Efficiencies of a hexagonal template bank placement to search for gravitational waves from inspiralling compact binaries in ground-based detectors.

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Introduction

The search for gravitational waves emitted by inspiralling compact binaries in the data from ground based detectors such as LIGO is based on matched filtering, which makes use of accurate theoretical waveforms (i.e., templates) that are available.

The signals we are searching for can be emitted by a wide variety of binary systems such as binary neutron stars (BNS), binary black hole (BBH), primordial black holes (PBH) and/or binary black hole neutron star (BHNS). Therefore, signals can be very different from each other, both in amplitude and duration (e.g., see Figure 1).

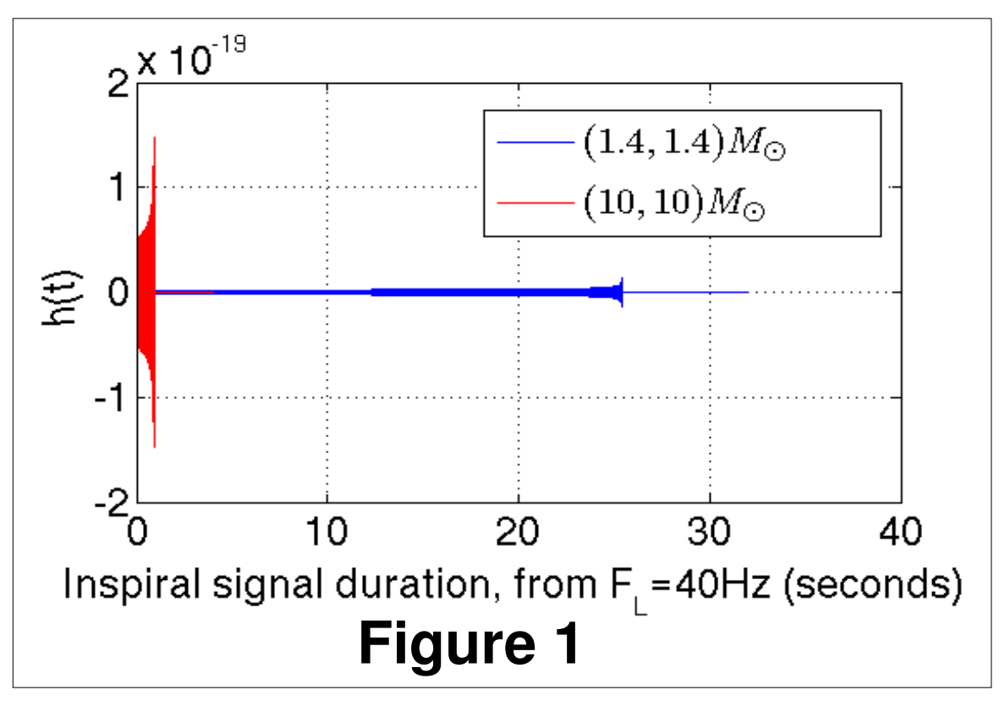

Figure 1

Because the parameters of the signals (i.e., component masses) are unknown, we filter the data with a discrete set of templates, known as **template bank**. We present an original **template bank placement** for the search of non-spinning compact binaries that is currently used to search for inspiralling compact binaries in LIGO data (S5). This template bank is shown to be **efficient** for any type of binary systems belonging to the list above (BBH, BNS...).

We present the template bank placement that was developed to optimally cover the parameter space. We performed exhaustive simulations so as to test and validate the proposed bank. We also show that the same template bank can be used with various physical template families such as EOB, or Pade at 2PN order.

