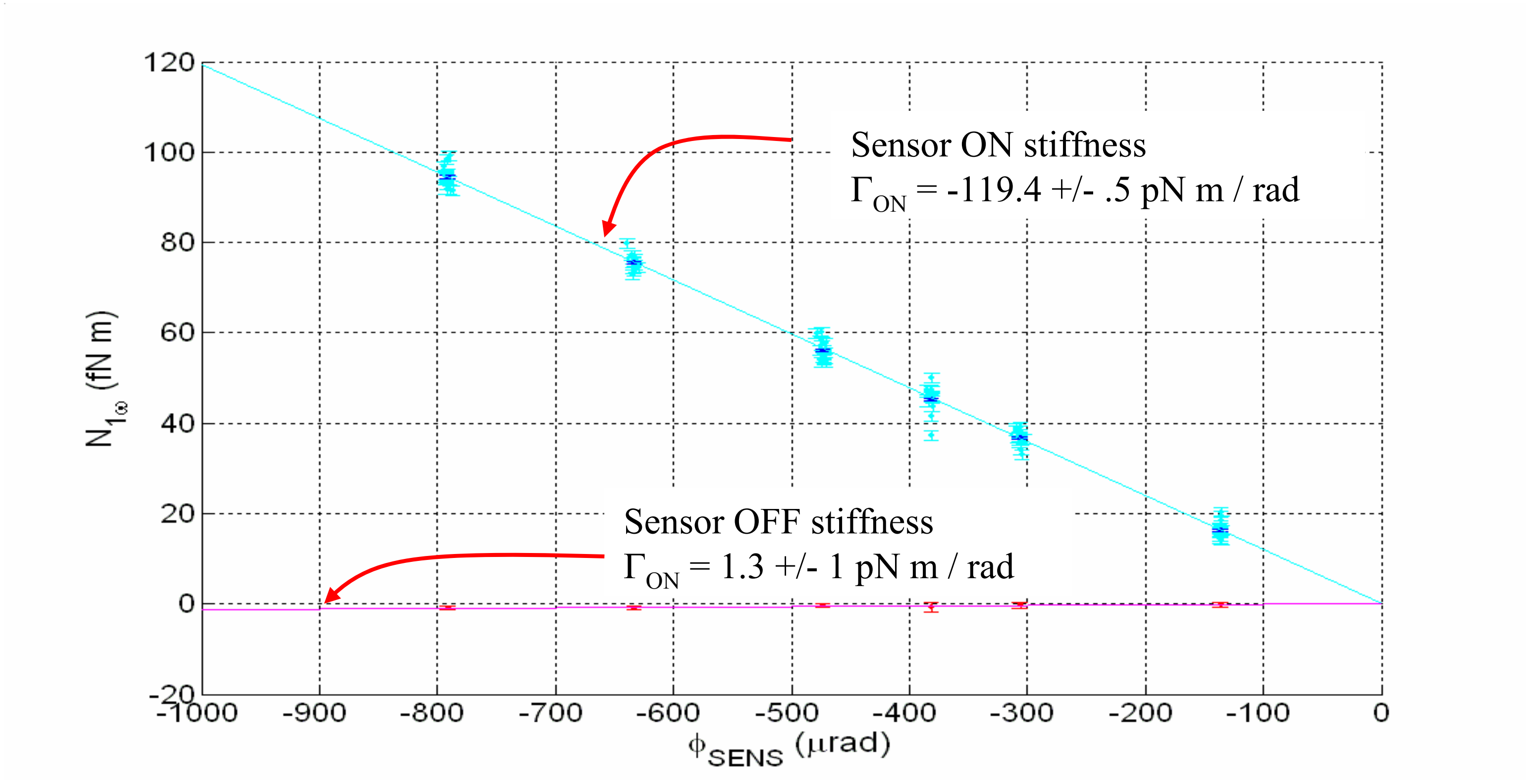
- 4 mm x-sensing gaps
- Mo / Au-coated Shapal

Hollow TM suspended as torsion pendulum

→ Rotational measurements of differential electrostatic forces



Electrostatic stiffness from stray electrostatic fields



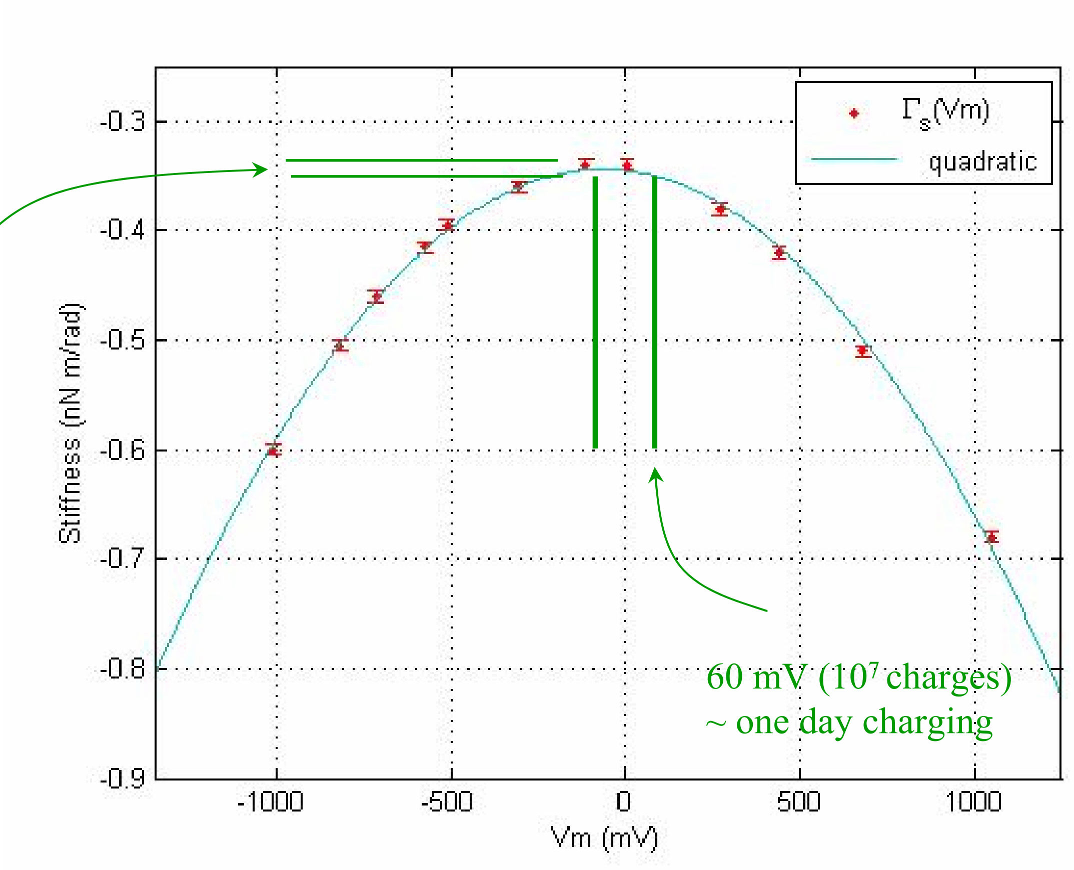
Sensor ON stiffness
Γ_{ON} = -119.4 +/- .5 pN m / rad

Sensor OFF stiffness
Γ_{ON} = 1.3 +/- 1 pN m / rad

- Stiffness from 100 kHz sensor bias roughly as modeled (30% below infinite plate prediction)
- Sensor OFF stiffness negligible → stiffness from patch charges not important for LISA!!
  → measurement confirmed recently in translation (4-mass pendulum)

  → Benefit of 4 mm gaps, $\Gamma \sim d^{-3}$
- RMS patch voltage differences on ~ 4 mm domains no more than 50 mV

Electrostatic stiffness from TM charging



< 1% total
LISA stiffness

60 mV (10^7 charges)
~ one day charging

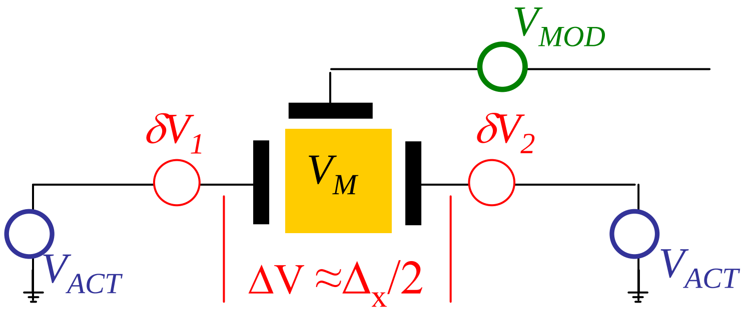
- As expected from electrostatic model (roughly 30% below infinite plate model)

- Note: minimum magnitude obtained for V_{TM} ~ 60 mV (NOT 0 V)
  → DC biases effect charge measurement and stiffness

Noise source: DC biases and charge shot noise

Fluctuating test mass charge (cosmic ray shot noise)
forced by stray DC electrostatic “patch” fields

$$F = -\frac{C}{d} V_M \; \Delta V$$



$$S_a^{1/2}(f) \sim 6\ \mathrm{fm/s^2}/\sqrt{\mathrm{Hz}} \left(\frac{\lambda_{eff}}{800/s}\right)^{1/2} \left(\frac{\Delta_x}{100\ \mathrm{mV}}\right) \left(\frac{10^{-4}\ \mathrm{Hz}}{f}\right)$$

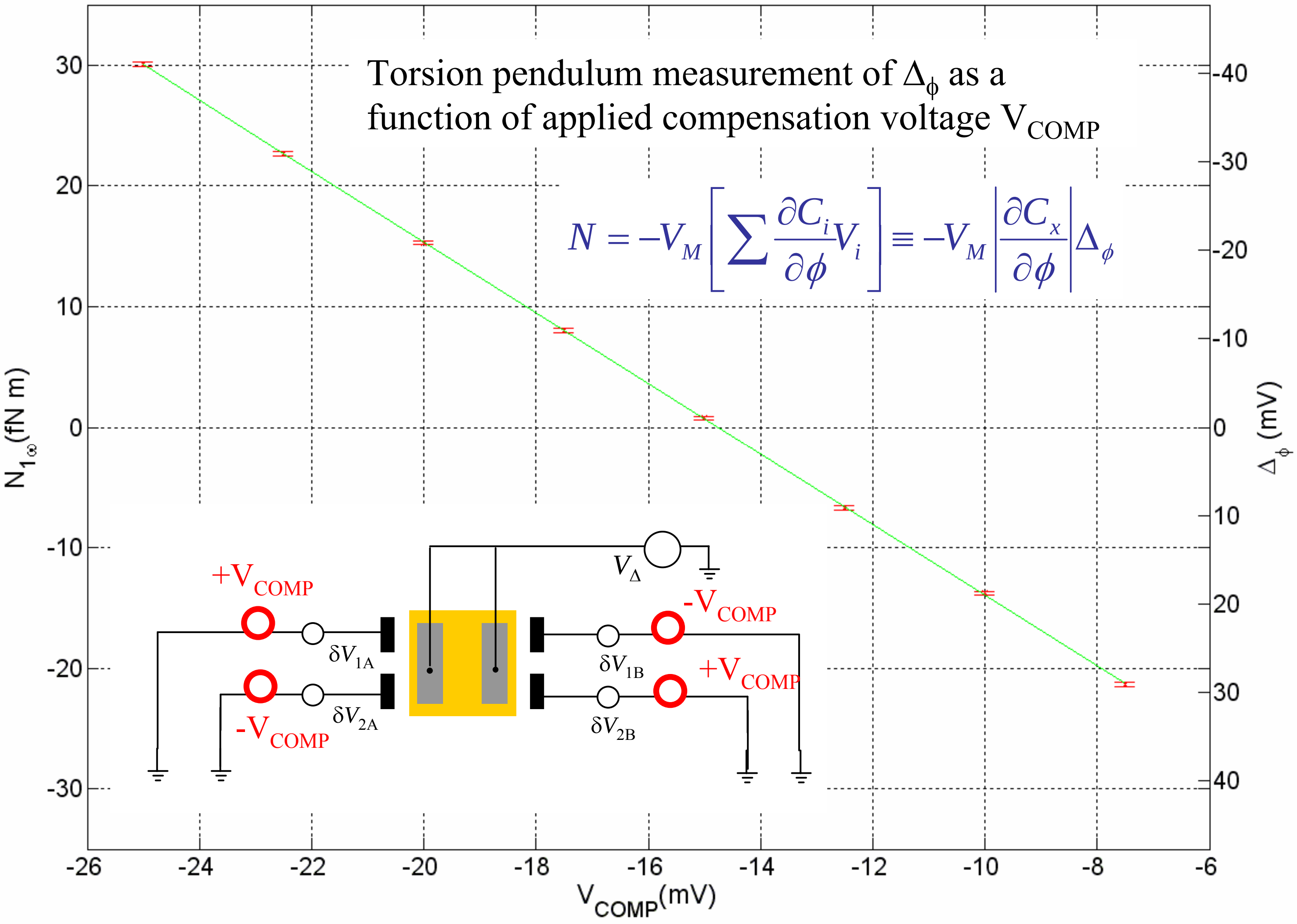
$$\left[\propto \frac{1}{d}\right]$$

- λ_{eff} ~ 800 e/s (H. Araujo, LISA Symposium 2004) includes +/-, different charge number

Charge feels integrated effect from all patch fields

- Can be measured by applying a coherent TM bias (simulated charge)
- Can be cancelled by application of correct compensation voltage

DC Bias: measurement and compensation

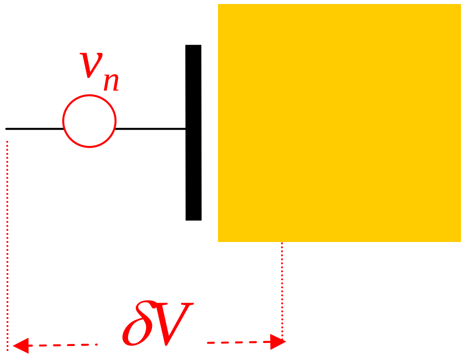


Torsion pendulum measurement of Δ_ϕ as a function of applied compensation voltage V_{COMP}

$$N = -V_M \left[\sum \frac{\partial C_i}{\partial \phi} V_i \right] \equiv -V_M \left| \frac{\partial C_x}{\partial \phi} \right| \Delta_\phi$$

- DC biases compensated with V_{COMP} = +15 mV (intrinsic Δ_ϕ = -60 mV)
- Sub-mV measurement possible in 15 minutes integration
- Compensation possible to DAC resolution, in flight
- Random charging should not be problematic under normal conditions

Noise source: in-band voltage noise mixing with DC bias



$$F \approx -\frac{C}{d}\,\delta V\ v_{\mathrm{n}}$$

Voltage noise: v_{n}

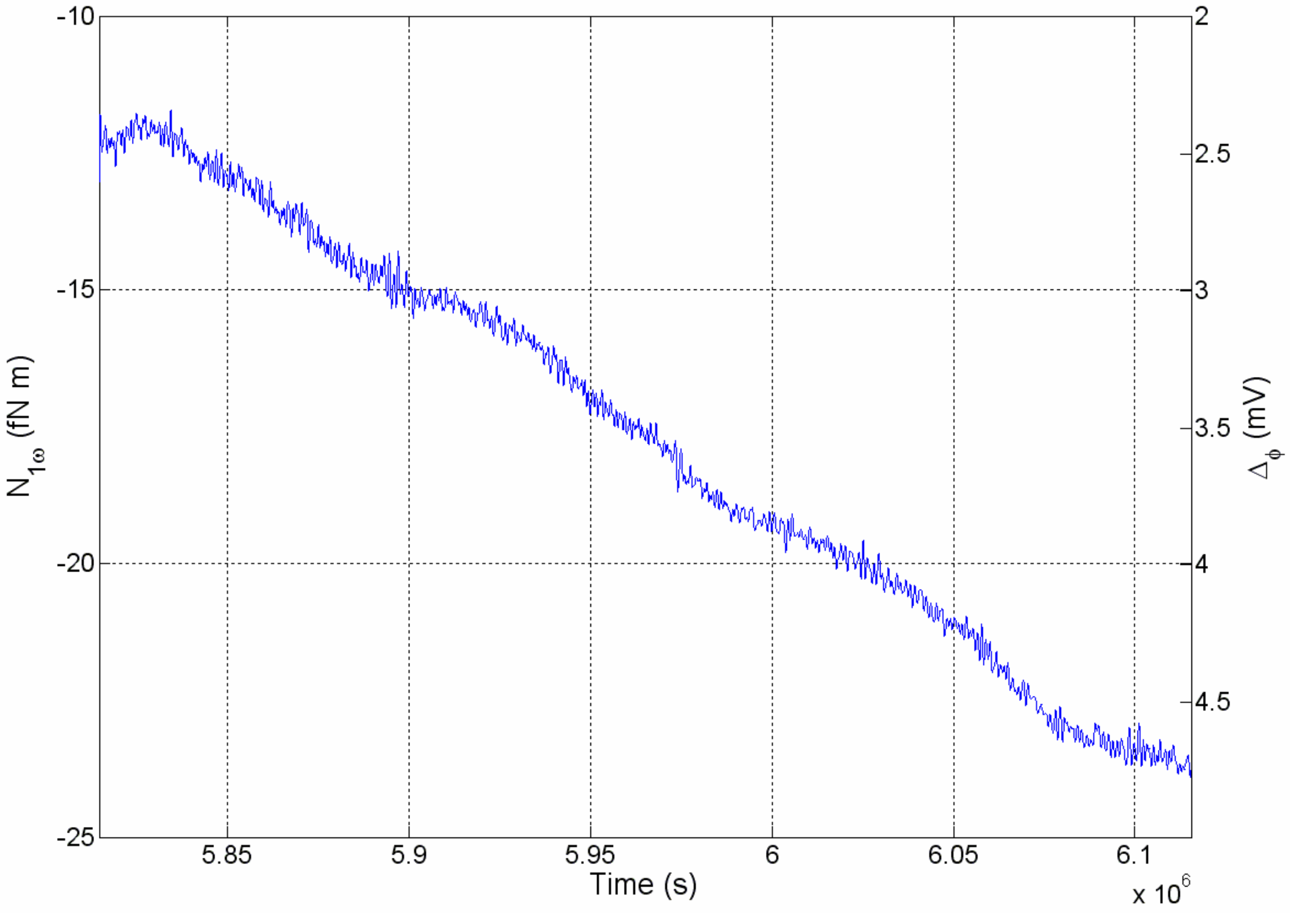
- Actuation amplifier noise (electronics)
- Thermal voltage fluctuations (δ)
- Drifting (not Brownian) DC bias $S_{\delta V}^{1/2}$

