Stochastic Gravitational-Wave Background Limits from LLO-ALLEGRO Correlations

John T. Whelan (for the LIGO Scientific Collaboration & the ALLEGRO Group)
Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Potsdam, Germany
`john.whelan@ligo.org`

Abstract

Data from the interferometer (L1) at the LIGO Livingston Observatory (LLO) and the ALLEGRO resonant bar detector (A1), taken during LIGO's fourth science run, were examined for cross-correlations indicative of a stochastic gravitational-wave background in the frequency range 850-950 Hz, with most of the sensitivity arising between 905 Hz and 925 Hz. ALLEGRO was operated in three different orientations during the experiment to modulate the relative sign of gravitational-wave and environmental correlations. No statistically significant correlations were seen in any of the orientations, and the results were used to set a Bayesian 90% confidence level upper limit of $\Omega_{\text{gw}}(f) \leq 1.02$*, which corresponds to a gravitational wave strain at 915 Hz of* $1.5 \times 10^{-23}\,\text{Hz}^{-1/2}$*. In the traditional units of* $h_{100}^2 \Omega_{\text{gw}}(f)$*, this is a limit of* 0.53*, two orders of magnitude better than the previous direct limit at these frequencies. The method was also validated with successful extraction of simulated signals injected in hardware and software. These results are published as [1].*

Background/Motivation for LLO-ALLEGRO Search

Searches for an isotropic stochastic background seek to measure the strength of a GW spectrum $S_{\text{gw}}(f)$ defined by

$$\langle \widetilde{h}_1^*(f) \widetilde{h}_2(f') \rangle = \frac{1}{2} \delta(f - f')\, S_{\text{gw}}(f)\, \gamma_{12}(f) \qquad (1)$$

using an Optimally filtered cross-correlation statistic

$$Y = \int_{f_{\text{min}}}^{f_{\text{max}}} df\, \widetilde{s}_1^*(f)\, \widetilde{Q}(f)\, \widetilde{s}_2(f) \qquad (2)$$

with optimal filter

$$\widetilde{Q}(f) \propto \frac{S_{\text{gw}}(f) \gamma_{12}(f)}{P_1(f) P_2(f)} \qquad (3)$$

Initial analyses assume $S_{\text{gw}}(f)$ or $\Omega_{\text{gw}}(f)$ constant across band, where

$$\Omega_{\text{gw}}(f) = \frac{f}{\rho_{\text{crit}}} \frac{d\rho_{\text{gw}}}{df} \propto f^3 S_{\text{gw}}(f) \qquad (4)$$

The optimally filtered CC method has Ω_{gw} sensitivity

$$\sigma_\Omega \propto \left(T \int \frac{df}{f^6} \frac{\gamma_{12}^2(f)}{P_1(f) P_2(f)} \right)^{-1/2} \qquad (5)$$

Significant contributions come from frequencies where detector noise power spectra $P_1(f)$, $P_2(f)$ are small or the overlap reduction function

$$\gamma_{12}(f) = d_{1ab} d_2^{cd} \frac{5}{4\pi} \iint_{S^2} d^2\Omega_{\hat{\mathbf{n}}}\; P^{\text{TT}}{}^{ab}_{cd}(\hat{\mathbf{n}}) e^{i2\pi f \hat{\mathbf{n}} \cdot \Delta\vec{\mathbf{r}}/c} \qquad (6)$$