Previous work (square bank)

In a previous work [1], we developed and tested a template bank (square bank) which is based on a **metric** defined on the signal manifold, where we consider both signal and template as part of the same physical template family that is the **Stationary Phase Approximation (SPA)** model. Using this metric, we can estimate the distance between two infinitesimally separated normalized templates :

$$\begin{aligned} ||h(\vartheta^\mu + d\vartheta^\mu) - h(\vartheta^\mu)||^2 &= ||h_\mu d\vartheta^\mu||^2 \\ &= (h_\mu, h_\nu)\, d\vartheta^\mu d\vartheta^\nu \\ &\equiv g_{\mu\nu} d\vartheta^\mu d\vartheta^\nu, \end{aligned}$$

The templates are chosen such that any signal filtered with the template bank has a match larger than a **minimal match, MM**, given by

$$MM = \min_{\vartheta^\mu} \max_{\delta\vartheta^\mu_i} (h(\vartheta^\mu), h(\vartheta^\mu + \delta\vartheta^\mu_i)),$$

We intensively tested the square bank in the context of BNS, BBH, BHNS, and PBH signals, and for various design sensitivity curves as LIGO, advanced LIGO, VIRGO and GEO. The drawback of the square bank, as expected, is that it is **over-efficient** (i.e., matches are systematically greater than MM, as shown in Fig.2 and 3). See [1] for more details on the placement itself or interpretation of the simulations that were made. The square template bank was used to search for PBH and BNS in S1, S2, S3 and S4 LIGO searches [2,3,4,5].

In Fig. 2 and 3, we show two figures of merit that show the efficiency of the square template bank to filter BNS signals. Both signals and templates are based on SPA model. We used 4 design sensitivity curves and 10,000 injections for each of them. Figure 2 shows the cumulative distribution of matches (left) and matches versus total mass (right).

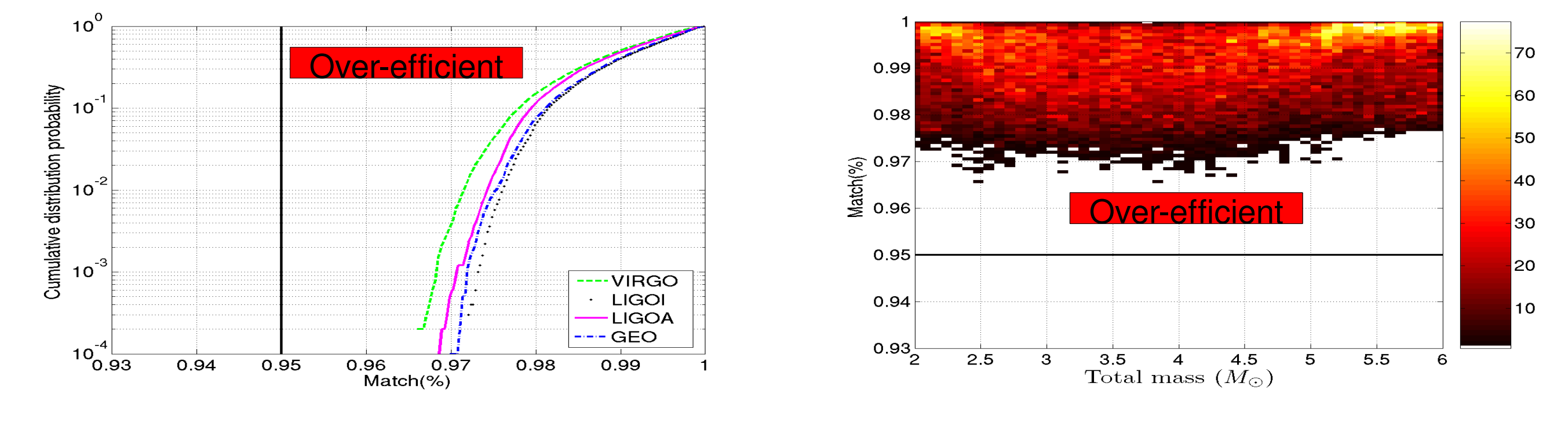

Figure 2. Efficiency of the square bank. BNS templates and injections based on SPA model.