DC voltage difference: δV

- Test mass charge
- Residual unbalanced patch effects

LISA goal $v_{\mathrm{n}} \approx 20\ \mu\mathrm{V/Hz}^{1/2}$ at 0.1 mHz

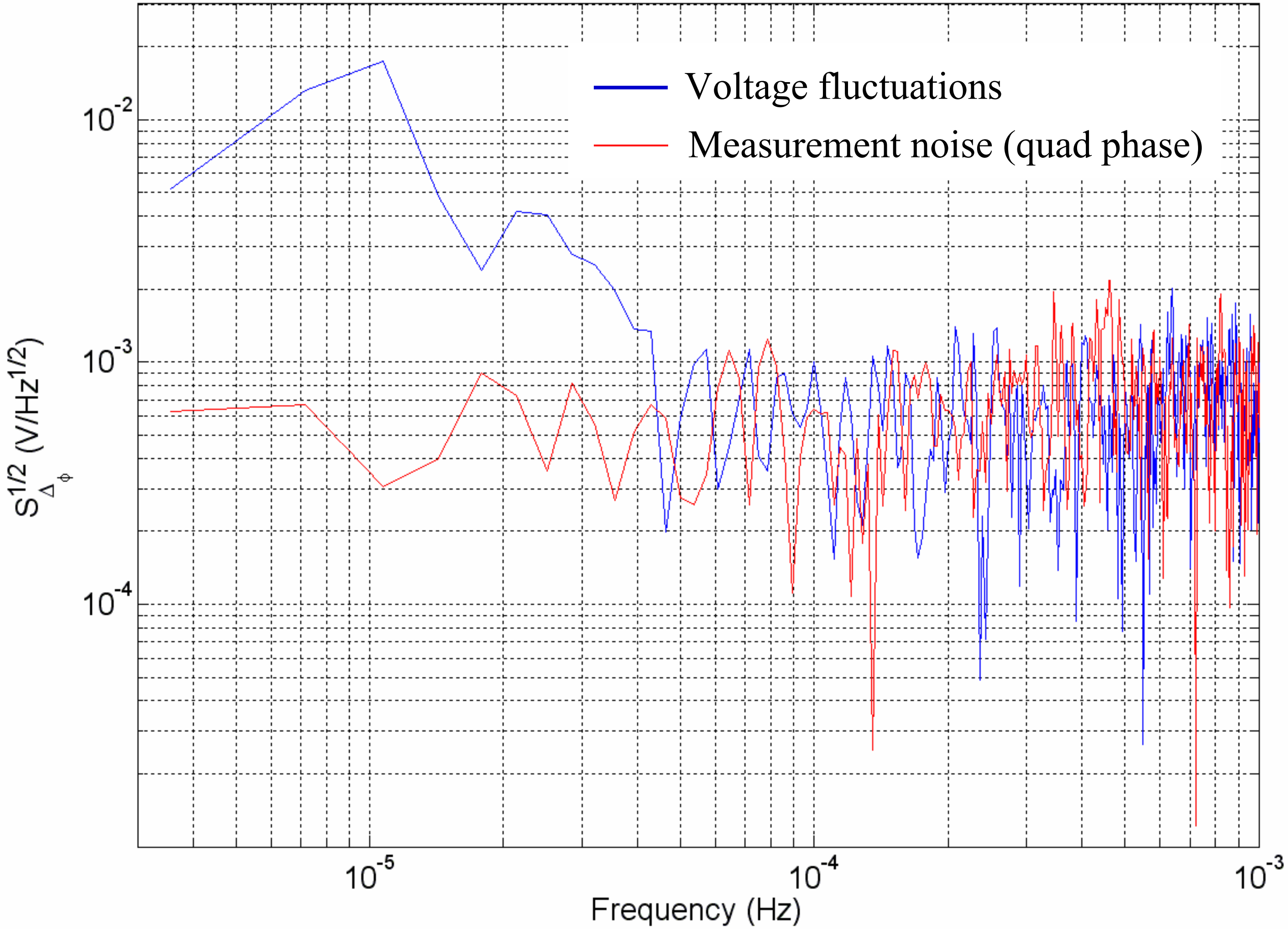
Stability in measured stray "DC" biases



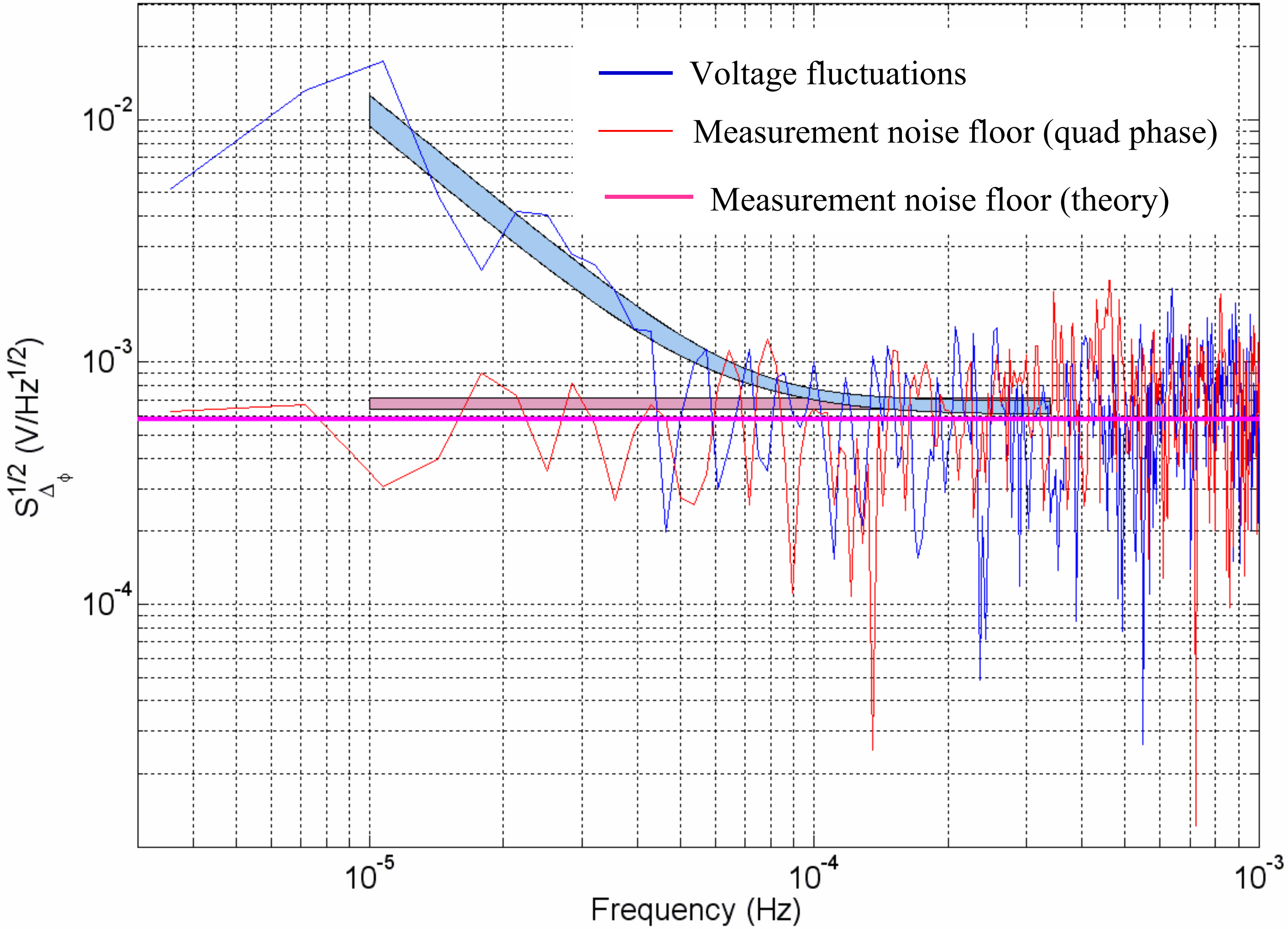
$$\Delta_\phi \equiv \frac{\sum_i \frac{\partial C_i}{\partial \phi} \delta V_i}{\left| \frac{\partial C_x}{\partial \phi} \right|}$$

- Rotational DC bias imbalance Δ_ϕ measured over several days
- "DC" biases drift away from (compensated) null over time
- Need to consider noise in "DC" biases

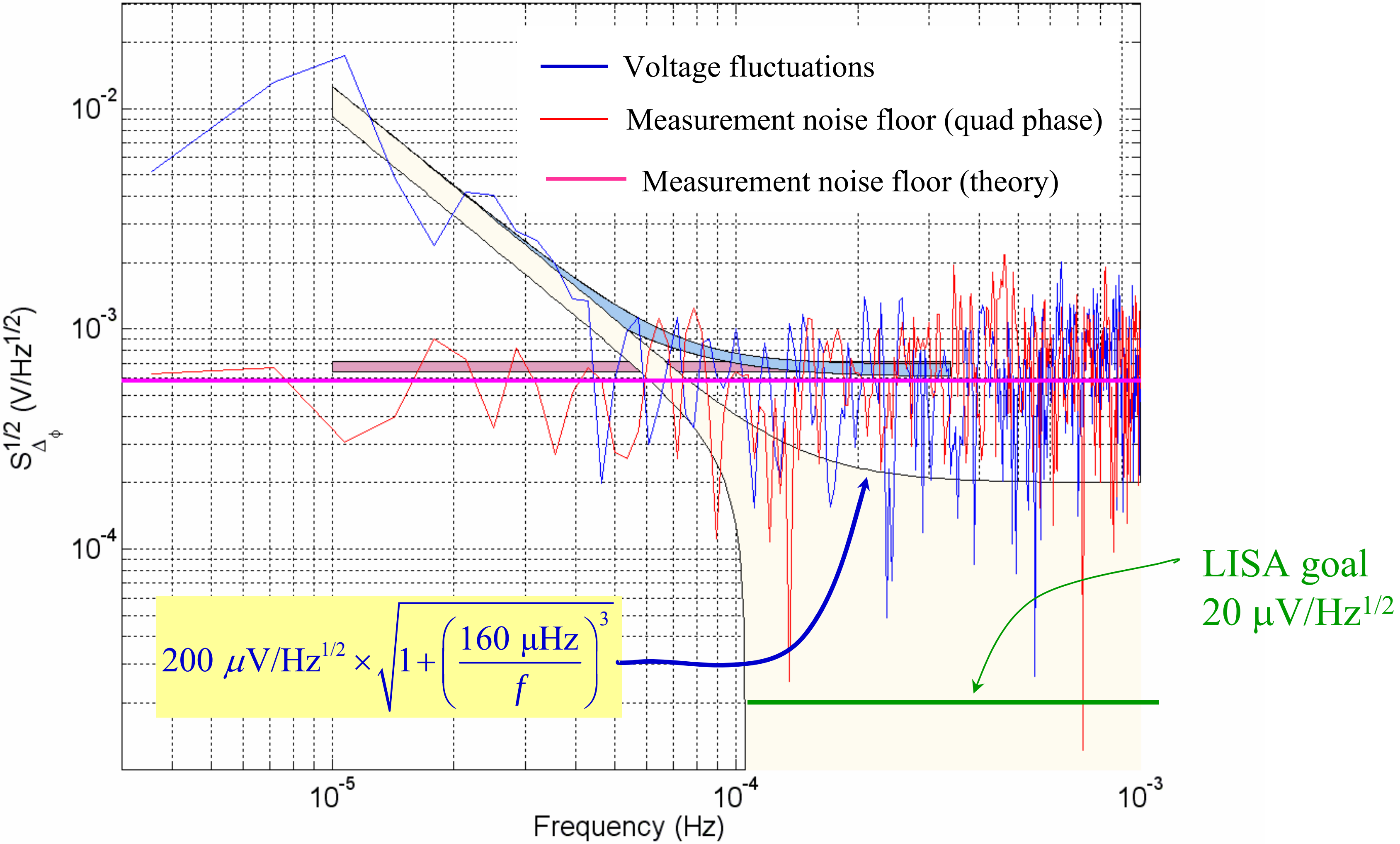
Measured noise in stray “DC” biases



Measured noise in stray “DC” biases



Measured noise in stray “DC” biases

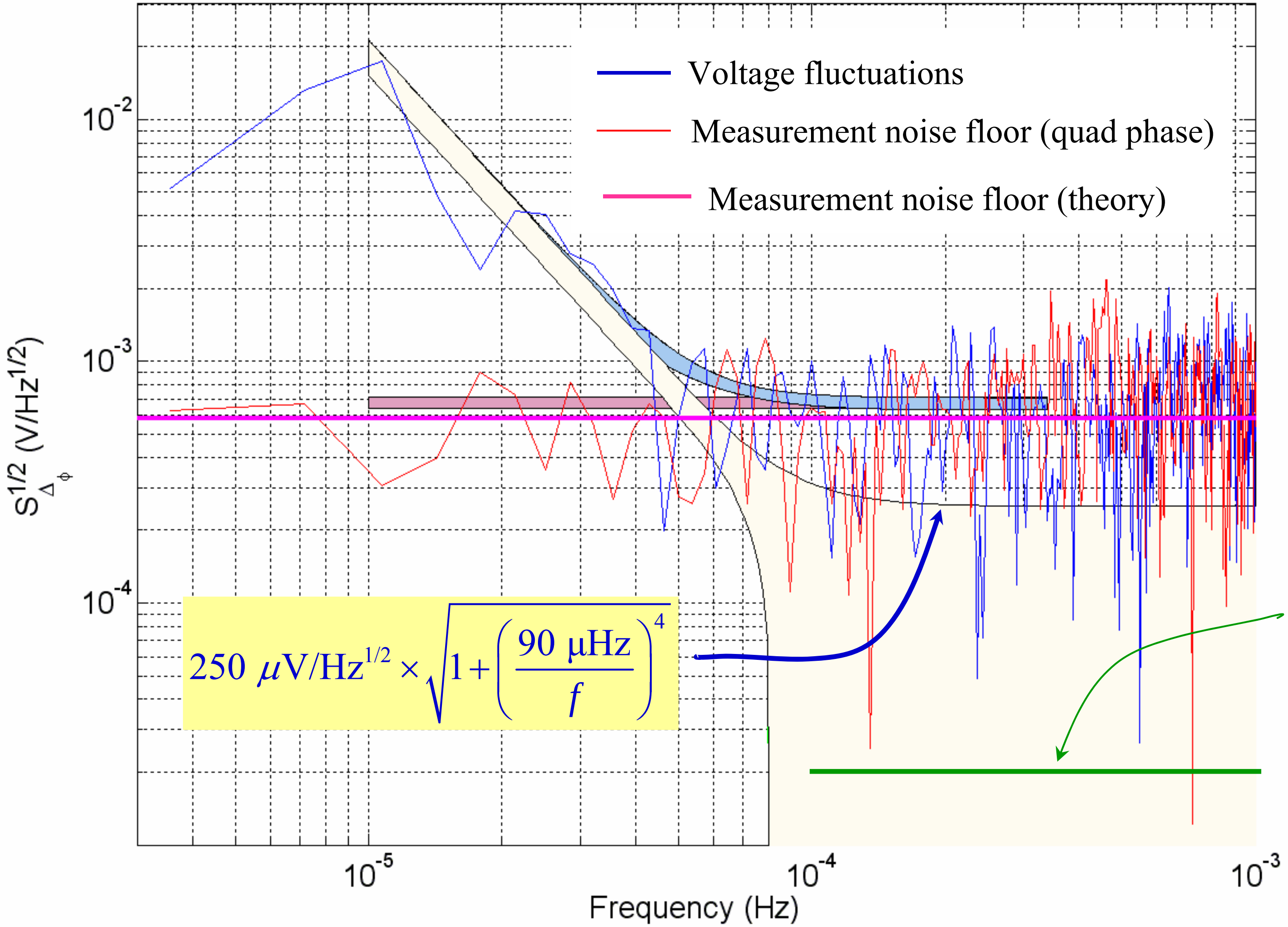


$$200\ \mu\mathrm{V/Hz}^{1/2} \times \sqrt{1+\left(\frac{160\ \mu\mathrm{Hz}}{f}\right)^3}$$

