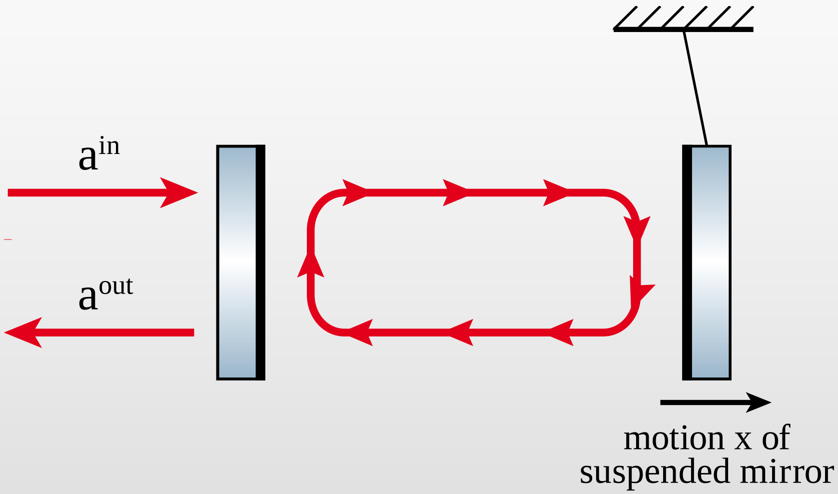
Improving Advanced LIGO sensitivity using a local readout scheme

Henning Rehbein, Helge Müller-Ebhardt, Kentaro Somiya, Chao Li, Roman Schnabel, Karsten Danzmann, Yanbei Chen

Max-Planck-Institut für Gravitationsphysik (AEI)
Institut für Gravitationsphysik, Leibniz Universität Hannover

Dynamics of Single Cavity

Equations of motion of suspended mirror



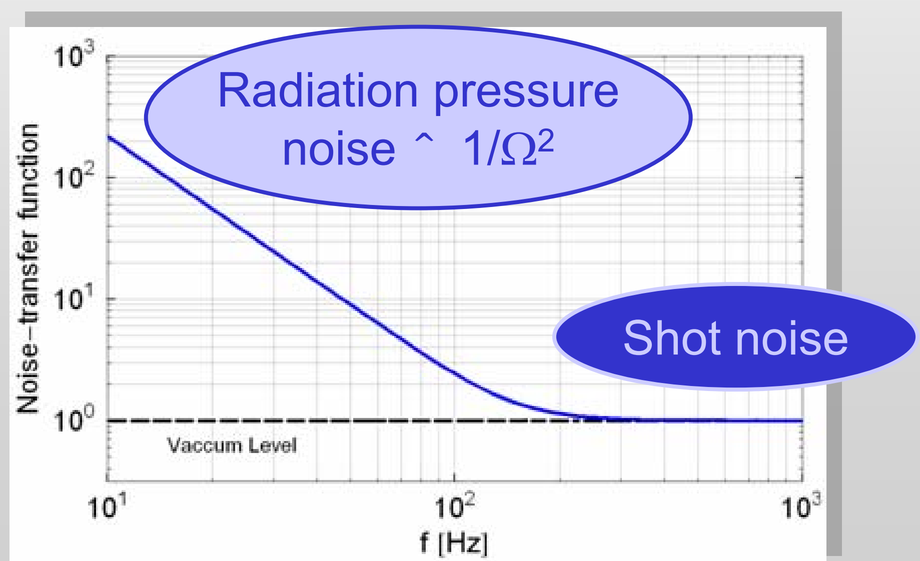
$$(m\ddot{x} = \text{Force})$$

Suspended mirror is subject to fluctuating radiation pressure force $F_0(\Omega)$ and possible GW forces

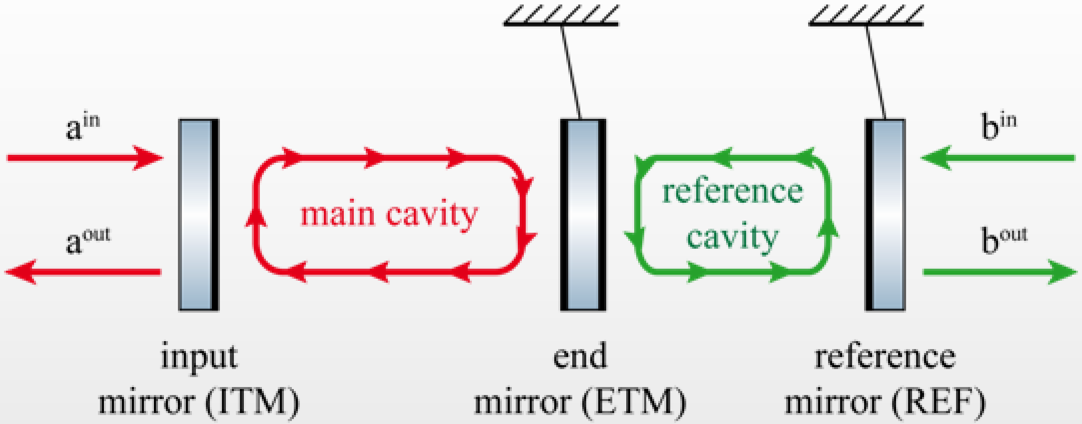
$$\hat{x}(\Omega) = R_{xx}(\Omega)\hat{F}_0(\Omega) + \text{GW Force}$$

$$R_{xx}(\Omega) \approx \frac{1}{m\Omega^2}$$

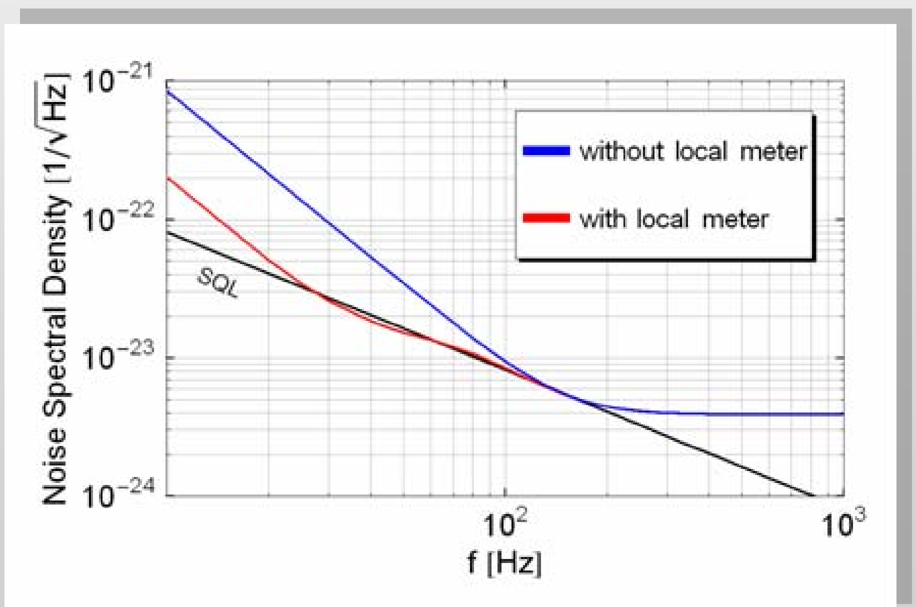
$$\hat{F}_0(\Omega) \sim \frac{1}{\gamma - \mathrm{i}\Omega}\,\hat{a}_1^{\mathrm{in}}$$