(geometric correction) is near ± 1

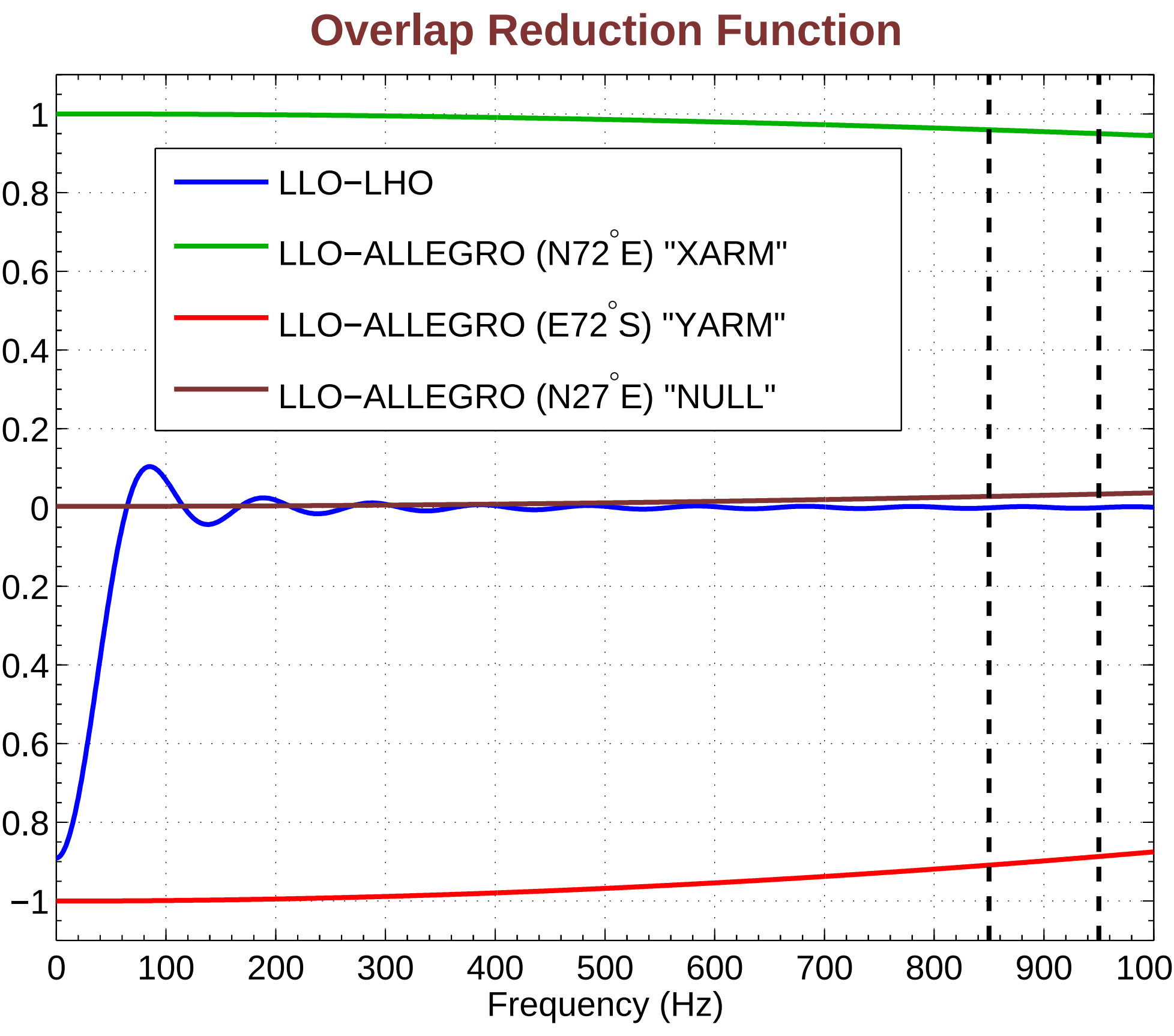

Figure 1: *Overlap reduction functions for LLO-ALLEGRO, with LLO-LHO shown for reference.*

LLO and ALLEGRO are only ~ 40 km apart, so they are still sensitive @ 900 Hz. The response is different for XARM, YARM and NULL orientations. ALLEGRO ran in all 3 orientations during LIGO S4 Run (2005 Feb 22-Mar 23).
L1 data were digitally downsampled from 16384 Hz to 4096 Hz, while A1 data were heterodyned @ 904 Hz & sampled @ 250 Hz. Differently-sampled data were correlated in frequency domain, using the method written up in [2]. The heterodyning means that the CC statistic is complex. The real part is Gaussian-distributed about the SGWB strength; the imaginary part is Gaussian-distributed about 0.