LISA goal
20 μV/Hz$^{1/2}$

- No excess voltage fluctuation noise observed above 0.1 mHz
- 1σ-limit of measurement: 200 μV/ Hz$^{1/2}$ white noise near 0.2 mHz
- fit to $1/f^{3/2}$ excess at lower frequencies

Measured noise in stray “DC” biases

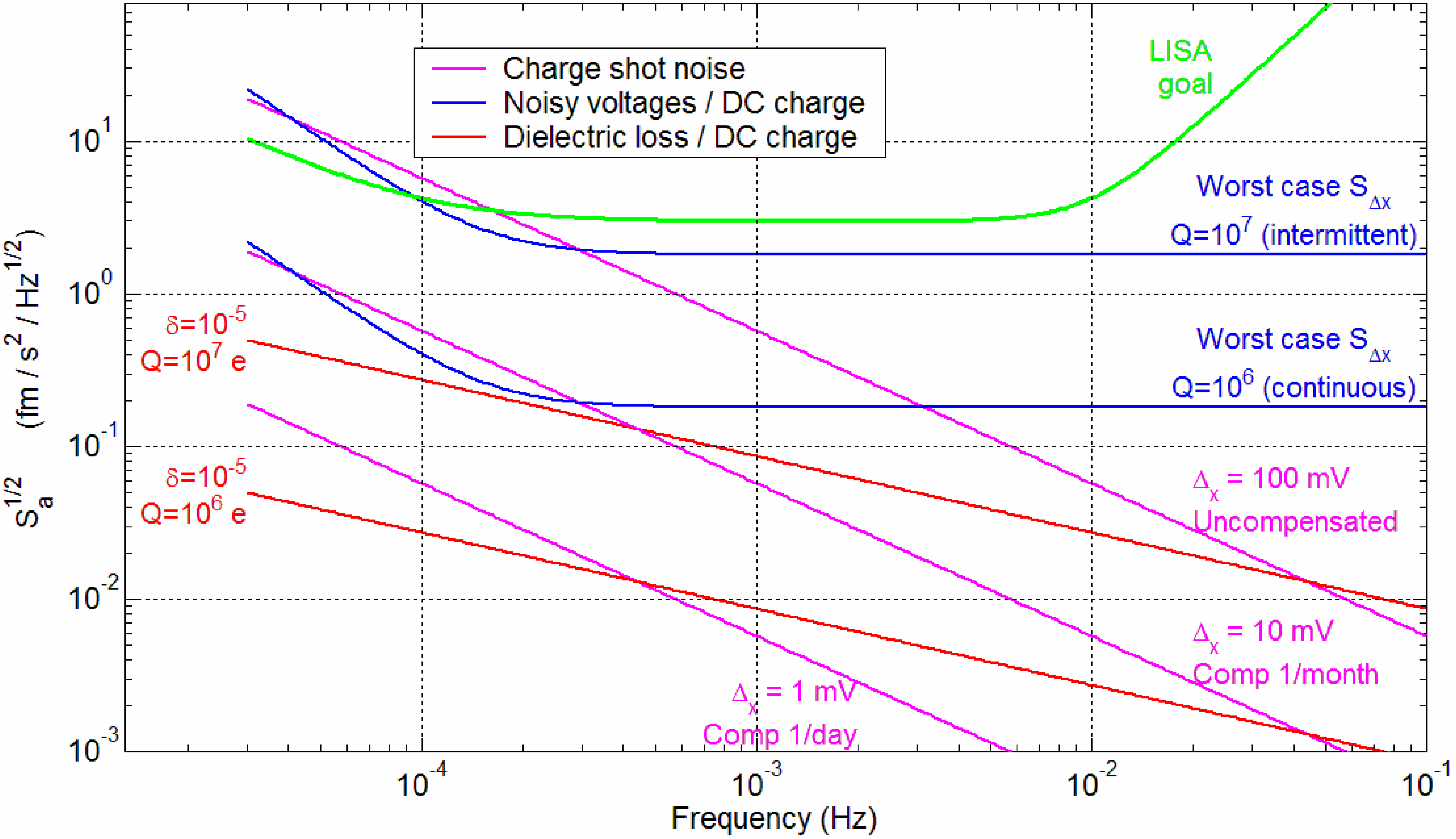


$$250\ \mu\text{V/Hz}^{1/2} \times \sqrt{1+\left(\frac{90\ \mu\text{Hz}}{f}\right)^4}$$

LISA goal
20 μV/Hz$^{1/2}$

- fitting low frequency excess noise to $1/f^2$

Noise budget for charge – stray voltage interaction



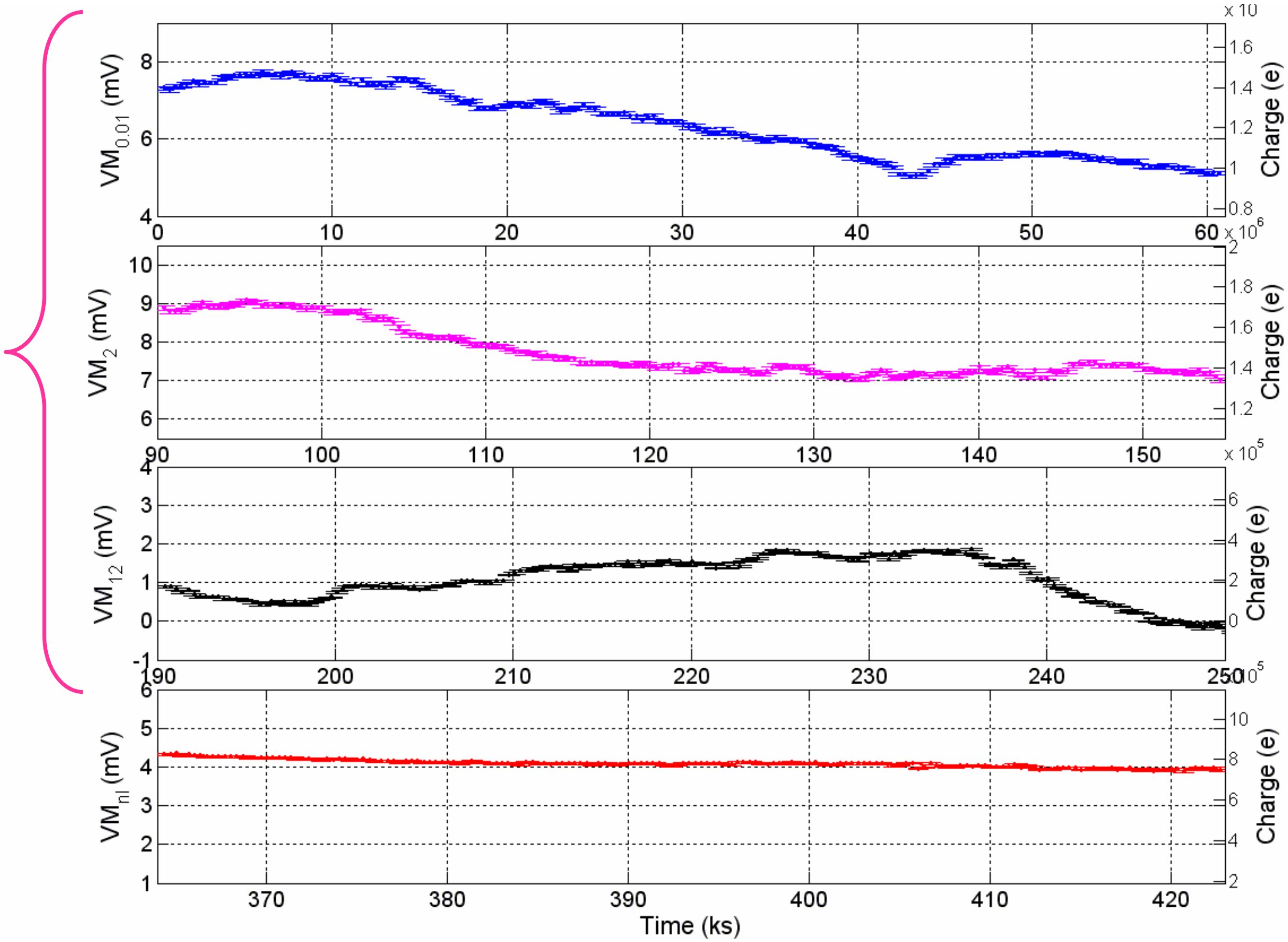
NB: “worst case” for stray voltage fluctuations is measurement limited (true noise likely falls off with increasing frequency)

Continuous discharging tests

[UV discharging tests in collaboration with Imperial College]

- Use two UV lamps, one to charge (+12000 e/s) and one to discharge (-12000 e/s)
- (open loop) charge constant within several mV over 20 hour measurements
- Last measurement in absence of UV light demonstrates charge measurement resolution

With UV!!

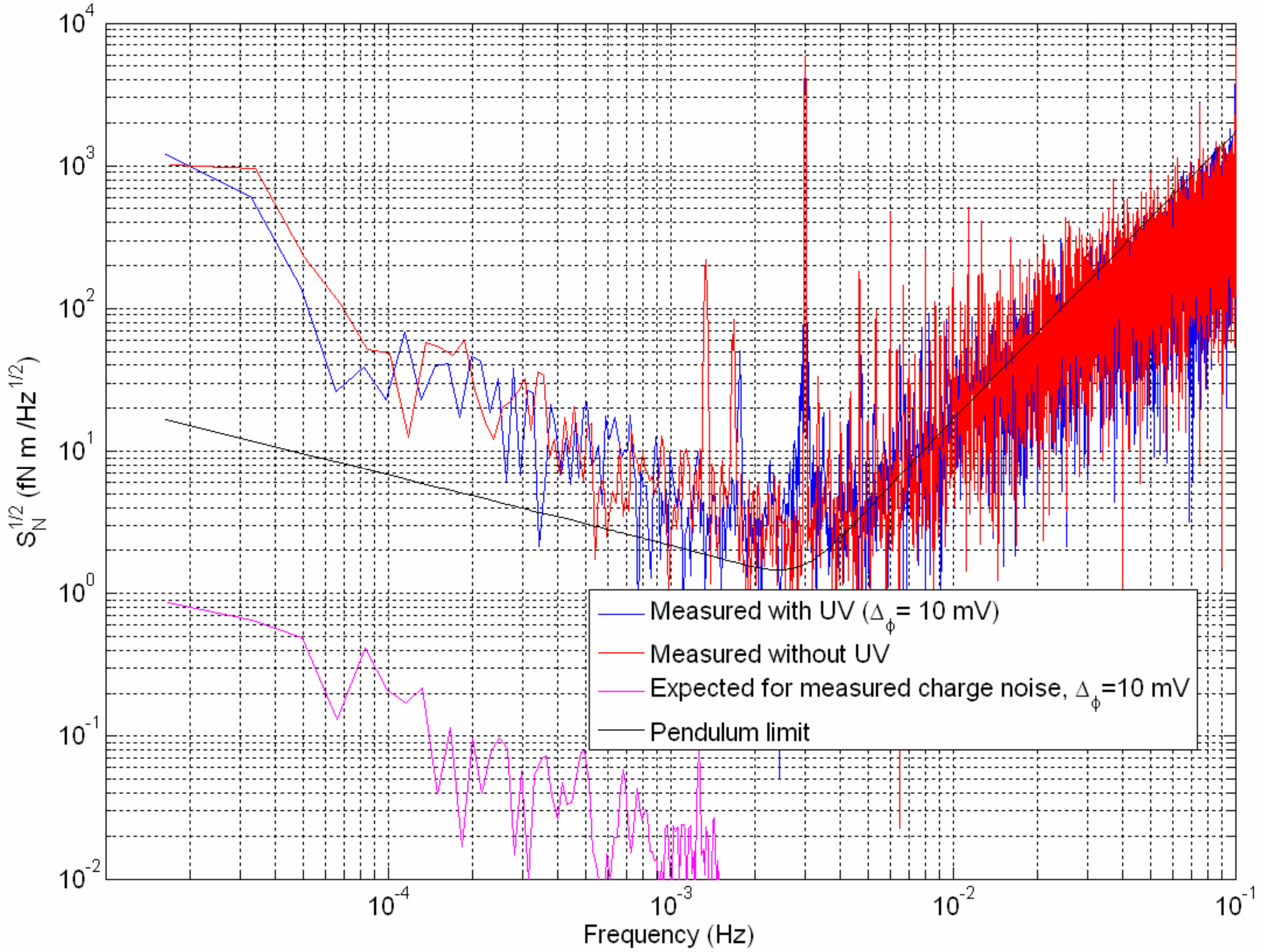


Without UV!!

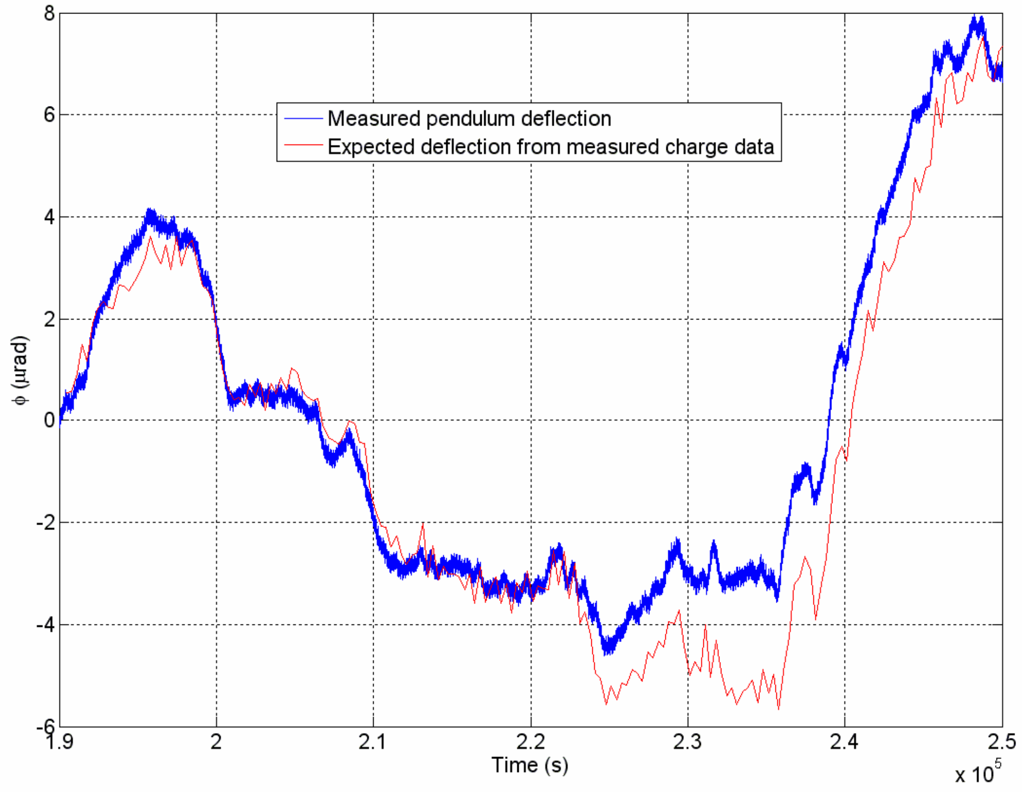
Continuous discharging tests

- create large TM charge fluctuations (20x LISA value) with net current zero by double UV illumination $\lambda_{EFF} > 20000$ e/s

- no net increase in torque noise observed (resolution of roughly 5x LTP goal at 1 mHz)



Experimental verification of random charge force noise model



Torque noise with:

- large charge fluctuations produced by UV illumination

$\lambda_{EFF} > 20000$ e/s

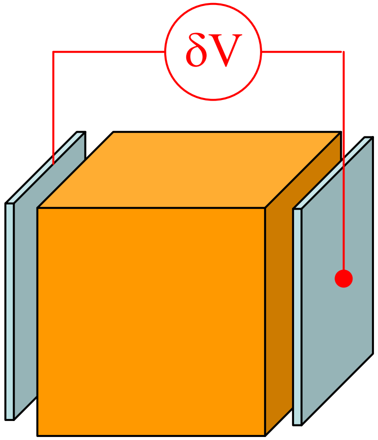
- large applied rotational DC bias

$\Delta_\phi = 12$ V
(+/- 3 V on electrodes)