Adaptive Hexagonal Placement (1/2)

In order to avoid the over-efficiency effect of the square lattice, we developed a hexagonal placement that reduces the size of the previous bank by 40% while keeping matches above the requested minimal match. The main difficulty of the placement is that distances between templates, which are defined by the metric components, are not constant over the parameter space. The orientation of the ambiguity function, is not constant either.

The placement, in a nut-shell, works as follows. We represent each template by an ellipse, for which any signal within its boundary is guarantee to have a match higher than MM. We use a hexagonal placement, so each template has 6 neighbors. Because the metric is not constant in the parameter space, we place templates on a constant lattice. We start to lay a single template (Fig 3.a), and we assume local flatness around it. Therefore, 6 new templates are placed at the same distance in 6 directions (hexagonal lattice) as in Fig.3.b. Then, we loop over the 6 new templates so as to compute the metric for each of them. Each new template extend in at most 6 directions, however, template have already been placed, therefore, in general, the number of new templates is restricted to 3, as in Fig3.e. The placement evolves until the parameter space is filled. The tricky part of the placement is that connections are set (fig3.c) between templates so that 2 templates can not be placed at the same location. The placement evolves like a snail shell until the whole parameter space is covered. See [6] for more details on the placement.

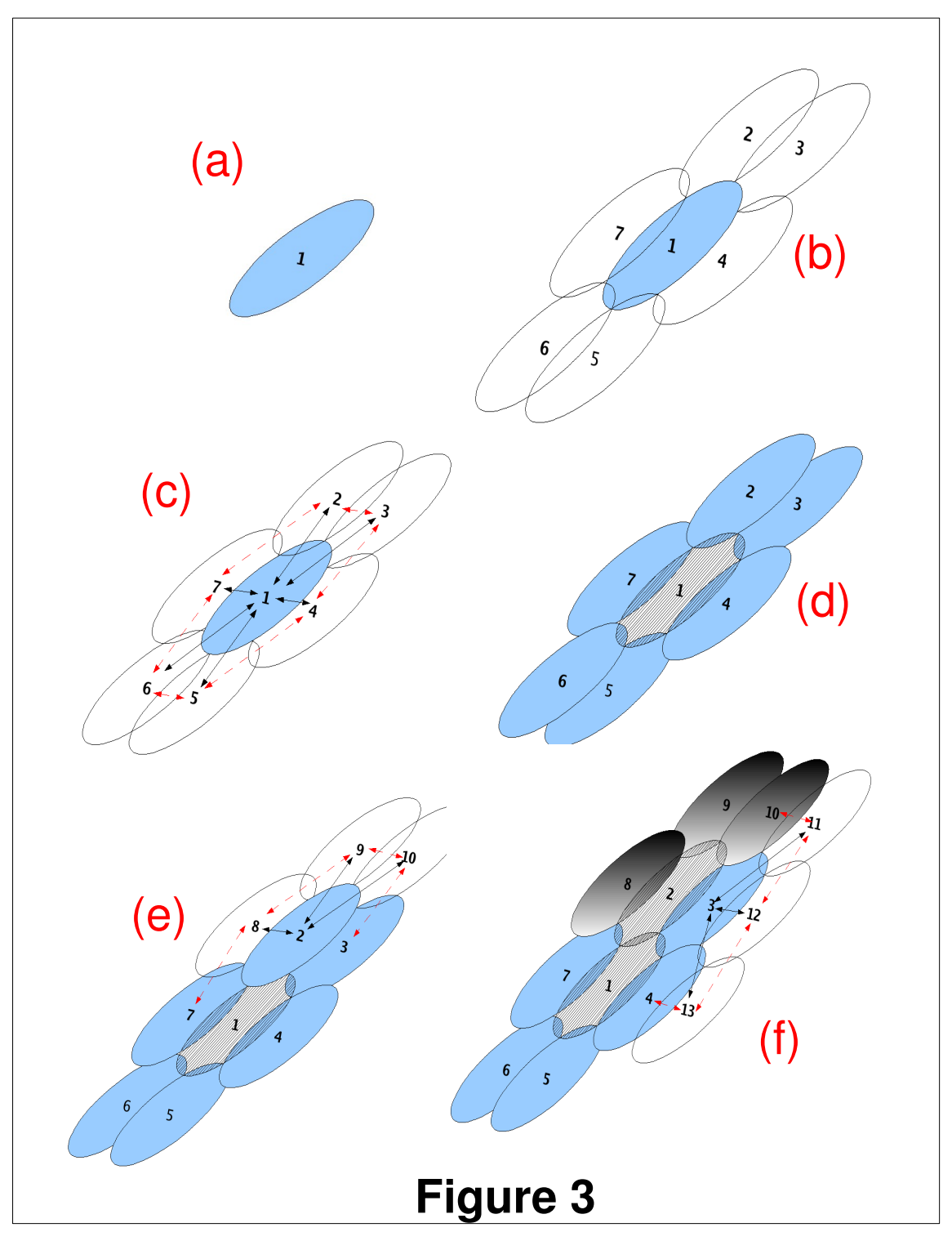

Figure 3

Adaptive Hexagonal Placement (2/2)

In Fig. 4, we plot an instance of the square and hexagonal placement for the same part of the parameter space. For convenience, we modified the metric components so that ellipses have small eccentricities. We can see that the hexagonal placement requires less templates to cover the same area.

The square placement aligns templates along the axis of the two chirp times parameters whereas the hexagonal placement aligns templates along the eigen-directions.

Table I gives the reduction of template bank size between the square and hexagonal placements, which is about 40% (expectation was about 30%).

The computational cost is of the order of a few seconds, which is negligible as compared to time spent in reading the data. Table II includes the time needed to write the bank which is about 6 seconds in the BBH case.

