Detection and reconstruction of burst signals with networks of gravitational wave detectors

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- likelihood analysis
- network parameters
- constraints/regulators
- network coherent energy
- reconstruction of GW signals
- summary

Objectives of Coherent Burst Analysis

- Combine measurements from several detectors
  - confident detection, elimination of instrumental/environmental artifacts
  - reconstruction of source coordinates
  - reconstruction of GW waveforms
- Detection & reconstruction methods should handle
  - arbitrary number of co-aligned and misaligned detectors
  - variability of the detector responses as function of source coordinates
  - differences in the strain sensitivity of detectors
- Extraction of source parameters
  - confront measured waveforms with source models
- For burst searches matched filters do not work
  - need robust model independent detection algorithms

Coherent network analysis

Combine data, not triggers

- Guersel,Tinto, PRD 40 v12,1989
  - reconstruction of GW signal for a network of three misaligned detectors
- Likelihood analysis: Flanagan, Hughes, PRD57 4577 (1998)
  - likelihood analysis for a network of misaligned detectors
- Two detector paradox: Mohanty et al, CQG 21 S1831 (2004)
  - state a problem within likelihood analysis
- Constraint likelihood: Klimenko et al, PRD 72, 122002 (2005)
  - address problem of ill-conditioned network response matrix
  - first introduction of likelihood constraints/regulators
- Penalized likelihood: Mohanty et al, CQG 23 4799 (2006).
  - likelihood regulator based on signal variability
- Maximum entropy: Summerscales at al, to be published
  - likelihood regulator based on maximum entropy
- Rank deficiency of network matrix: Rakhmanov, CQG 23 S673 (2006)
  - likelihood based in Tickhonov regularization
- GW signal consistency: Chatterji et al, PRD 74 082005(2006)
  - address problem of discrimination of instrumental/environmental bursts
- Several Amaldi7 presentations and posters by I.Yakushin, S. Chatterj, A.Searle and S.Klimenko

Likelihood

- **Likelihood for Gaussian noise with variance s^2_k and GW waveforms h_+, h_x : $x_k[i]$ – detector output, F_k – antenna patterns**

$$L = \sum_i \sum_k \frac{1}{2s_k^2}\left[x_k^2[i] - \left(x_k[i] - x_k[i]\right)^2\right]$$

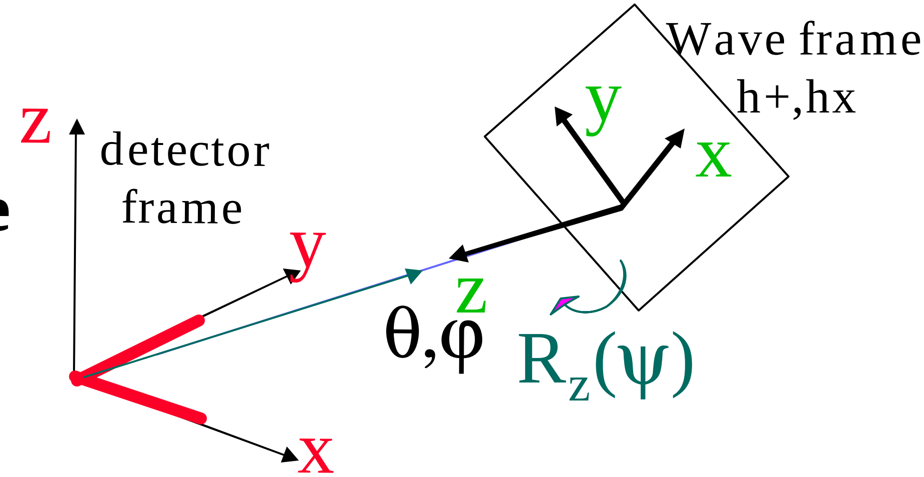
detector response - $x_k = h_+ F_{+k} + h_\times F_{\times k}$

- **Find solutions by variation of *L* over un-known functions h_+, h_x**
  (Flanagan & Hughes, PRD 57 4577 (1998))

- **“Matched filter” search in the full parameter space**
  - **good for un-modeled burst searches, but…**
  - **number of free parameters is comparable to the number of data samples**
  - **need to reduce the parameter space → constraints & regulators**
    (Klimenko et al , PRD 72, 122002, 2005)

Network response matrix

$$\vec{X} = \left\{ \frac{x_k}{s_k} \right\}, \quad \vec{F}_{+,\times} = \left\{ \frac{F_{+,\times k}(\Psi_{DPF})}{s_k} \right\}$$



- **Dominant Polarization Frame**

$$\vec{F}_+ \cdot \vec{F}_\times = 0$$

(all observables are $R_Z(Y)$ invariant)

- **solution for GW waveforms satisfies the equation**

$$\begin{bmatrix} \vec{X} \cdot \vec{F}_+ \\ \vec{X} \cdot \vec{F}_\times \end{bmatrix} = \begin{bmatrix} |F_+|^2 & 0 \\ 0 & |F_\times|^2 \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix} = g \begin{bmatrix} 1 & 0 \\ 0 & e \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix}$$

- ***g* – network sensitivity factor**
- ***e* – network alignment factor**

network response matrix

(PRD 72, 122002, 2005)