- Observe low frequency excess in torque noise, in quantitative agreement with random charge model and measured charge fluctuations:

$$N \approx -V_M \left[\sum \frac{\partial C_i}{\partial \phi} V_i \right] \approx -\frac{Q_{TM}}{C_{TOT}} \left| \frac{\partial C_x}{\partial \phi} \right| \Delta_\phi$$

Effect of "self-interacting" fluctuating inhomogeneous DC biases



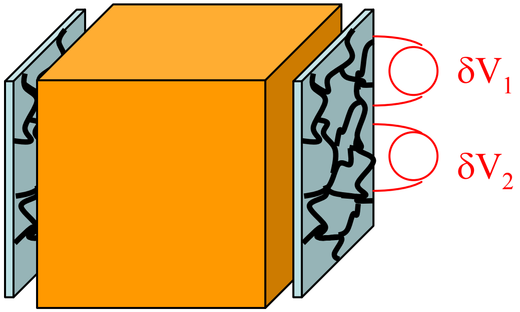
Average DC bias imbalance

- Couples to TM charge
- Balancing δV eliminates charge coupling
- Remove charge, immune to fluctuations in δV

$$S_F^{1/2} = \sqrt{\sum_i \left(\frac{\partial C_i}{\partial x}\right)^2 \delta V_i^{\,2} S_{\delta V_i}}$$

$$\left\langle S_F^{1/2} \right\rangle \approx \sqrt{\frac{N}{4}}\, S_{\Delta_x}^{1/2} \sqrt{\left\langle \Delta_x^2 \right\rangle} \left| \frac{\partial C_x}{\partial x} \right|$$

[N = # domains / electrode]



True electrostatic potential distribution

- Balancing average δV eliminates coupling to TM charge
- **individual domain voltages cannot be compensated**
- **force noise source independent of TM charge**

- Not much data, model dependent!
- Could be worse than $Q_{TM} * S_{\Delta x}^{1/2}$ ($Q_{TM} = 10^7\, e$) by a factor of several

Low frequency electrostatic force noise: conclusions

Experimental data suggest:

- Integrated average DC bias imbalances (Δ_x) of order 100 mV
- Stiffness not likely to be an issue (4 mm gaps!)
- Compensation of (Δ_x) to < mV level → random charging problem curable
- Low frequency drift / fluctuations
    - Need to correct periodically (or continuously) DC bias compensation
    - For f > 0.1 mHz → no excess noise in $S_{\Delta X}$ observed at 200 $\mu V/Hz^{1/2}$ level (still above LISA goal)
    - lower frequency excess observed, not yet understood

    →threatens LISA acceleration goals (in worst case) only at lowest frequencies

- continuous measurement / discharge help reduce noise

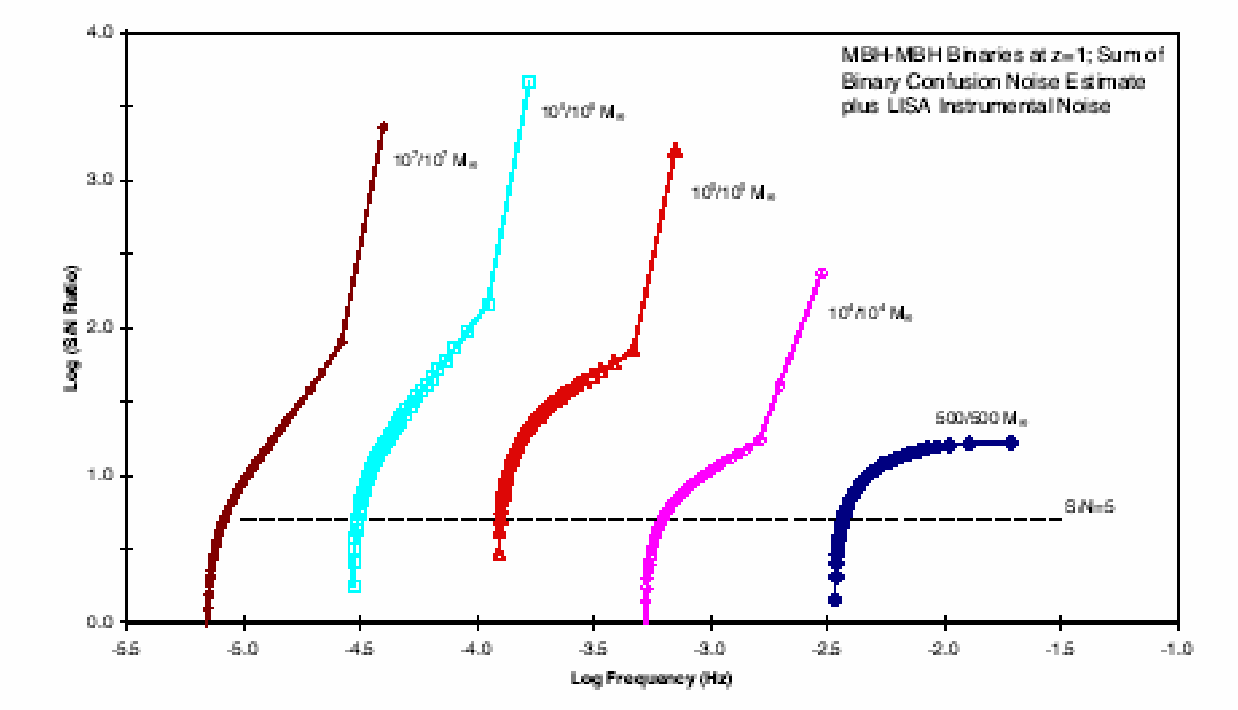
  → Appears possible without introducing force noise

- Interaction between local DC biases and their own fluctuations needs to be understood better

Extra slides

Purity of free-fall critical to LISA science

Example: massive black hole (MBH) mergers
Integrated SNR at 1 week intervals for year before merger

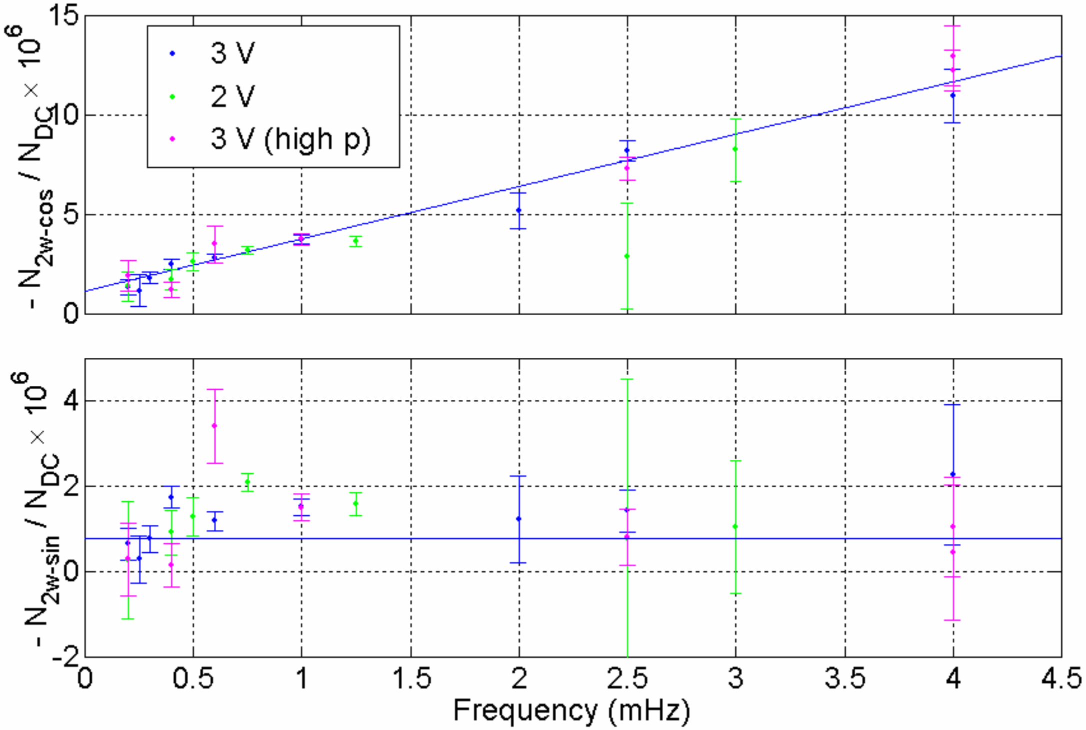


Assuming LISA goal:

$S_a^{1/2} < 3\ \mathrm{fm/s^2/Hz^{1/2}}$
at 0.1 mHz

Acceleration noise at and below 0.1 mHz determines how well, how far, and how early we will see the most massive black hole mergers.

Dielectric Loss Angle Measurement Results



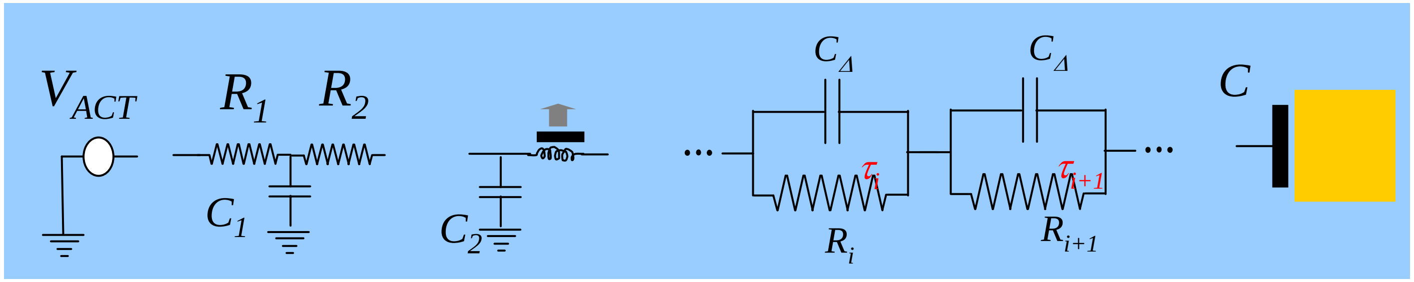
- 2ω cosine torque frequency dependence → ohmic delay time $\tau \approx 0.3$ ms (agrees with calculated value)

- 2ω sine + cosine intercept values → $\delta \approx 10^{-6}$

- likely not a problem for LISA!!

Electrodes 2W/1E	Averaged sine data		Linear fitted cosine data		
	δ (/10^{-6})	χ^2	**τ (ms)**	δ (/10^{-6})	χ^2
3 V (p ≈ 5.e-8 mBar)	.79 ± .07	1.8	.33 ± .02	1.06 ± .16	.86
2 V (p ≈ 5.e-8 mBar)	1.08 ± .09	1.36	.23 ± .05	1.48 ± .31	1.27
3 V (p ≈ 4.e-5 mBar)	.73 ± .14	2.25	.36 ± .03	.60 ± .27	1.27

Electrostatic noise source: thermal voltage noise from dissipation



Characterize surface + circuit dissipation with a capacitive loss angle δ:

$$v_n = \sqrt{4k_BT\frac{\delta}{\omega C}}$$

Thermal voltage noise mixing with DC voltages to produce force noise

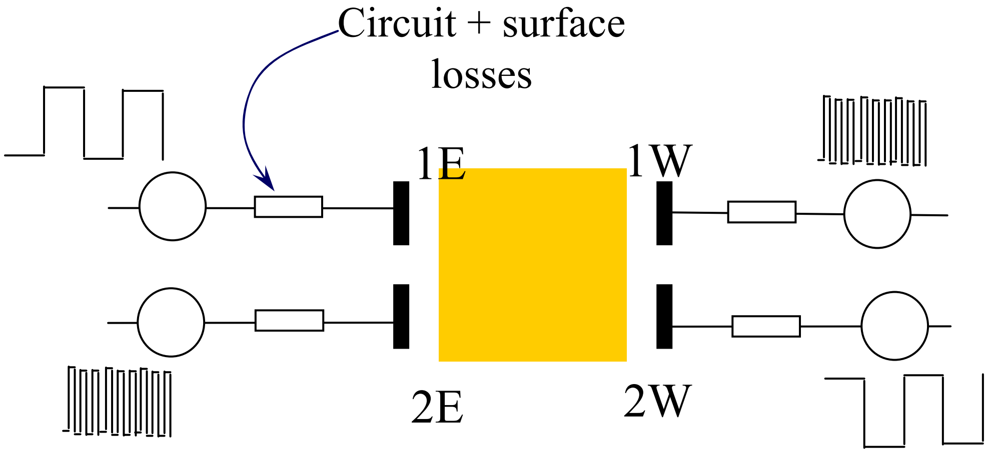
Equivalent

Thermal force noise generated by electrostatic dissipation (imaginary spring constant)

$$S_a^{1/2}(f) \sim .3\times10^{-15}\ \text{fm/s}^2/\sqrt{\text{Hz}}\left(\frac{\delta}{10^{-5}}\right)^{1/2}\left(\frac{10^{-4}\ \text{Hz}}{f}\right)^{1/2}\left(\frac{Q_M}{10^7\ \text{e}}\right)$$

LISA requires δ < 10^{-5}

New technique to measure δ

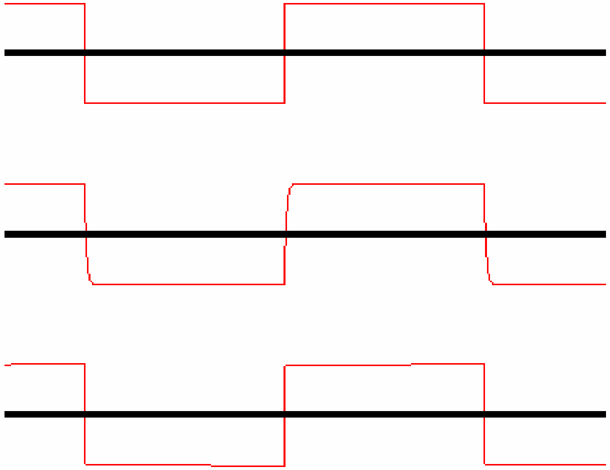


Force (torque) quadratic in voltage

$$F \propto V^2$$

perfect square wave voltage produces only DC force (torque)

Electrode voltage:

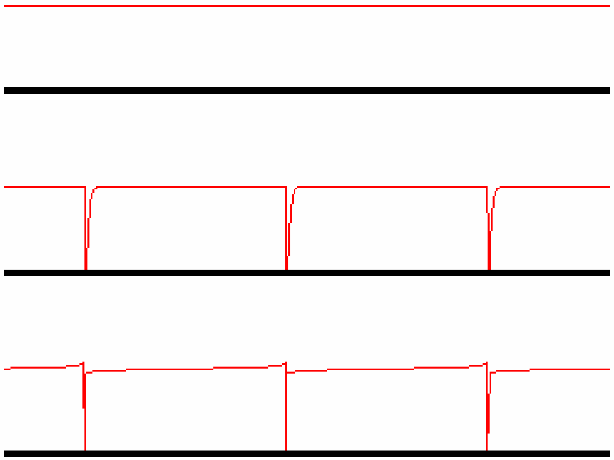


No losses

Ohmic delay

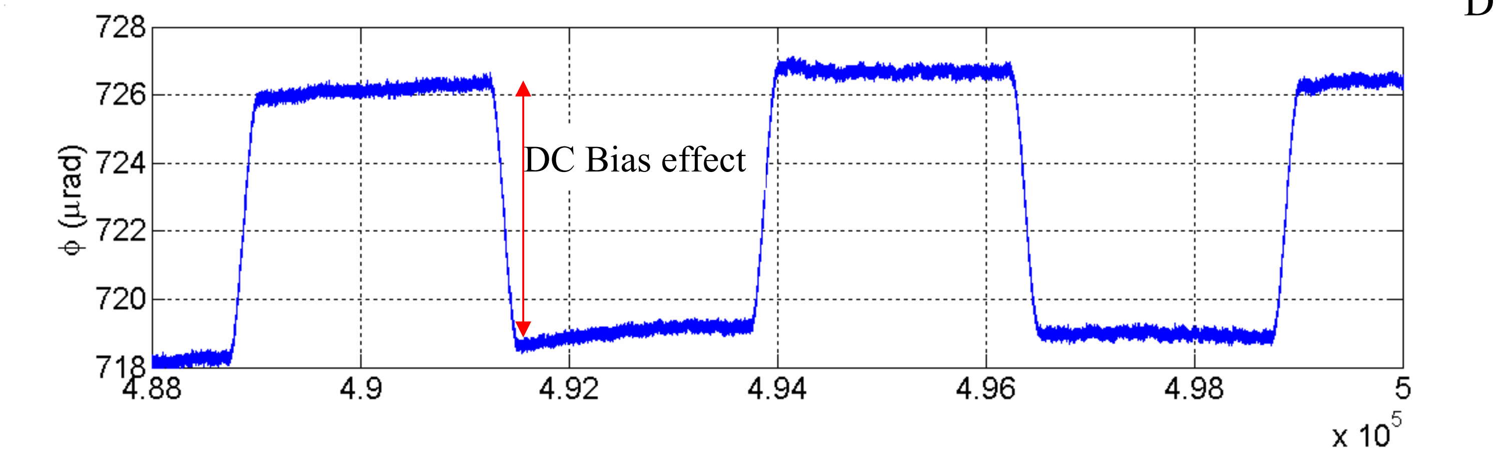
δ constant

Force:



