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Quantum Noise in differential-type GW Interferometer and Signal Recycling

(arXiv:0705.4668 [gr-qc])

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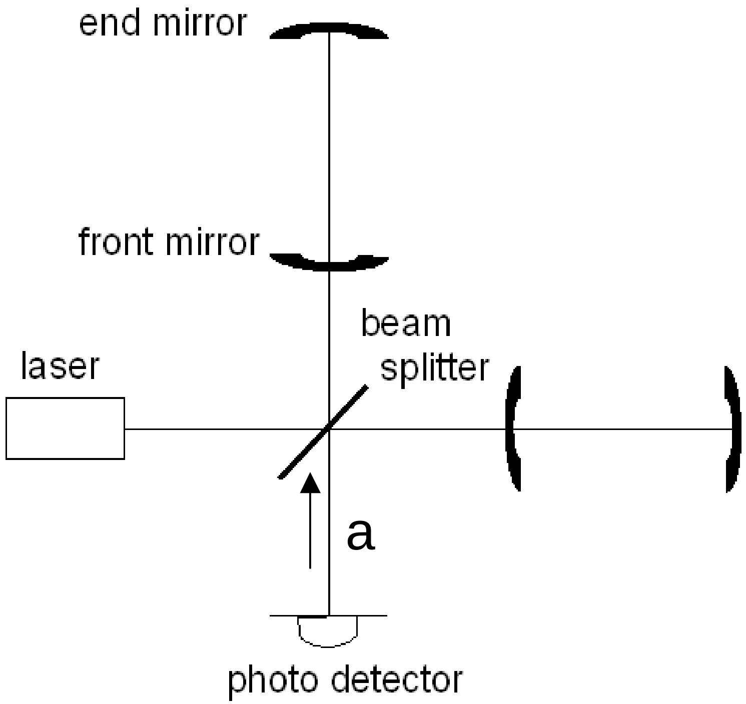
Introduction1 (Quantum noise)

Quantum noise in a recombined-type FPMI (FPMI : Fabry Perot Michelson Interferometer)

$$S_h = \frac{h_{SQL}^{2}}{2}\left(\frac{1}{K} + K\right)$$

[Kimble et al. 2001]

- A vacuum fluctuation "a" is injected from the dark port and creates quantum noise.
- The 1st term : shot noise
  The 2nd term : radiation pressure noise
- h_{SQL} is the standard quantum limit (SQL).
- K is proportional to laser power,
  so, there exists minimum achievable noise
  with fixed laser power.



Introduction2 (Signal Recycling)

Quantum noise
in a recombined-type SR FPMI

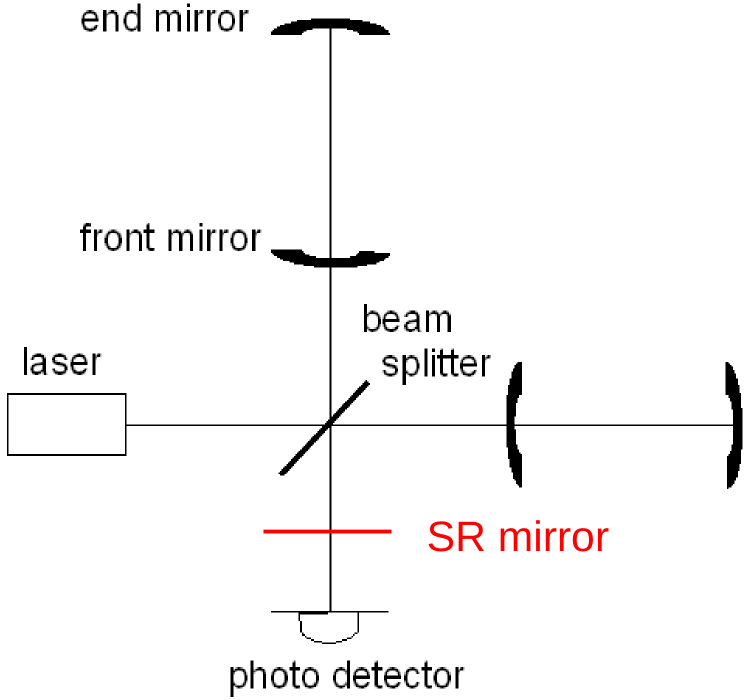
(SR : Signal Recycling)

$$S_h(\Omega) = \frac{h_{SQL}{}^2}{2\tau^2 K} \frac{(C_{11}\sin\zeta + C_{21}\cos\zeta)^2 + (C_{12}\sin\zeta + C_{22}\cos\zeta)^2}{|D_1 \sin\zeta + D_2\cos\zeta|^2}$$

[Buonnano & Chen 2001]