Monitoring End-Mirror Displacement



Radiation pressure induced position fluctuations of the ETM caused by the main cavity field are measured by a reference cavity.



Detection strategy:

-Output b^{out} can be feed back to ETM

- Optimal noise spectral density can by computed form outputs a^{out} and b^{out}

Heidmann et. al. [quant-ph/0311167]

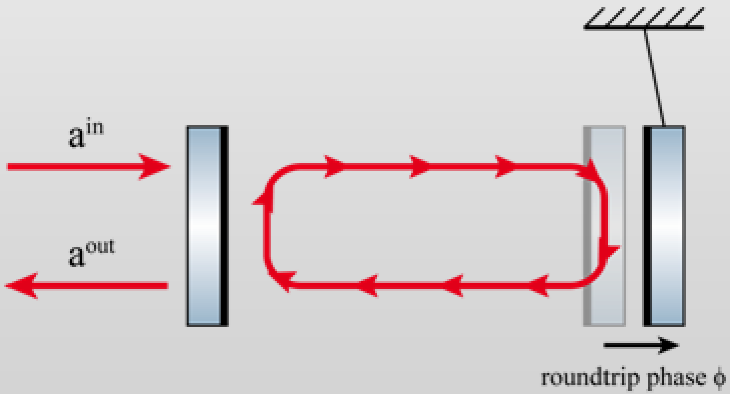
Detuned Cavity

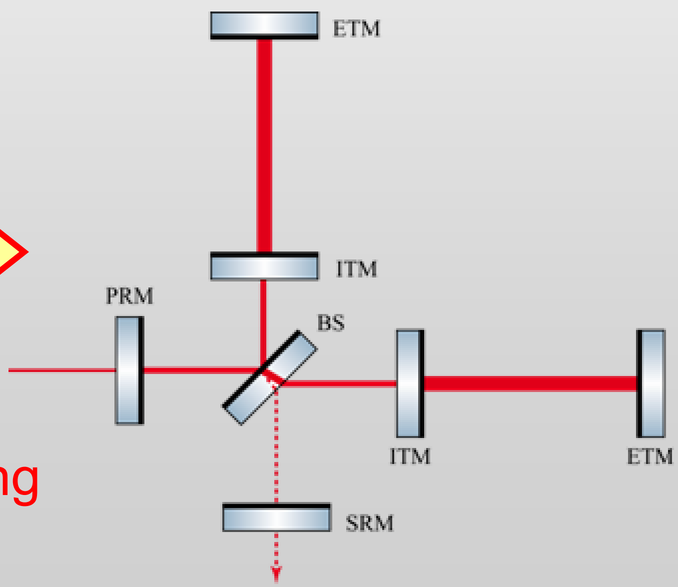
Detuning the cavity by ϕ modifies equation of motion:

$$\hat{x}(\Omega) = R_{xx}\left(\hat{F}_0(\Omega) + R_{FF}(\Omega)\hat{x}(\Omega)\right) + \text{GW Force}$$

Radiation pressure

Harmonic restoring force (optical spring)

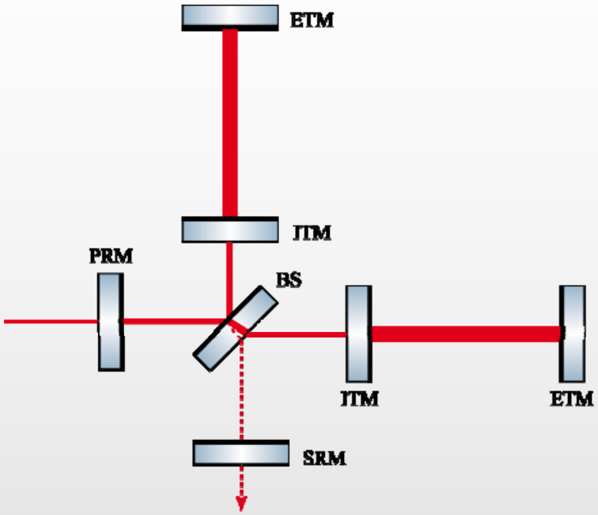


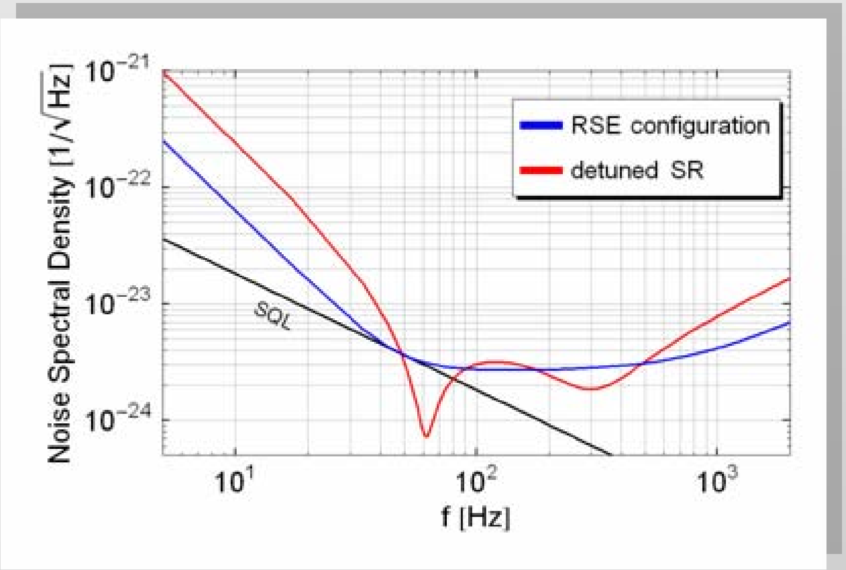


Is the local meter still useful for reducing radiation pressure effects?