S4 Data

orientation	azimuth	$\gamma(850\,\text{Hz})$	$\gamma(950\,\text{Hz})$	T after cuts
YARM	N$108°$W	-0.9087	-0.8867	114.7 hr
XARM	N$18°$W	0.9596	0.9498	181.2 hr
NULL	N$63°$W	0.0280	0.0340	114.7 hr

Table 1: *The orientations in which ALLEGRO was operated during S4.*

Data were analyzed in three orientations, as shown in Table 1. The spectra of L1 and A1 in the sensitive XARM and YARM times is illustrated in Fig. 2. The sensitive frequency band is mostly determined by the A1 noise curve.

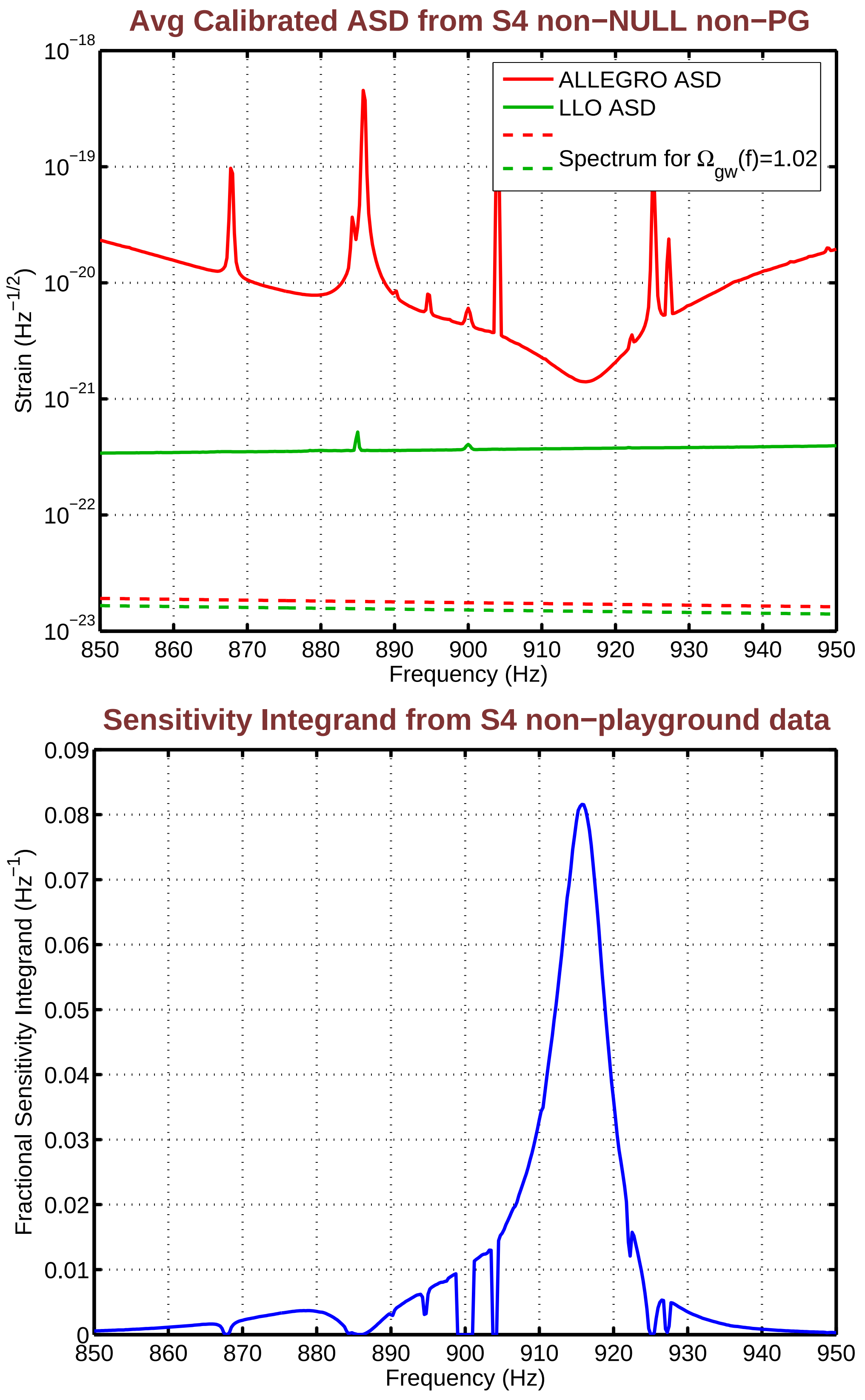

Figure 2: *Amplitude spectral densities for L1 and A1 in S4, and sensitivity integrand for S4 L1-A1 cross-correlation measurement. Most of the sensitivity comes from 905–925 Hz.*

Cross-Correlation Results

We searched for a constant (across our band) $\Omega_{\text{gw}}(f) \equiv \Omega_R$ (assuming $H_0 = 72\,\text{km/s/Mpc}$). The results of the cross-correlation analysis are shown in Table 2.

		Ω_R	
Type	T_{eff} (hrs)	Point Estimate	Error Bar
XARM	181.2	$0.61 + 0.25i$	0.56
YARM	114.7	$-0.47 + 0.47i$	0.90