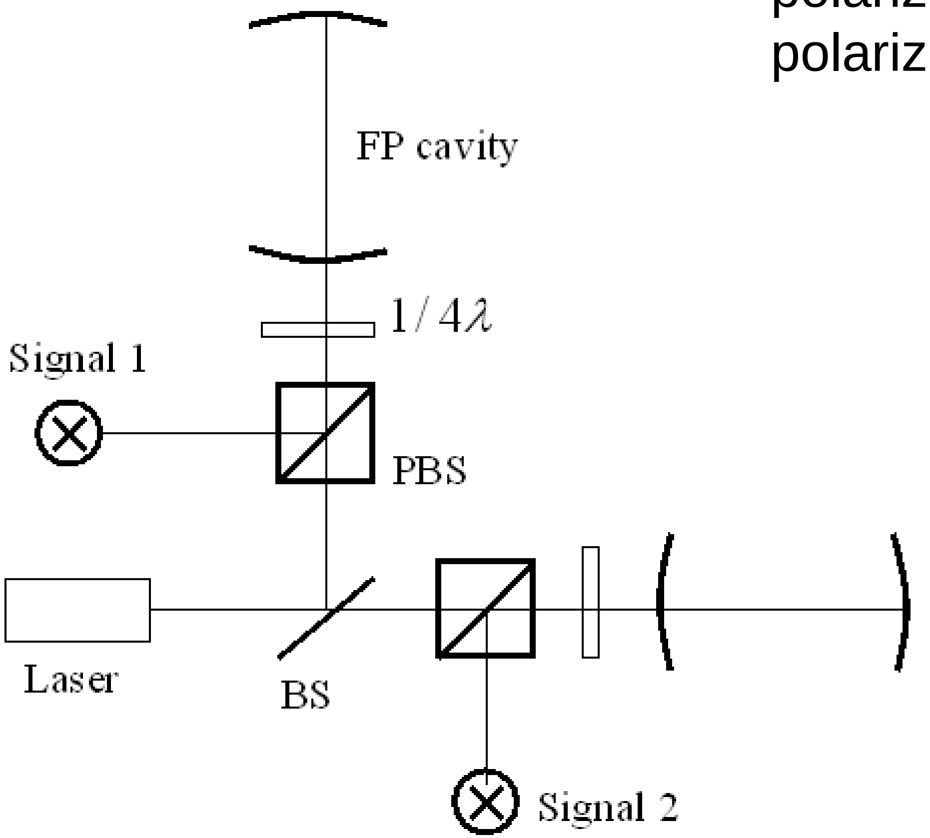
- An additional mirror at the dark port reflects the output signal and creates dynamical correlation between the mirror and the optical field.

- As a result, the quantum correlation allows the sensitivity to overcome SQL and produces two dips on the noise curve.



Differential-type FPMI

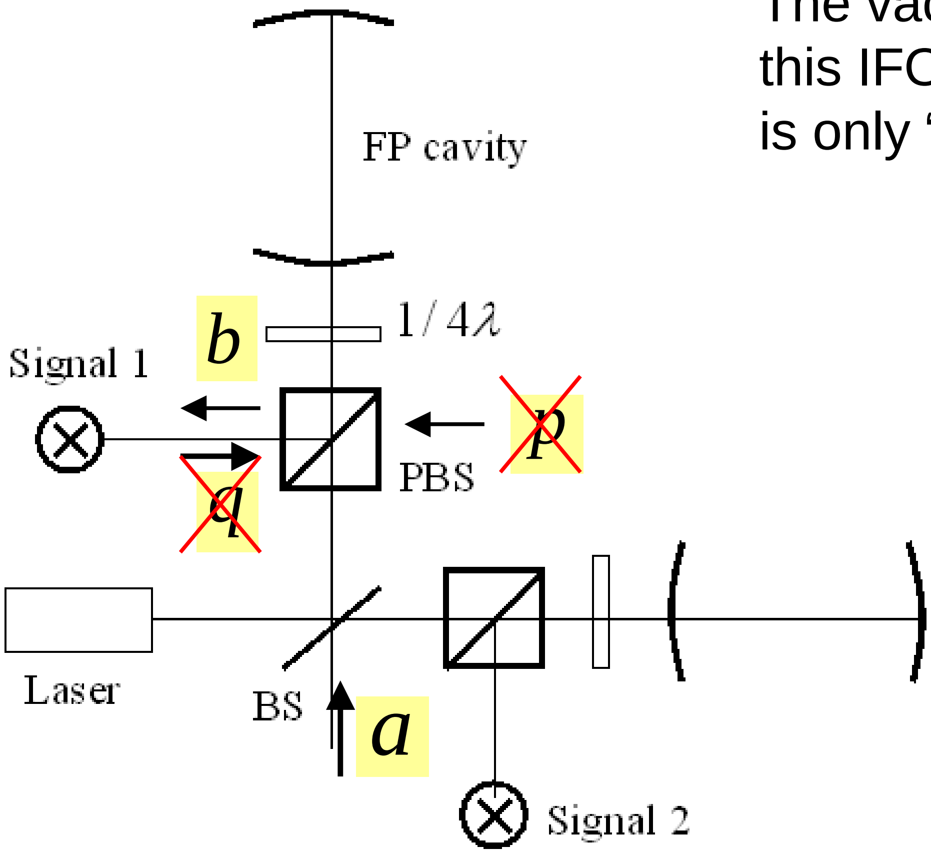
- Signals are detected in each arm.
- PBS transmits the light with horizontal polarization and reflects that with vertical polarization.



assumption

1. Cavity's end mirrors are completely reflective.
2. No phase shift for the carrier light other than in the FP cavity (tuned).
3. Negligible phase shift for the sideband except in FP cavity.
4. All optics are lossless.

