A Radiometer for a Stochastic Background of Gravitational Radiation

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LIGO Scientific Collaboration

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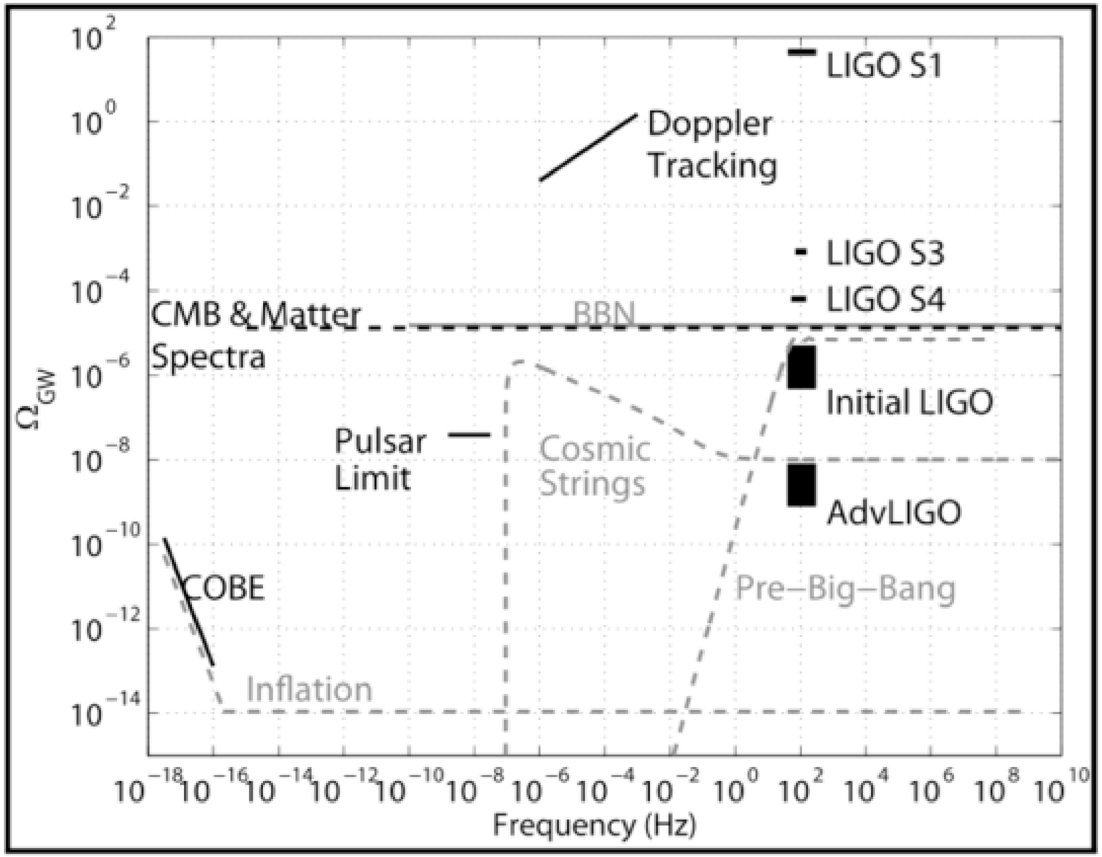
A Stochastic Background of Gravitational Waves

Signals from cosmological sources and astrophysical foregrounds



Characterized by
gravitational wave spectrum

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}}{df}$$

Approximate by power law in LIGO band

$$\Omega_{GW}(f) = \Omega_\alpha \left(\frac{f}{100\ \mathrm{Hz}}\right)^{\alpha}$$

Also see presentations by
J. Whelan and B. Whiting

Isotropic Search: Average over the whole sky

Cross Correlation Estimator

$$Y = \int_{-\infty}^{+\infty} df\ \tilde{s}_1^*(f)\ \tilde{s}_2(f)\ \tilde{Q}(f)$$

With a variance

$$\sigma_Y^2 \approx \frac{T}{2} \int_0^{+\infty} df\ P_1(f)\ P_2(f)\ |\ \tilde{Q}(f)\ |^2$$

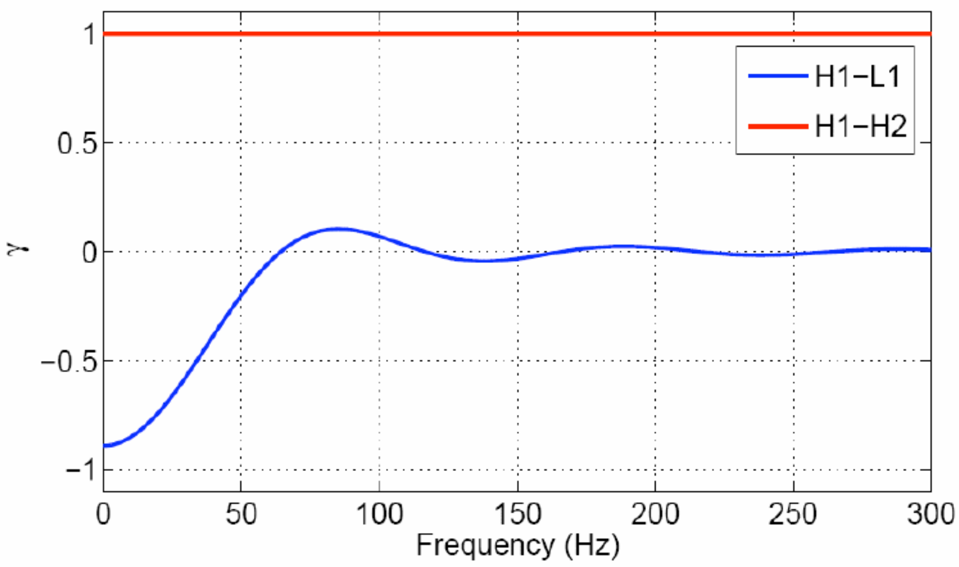
Using an Optimal Filter

$$\tilde{Q}(f) = \frac{1}{N} \frac{\gamma(f)\ \Omega_t(f)}{f^3\ P_1(f)\ P_2(f)}$$

Assuming a Source Strain Spectrum

$$\Omega_t(f) = \Omega_\alpha(f/100\ \mathrm{Hz})^\alpha$$

γ(f) = Overlap Reduction Function



S4 Science Run, H1L1+H2L1:

H0 = 72 km/s/Mpc

51-150 Hz (includes 99% of inverse variance

$\Omega_0 \pm \sigma\Omega$ = (-0.8 ± 4.3) × 10-5

90% Bayesian UL: **6.5 × 10-5**

Also see presentation by Nick Fotopoulos

GW Radiometer Motivation

- Stochastic GW Background due to Astrophysical Sources?
  - Not isotropic if dominated by nearby sources
  - ➔ Do a ***Targeted Stochastic Search*** with LIGO

- Source position information from
  - Signal time delay between different sites (sidereal time dependent)
  - Sidereal variation of the single detector acceptance

➔ Time-Shift and Cross-Correlate!