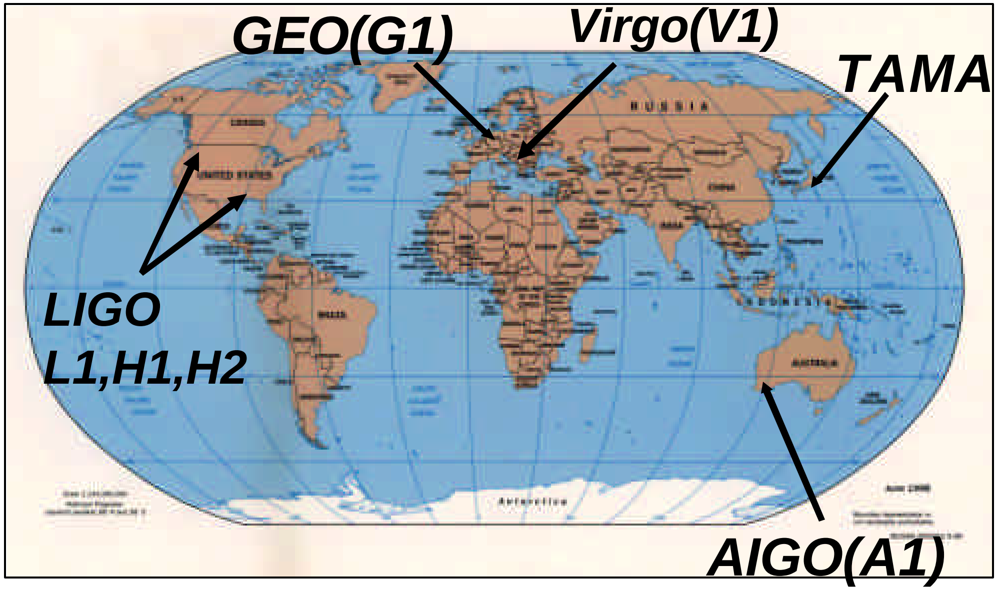
Global Network of GW detectors

$$|\vec{F}_{+,\times}|^2 = \sum_k \frac{F^2_{+,\times k}}{s^2_k}$$

$$g = |\vec{F}_+|^2, \quad e = \frac{|\vec{F}_\times|^2}{|\vec{F}_+|^2}$$

detector: L1:H1:H2:G1:V1:A1

σ^2_k: 1 : 1 : 4 : 10 : 1 : 1



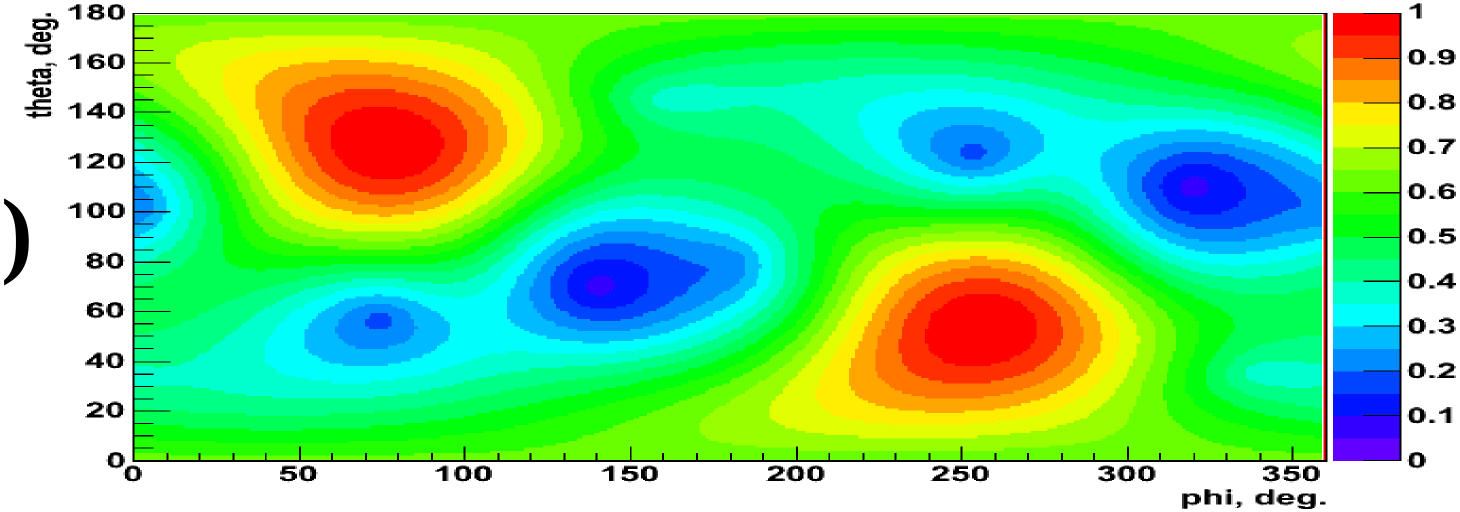
g_a and ε_a are averaged over the sky

network	g_a	ε_a,%	θ,ϕ	rejection of glitches
single IFO	1	0	-	-
H1/H2	1.4	0	-	H1-H2 consistency (correlated noise?)
H1/H2/L1	2.3	2.7	ring	waveform consistency
H1/H2/L1/G1	2.4	4.8	ring-point	waveform consistency
H1/H2/L1/G1/V1	3.1	16.5	ring-point	waveform consistency