Vacuum fluctuations injected into the IFO



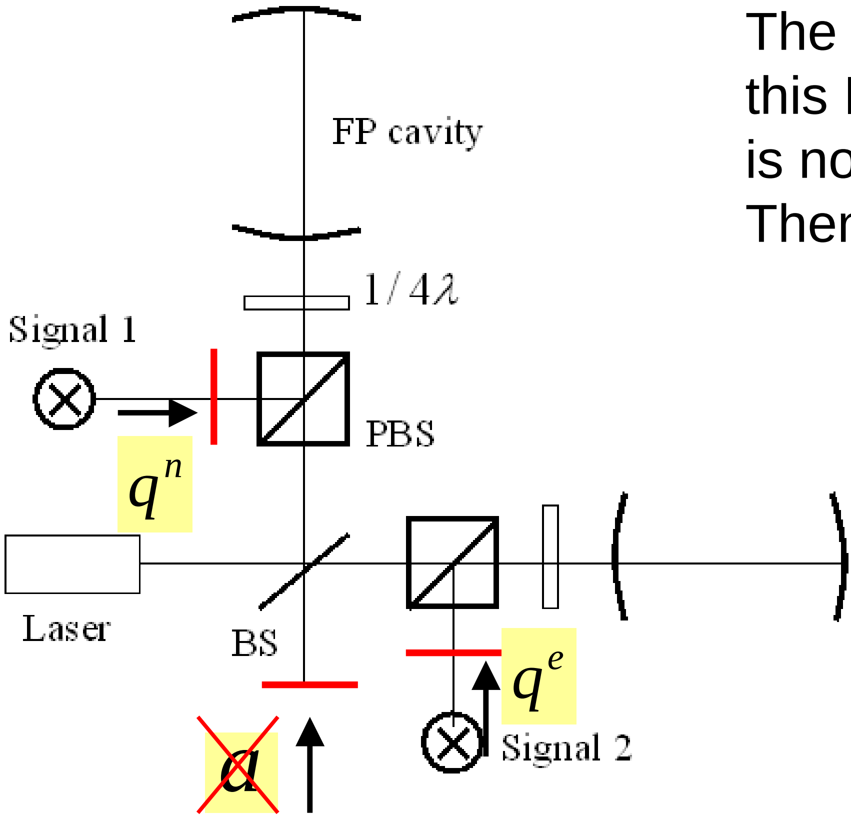
The vacuum fluctuation that is injected into this IFO and contributes to the sensitivity, is only “a”.

What we want to know is
the relation between “a” and “b”.
(input-output relation)

$$S_h(\Omega) = \frac{h_{SQL}^{\ \ 2}}{2}\left(\frac{1}{K} + K\right)$$

This is the same as the spectral density of a reconbined-type FPMI.

Differential-type SR FPMI



The vacuum fluctuation that is injected into this IFO and contributes to the sensitivity, is not “a” but “q” in this configuration. Then, “a” should be replaced with “a’ ”

$$\mathbf{a} \longrightarrow \mathbf{a}' = \frac{1}{\sqrt{2}} e^{2i\beta} \Delta \mathbf{q}$$

$$\Delta \mathbf{q} \equiv \mathbf{q}^n - \mathbf{q}^e$$

Important parameters which change the dynamics of the system.

Detuned phase in the SR cavity $\phi \equiv [\omega_0 \ell_s / c]_{\mathrm{mod}(2\pi)}$

Detuned phase in the darkport cavity $\theta \equiv [\omega_0 \ell_d / c]_{\mathrm{mod}(2\pi)}$