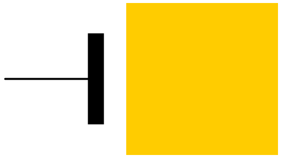
Measurement of dielectric losses: new direct measurement technique

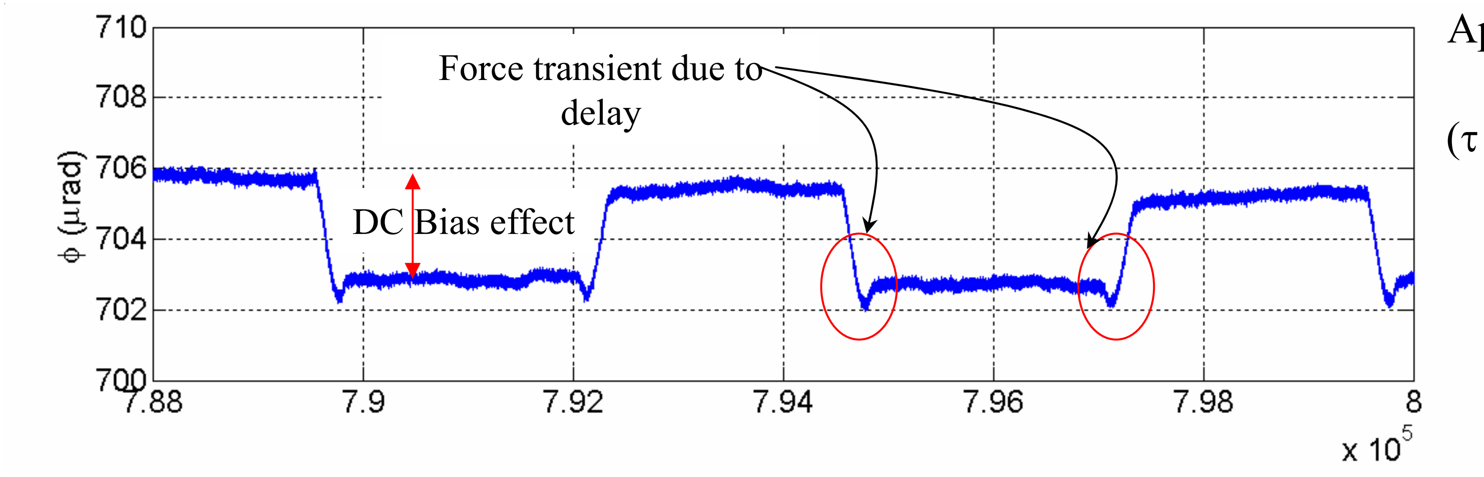
Application of perfect square wave yields constant force
Any lossy element creates delays and thus force transients



DC Bias effect

Direct application
(f = .4 mHz)

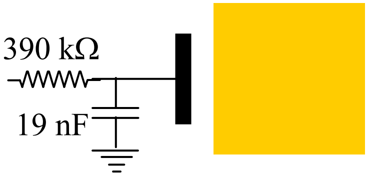




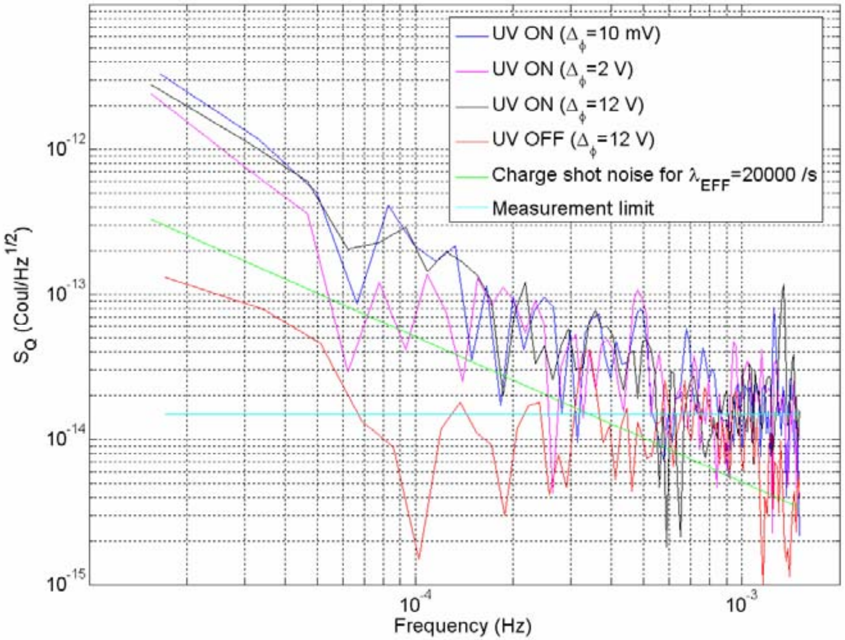
Force transient due to delay

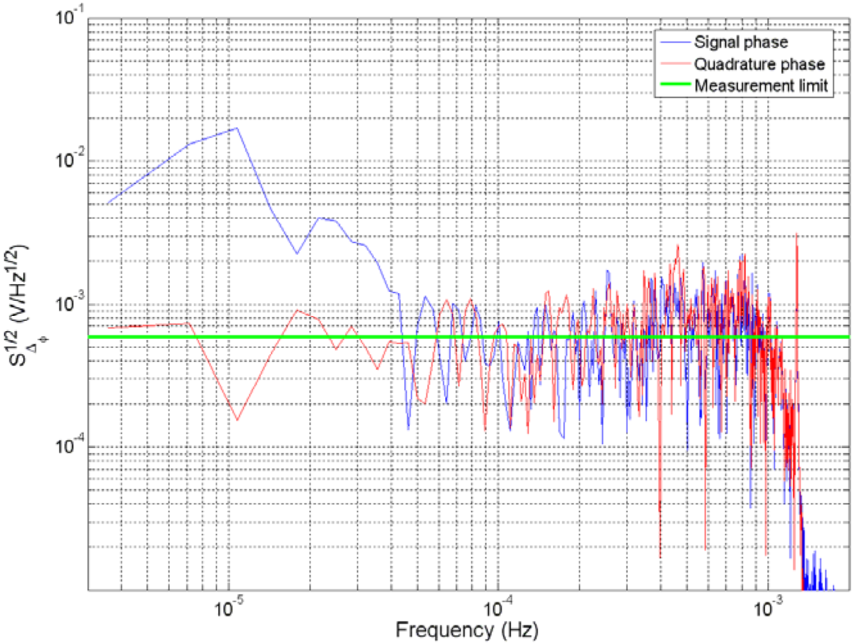
DC Bias effect

Application through an ohmic delay
($\tau \approx 7$ ms, $\delta \approx 2\ 10^{-5}$)



In-flight continuous measurement and compensation of Q, Δ_x





Continuous charge measurement

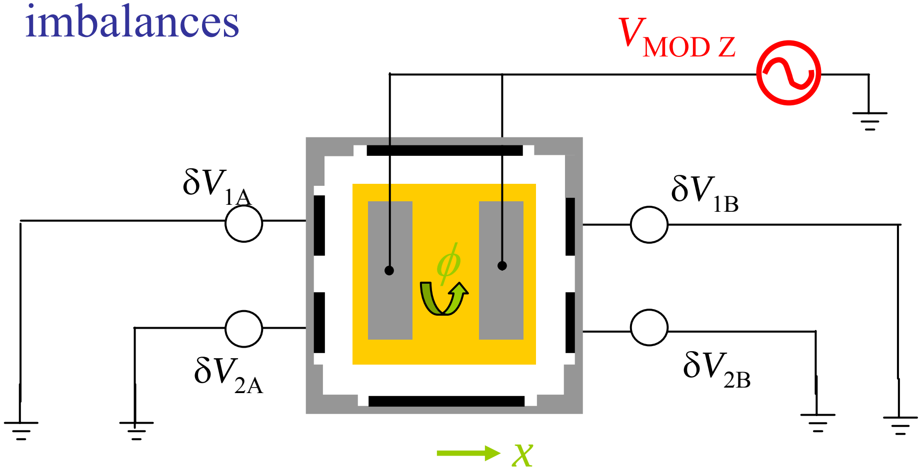
- Sufficient to see charge fluctuations below 0.1 mHz
- Allow “closed loop” continuous charge control to maintain $Q_{TM} < 10^{-6}\ e$
- No disturbance on interferometry axis

Continuous measurement of Δ_x

- Sufficient to measure and compensate low frequency charge fluctuations
- Maintain low Δ_x, reduce low frequency $S_{\Delta x}$
- Demands a force signal on critical interferometry axis

DC Bias measurement and compensation (in lab and in flight)

- Applied oscillating TM bias simulates TM “charge”
- Excites torque and force proportional to integrated rotational and translational DC bias imbalances



$$N = -V_M\left[\sum \frac{\partial C_i}{\partial \phi} V_i\right] \equiv -V_M\left|\frac{\partial C_x}{\partial \phi}\right|\Delta_\phi$$

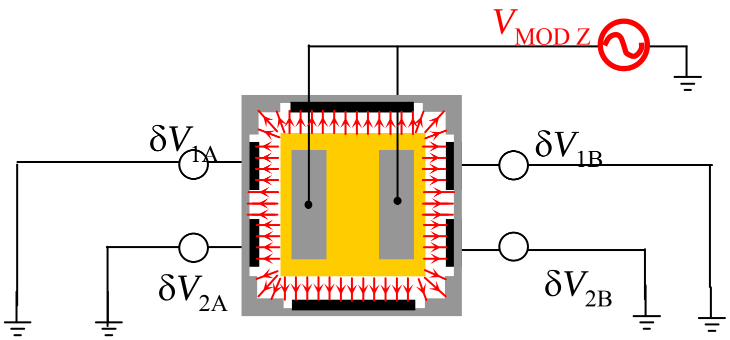
$$F = -V_M\left[\sum \frac{\partial C_i}{\partial x} V_i\right] \equiv -V_M\left|\frac{\partial C_x}{\partial x}\right|\Delta_x$$

Δ_ϕ and Δ_x :

- “averaged” rotational and translational DC bias imbalances
- couple directly to TM charge to produce a torque (force)
- With torsion pendulum, measure and compensate Δ_ϕ
- Δ_ϕ statistically similar to translational imbalance Δ_x

NB: for spatially uniform DC biases: $\Delta_x = \delta V_{1B} + \delta V_{2B} - \delta V_{1B} - \delta V_{2B}$
$\Delta_\phi = -\delta V_{1B} + \delta V_{2B} - \delta V_{1A} + \delta V_{2A}$

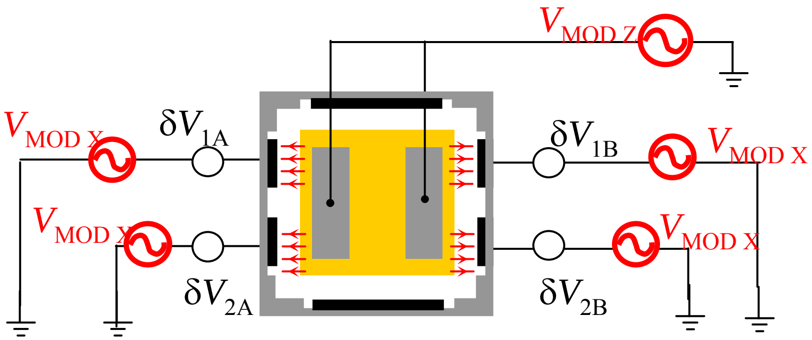
Different applied modulated E-fields
→ Distinguishing DC bias contributions



Modulated ΔV between TM and whole sensor

→ sensitive to sum of all DC biases, (as with TM charge)

$$N = -V_M\left[\sum \frac{\partial C_i}{\partial \phi} V_i\right] \equiv -V_M\left|\frac{\partial C_x}{\partial \phi}\right|\Delta_\phi$$



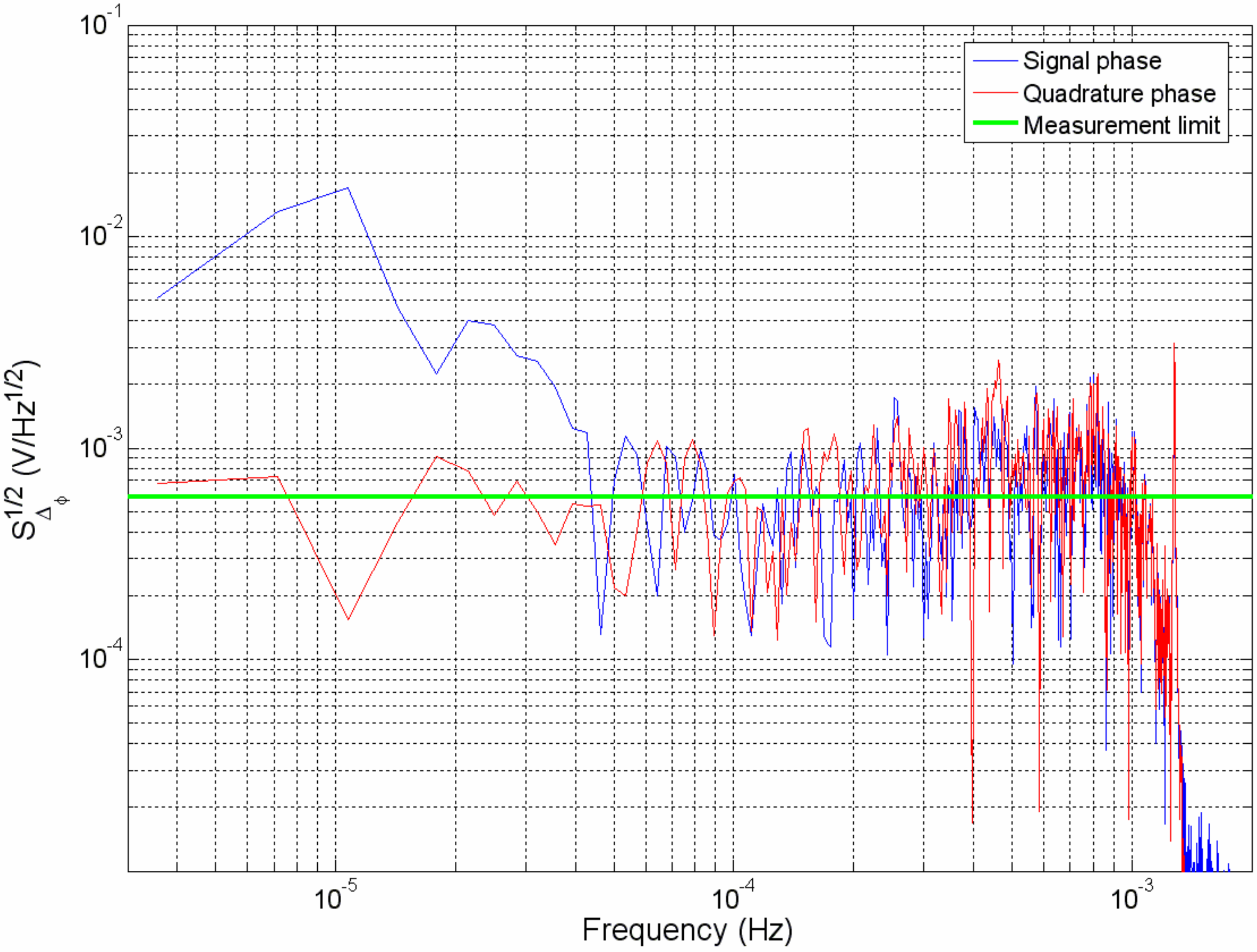
Modulated ΔV only between TM and x-electrodes

→ sensitive only to x-electrode DC biases

$$N = -V_M\left[\sum_{i(x\,\text{el})} \frac{\partial C_i}{\partial \phi} V_i\right] \equiv -V_M\left|\frac{\partial C_x}{\partial \phi}\right|\Delta_{\phi(x\,\text{el})}$$