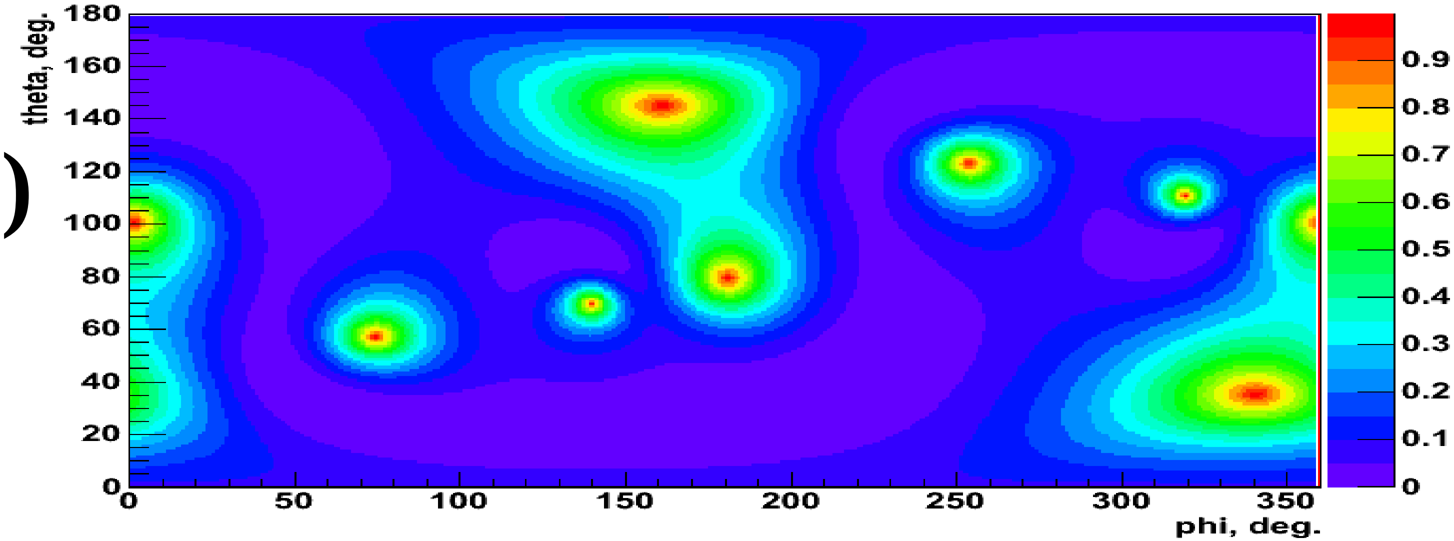
L1/H1/V1 network

- **For better reconstruction of waveforms (and source parameters) more coverage on the second polarization is desirable**

g(q,f)

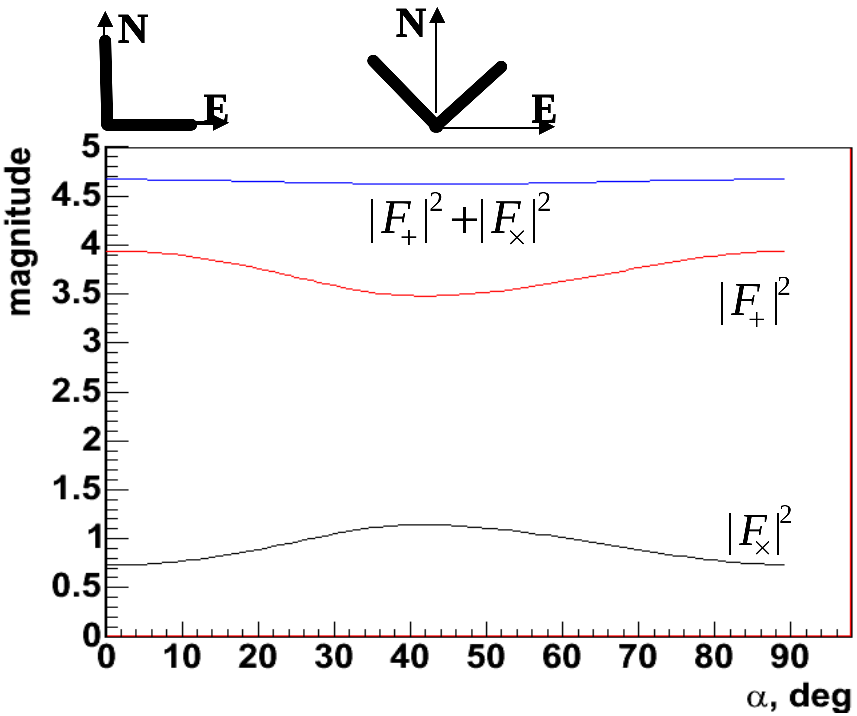


e(q,f)