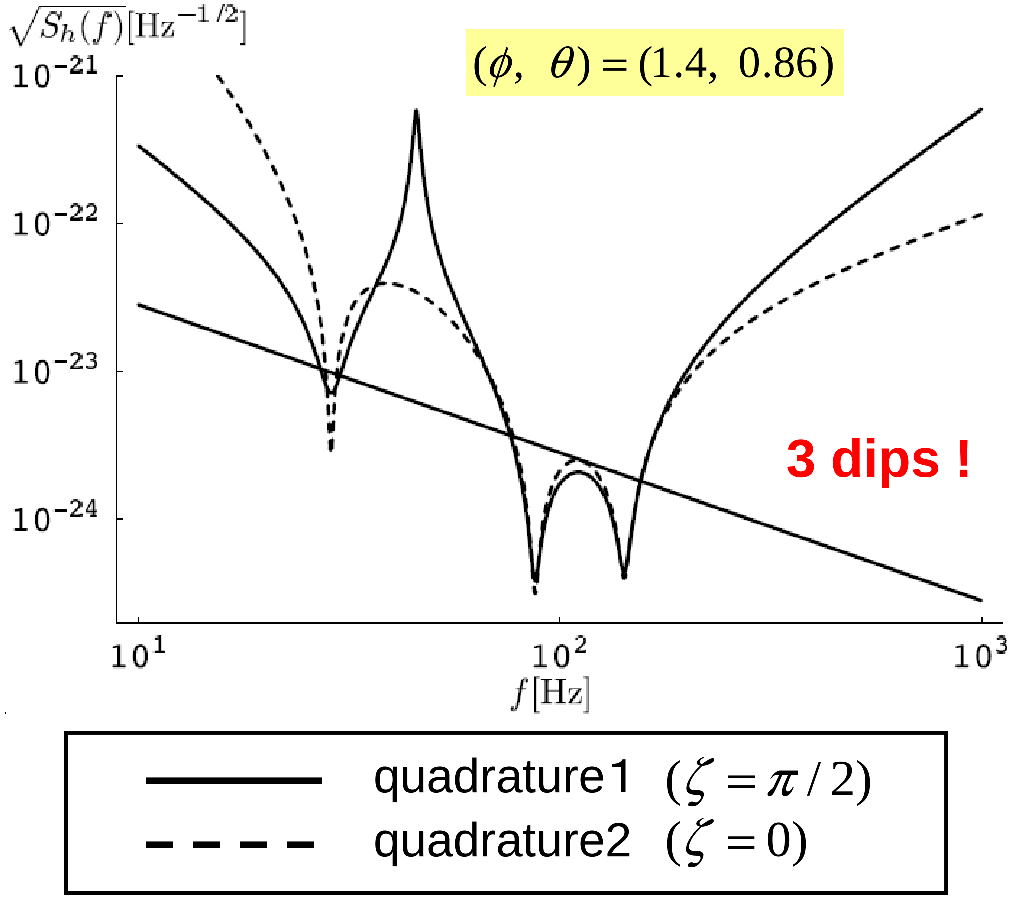
Input –output relation

$$
\begin{aligned}
\Delta \mathrm{b} &= \frac{1}{M}\left[e^{4i\beta}\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22}\end{pmatrix}\Delta \mathrm{q} + 2\tau\sqrt{K}e^{i\beta}\begin{pmatrix} D_1 \\ D_2\end{pmatrix}\left(\frac{h}{h_{SQL}}\right)\right] \\
M &= 1+\rho^2 e^{8i\beta} \\
&\quad -2\rho\, e^{4i\beta}\left[cos2(\theta+\phi)+\frac{K}{2}\left\{(1+\rho^2)\ sin2(\theta+\phi)+(e^{-2i\beta}+\rho^2 e^{2i\beta})\ sin2\theta+2\rho\ cos2\beta\ sin2\phi\right\}\right] \\
C_{11} &= (1+\rho^2)\ cos2(\theta+\phi)-2\rho\ cos4\beta \\
&\quad +\frac{K}{2}\left[(1+\rho^2)^2\ sin2(\theta+\phi)-\tau^4\ sin2\theta+2\rho\ cos2\beta\{(1+\rho^2)\ sin2\phi+2\rho\ sin2\theta\}\right] \\
C_{22} &= (1+\rho^2)\ cos2(\theta+\phi)-2\rho\ cos4\beta \\
&\quad +\frac{K}{2}\left[(1+\rho^2)^2\ sin2(\theta+\phi)+\tau^4\ sin2\theta+2\rho\ cos2\beta\{(1+\rho^2)\ sin2\phi+2\rho\ sin2\theta\}\right] \\
C_{12} &= -\tau^2\left[sin2(\theta+\phi)+K\ sin\phi\ \{(1+\rho^2)\ sin(2\theta+\phi)+2\rho\ cos2\beta\ sin\phi\}\right] \\
C_{21} &= \tau^2\left[sin2(\theta+\phi)-K\ cos\phi\ \{(1+\rho^2)\ cos(2\theta+\phi)+2\rho\ cos2\beta\ cos\phi\}\right] \\
D_1 &= -\left[(1+\rho^2 e^{6i\beta})\ sin\phi+2\rho\ e^{3i\beta}\ cos\beta\ sin(2\theta+\phi)\right] \\
D_2 &= -\left[(-1+\rho^2 e^{6i\beta})\ cos\phi+2i\rho\ e^{3i\beta}\ sin\beta\ cos(2\theta+\phi)\right]\ .
\end{aligned}
$$

Sensitivity curves

$$S_h(\Omega) = \frac{h_{SQL}{}^2}{2\tau^2 K} \frac{(C_{11}\sin\zeta + C_{21}\cos\zeta)^2 + (C_{12}\sin\zeta + C_{22}\cos\zeta)^2}{|D_1 \sin\zeta + D_2 \cos\zeta|^2}$$

(ζ : homodyne detection angle)



Arm length $3km$

Mirror mass $30\,kg$

Laser wavelenth $1.064\,\mu m$

Laser power

$I_0 = I_{SQL} = 2162\,W$

FP cavity's Mirror transmissivity

$T = 0.14$

SR mirror reflectivity

$\rho = 0.98$

Decomposition of spectral density

$q^n - q^e$

$$\Delta \mathbf{b} = \frac{1}{M}\left[e^{4i\beta}\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}\Delta \mathbf{q} + 2\tau\sqrt{K}e^{i\beta}\begin{pmatrix} D_1 \\ D_2 \end{pmatrix}\frac{h}{h_{SQL}}\right]$$

$b^n - b^e$

Noise

GW signal

normalization

$$\mathcal{O}_i(\Omega) = \mathcal{Z}_i(\Omega) + R_{xx}\mathcal{F}_i(\Omega) + Lh(\Omega)$$

$i = 1,2$

shot $\propto 1/\sqrt{I_0}$

Rad. Pressure $\propto \sqrt{I_0}$

GW

$$R_{xx}(\Omega) = -\frac{4}{m\Omega^2}$$

$$S_h = \frac{1}{L^2}[S_{\mathcal{Z}\mathcal{Z}} + R_{xx}^2 S_{\mathcal{F}\mathcal{F}} + 2R_{xx}S_{\mathcal{Z}\mathcal{F}}]$$

Analyzing each term can give the interpretation of the dips.