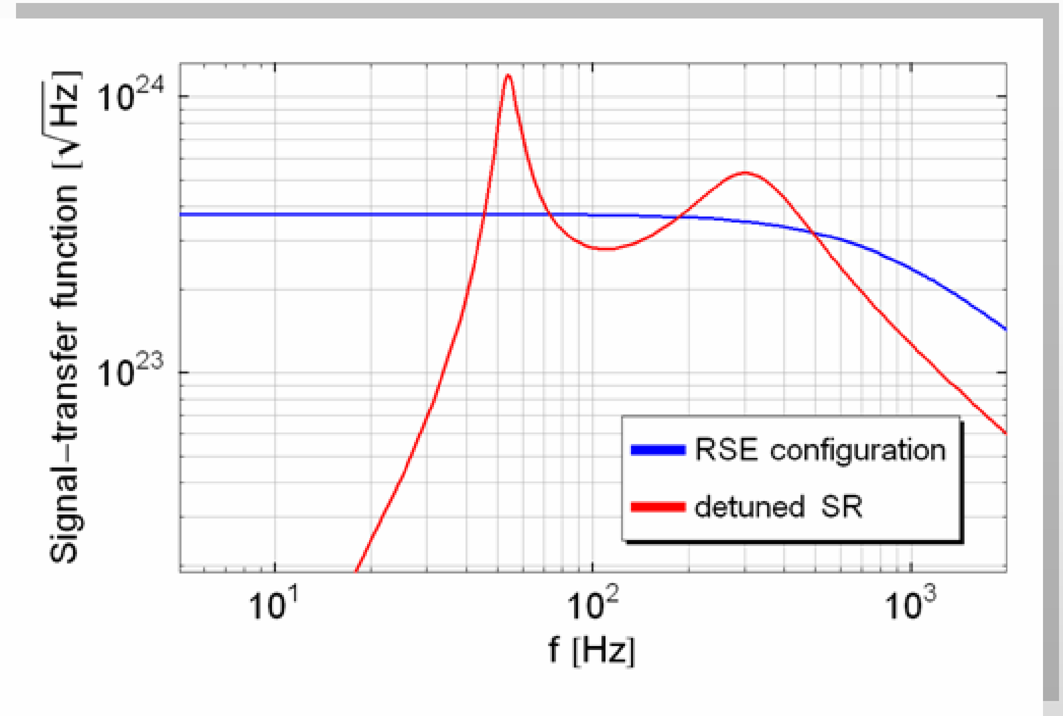
Detuned SR Interferometer

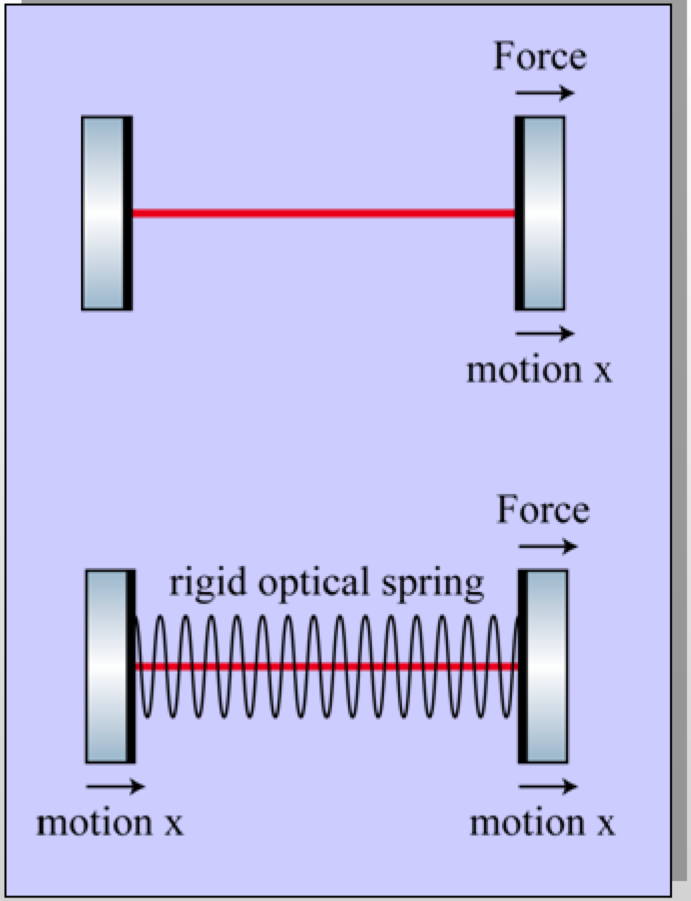
- **<u>Remember:</u> Detuned SR Interferometer is equivalent to single detuned cavity**
- Detuning is moved to signal recycling cavity
- Additional resonances:
  - optical resonance
  - optomechanical resonance
- Gain of sensitivity at the two resonances
- Loss of sensitivity below optomechanical resonance frequency