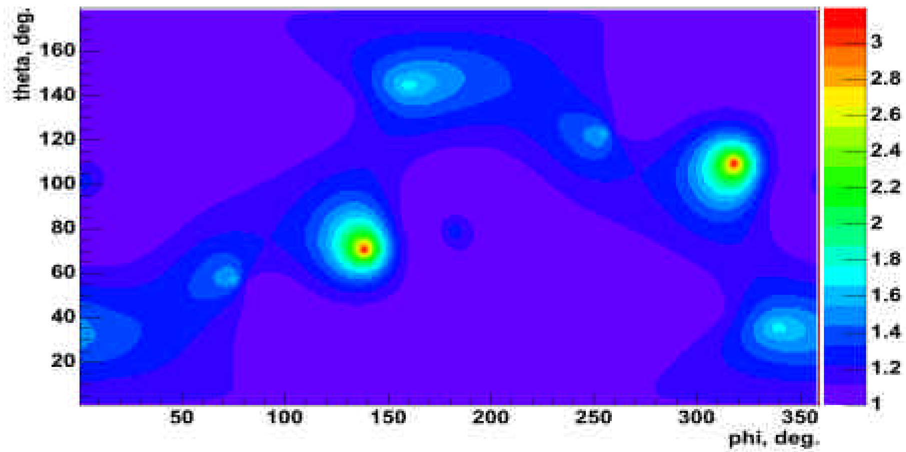


Optimal orientation of AIGO arms

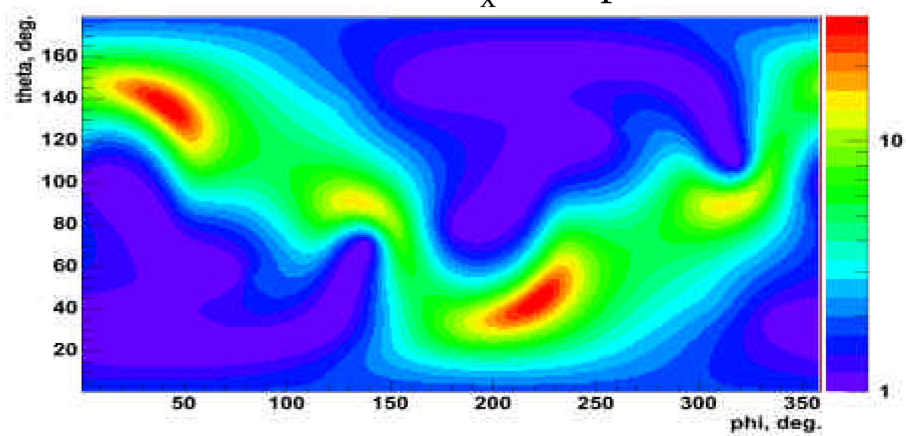
- **AIGO is almost antipodal to LIGO (lat: 121.4, long: -115.7)**



enhancement of F_+ component



enhancement of F_x component



network	g_a	ε_a,%
H1/H2/L1/G1/V1	**3.1**	**16.5**
H1/H2/L1/G1/V1/A1	**3.5**	**33.0**

- **significant improvement in the detection of the second polarization**

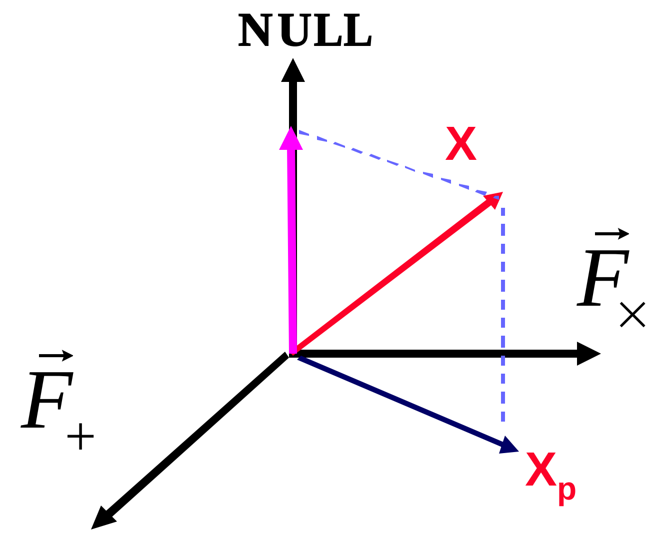
Network projections

$$\vec{F}_+ \cdot \vec{F}_\times = 0$$

- **Likelihood ratios**

$$L_+ = \frac{\left(\vec{X}\cdot\vec{F}_+\right)^2}{|F_+|^2} = X^T P_+ X, \quad P_{ij} = \frac{F_{+i}F_{+j}}{|F_+|^2} = \mathrm{e}_{+i}\mathrm{e}_{+j}$$

$$L_\times = \frac{\left(\vec{X}\cdot\vec{F}_\times\right)^2}{|F_\times|^2} = X^T P_\times X, \quad P_{ij} = \frac{F_{\times i}F_{\times j}}{|F_\times|^2} = \mathrm{e}_{\times i}\mathrm{e}_{\times j}$$

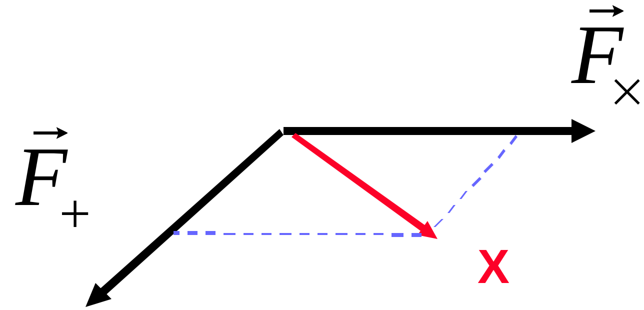


- **regulators are introduced to construct P_x when $|F_x| \to 0$**

  hard, soft, Tikhonov, etc..