The number of dips (low laser power limit)

$$S_h = \frac{1}{L^2}[S_{\mathcal{ZZ}} + R_{xx}^2 S_{\mathcal{FF}} + 2R_{xx}S_{\mathcal{ZF}}] \xrightarrow{I_0/I_{SQL} \to 0} S_h = S_{ZZ}/L^2$$

$S_{Z1Z1} = 0, \ S_{Z2Z2} = 0$

$$1 - 6y + y^2 = (1+y)^2 \ cos2(\theta + \phi) \ . \qquad y \equiv \left(\frac{\Omega_{res}}{\gamma}\right)^2$$

$$\Rightarrow \quad y_s = \frac{3 + cos2(\theta+\phi) \pm 2\sqrt{2\{1 + cos2(\theta+\phi)\}}}{1 - cos2(\theta+\phi)} \ .$$

2 real solutions（ when $2(\theta+\phi) = \pi$, degenerated solutions）

⇒ 2 dips on the noise curve

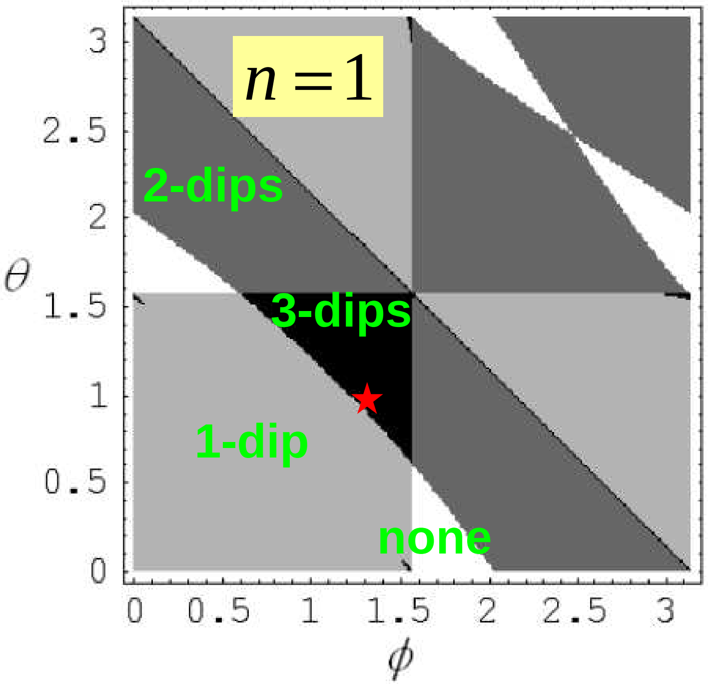
The number of dips (general case)

$S_h = 0$ ⇒ $\sqrt{\bar{S}_{\mathcal{Z}_i\mathcal{Z}_i}} = -R_{xx}\sqrt{\bar{S}_{\mathcal{F}_i\mathcal{F}_i}}$

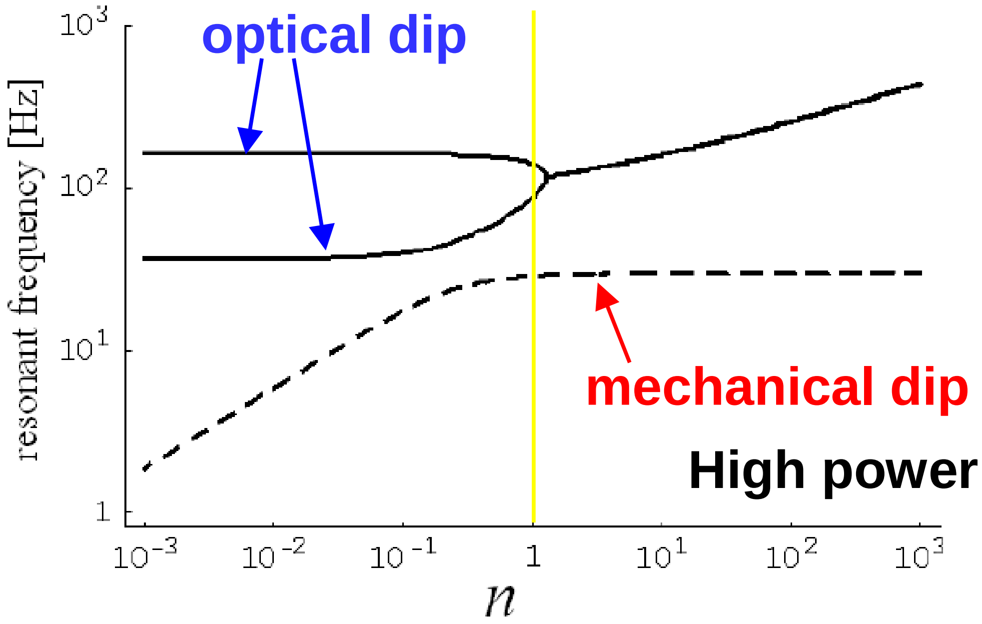
$$y\left[(1+y)^2\, cos2(\theta+\phi) - (1-6y+y^2)\right] = 2n\left[(1+y)\, sin2(\theta+\phi) + (1-y)\,(sin2\theta + sin2\phi)\right]$$

$$y \equiv \left(\frac{\Omega_{res}}{\gamma}\right)^2 \qquad n \equiv I_0/I_{SQL}$$