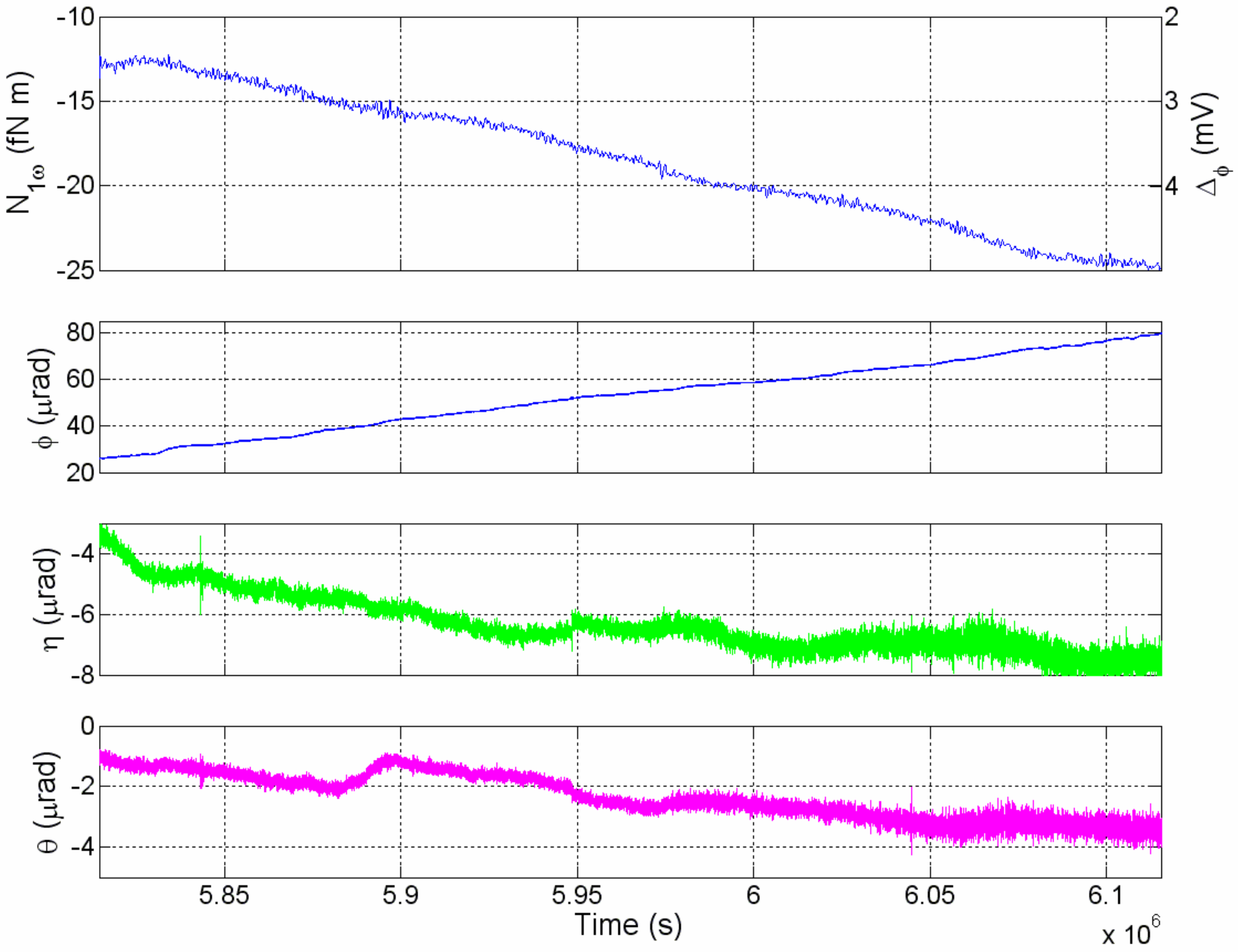
- Can distinguish and compensate DC bias contributions from different electrodes
- As DC biases arise in electrodes and guard ring surfaces, cannot simultaneously compensate both overall DC bias (Δ_ϕ or Δ_x) and individual electrode DC biases (δV_i)
- **True intrinsic DC bias values are important**

Measured noise in stray “DC” biases

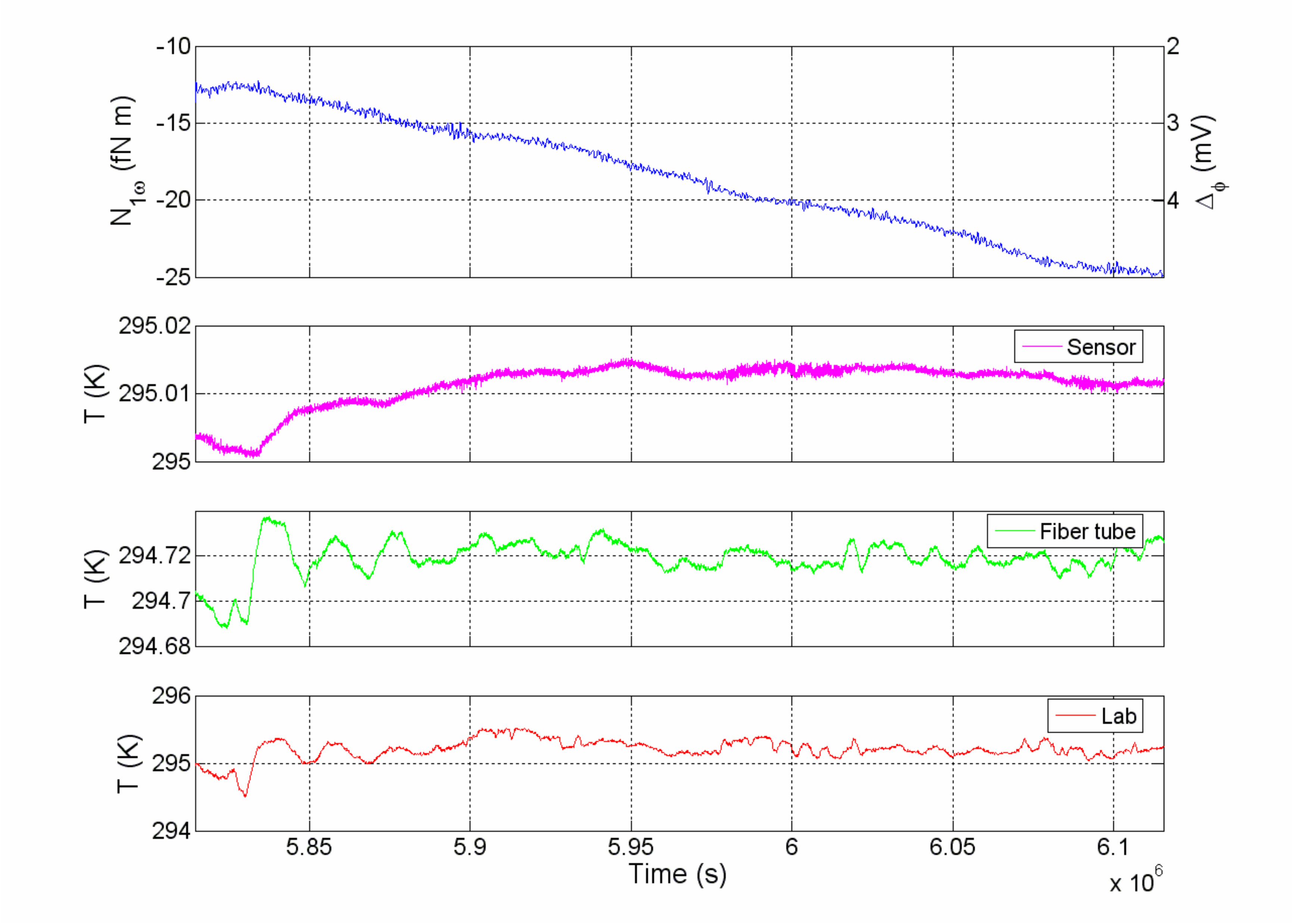


- Excess noise in Δ_ϕ observed below 50 μHz
- Measurement limit (roughly 600 μV/Hz$^{1/2}$) factor 30 - 50 above LISA goal

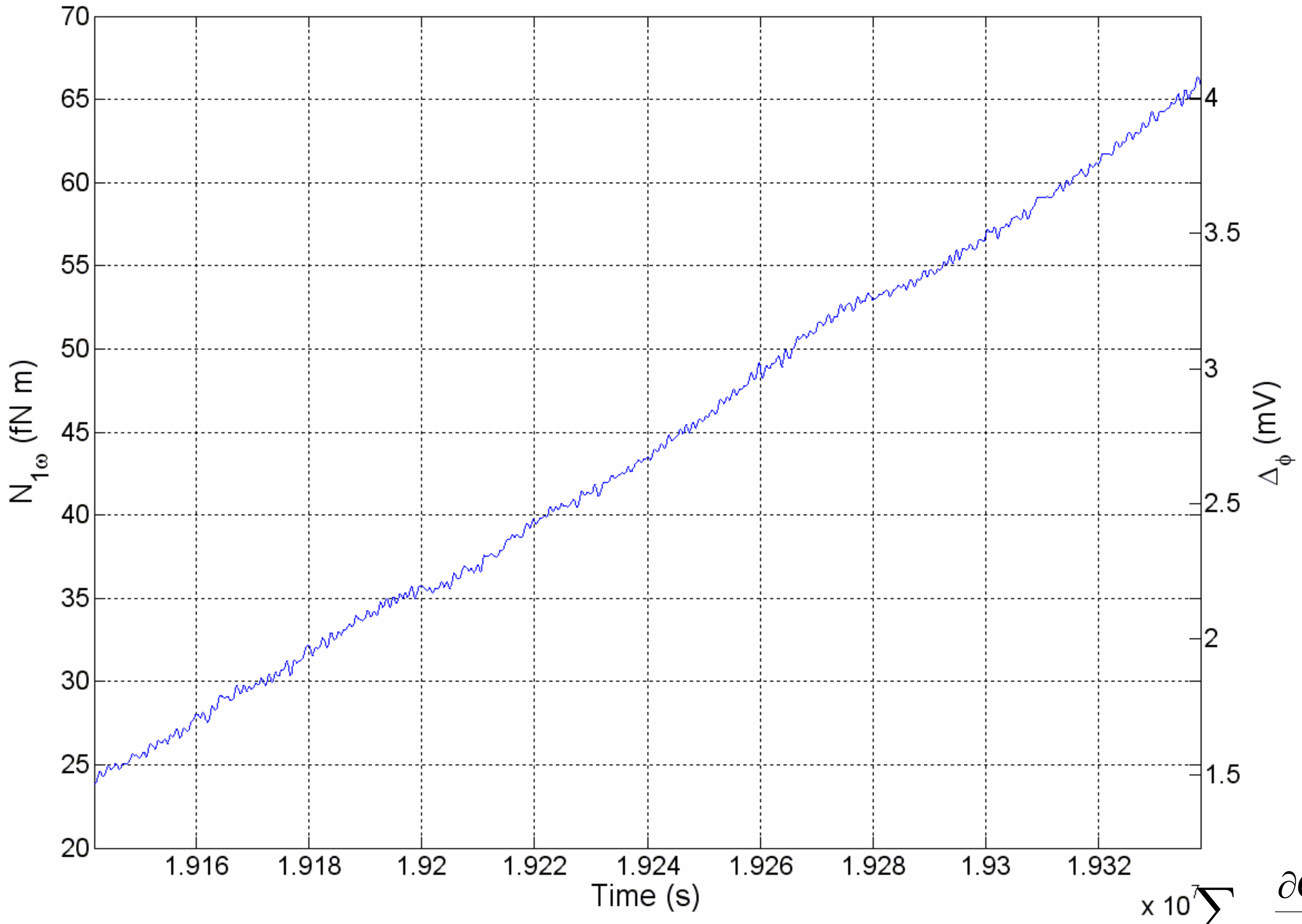
DC Bias measurement fluctuation correlations with TM motion



DC Bias measurement fluctuation correlations with TM motion



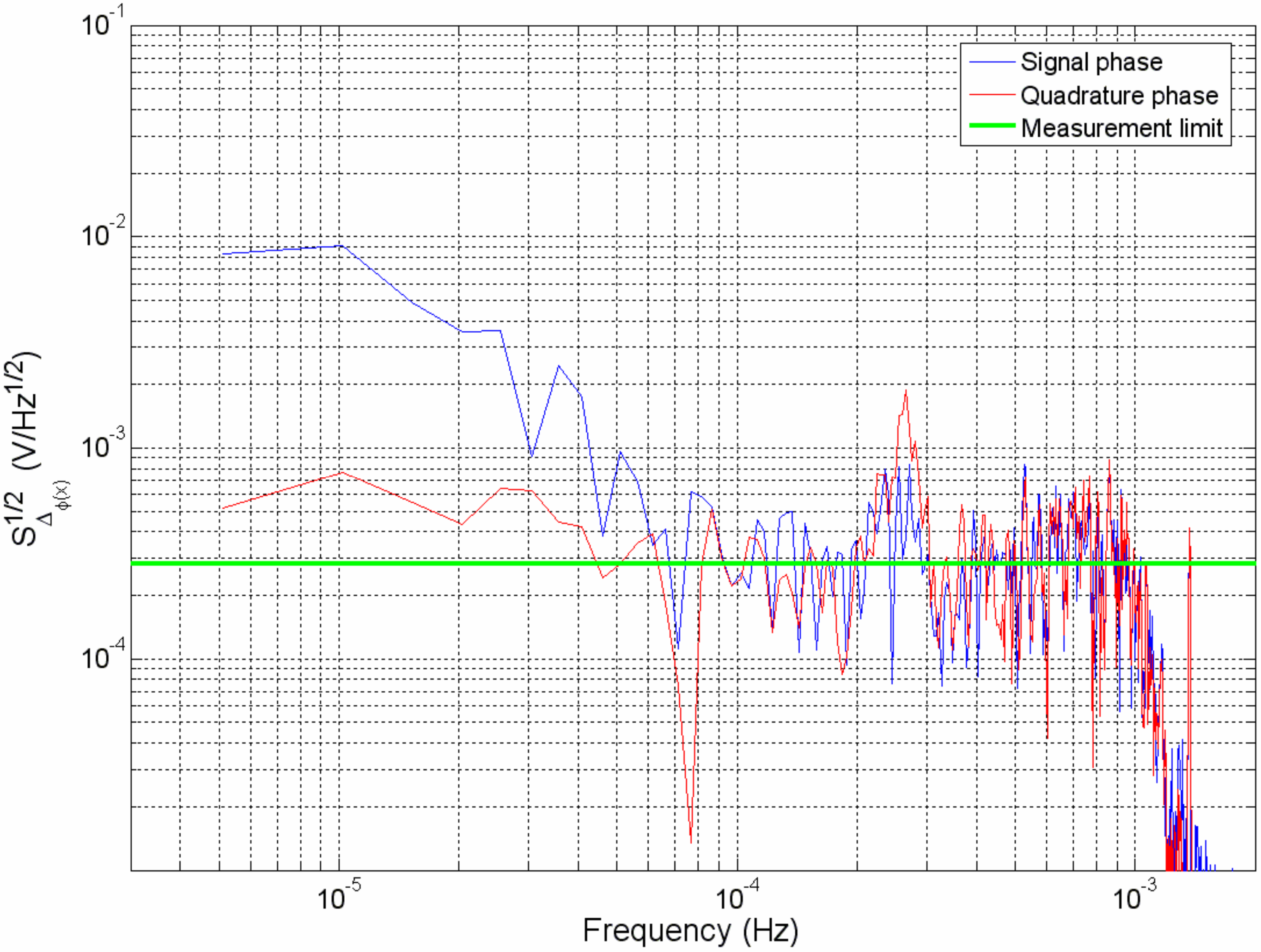
Stability of x-electrode DC biases



$$\Delta_{\phi(x)} \equiv \frac{\sum_{x\ \text{electrodes}} \frac{\partial C_i}{\partial \phi} \delta V_i}{\left| \frac{\partial C_x}{\partial \phi} \right|}$$

Measurement of $\Delta_{\phi(x)}$ using V_{COMP}= + 20 mV

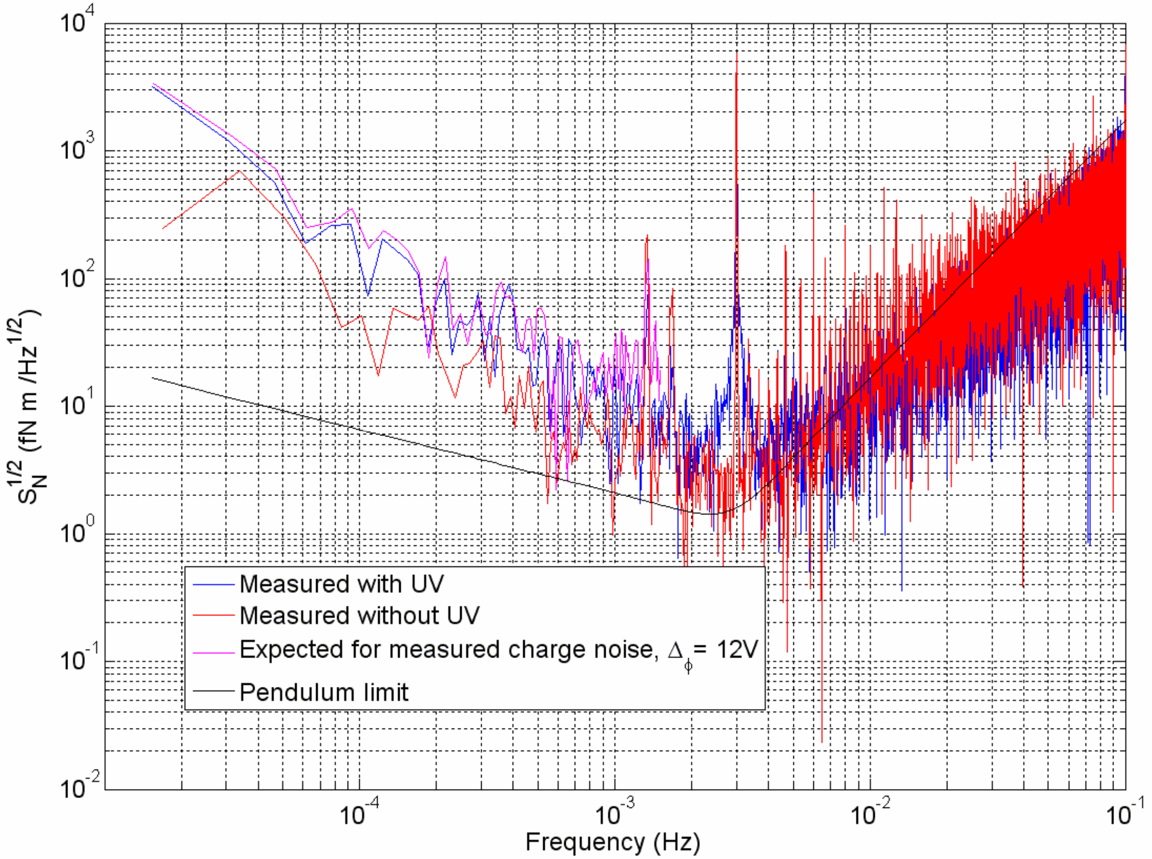
Noise in x-electrode DC biases



Measurement of $\Delta_{\phi(x)}$ using $V_{COMP} = +20$ mV

Experimental verification of random charge force noise model

[UV discharging tests in collaboration with Imperial College]