Signal and Noise Transfer

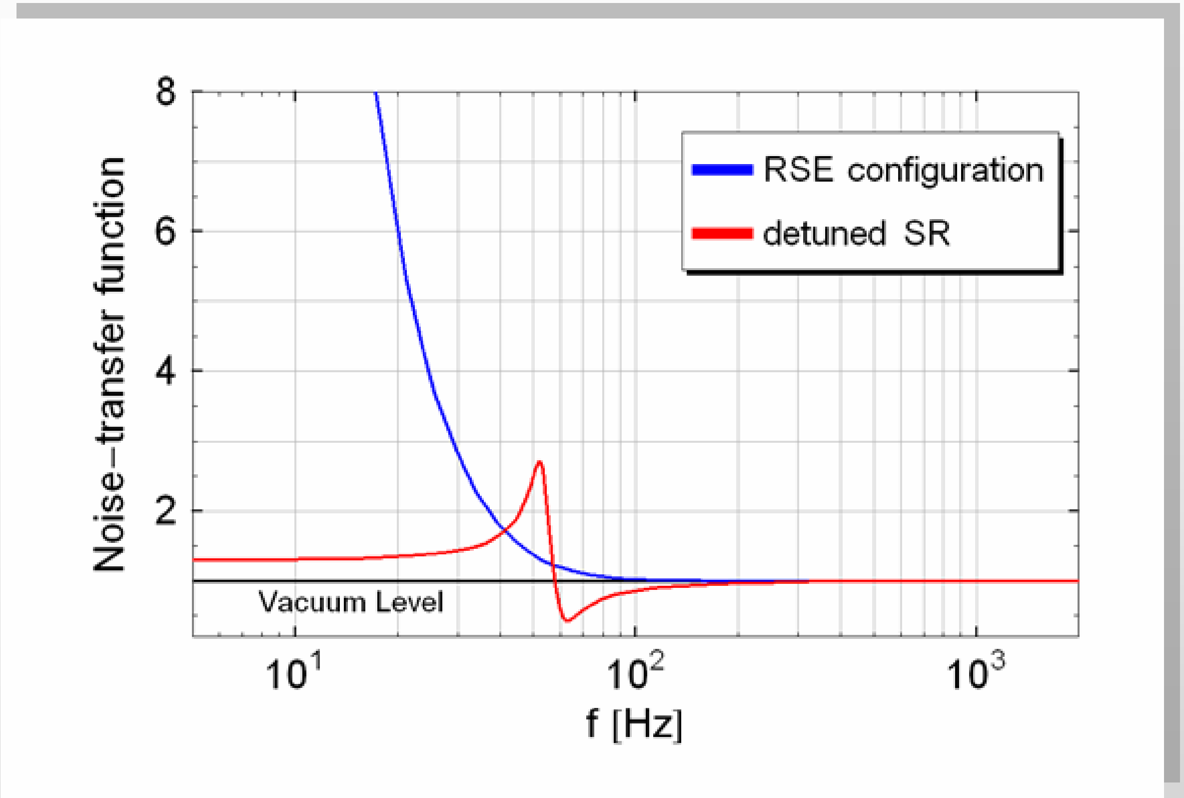




Mainly the rigid optical spring suppresses the response of the interferometer's differential mode to GW waves → cf. *optical bar*

[Braginski, Gorodetsky & Khalili 1997;
Braginski & Khalili 1999; Khalili 2002]

Signal and Noise Transfer

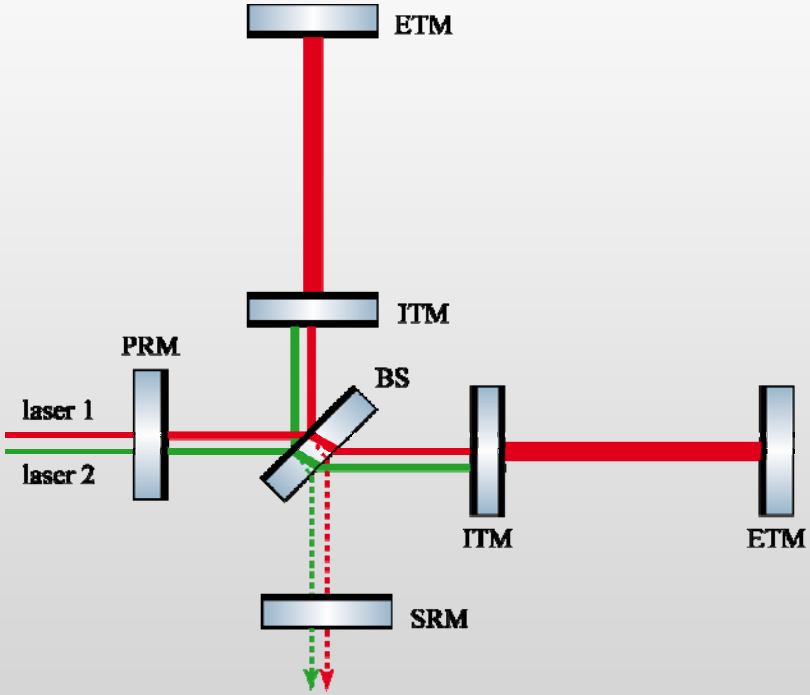


Radiation pressure noise of SR interferometer is also suppressed by the optical spring

Further suppression of radiation pressure noise by a local meter is not necessary for detuned SR interferometers! But…

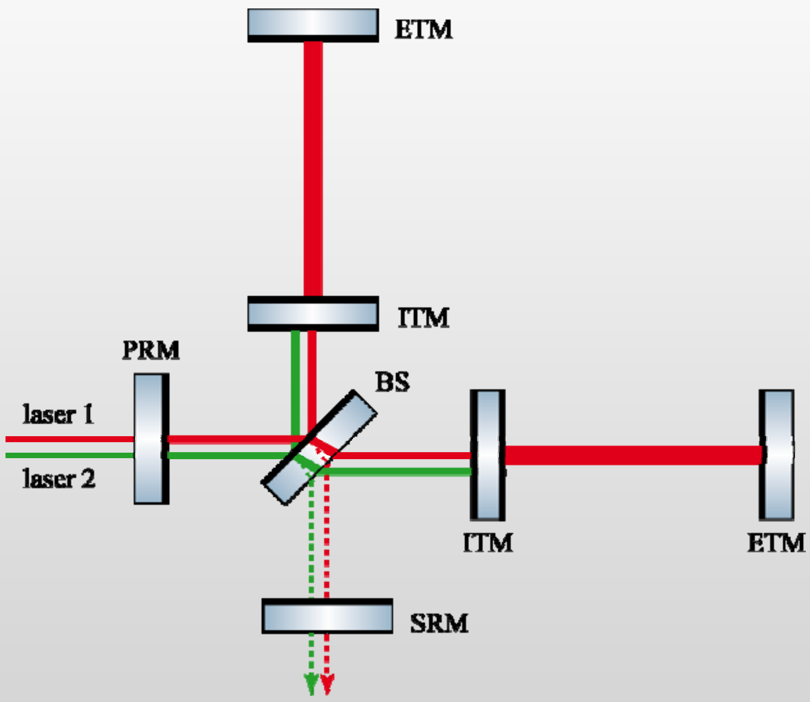
Local Readout Scheme (1)

- Reading out the ITMs' motion by injection of second laser beam
- Additional laser beam should not resonate in arm cavities
- At frequencies below optomechanical resonance the local meter views bar formed by rigidly connected ITM and ETM
- Effective mass sensed by local meter is twice that of ITM or ETM (assuming equal masses)
- Recover low-frequency response of detuned SR interferometer
- Extension to stand-alone optical bar or SR schemes



Local Readout Scheme (2)