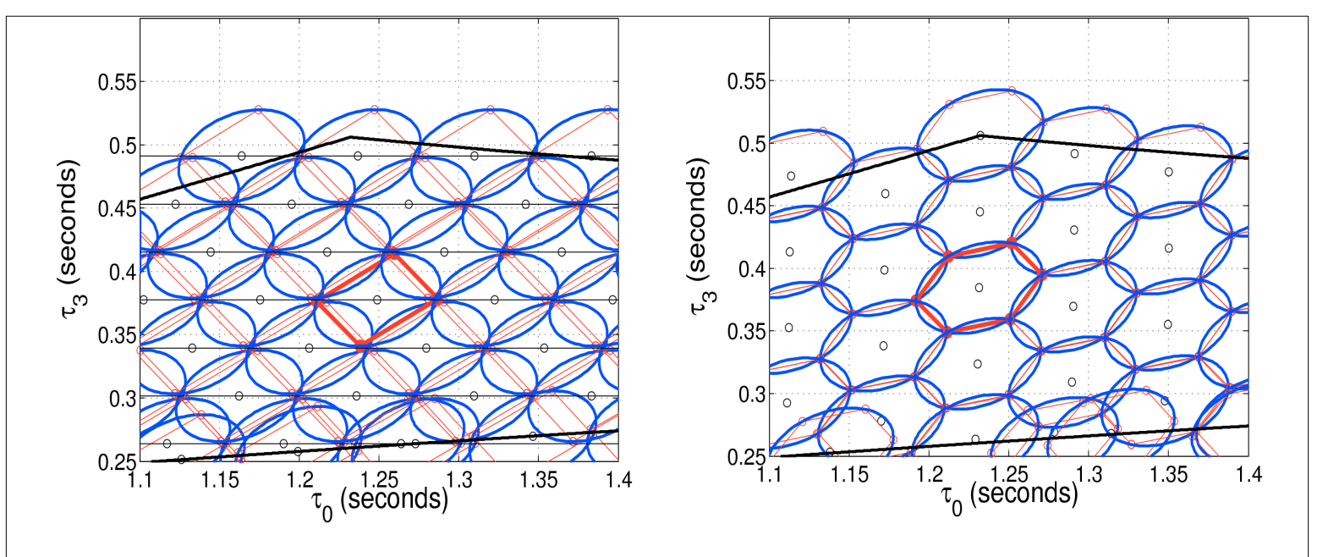

Figure 4

	EGO	GEO 600	LIGO-I	LIGO-A	Virgo	average
BBH	36	47	40	31	34	37.6
BHNS	32	33	27	30	29	30.2
BNS	40	49	49	40	37	43.0
PBH	48	51	46	46	45	47.2
average	39	45	40.5	36.75	36.25	39.5

Table I. Bank size gain by using the hexagonal bank

$m_{\rm min}$	$m_{\rm max}$	$N_{\rm square}$	Time(s)	$N_{\rm hexa}$	Time(s)
0.5	30	182136	25.0	124652	9.5
1	3	10187	7.5	7251	6.3
1	30	34095	9	24501	7
3	30	2422	6.3	1764	6.1

Table II. Computational cost.

Simulations

We tested the bank by performing 100,000 SPA injections for 5 different design sensitivity curves, and using the same template family for the template bank.

The Fig.5.a shows the BNS results. Injections are found with matches above the requested minimal match. We can compare the results with the one obtained in Fig 2. The hexagonal placement is not over-efficient, as required.

The Fig. 5.b shows the results for PBH injections. The difficulty here is that the number of templates is rather large (>100,000).

The BHNS injections shows also that matches are found above MM (Fig 5 c).

In the case of BBH, some injections are below the requested MM, when total mass is rather high (Fig 5.d.). This corresponds to short template duration, where the metric computation is not a good approximation anymore. Most of the BBHs are still recovered with match above MM.

Table II summarizes the template bank sizes needed for different parameter space and design sensitivity curves.

Bank size	EGO	GEO 600	LIGO-I	LIGO-A	Virgo
BBH	4109	838	532	1712	3283
BHNS	71478	12382	7838	27511	57557
BNS	16036	3576	2319	6969	12958
PBH	205439	41354	26732	84154	167725