non-NULL	295.8	$0.31 + 0.31i$	0.48
NULL	88.2	$10.96 - 43.89i$	28.62
all	384.1	$0.31 + 0.30i$	0.48

Table 2: *Cross-correlation results*

No correlation was observed, so we convert the point estimate of $0.31 + 0.30i$ & errorbar of 0.48 into an upper limit. We used a formal prior on $\Omega_{\text{gw}}(915\,\text{Hz})$ from the Explorer-Nautilus result[3]: uniform on $[0, 115]$. We marginalized the likelihood function over the calibration uncertainty (5% amplitude & $2°$ phase for L1; 10% amplitude & $3°$ phase for A1) to get a posterior. This corresponded to a 90% CL Bayesian upper limit of $\Omega_R < 1.02$ i.e., $\sqrt{S_{\text{gw}}(915\,\text{Hz})} < 1.5 \times 10^{-23}\,\text{Hz}^{-1/2}$; this was a $100\times$ improvement on the previous limit of $\Omega_{\text{gw}}(907\,\text{Hz}) < 115$ [$h_{100}^2 \Omega_{\text{gw}}(907\,\text{Hz}) < 60$] [3].

Software Injections

We added simulated signals of strength $\Omega_R = 1.9$, 3.9, 9.6, 19 into a 9% subset of the data, and verified that our pipeline recovers them at the expected strength, as summarized in Fig. 3.