Symbol	Physical meaning	Value
m	Single mirror mass	40 kg
m_{BS}	BS mass	40 kg
$\omega_0^{(1)}$	Light frequency 1st laser	1.8×10^{15} 1/s
$P^{(1)}$	Circulating power 1st carrier	100-800 kW
$L^{(1)}$	Large scale ifo arm length	4 km
ρ_{PR}	PRM reflectivity	$(0.5)^{1/2}$
ϕ	Detuning for 1st carrier	$0-\pi$
ρ_{SR}	SRM reflectivity	$(0.93)^{1/2}$
γ_0	Cavity half bandwidth 1st carrier	2π 15 Hz
$\zeta^{(1)}$	Det. angle for 1st carrier	$0-\pi$
$\omega_0^{(2)}$	Light frequency 2nd laser	1.8×10^{15} 1/s
$P^{(2)}$	Circulating power 2nd carrier	0-16 kW
$L^{(2)}$	Local meter arm length	15 m
$\lambda^{(2)}$	Detuning for 2nd carrier	0 Hz
$\varepsilon^{(2)}$	Cavity half bandwidth 2nd carrier	2π 4 kHz
$\zeta^{(2)}$	Det. angle for 2nd carrier	0



Combined Sensitivity

The output can be written in the compact form

$$\hat{y}^{(1)} = \vec{n}_1^T \; \vec{\nu} + s_1 \; h \text{ and } \hat{y}^{(2)} = \vec{n}_2^T \; \vec{\nu} + s_2 \; h$$

and the combined output is given by

$$\hat{y} = K_1(\Omega) \; \hat{y}^{(1)} + K_2(\Omega) \; \hat{y}^{(2)}$$

with filter functions $K_1(\Omega)$ and $K_2(\Omega)$. The resulting total noise spectral density

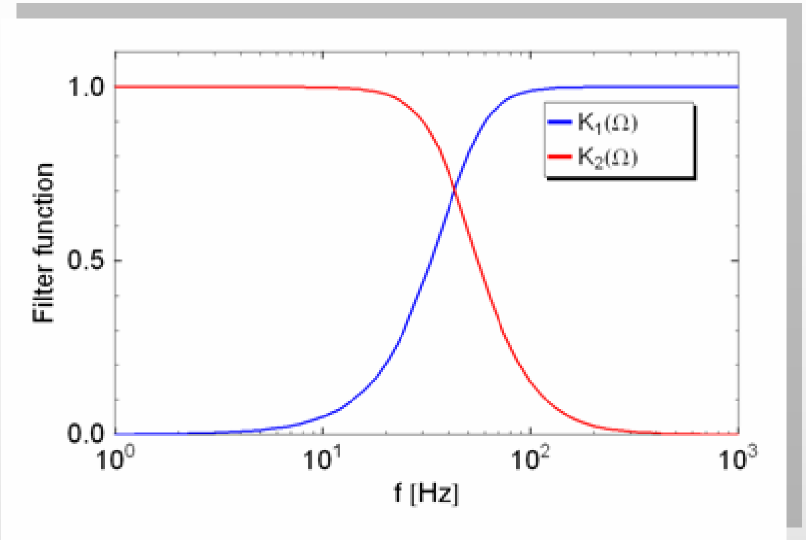
$$S_h(\Omega) = \frac{\begin{pmatrix} K_1(\Omega) & K_2(\Omega) \end{pmatrix} \begin{pmatrix} \vec{n}_1^T \vec{n}_1^* & \vec{n}_1^T \vec{n}_2^* \\ \vec{n}_2^T \vec{n}_1^* & \vec{n}_2^T \vec{n}_2^* \end{pmatrix} \begin{pmatrix} K_1^*(\Omega) \\ K_2^*(\Omega) \end{pmatrix}}{\begin{pmatrix} K_1(\Omega) & K_2(\Omega) \end{pmatrix} \begin{pmatrix} s_1 s_1^* & s_1 s_2^* \\ s_2 s_1^* & s_2 s_2^* \end{pmatrix} \begin{pmatrix} K_1^*(\Omega) \\ K_2^*(\Omega) \end{pmatrix}}$$

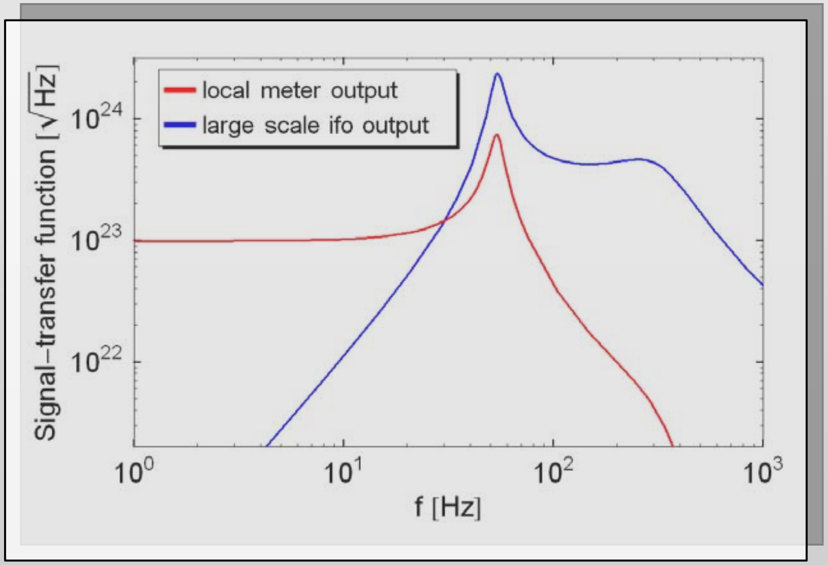
has to be minimized.

$$\vec{\nu}^T = (a_1^{(1)}, a_2^{(1)}, a_1^{(2)}, a_2^{(2)}, \hat{\xi}_{\rm ITM}^{\rm cl}, \hat{\xi}_{\rm ETM}^{\rm cl}, \hat{\xi}_{\rm BS}^{\rm cl})$$