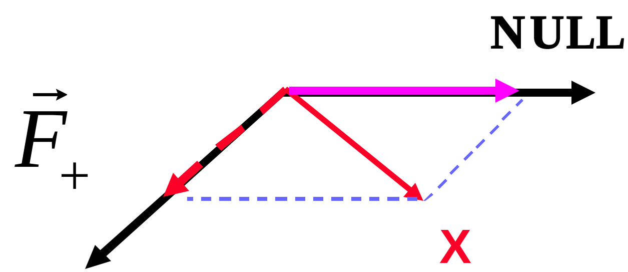
Two detector case

- **Misaligned detectors**
  - **no null space**
  - $e \ll 1$ **for significant fraction of the sky**
  - **L=const(**θ,ϕ**)**



$$L_+ + L_\times = x_1^2 + x_2^2$$

- **Aligned detectors (H1H2)**
  - e **= 0**
  - **only one projection P+**



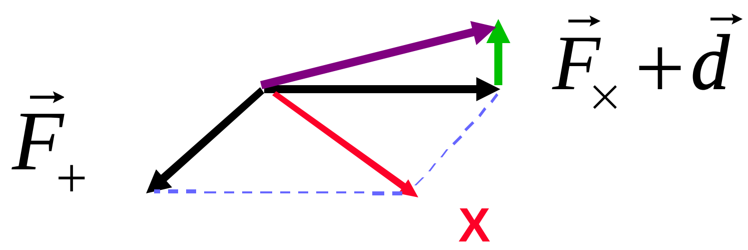
- **The discontinuity between aligned and misaligned cases can be resolved with regulators:** $|F'_\times|^2 = |F_\times|^2 + \delta$

Treatment of regulators

- regulators can not be arbitrary - they should preserve the orthogonality of the network vectors F_+ and F_x. Otherwise the projections P_+ and P_x can not be constructed.
- regulators can be introduced in two (equivalent) ways by adding small non-zero vector d to F_x

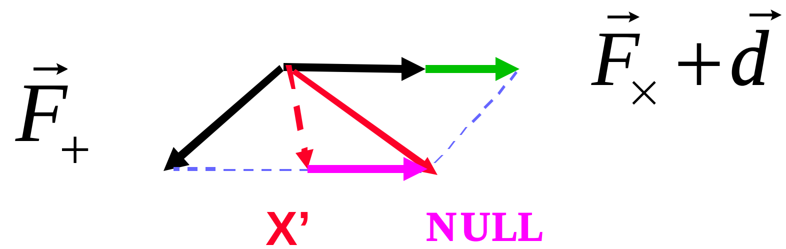
➢ **"dummy detector"**

$$\vec{F}'_+ = \{F_{+1}, F_{+2}, 0\}$$
$$\vec{F}'_\times = \{F_{\times 1}, F_{\times 2}, d\}$$
$$\vec{X} = \{x_1, x_2, 0\}$$



➢ **split the X-axis**

$$|F'_\times|^2 = \left(|F_\times| + |d|\right)^2$$



In-coherent/Coherent Energy

$$L_+ = \sum_{i,j} x_i x_j P_{ij,+} = E_{+(i=j)} + C_{+(i\neq j)}$$

$$L_\times = \sum_{i,j} x_i x_j P_{ij,\times} = E_{\times(i=j)} + C_{\times(i\neq j)}$$

- **quadratic forms C_+ & C_x depend on time delays between detectors and carry information about θ,ϕ – sensitive to source coordinates**
- **properties of the likelihood quadratic forms**