Torque noise excess with:

- large TM charge fluctuations produce by UV illumination

$$\lambda_{EFF} > 20000 \text{ e/s}$$

- large applied rotational DC bias

$$\Delta_\phi = 12 \text{ V}$$

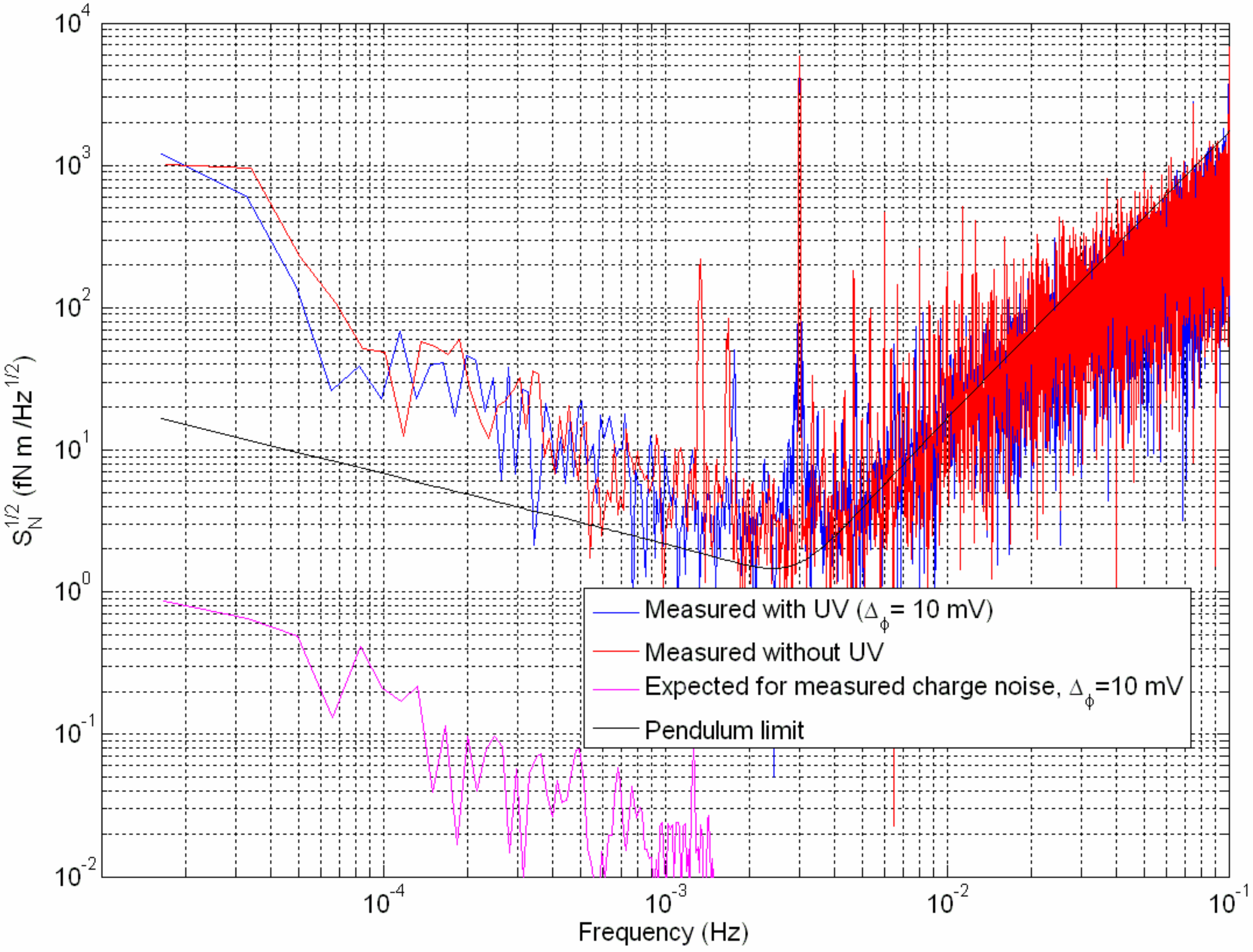
- Observe low frequency excess in torque noise, in quantitative agreement with random charge model and measured charge fluctuations:

$$N \approx -V_M \left[\sum \frac{\partial C_i}{\partial \phi} V_i \right] \approx -\frac{Q_{TM}}{C_{TOT}} \left| \frac{\partial C_x}{\partial \phi} \right| \Delta_\phi$$

Continuous discharging tests

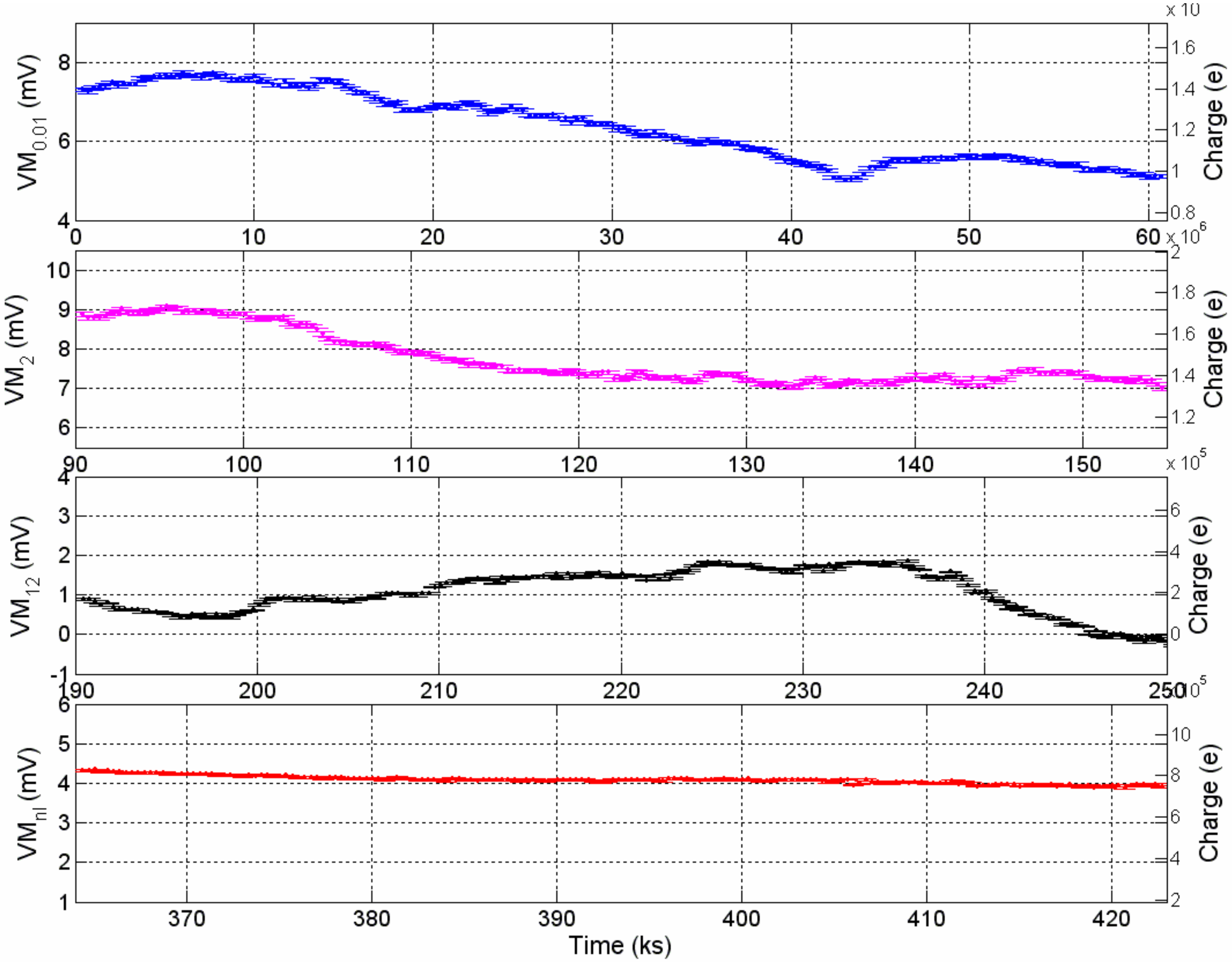
- create large TM charge fluctuations (20x LISA value) with net current zero by double UV illumination $\lambda_{EFF} > 20000$ e/s

- no net increase in torque noise observed (resolution of roughly 5x LTP goal at 1 mHz)



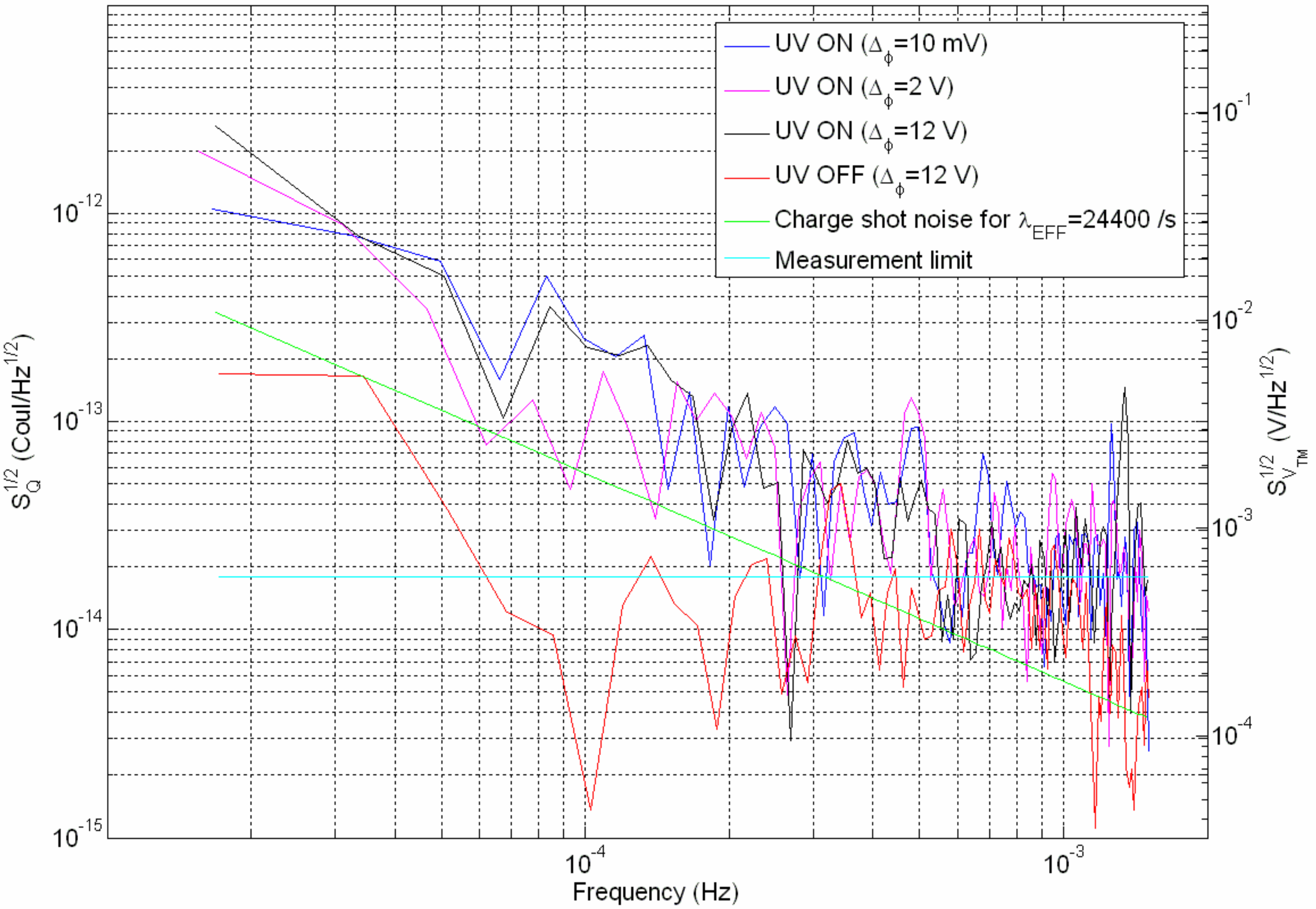
Continuous discharging tests

- Use two UV lamps, one to charge (+12000 e/s) and one to discharge (-12000 e/s)
- (open loop) maintain charge constant within several mV (within 10 mV of 0) over 20 hour measurements
- Last measurement in absence of UV light demonstrates charge measurement resolution