Filter Functions

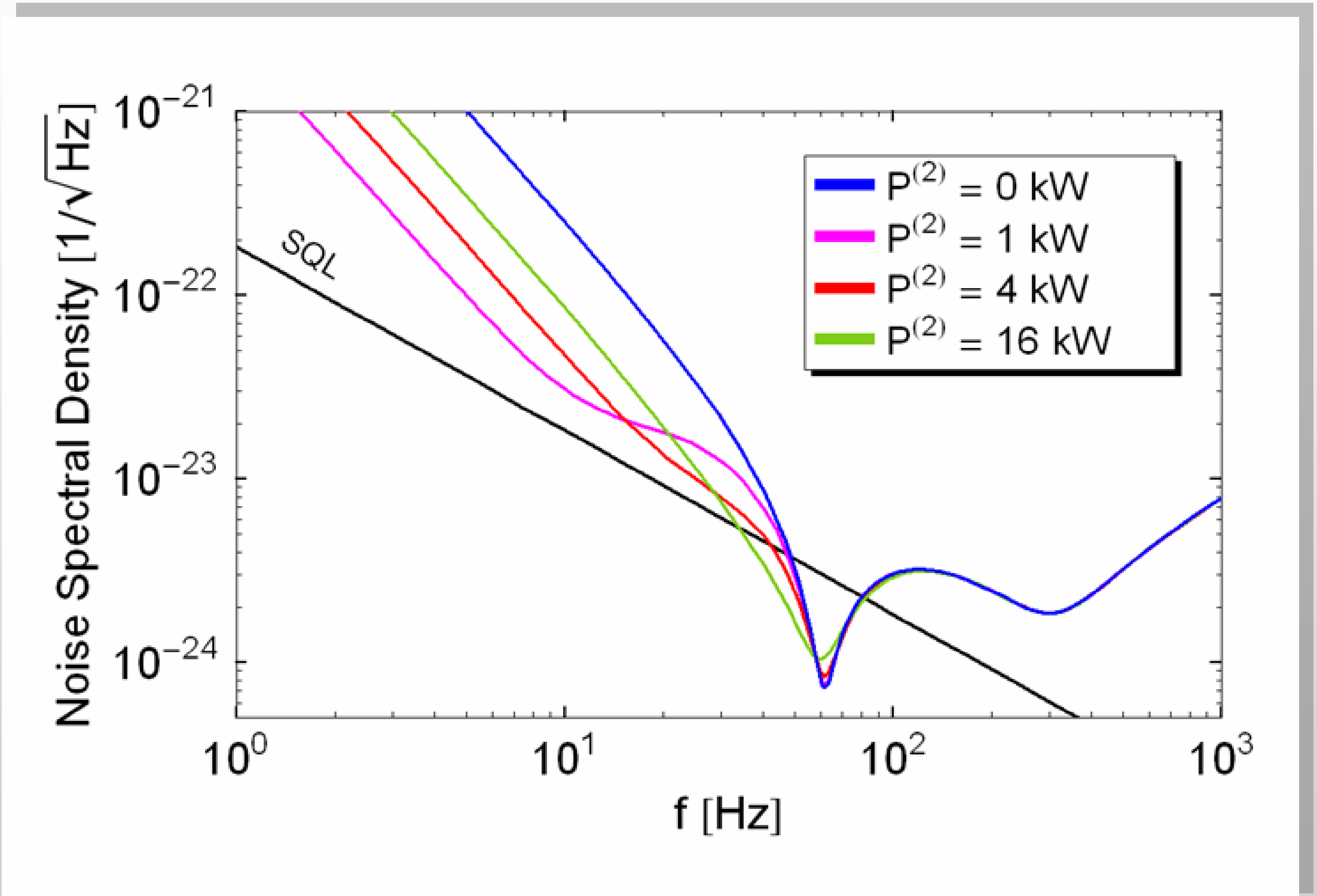
- Optimal filter functions
  - minimize noise spectral density
  - are close to step functions
- Signal transfer function of large scale interferometer below optomechanical resonance decreases rapidly but…
- …signal transfer of local meter stays constant





Combination of locally sensed optical bar scheme and SR interferometer

Noise Spectral Density



Astrophysical Optimization

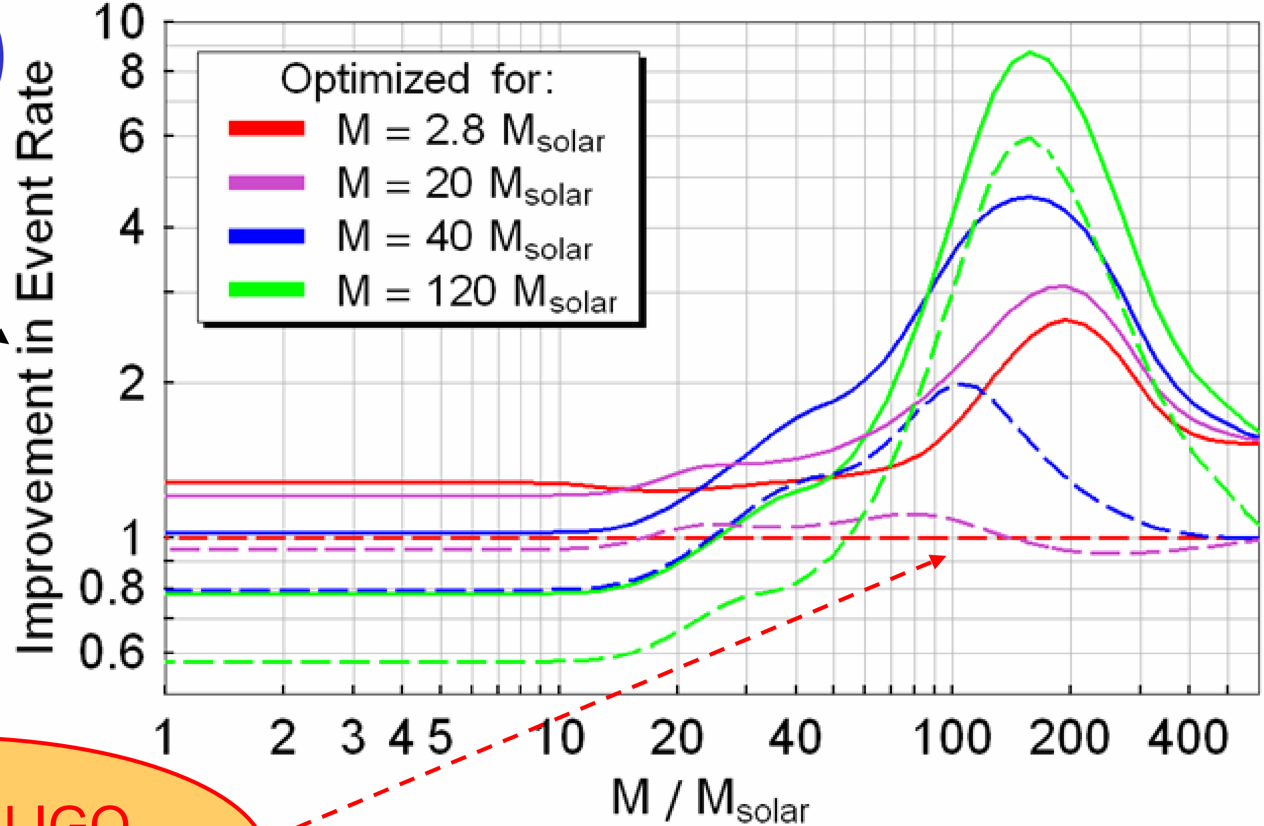
- Optimization with respect to binary systems with total mass M and reduced mass μ
- Observable distance for a given SNR ρ_0 and averaged binary orientation has to be maximized:

$$D = \sqrt{\frac{2}{15}} \frac{G^{5/6}\mu^{1/2}M^{1/3}}{\pi^{2/3}c^{3/2}\rho_0} \sqrt{\int_{f_{\min}}^{f_{\max}} \mathrm{d}f \, \frac{f^{-7/6}}{S_h(f)}}$$