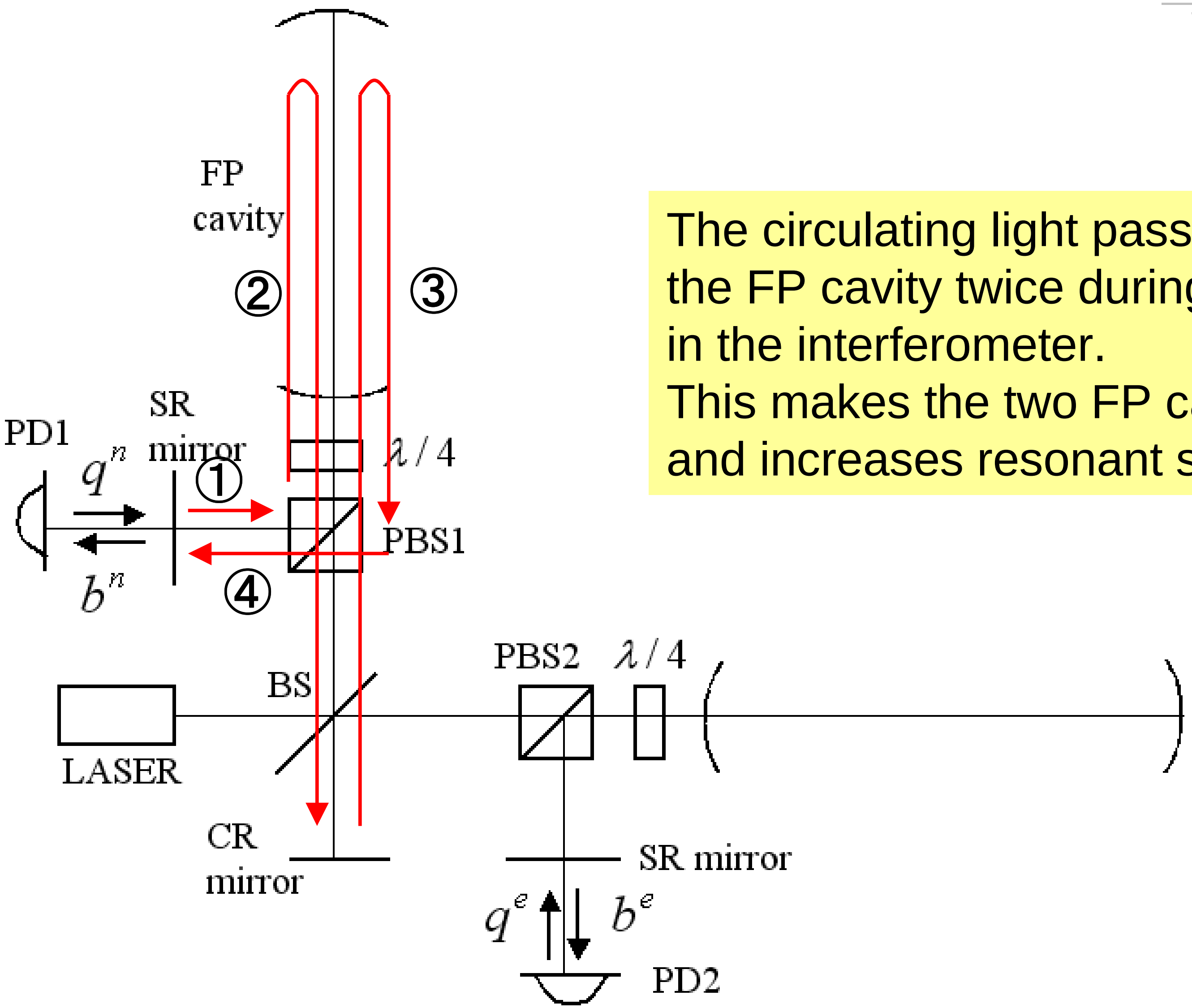
Number of dips



laser power dependence of resonant frequencies



The reason for the increase of optical dips1



The circulating light passes through the FP cavity twice during the light trip in the interferometer.
This makes the two FP cavities coupled and increases resonant solutions.