Table III. Bank sizes

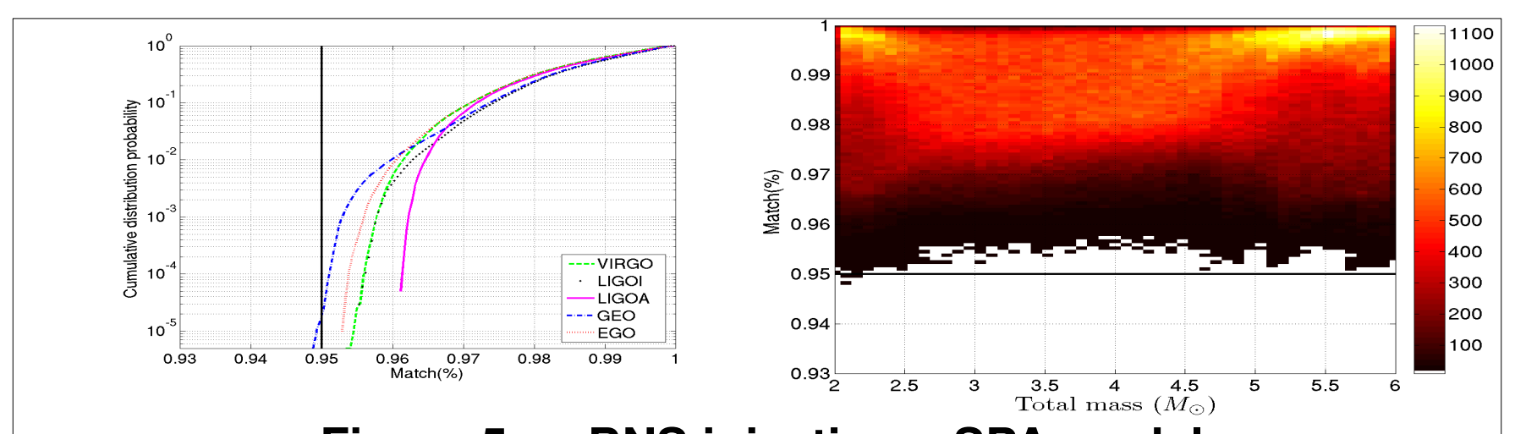

Figure 5 a, BNS injections, SPA model

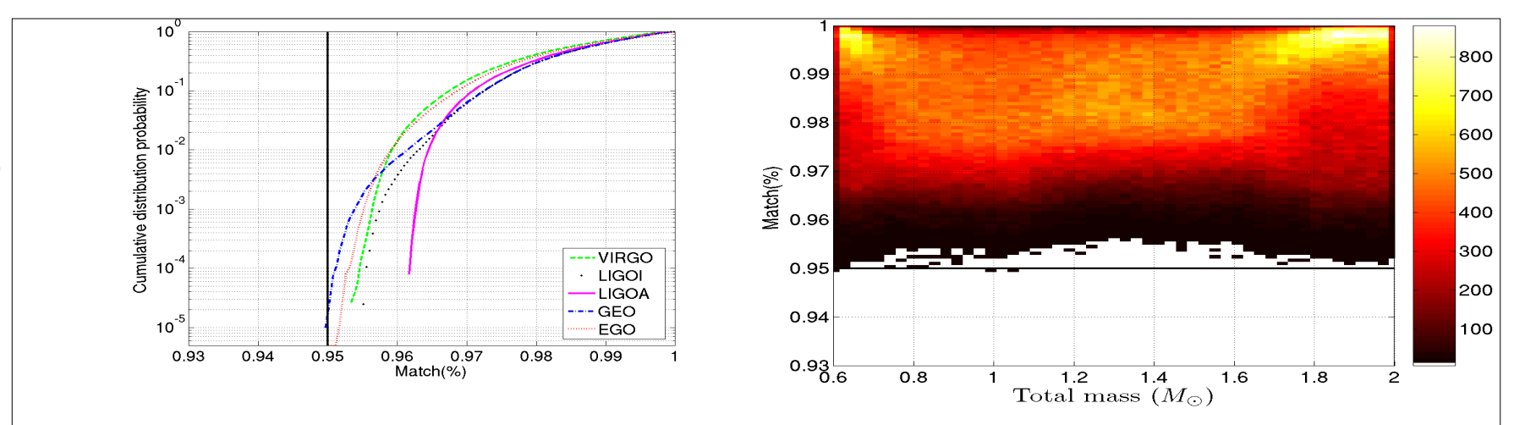

Figure 5 a, PBH injections, SPA model