➔ Effectively a Radiometer for Gravitational Waves

The Radiometer

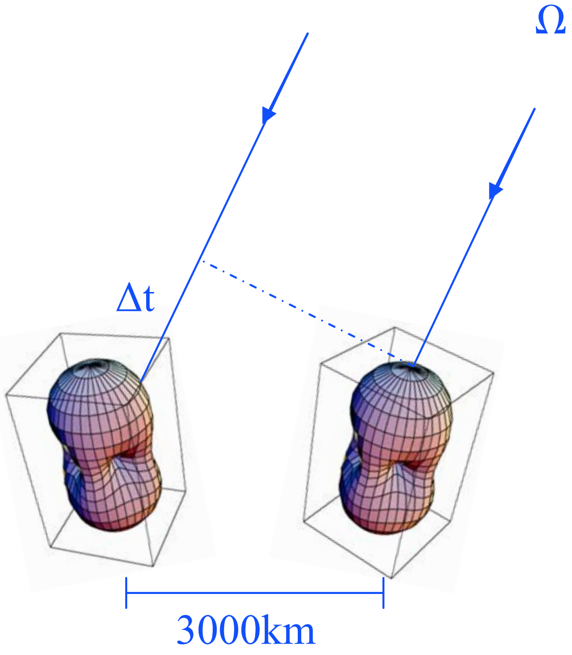
$$Y(\Omega) = \int dt \int dt' \, s_1(t) s_2(t') \tilde{Q}_{t_{sidereal}\,\Omega}(t-t')$$

$$\sigma^2(\Omega) \approx \frac{T}{4}(Q_{t_{sidereal}\,\Omega}, Q_{t_{sidereal}\,\Omega})$$

$$Q_{t_{sidereal}\Omega}(f) \propto \frac{H(f)\,\gamma_{t_{sidereal}\Omega}(f)}{P_1(f)P_2(f)}$$

$H(f)$ is the source spectrum

Detailed description in gr-qc/0510096



LIGO’s Fourth Science Run (S4)

$$H_\beta(f) \propto \left(\frac{f}{100Hz}\right)^\beta$$

(β=-3 corresponds to scale-invariant primordial perturbation spectrum)

astro-ph/0703234
submitted to PRD

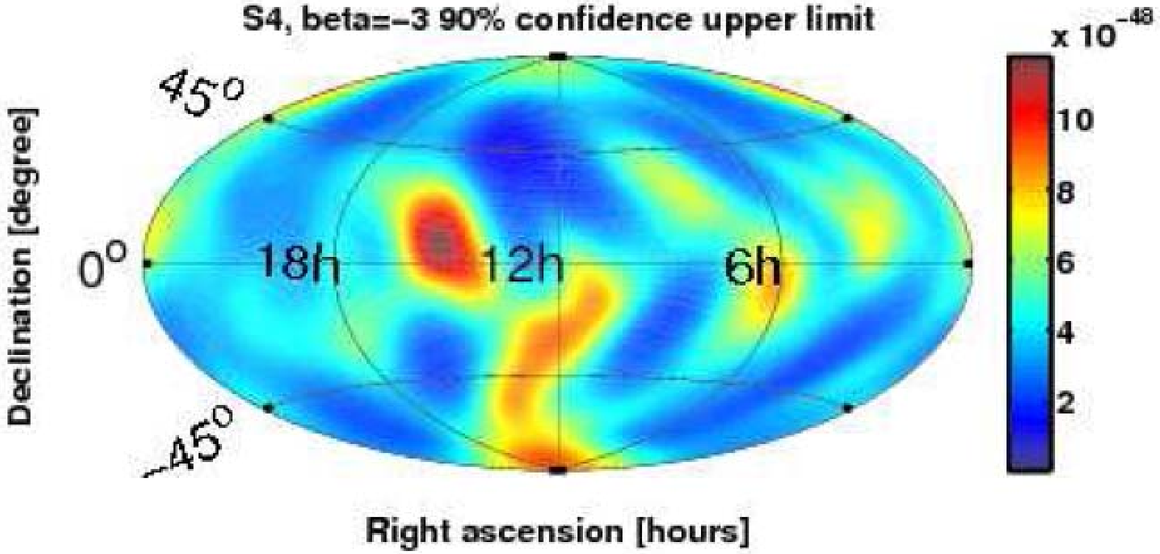


FIG. 6: **S4 Result:** Map of the 90% confidence level Bayesian upper limit on H_β for $\beta = -3$. The upper limit varies between $1.2 \times 10^{-48} \mathrm{Hz}^{-1} (100\ \mathrm{Hz}/f)^3$ and $1.2 \times 10^{-47} \mathrm{Hz}^{-1} (100\ \mathrm{Hz}/f)^3$, depending on the position in the sky. All fluctuations are consistent with the expected noise.

Upper Limit map
H(f)=const (β=0)