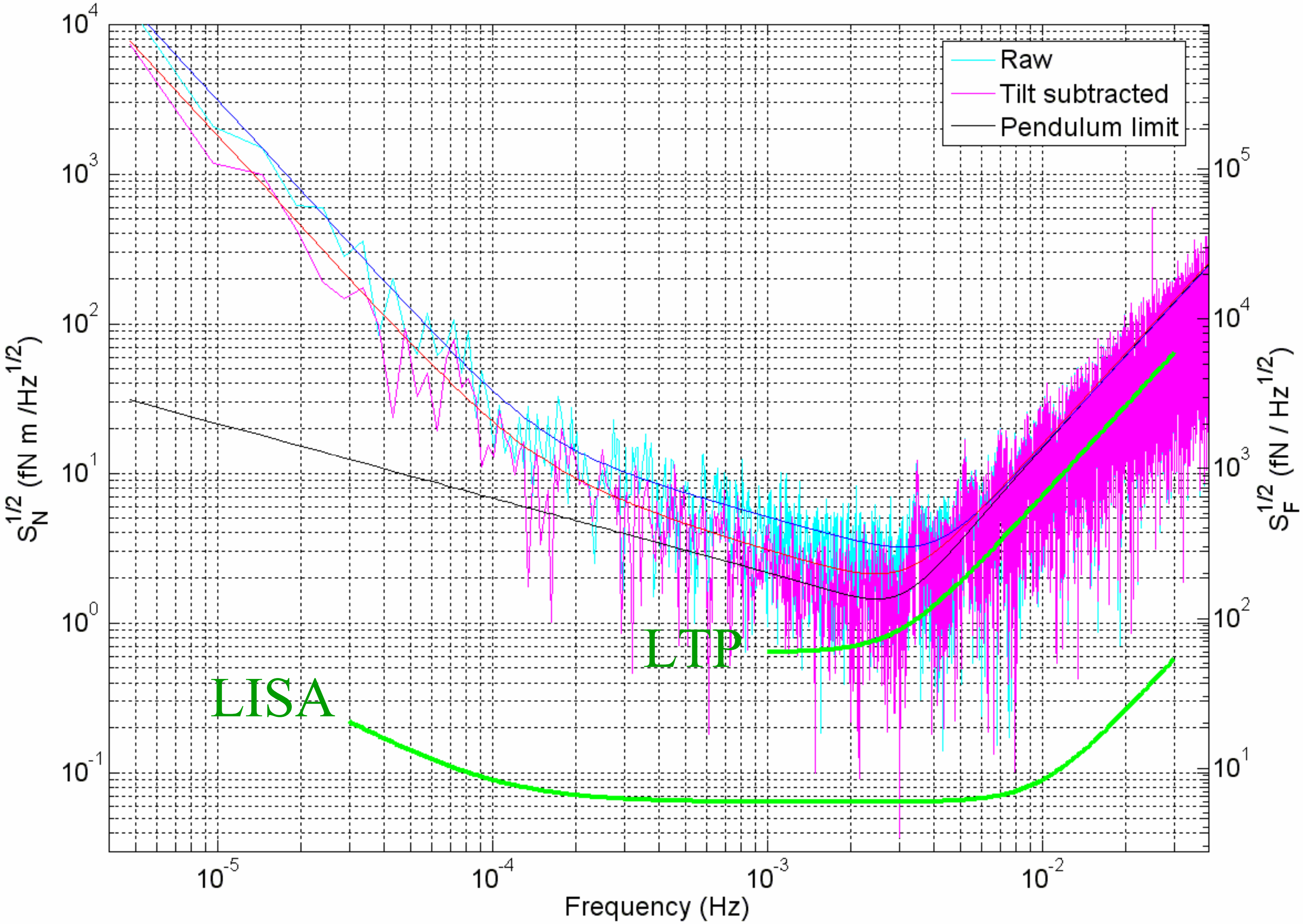


Continuous discharging tests

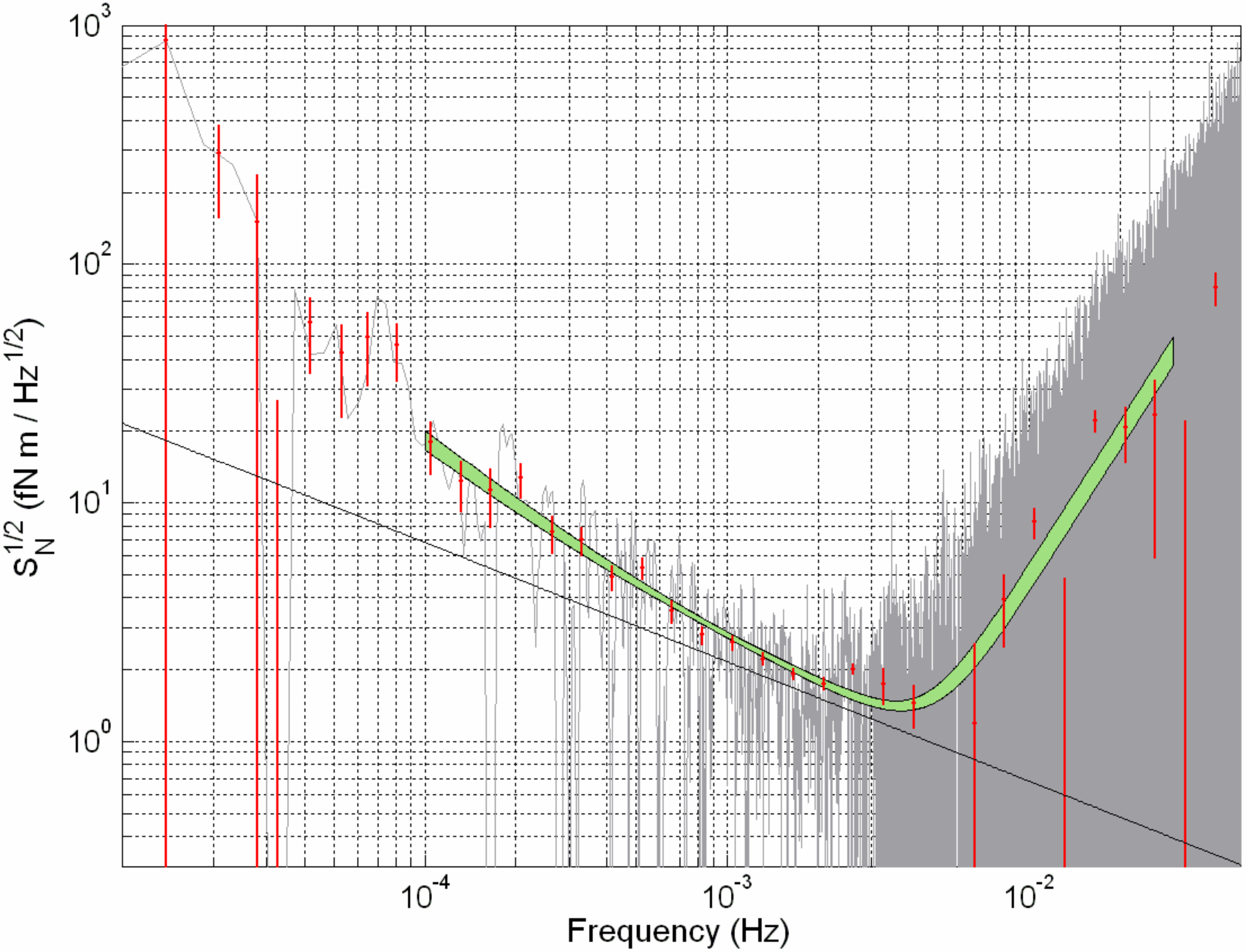
- measurement resolution (seen above 10^{-4} Hz in absence of UV) 10^5 e/Hz$^{1/2}$
- with UV light, measured charge noise roughly 3 x the minimum shot noise level, consistent with UV power fluctuations



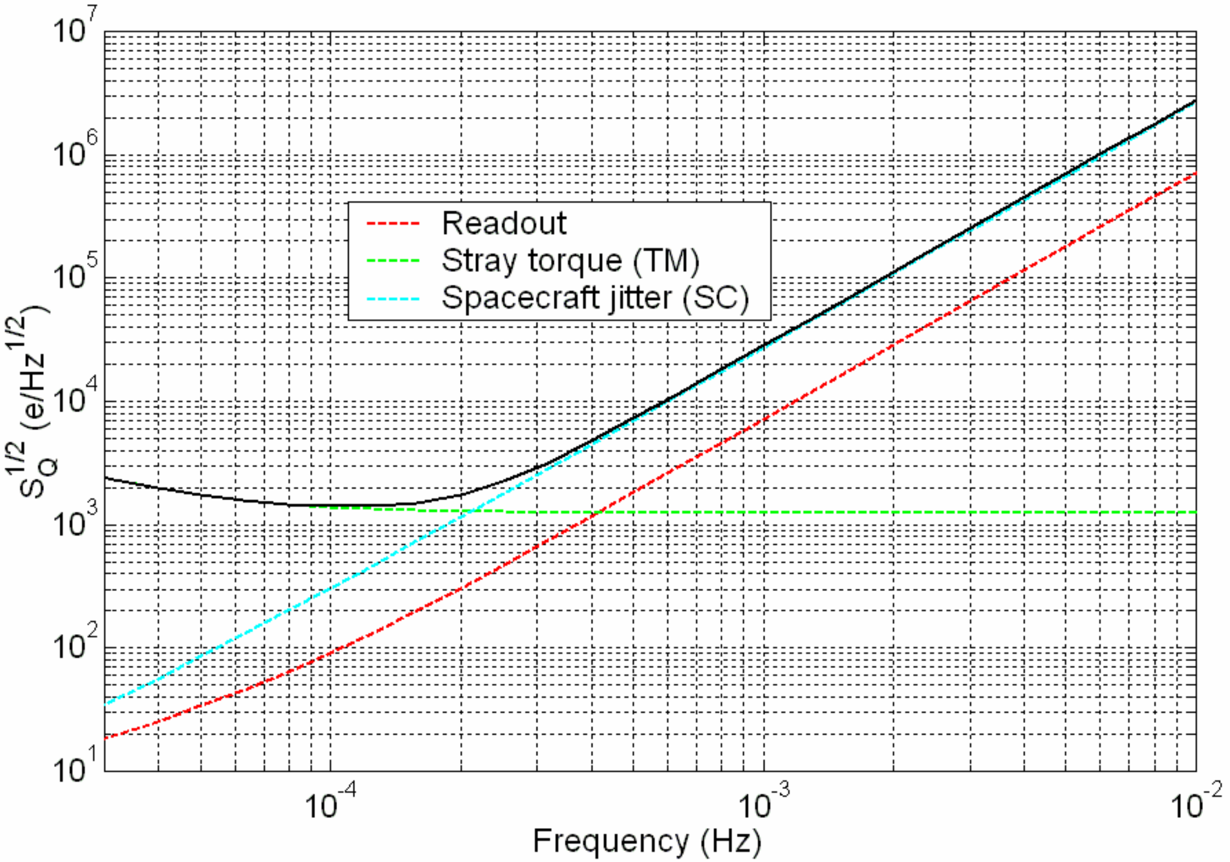
Sensor force noise upper limits from torsion pendulum noise data



- Factor of 50 above LISA goal at 1 mHz
- Factor of 300 above LISA goal at 0.1 mHz


10^3
10^2
10^1
10^0
10^{-4}
10^{-3}
10^{-2}
$S_N^{1/2}$ (fN m / Hz$^{1/2}$)
Frequency (Hz)

Charge measurement resolution: 1 mass config, η

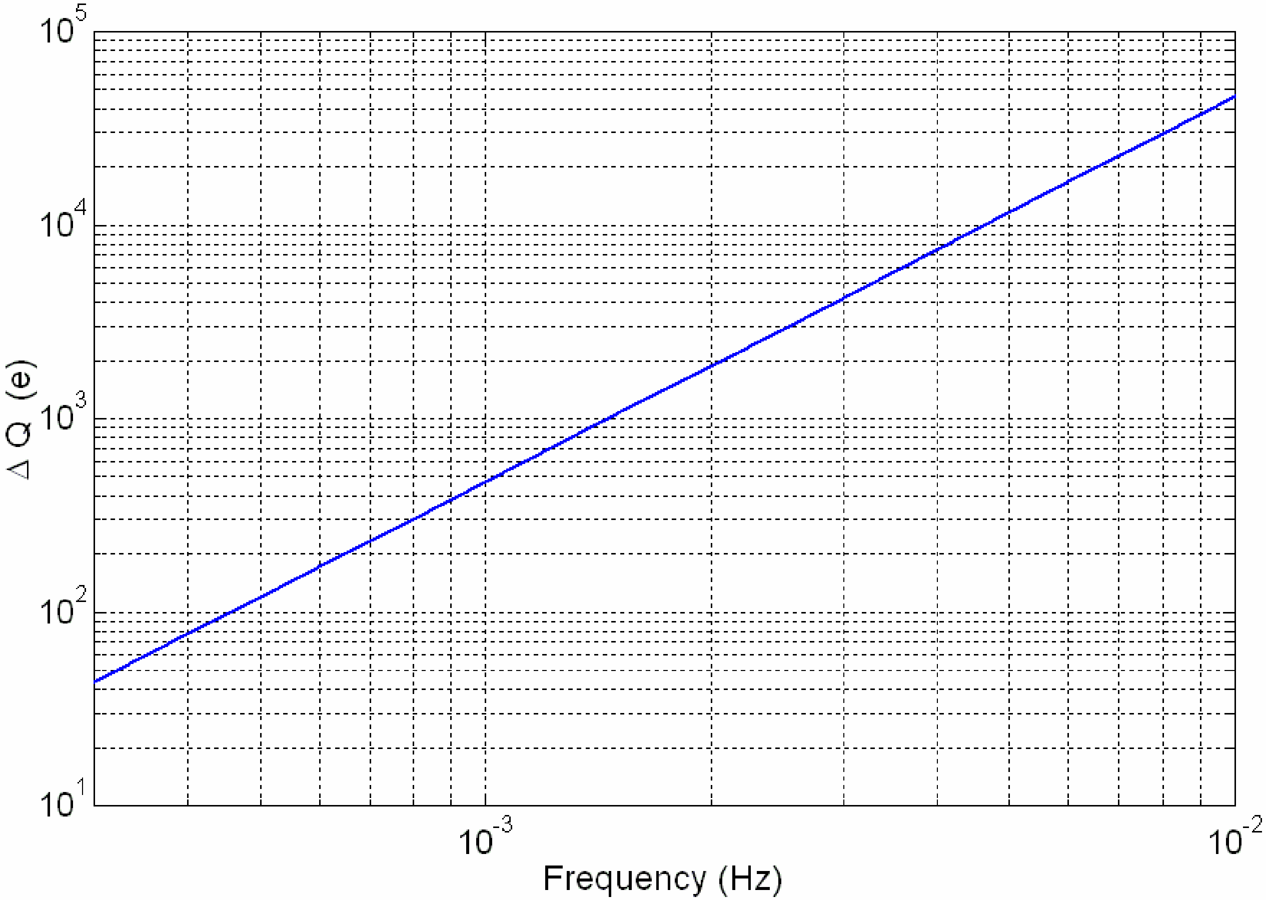


- Charge measurement noise as a function of measurement frequency
- Assumes 1 Volt measurement voltage
- Assuming stray torque noise with differential force noise similar to overall force noise budget (140 frad/s^2/Hz$^{1/2}$)

(not critical for charge measurement above 0.2 mHz)

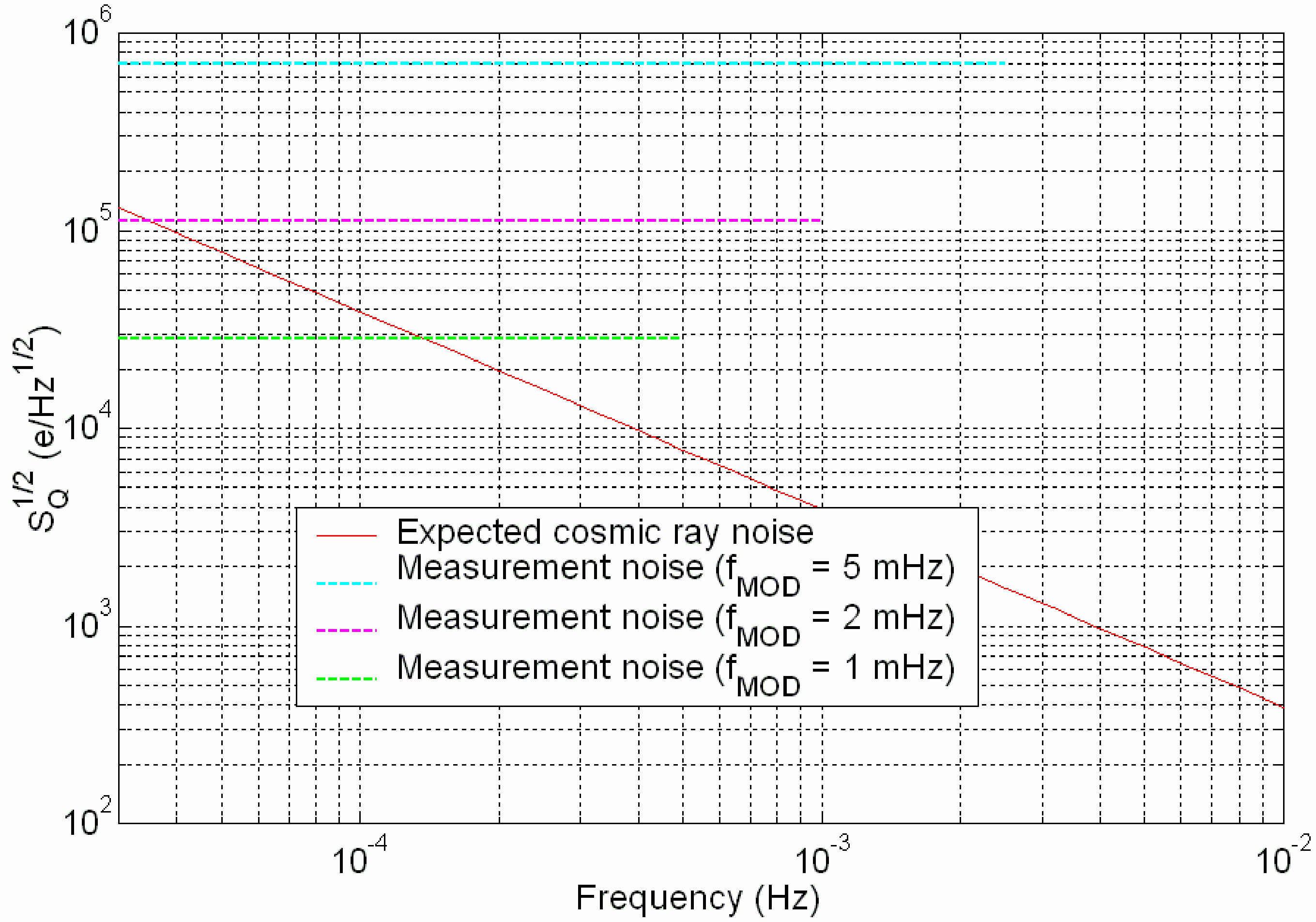
$$\frac{S_N^{1/2}}{I} = \frac{mS_{a_x}^{1/2}\left(\frac{L}{2}\right)}{I}$$

Charge measurement resolution: 1 mass config, η



- Charge measurement noise as a function of measurement frequency for **1 hour measurement time**
- Stiffness discharge threshold of 10^7 charges (60 mV, 2% change in likely x-stiffness)
- Assumes 1 Volt measurement voltage

Charge measurement resolution: 1 mass config, η



- charge measurement noise (continuous measurement) as a function of modulation frequency
- assume low-pass filtered torque signal, useful data only up to ***.5 f_{MOD}***
- can subtract noise related to TM charging and interaction with DC bias at low frequencies