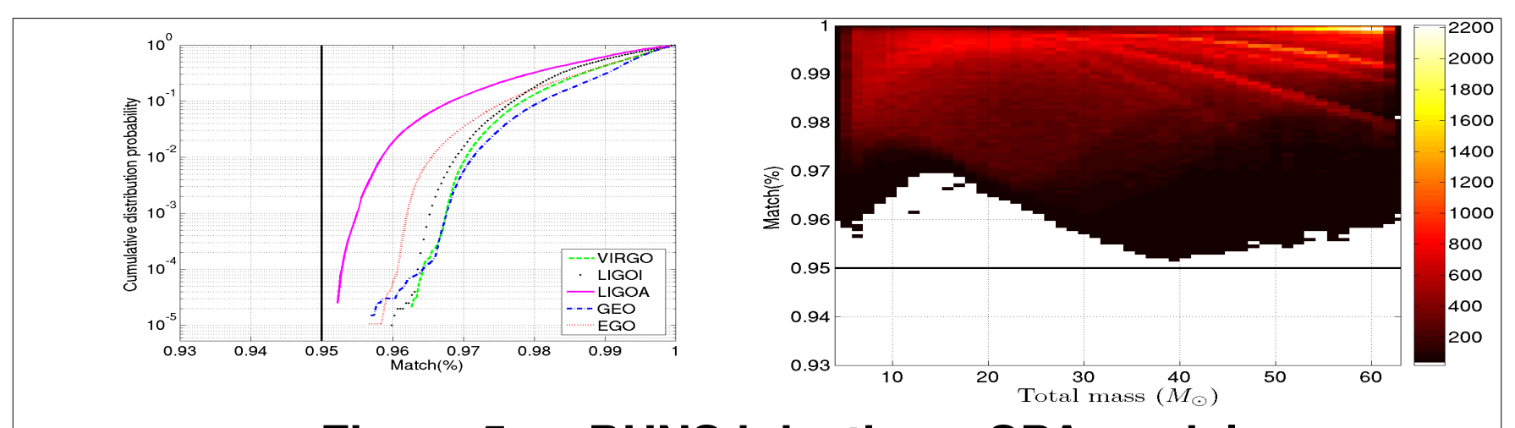

Figure 5 c, BHNS injections, SPA model

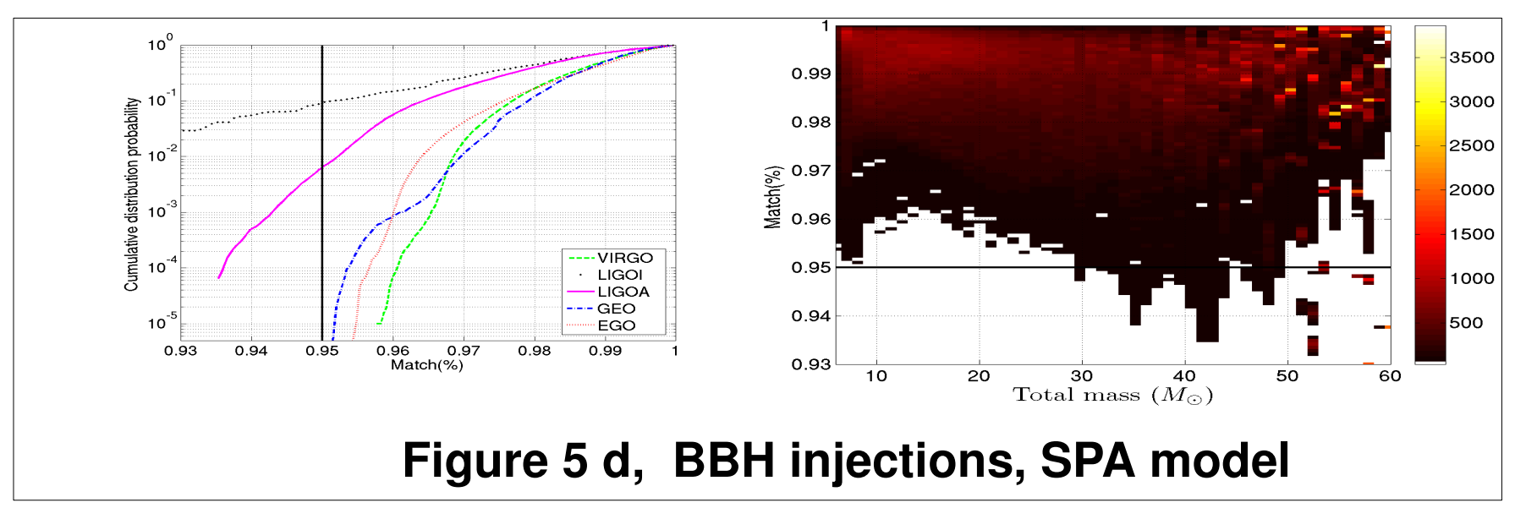

Figure 5 d, BBH injections, SPA model

Potential of the proposed template bank

The proposed hexagonal template bank was designed and tested to search for inspiralling compact binaries in the context of stationary phase approximant model at 2PN (the metric is based on this partcular model).