arbitrary network

$$\mathrm{cov}(L_+ L_\times) = 0$$

$$\mathrm{cov}(C_+ C_\times) = -\sum e_{+i}^2 e_{\times i}^2$$

$$\mathrm{cov}(E_+ E_\times) = \sum e_{+i}^2 e_{\times i}^2$$

2 detector network

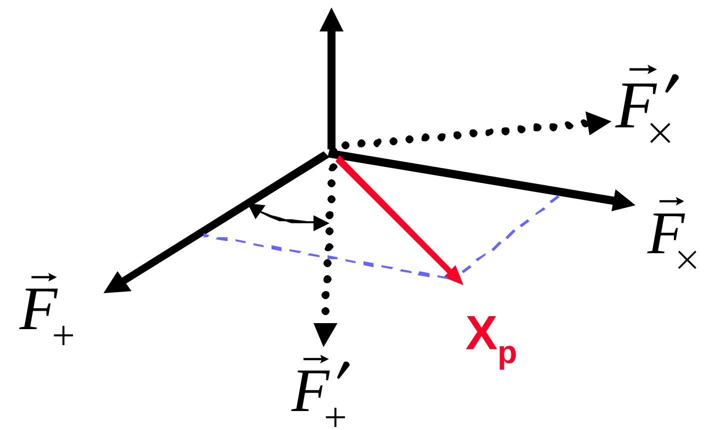
$$C_+ + C_\times = 0$$

$$E_+ + E_\times = x_1^2 + x_2^2$$

- **How is the coherent energy defined?**

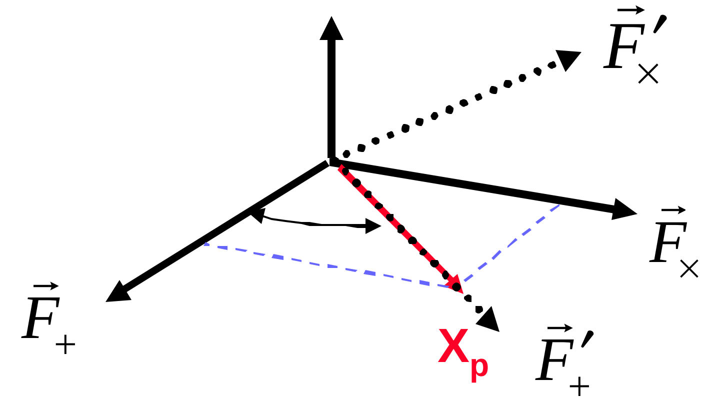
Rotation in the F_+F_x plane

- **L, null stream and reconstructed waveforms are invariant with respect to rotation of vectors F_+ and F_x in the detector plane**
- **But incoherent & coherent terms depend on the selection of the coordinate frame**



- **Define coherent energy in the frame where F'$_+$ is aligned with the projection X_p (*principle component frame*)**

$$L_+ + L_\times = C'_+ + E'_+$$



$$C'_+ = E'_\times = -C'_\times$$

- **Only in this frame, for two detector network Pearson's correlation can be defined**

$$r = \frac{1}{2}\frac{C'_{12}}{\sqrt{E_{11}E_{22}}} = \frac{\langle x_1 x_2\rangle}{\sqrt{\langle x_1^2\rangle\langle x_2^2\rangle}}, \quad \langle x_1\rangle = \langle x_2\rangle = 0$$