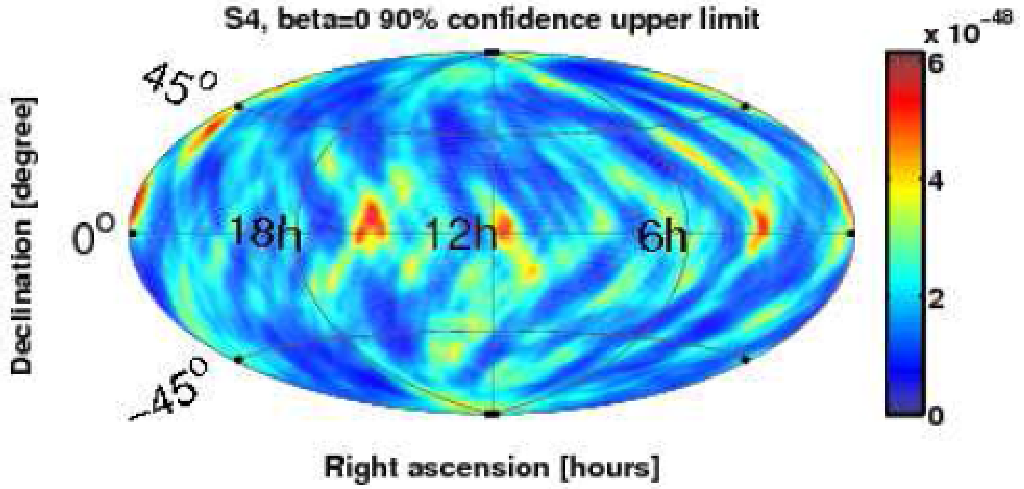
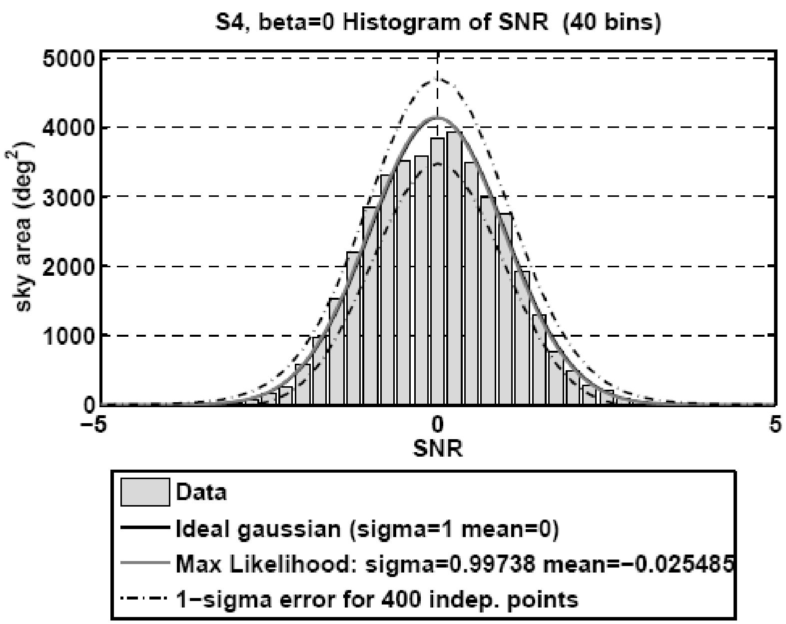
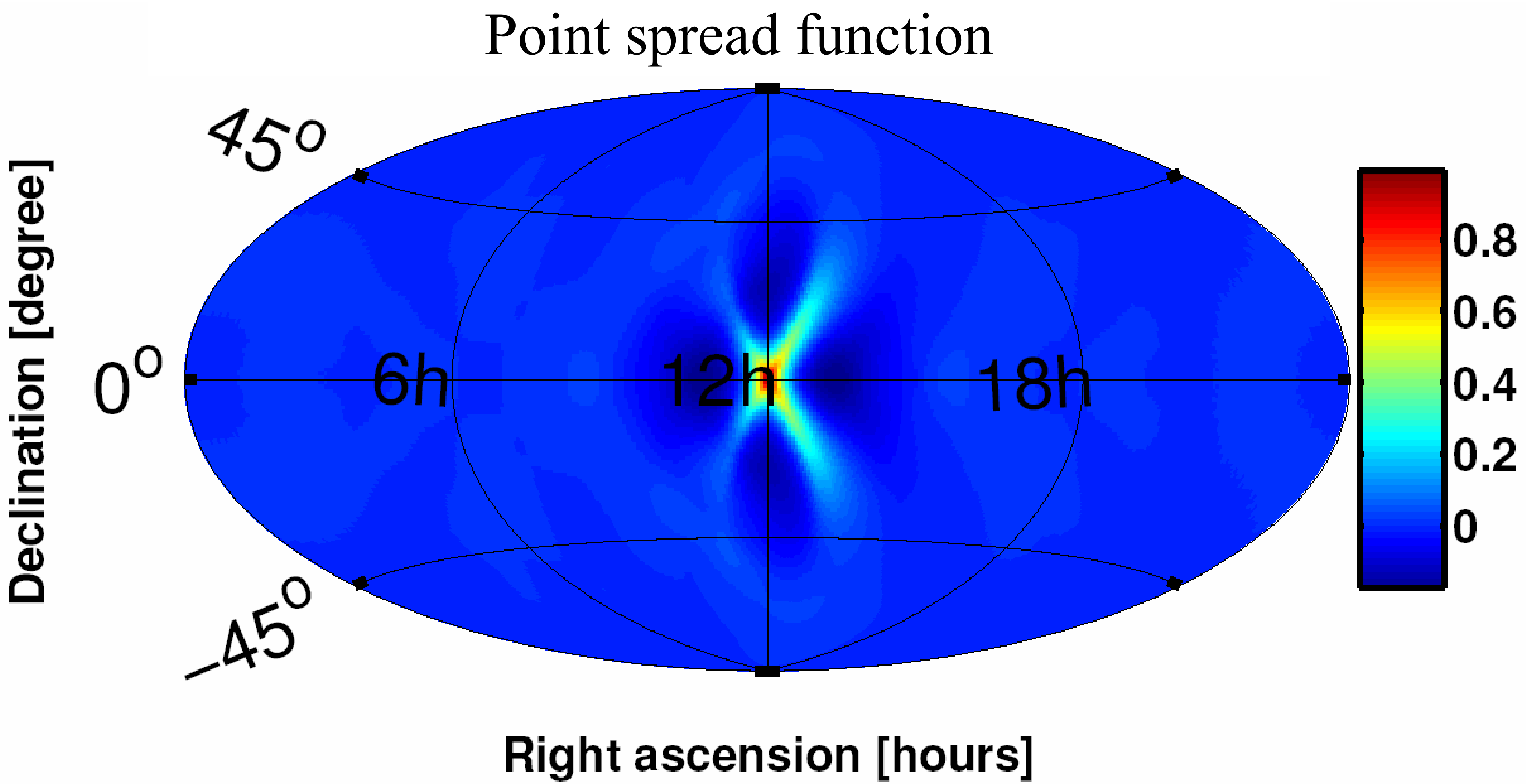


FIG. 7: **S4 Result:** Map of the 90% confidence level Bayesian upper limit on H_β for $\beta = 0$.The upper limit varies between $8.5 \times 10^{-49}\,\mathrm{Hz}^{-1}$ and $6.1 \times 10^{-48}\,\mathrm{Hz}^{-1}$ depending on the position in the sky.