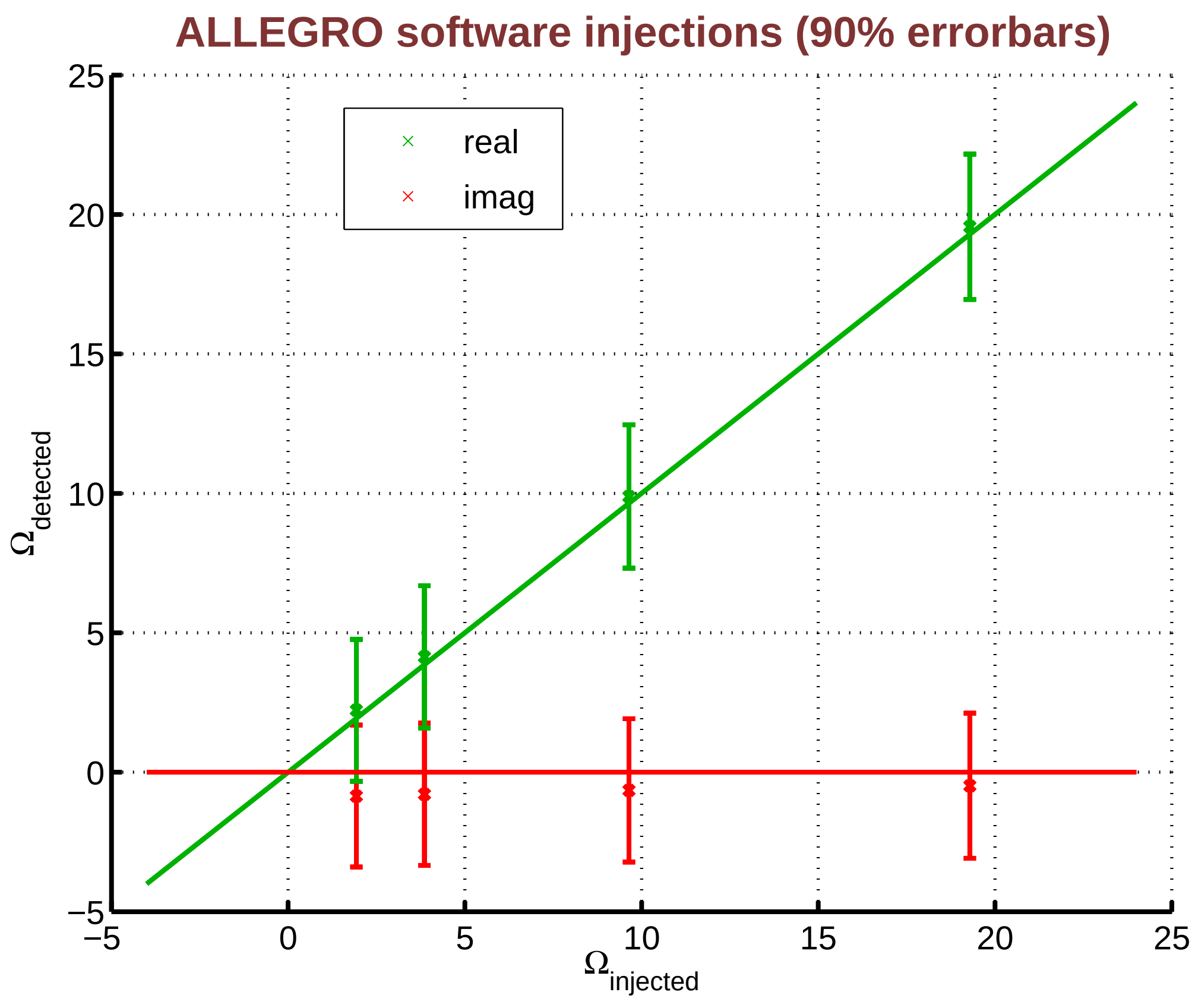

Figure 3: *Results of software injections*

Hardware Injections

We also injected stronger ($\Omega_{\text{gw}}(f) = 8100$) but shorter (1024 secs) signals directly into the LLO & ALLEGRO hardware. We simulated all three orientations in rounds "A" & "B". The simulated NULL signals had levels of correlation consistent with their (large, due to the simulated misalignment) statistical errorbars. The "plus" (simulated XARM) & "minus" (simulated YARM) signals were recovered at levels consistent with the calibration uncertainty, as illustrated in Fig. 4.