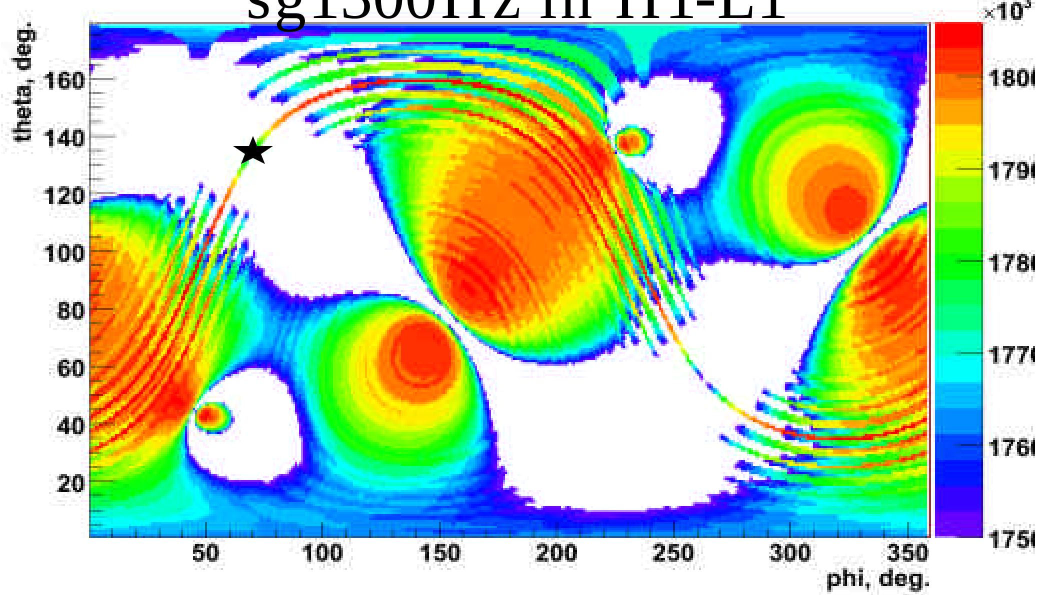
Coordinate reconstruction

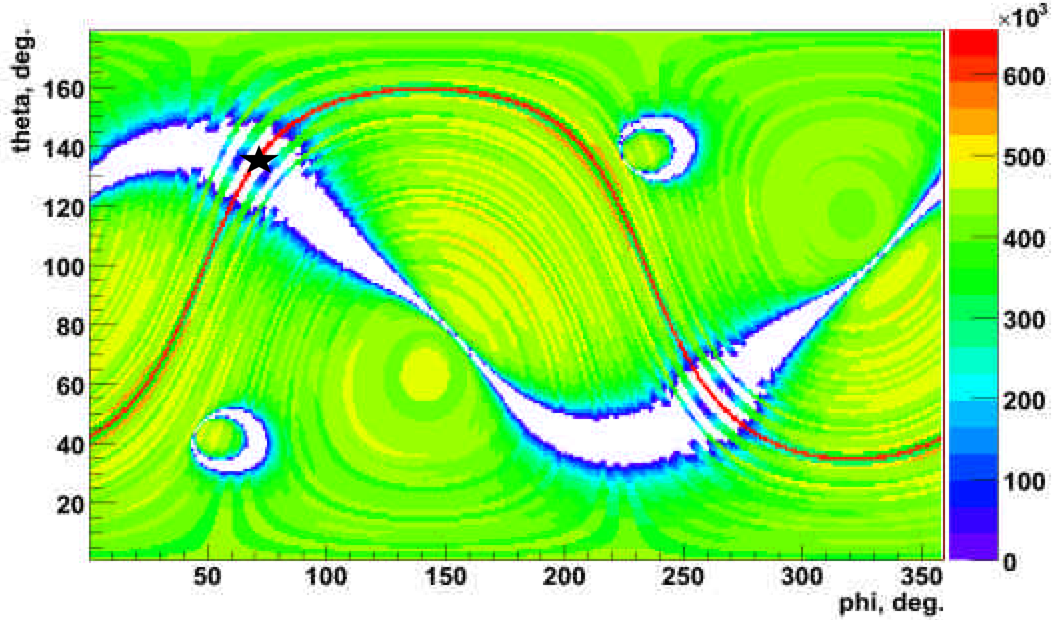
- **What statistic to use?**

- **Likelihood ratio**
  - very dependent on regulators
  - large bias

- **Correlated Energy**
  - sensitive to time delays
  - calculated in PCF
  - works with “right” regulator,
  - little dependence on regulator
  - small bias

sg1300Hz in H1-L1

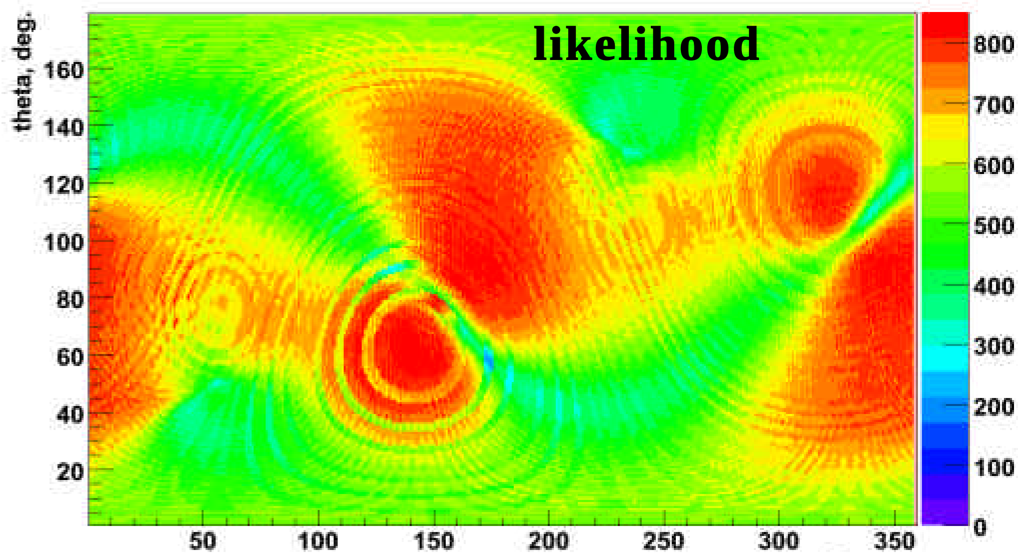




Reconstruction of source coordinates

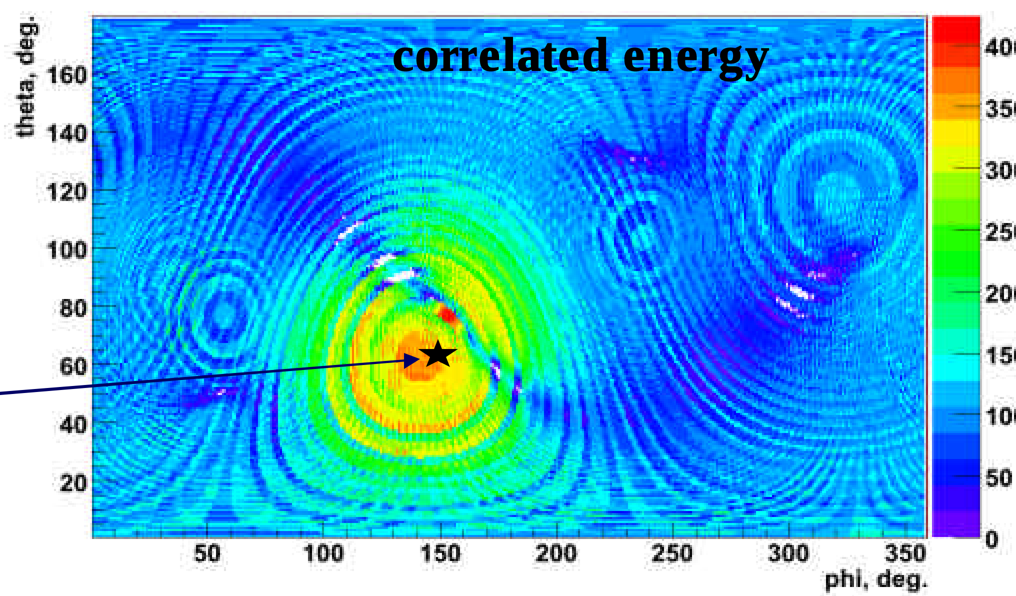
simulated sine-Gaussian
waveform: f=1304, q=9,