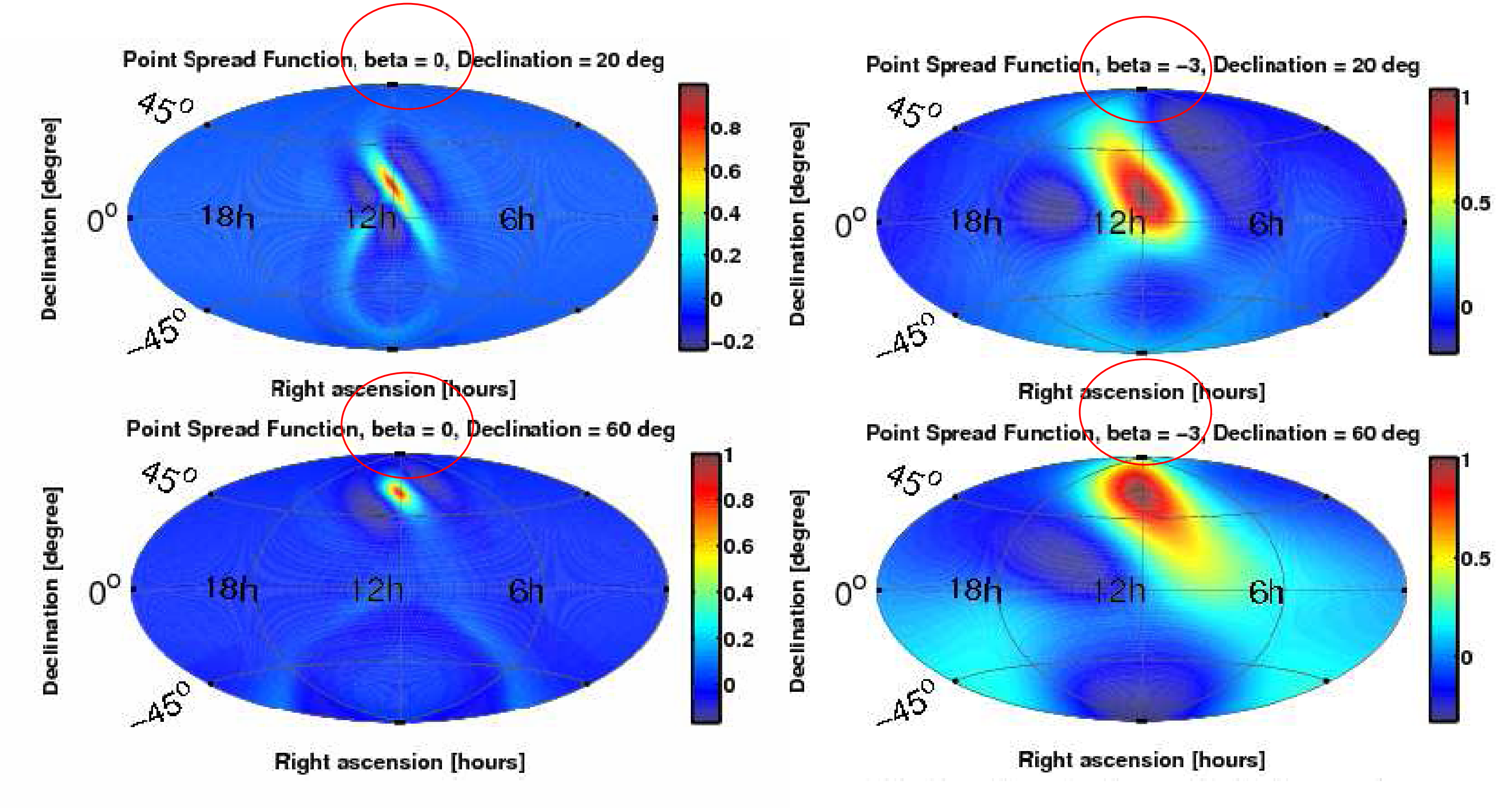


$$\boxed{H_{90\%} = (0.85 - 6.1)\times 10^{-48}\,\mathrm{Hz}^{-1}}$$

But the maps are convolved

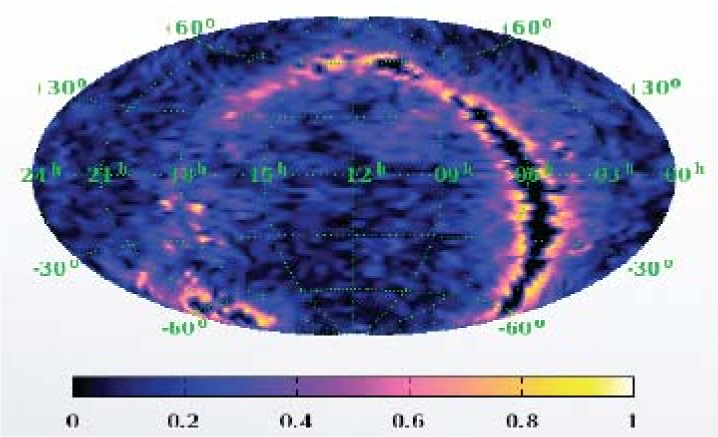


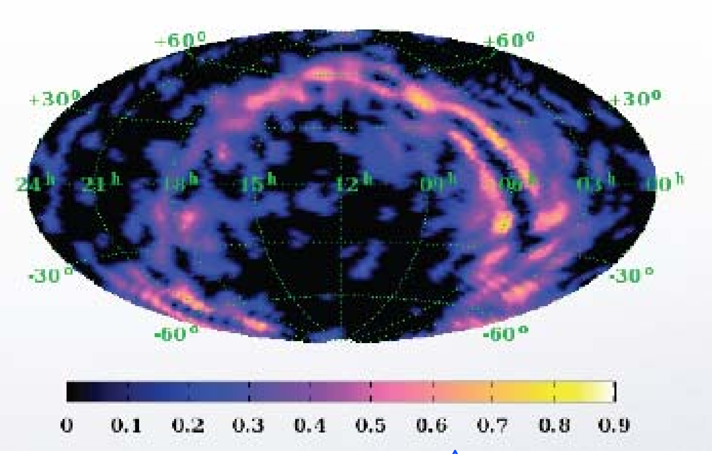
Point Spread Function



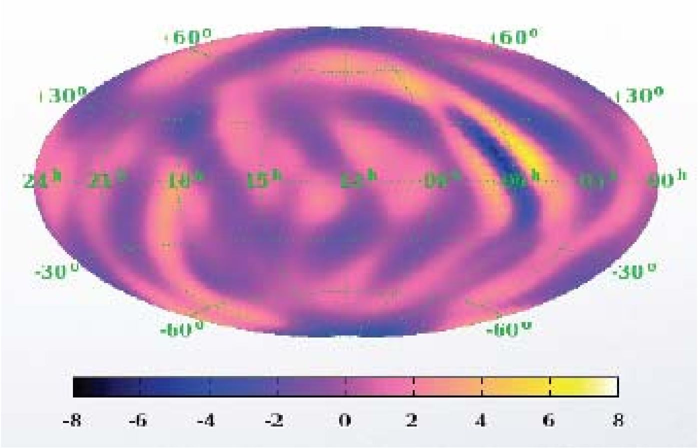
Deconvolving the map to get a Maximum Likelihood Estimation

Toy CMB map from Planck Simulator





Run through radiometer analysis to get “dirty” map; this convolves with point spread function



To deconvolve, invert the covariance matrix, which is large and varies with time & IFO sensitivity → computationally expensive

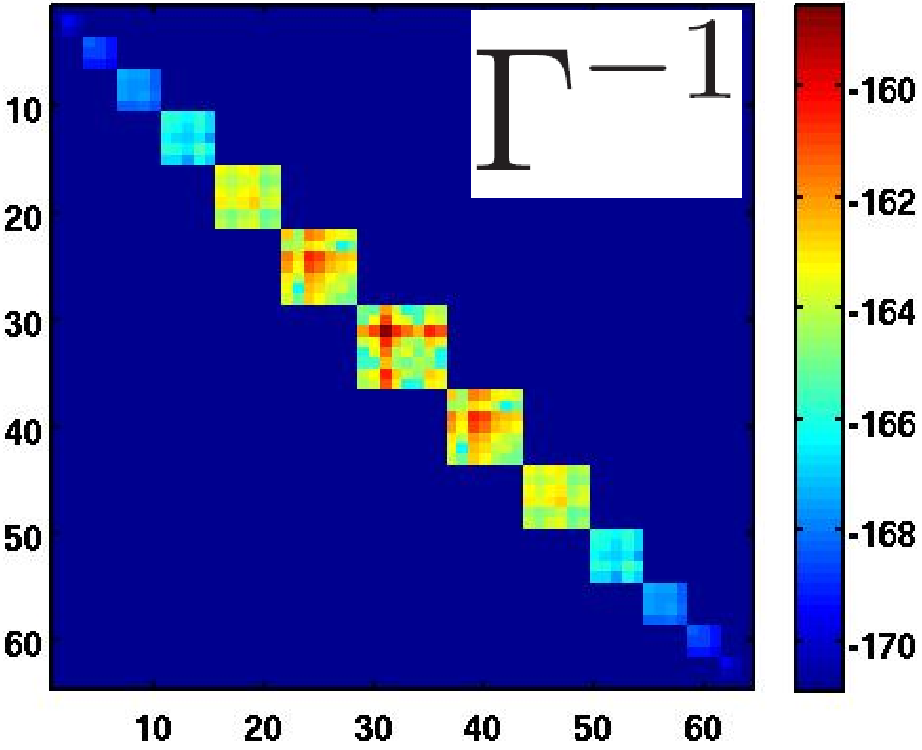
Or use a Spherical Harmonic Basis: Maximum Likelihood Estimation