There are other template families such as Pade resummation, Effective One Body. Therefore, we tested the template bank with those models as well (where both injections and templates are based on the same model). As we can see in Fig. 6, the proposed bank has also a remarkable good efficiency for the different models that have been used in this test.

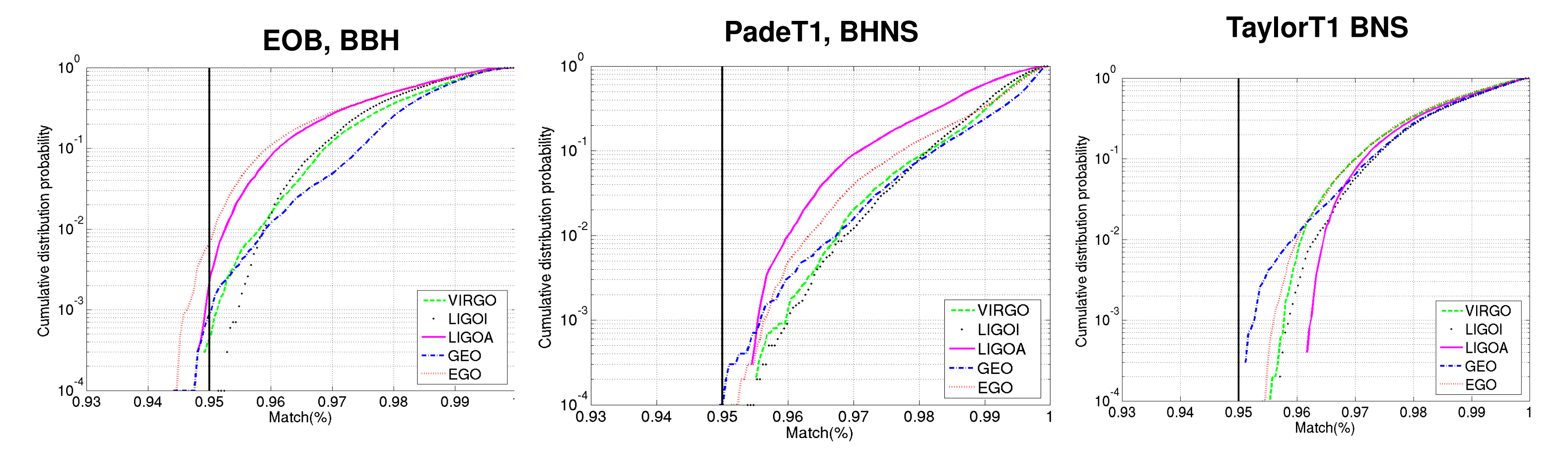

Figure 6

Therefore, we can use the proposed template bank to search for various physical template families. We will test it for various PN order, and for amplitude corrected waveforms.

Conclusion and perspectives

We developed a hexagonal template placement that is currently used to search for inspiralling compact binaries into LIGO S5 data.

The placement is close to optimality and fast. It uses an hexagonal placement that reduces by 40% the number of templates required to cover the astrophysically pertinent parameter space with respect to the template bank that was used in S1, S2, S3 and S4 LIGO data. The proposed template bank can be used to search for BNS, BBH, BHNS, and PBH binary searches.

We performed exhaustive simulations to show that efficiency of the template bank lies between unity and the requested minimal match. We also emphasizes the fact that many theoretical waveforms can be used with this template bank: we can search for signals based on EOB, Pade or Taylor models (time domain) in addition to the usual stationary phase approximation model.

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