	L1 / H1 / H2 / G1
hrss(10^{-21}):	2.5 / 1.3 / 1.3 / 1.6
SNR(a) :	24 / 16 / 8 / 5
F+ :	.25 / .13 / .13 / .16



real noise, average
amplitude SNR=14
per detector

injection

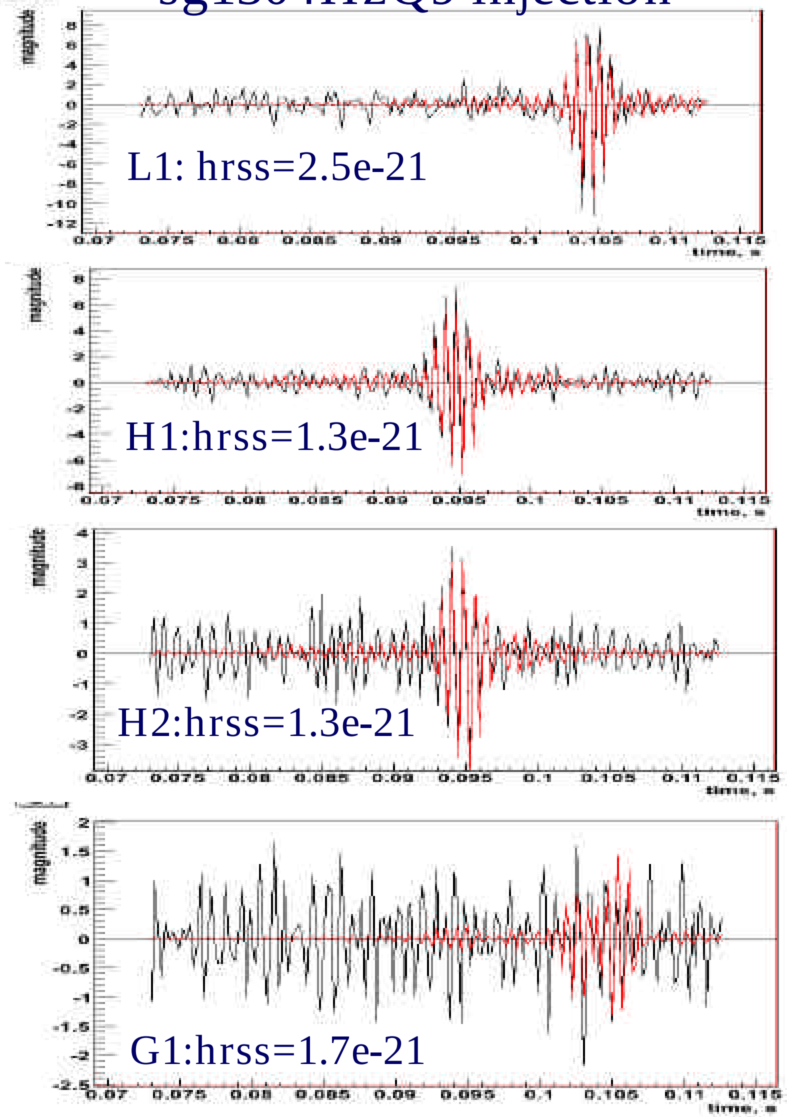


Reconstruction of burst waveforms

black
band-limited
time series

red
reconstructed
response

sg1304HzQ9 injection



- **If GW signal is detected, two polarizations and detector responses can be reconstructed and confronted with source models for extraction of the source parameters**

- **Figures show an example of LIGO glitch reconstructed with the coherent WaveBurst event display (A.Mercer et al.)**

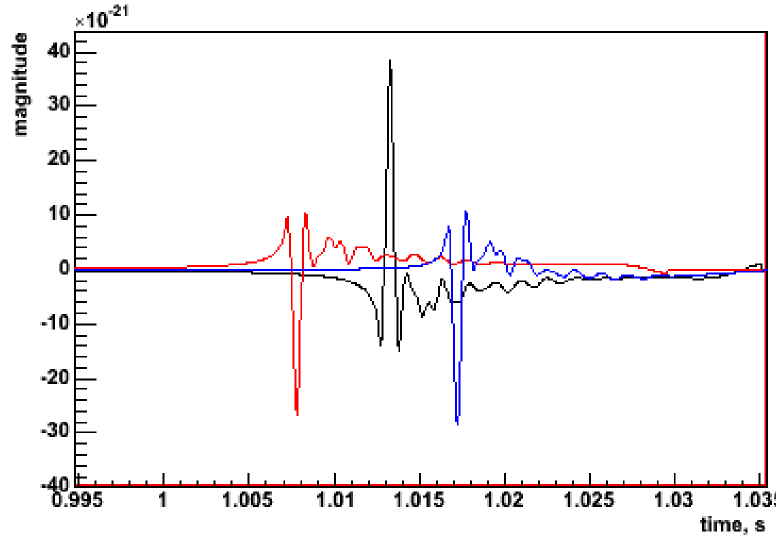
  → powerful tool for consistency test of coherent triggers.

Summary

- **Several GW detectors are now operating around the world forming a network**
  - **coherent network analysis addresses problems of detection and reconstruction of GW signals with detector networks**
- **Likelihood methods provide a universal framework for burst searches**
  - **likelihood ratio statistic is used for detection**
  - **constraints significantly improve the performance of coherent algorithms**
  - **GW waveforms can be reconstructed from the data**
  - **location of sources in the sky can be measured,**
  - **consistency test of events in different detectors can be performed**
- **New statistics based on coherent energy are developed for coordinate reconstruction and consistency tests.**

Reconstruction of source coordinates

simulated DFM-A1B2G1 waveform at θ=119, ϕ=149, L1 / H1 / V1



simulated noise, average SNR=160 per detector

Likelihood sky map
Signal detected at
θ=118, ϕ=149