- Rotational Symmetry➔ covariance =0 for m≠m’
- different l’s at the same m are correlated
- Symmetry broken due to diurnal sensitivity variations

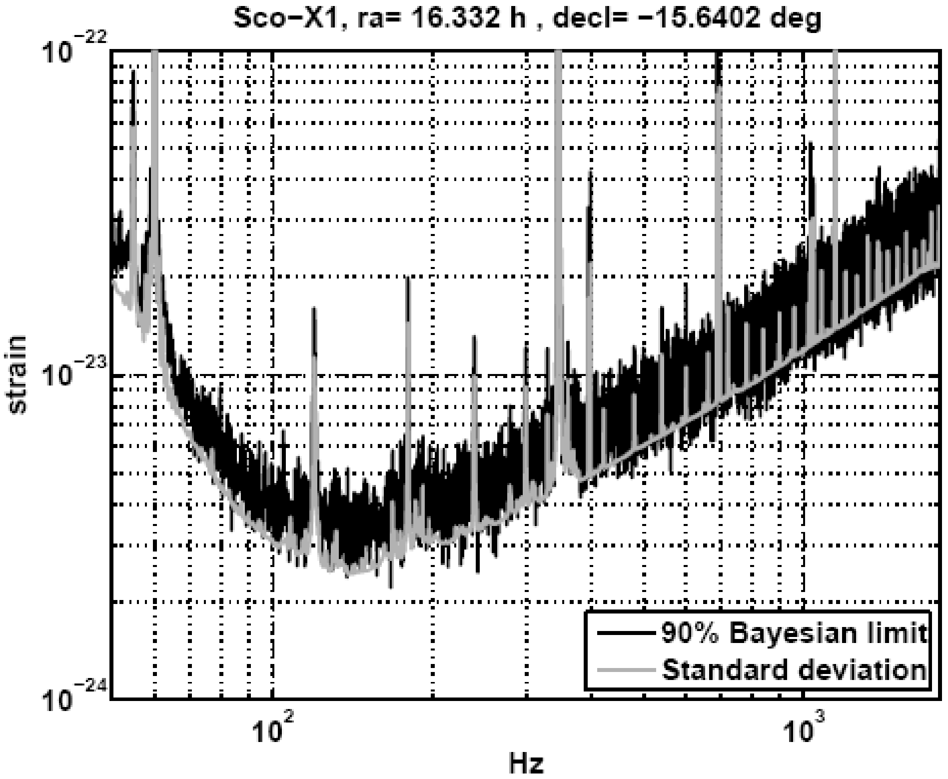
Advantage of a smaller (tens instead of thousands), block diagonal, covariance matrix→lower computational cost.

Being actively pursued by the stochastic analysis group.

Covariance matrix



Narrowband Radiometer
Sco-X1 (brightest LMXB), S4



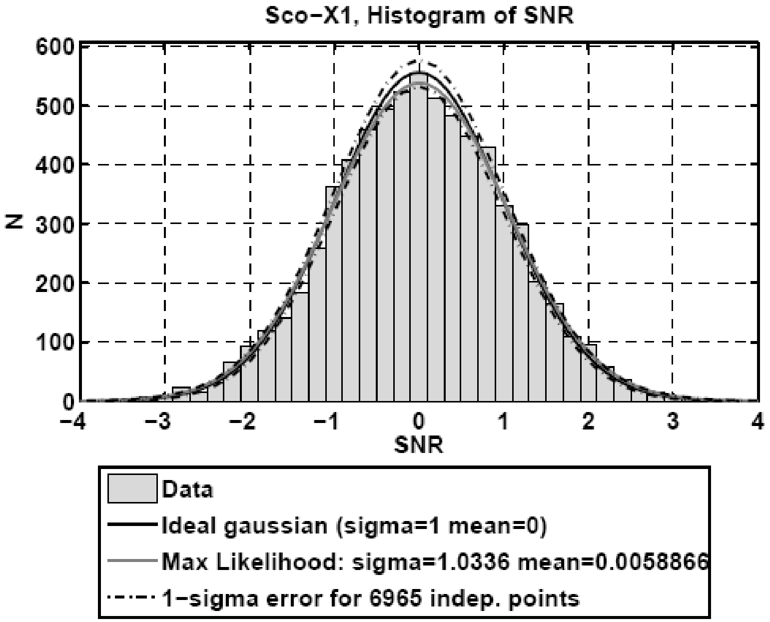


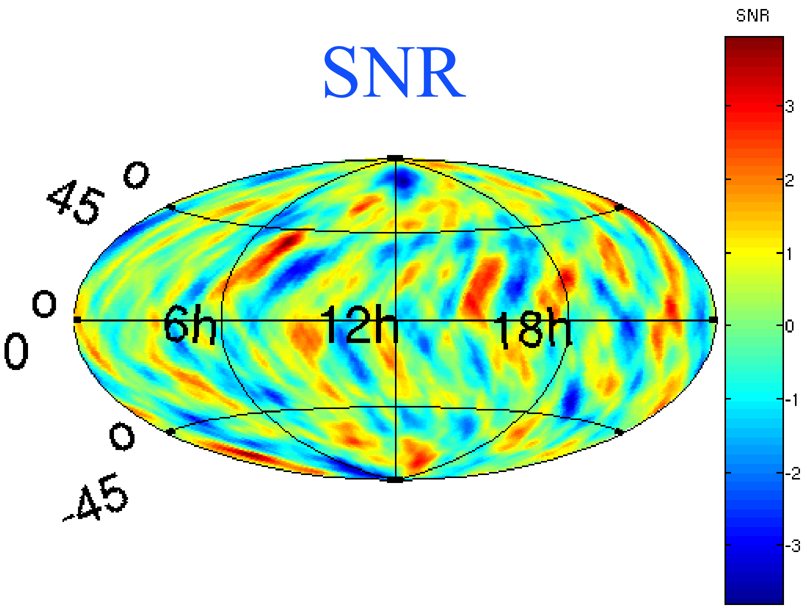
FIG. 9: **S4 Result for Sco-X1:** The 90% confidence Bayesian upper limit as a function of frequency - marginalized over the calibration uncertainty. The standard deviation (one sigma error bar) is shown in blue.