The reason for the increase of optical dips2

$S_{Z1Z1}=0,\ \ S_{Z2Z2}=0$

β : Effective phase shift in the FP cavity

recombined type

$$\cos 2\beta = \cos 2\phi$$

$$\longrightarrow \pm 2\beta + 2\pi\ m = 2\phi$$

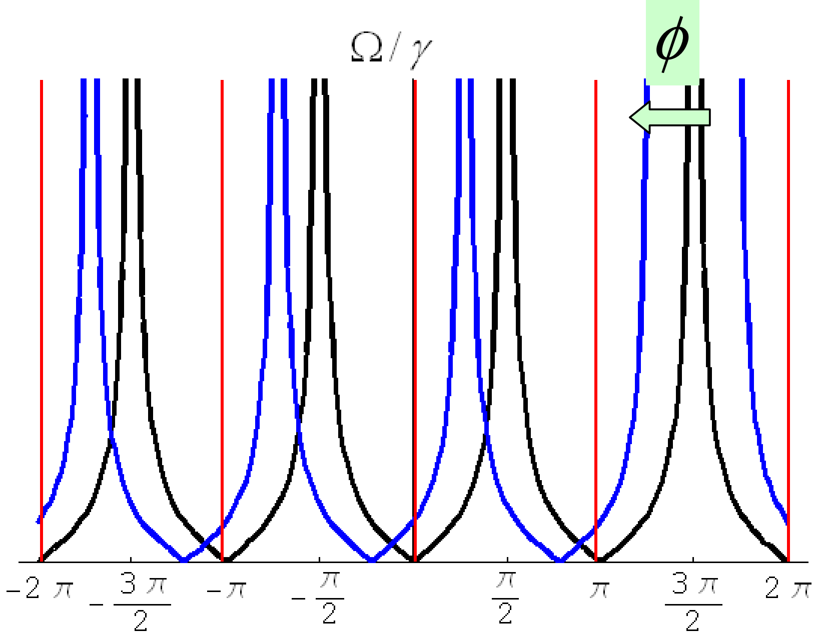
$$\longrightarrow \pm \arctan(\Omega/\gamma) - \phi = \pi\ m$$

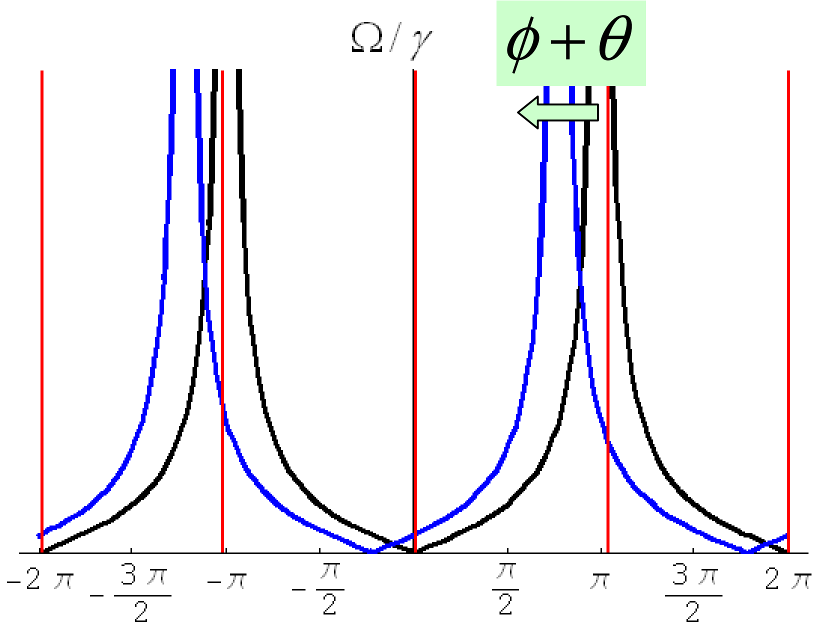
differential type

$$\cos 4\beta = \cos 2(\phi+\theta)$$

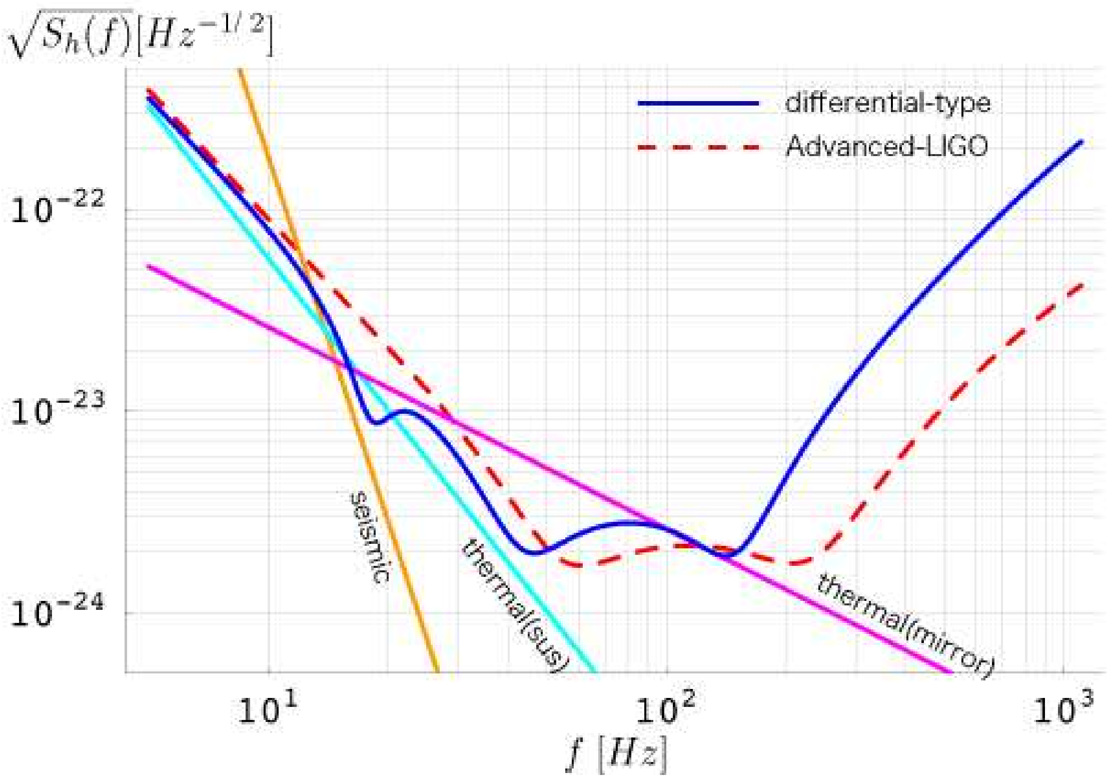
$$\longrightarrow \pm 4\beta + 2\pi\ m = 2(\phi+\theta)$$

$$\longrightarrow \pm 2\arctan(\Omega/\gamma) - (\phi+\theta) = \pi\ m$$





Comparison of Inspiral range with Adv-LIGO



Adv-LIGO(quantum)