- Classical noise cutoff: $f_{min} \sim$ 7 Hz
- Upper cutoff given by GW frequency at last stable circular orbit of the binary: $f_{max} \sim$ 4400Hz $(M_{\odot}/M)$

Comparison of Event Rates

Compared to Advanced LIGO

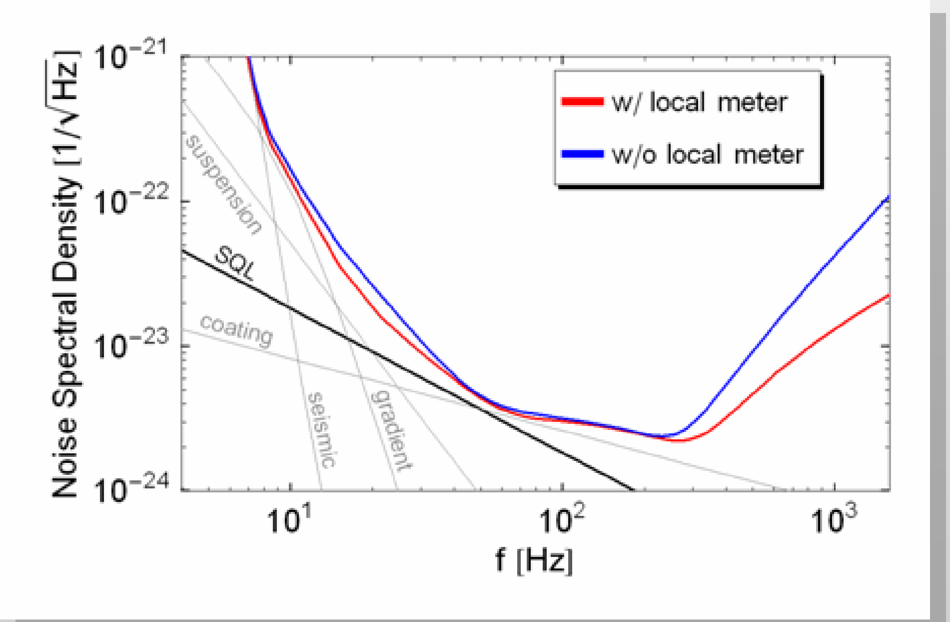


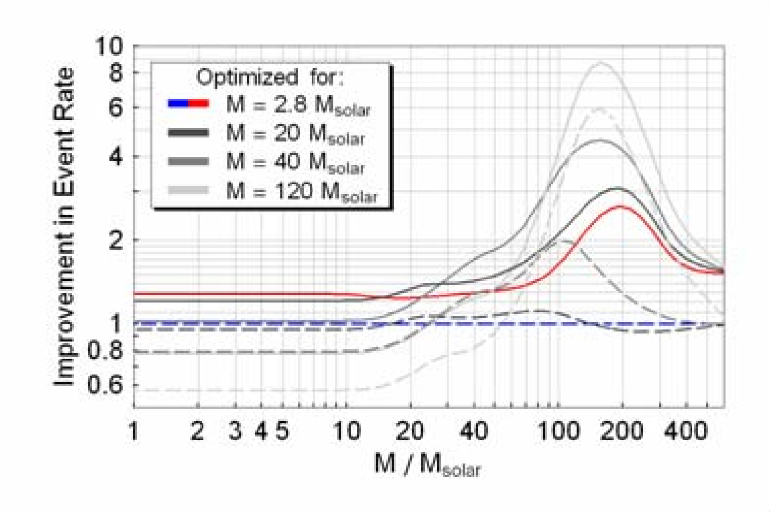
$\left(\frac{4}{3}\right)^{3/2}$

Advanced LIGO is optimized for 2.8 M.

Noise Spectral Densities with Classical Noise

M/M.	opt. parameter with local meter			opt. parameter without local meter			Improv.
	$P^{(1)}$ [kW]	ϕ [rad]	$\zeta^{(1)}$[rad]	$P^{(1)}$ [kW]	ϕ [rad]	$\zeta^{(1)}$[rad]	
2.8	800	0.48 π	0.7 π	800	0.48 π	0.49 π	29%
20	450	0.47 π	0.58 π	500	0.48 π	0.48 π	28%
40	150	0.45 π	0.43 π	150	0.45 π	0.46 π	33%
120	100	0.46 π	0.32 π	100	0.47 π	0.41 π	42%





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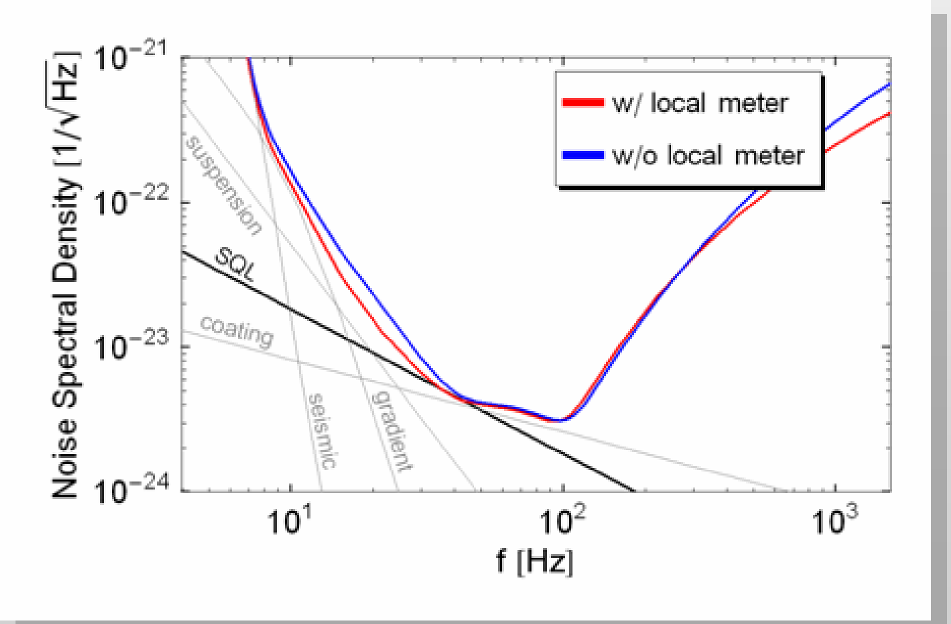