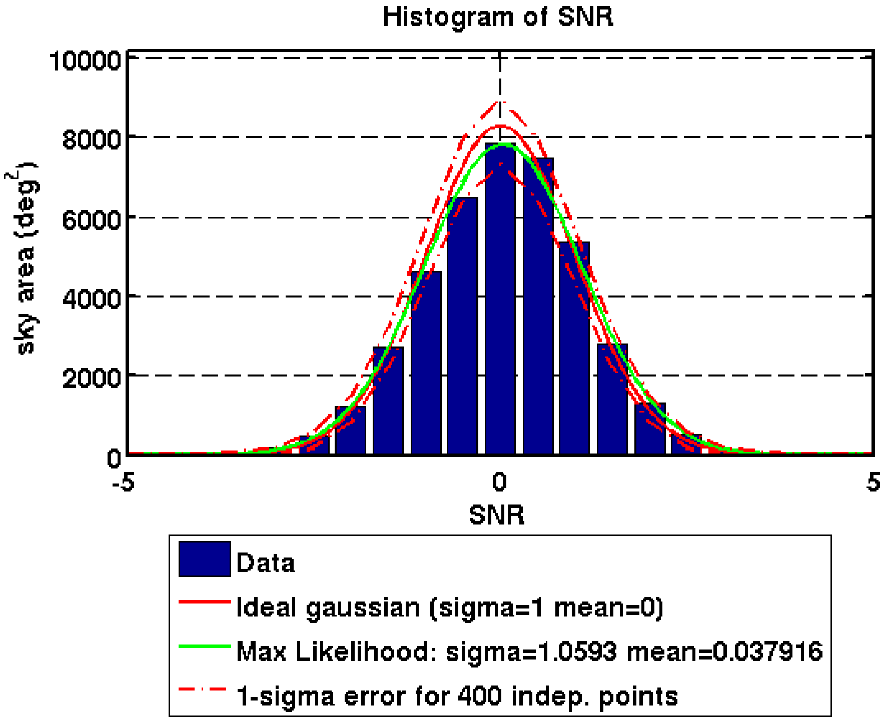
This is currently the best published limit on the gravitational wave flux coming from the direction of Sco-X1

Consistent with no signal

Blinded Data from S5

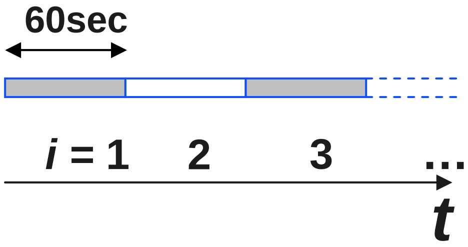
time shift the detector streams by more than the light travel time between detectors→no true gw signal remains



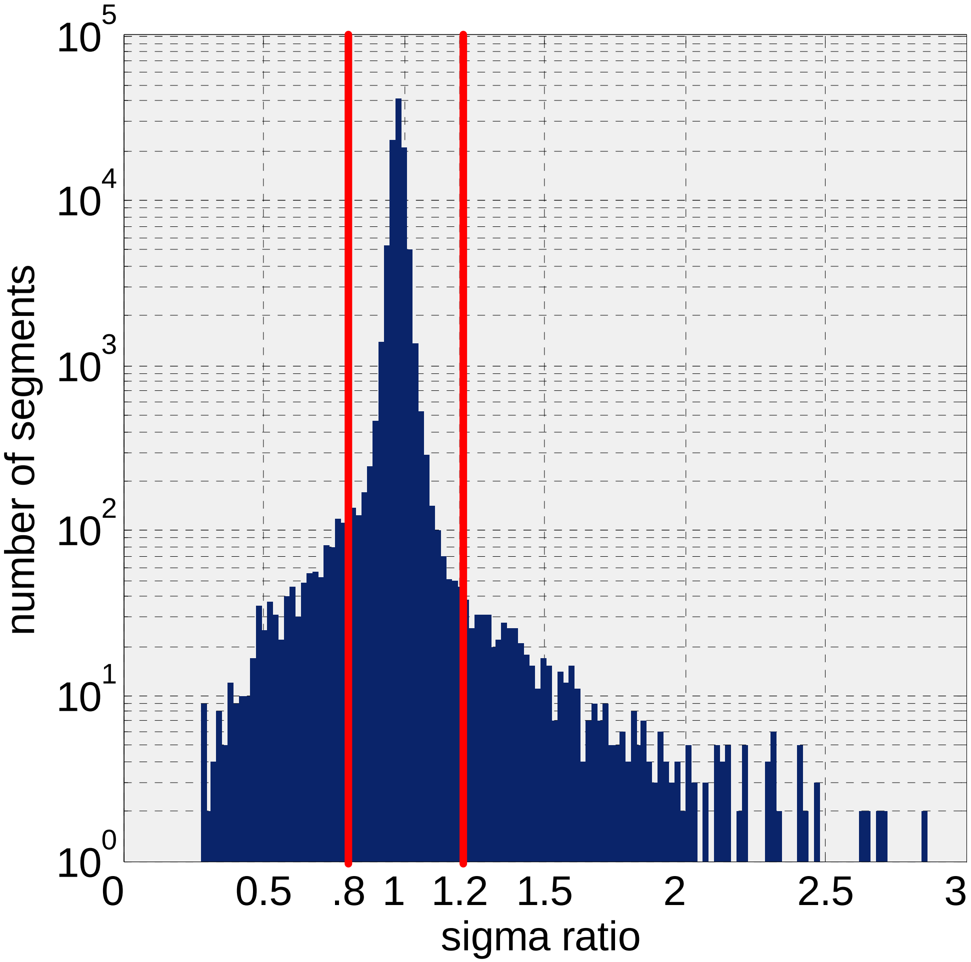


S5 result should be at least 10x better than S4 result

Detector Noise Non-stationarity: The Sigma Ratio Data Quality Cut

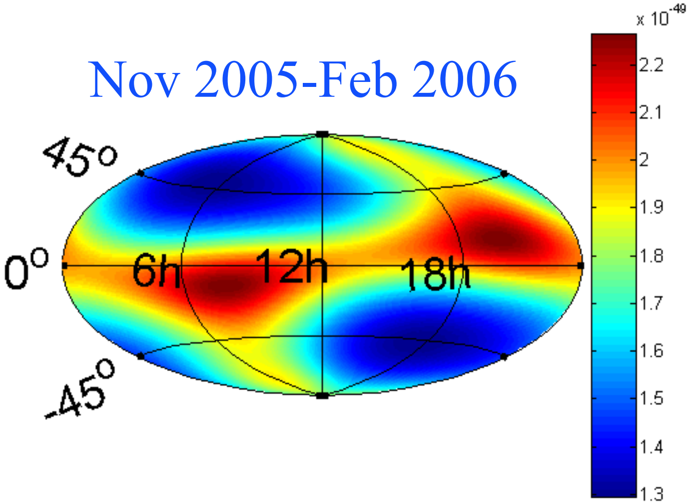


- Don’t include any 60 second segments whose PSD gives a 20% larger sigma than neighboring PSD’s to reduce effects of nonstationarity.
- This rejects 1.80% of the data
- 1st 4 months of S5 (blind)