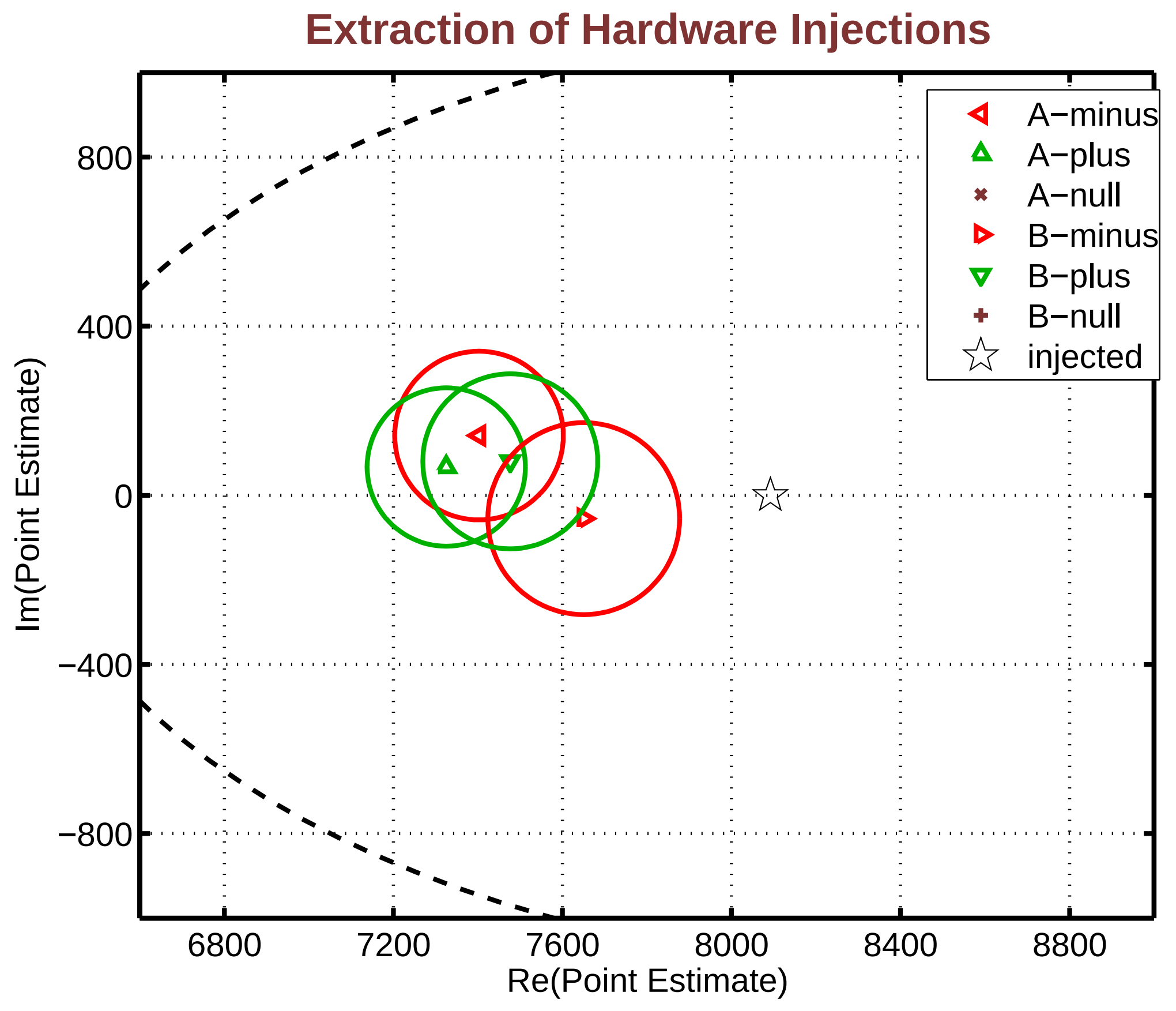

Figure 4: *Recovery of hardware injections. The colored circles are 90% CL statistical error bars around the measurements. The dashed black shape represents the calibration uncertainty around the injected signal strength. The point estimates of the null injections are outside the limits of this plot, but the whole plot is contained within their error circles.*

Summary

We performed a cross-correlation analysis of L1 and A1 data from LIGO's S4 run, looking for a stochastic GW background in the frequency band $\sim 850 - 950$ Hz. Data from 3 different orientations of ALLEGRO were used to modulate the expected response to an isotropic BG. No significant correlations were seen in 384.1 hours of data, so we set a 90% CL upper limit on $\Omega_{\text{gw}}(915\,\text{Hz})$ of 1.02. This is a $100\times$ improvement on the previous results at these frequencies[3]. The analysis was shown to extract long, low-amplitude simulated signals (software injections). Strong, short injections directly into the instrumental hardware were extracted consistent with the calibration uncertainty.

References

[1] B. Abbott et al (LSC), to appear in PRD; gr-qc/0703068
[2] JTW et al, *CQG* **22**, S1087 (2005); gr-qc/0506025
[3] P. Astone et al., *A&A* **351**, 811 (1999).

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