Differential SR

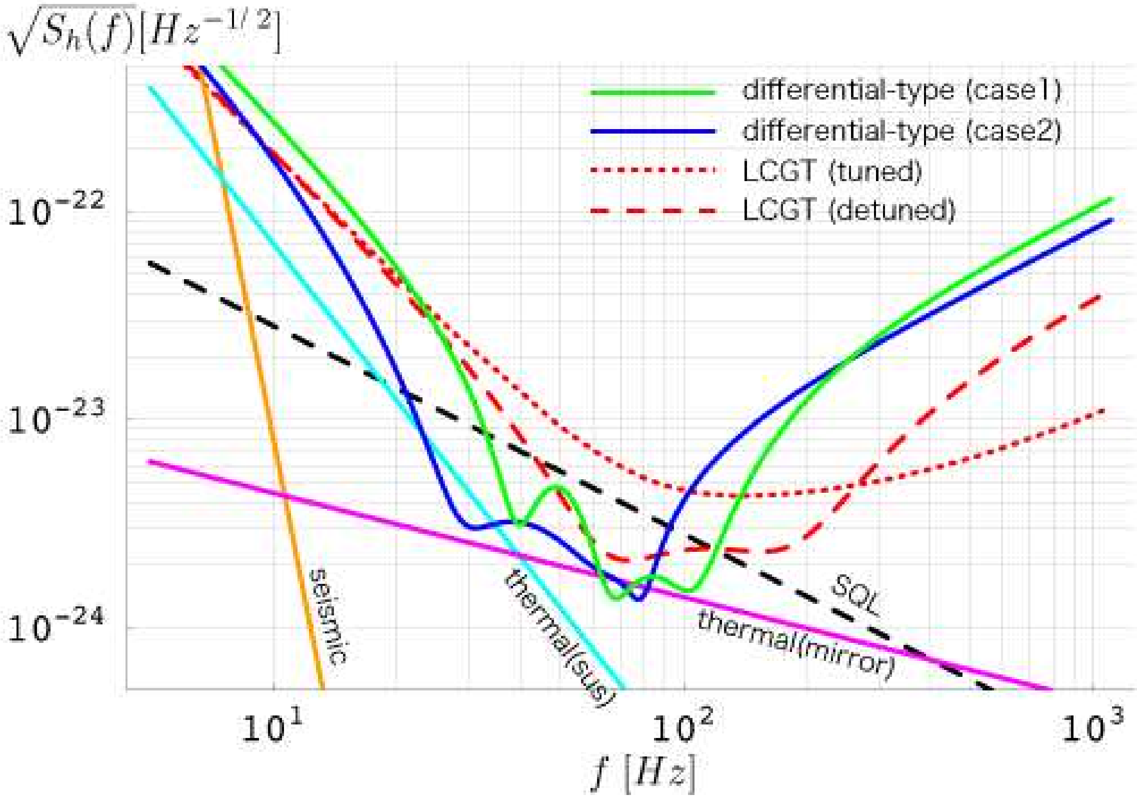
$(\phi = 1.09,\ \theta = 1.32)$

The ratio of SNR

	NS binary	BH binary ($50M_\odot$)	BH binary ($100M_\odot$)
Advanced-LIGO	1	1	1
Differential-type	0.90	1.05	1.24

Classical noise impairs the advantage of the differential-type SR.
But, smaller classical noise allows us to improve the sensitivity.

Comparison of Inspiral range with LCGT



LCGT(tuned, detuned)

Differential SR1

$(\phi = 0.13,\ \ \theta = 1.49)$

Differential SR2

$(\phi = 1.38,\ \ \theta = 0.61)$

Significant improvement

The ratio of SNR

	NS binary	BH binary ($50M_{\odot}$)	BH binary ($100M_{\odot}$)
LCGT (tuned)	1	1	1
LCGT (detuned)	1.25	1.56	1.17
Differential-type (case1)	1.30	1.87	1.81
Differential-type (case2)	1.43	2.28	2.94

Discussion and Summary

Practical issues

- Lock acquisition scheme
- The instability of the system
- The effect of optical losses
- The dynamic range of a photo detector
- Slightly high laser power (even if power recycling is done)

It's possible in our configuration.

Summary

- We considered **quantum noise in a differential-type FPMI** and applied **signal recycling** to it.
- There appears at most **3 dips** on the sensitivity curve.
- Then, applying differential-type signal recycling to a real IFO and making the third dip in low frequency, gives **the improvement of the SNR for binaries** by the factor **1.4 - 2.9**.
- It is important to reduce the classical noise level (thermal noise) in order to take advantage of the quantum technique.