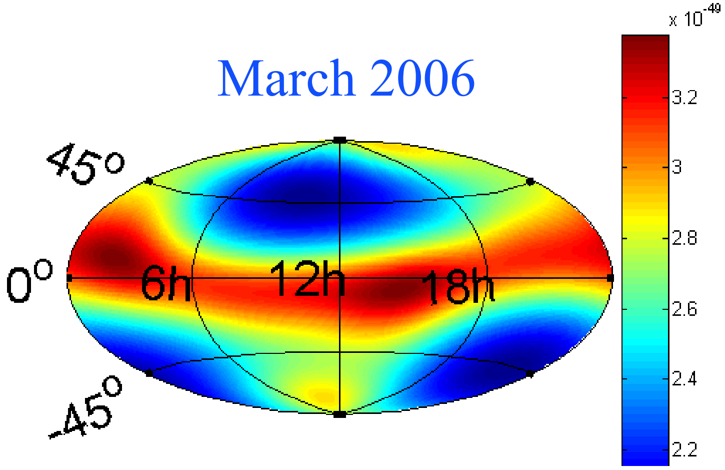
Theoretical Sigma Maps from 2006 (S5, blind)

Nov 2005-Feb 2006

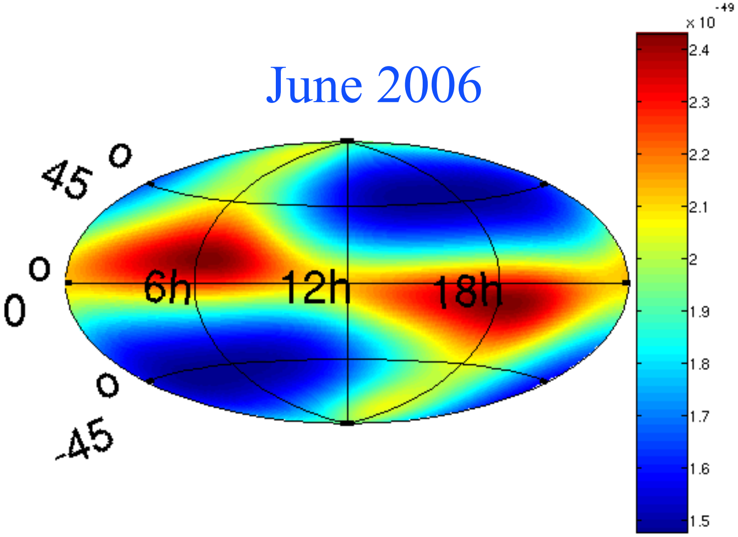


$$\sigma(\Omega) = (1.3 - 2.27)\times 10^{-49}\ \mathrm{Hz}^{-1}$$

March 2006



June 2006



Because the IFOs still work better at night, the region of best sensitivity moves across the sky as the earth goes around the sun

Another detection method: autocorrelation at each declination

- NOT a standard 2-point correlation function
    - Can't because of variability of point spread function with sky position
- X(ΔRA) at each declination (integral is over RA)
- Y_i's are point estimates of GW strain
- w_i's are statistical weights of each point on the sky (1st attempt: use reciprocal of theoretical sigma)
- Overall normalization is X(0)

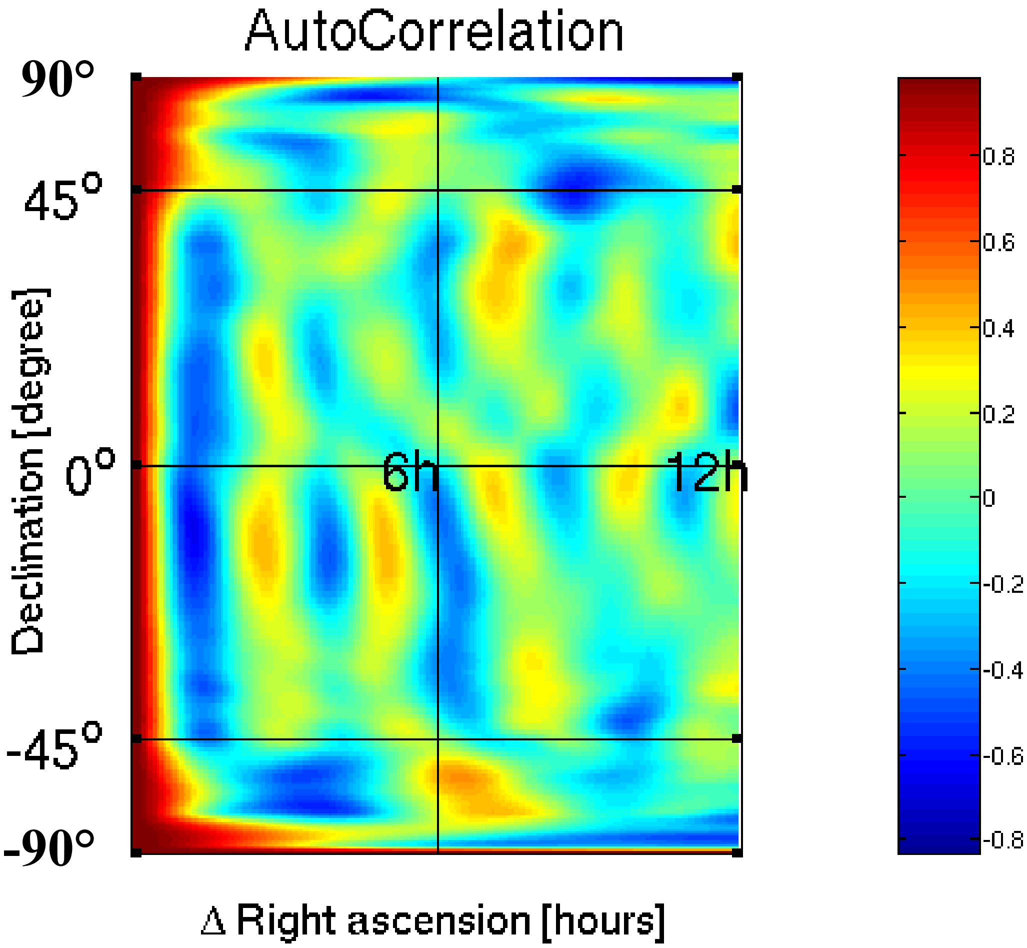
For all i,j separated by ΔRA

$$X(\Delta RA) = \sum_{i,j} w_i Y_i Y_j w_j$$

$$w_i = \frac{\langle \sigma \rangle_i}{\sigma_i}$$

Autocorrelation at each declination

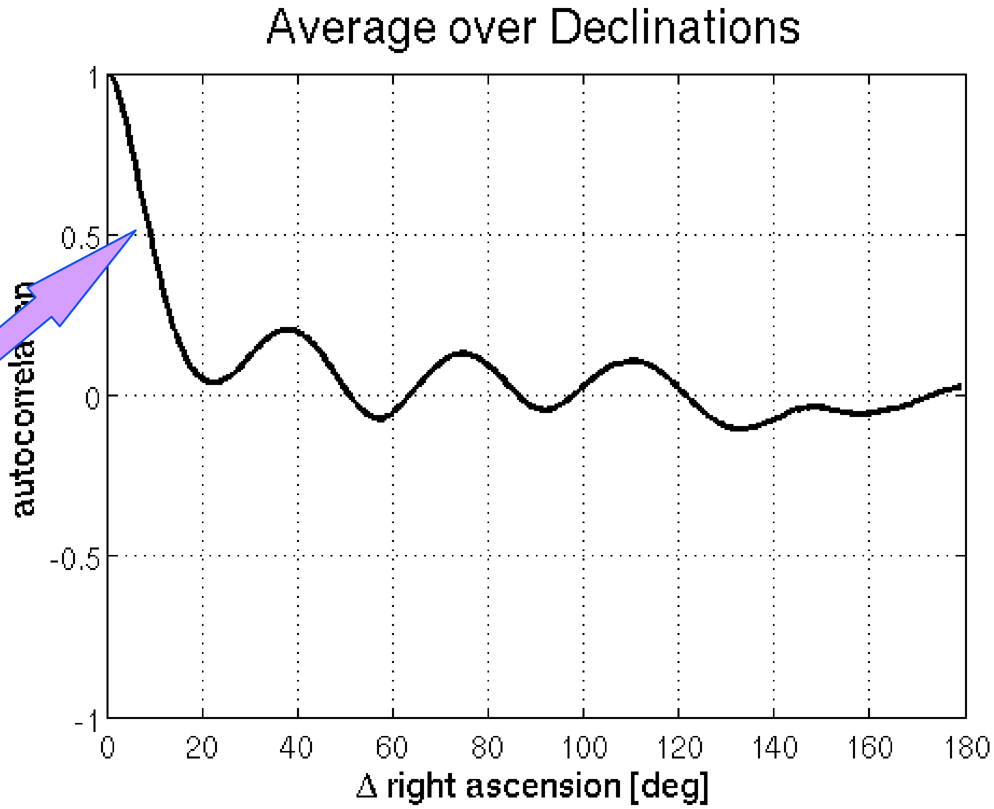
- Simulated data
- H(f) = const
- Working on a "detection metric"—what quantifies absence/presence of signal?



Average the autocorrelations at each declination, over all declinations

In the absence of signal, this can tell us about the resolving power of the instrument; consistent with other estimates.

$$\frac{\lambda}{D}\quad\frac{500\,Hz}{3000\,km} \Rightarrow 11^{\circ}$$



diffraction limited gw astronomy

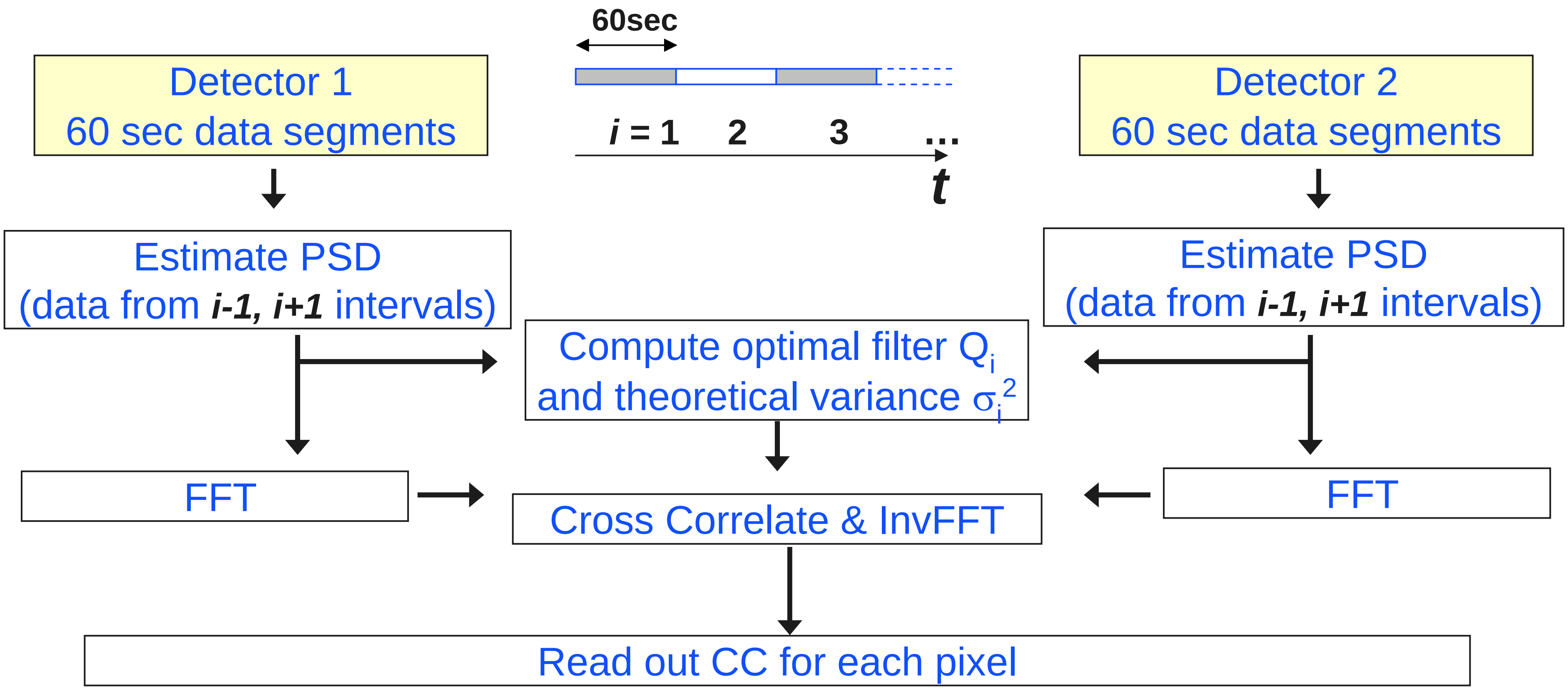
The Near Future

- Projected sensitivity increase and longer run time means we should surpass BBN bound during S5, for the isotropic analysis (integrating over the whole sky).
- Conduct narrowband searches from more directions (Virgo cluster, galactic plane)
- Continue development of maximum likelihood analysis, in both point-source (pixel) basis and spherical harmonic basis.

The End

The End

Data Analysis Flow



60sec

$i = 1 \quad 2 \quad 3 \quad \ldots$

t

Detector 1
60 sec data segments

Estimate PSD
(data from ***i*-1, *i*+1** intervals)

FFT

Compute optimal filter Q_i
and theoretical variance σ_i^2

Cross Correlate & InvFFT

Detector 2
60 sec data segments

Estimate PSD
(data from ***i*-1, *i*+1** intervals)

FFT

Read out CC for each pixel