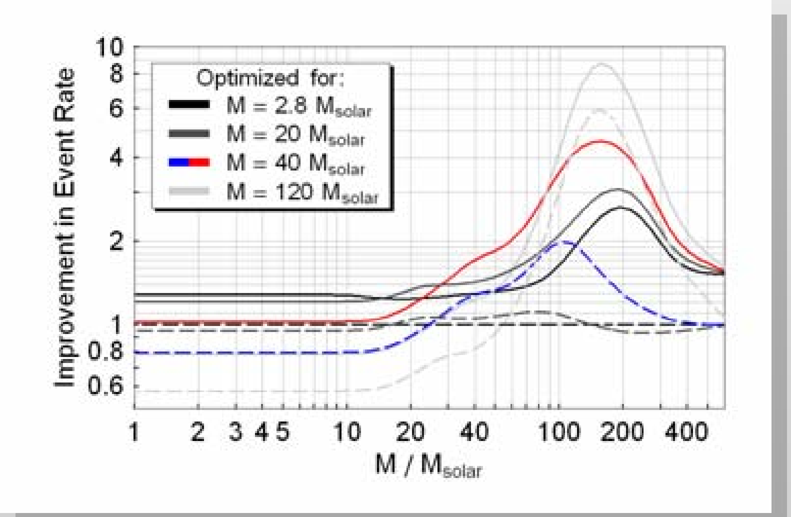

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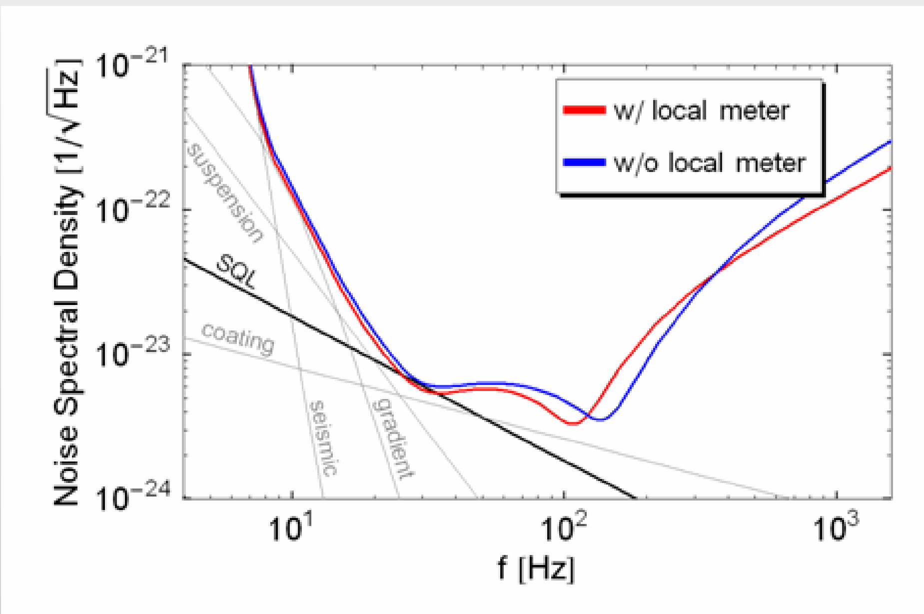
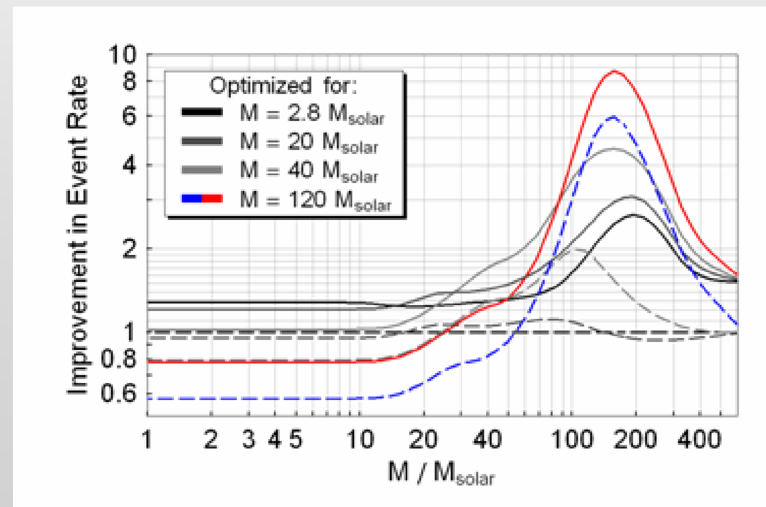
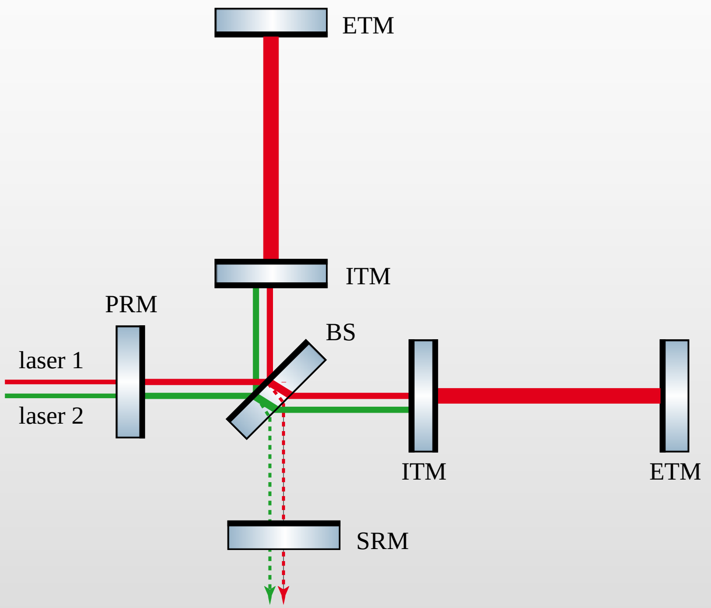




Conclusion and Outlook

- Locally reading out ITMs' motion by second laser beam improves sensitivity significantly
- Maintaining or improving sensitivity of Advanced LIGO even for reduced power in arm cavities



- Combinable with other QND schemes, e.g. injection of squeezed vacuum
- Our proposed upgrade for Advanced LIGO should be realizable with low effort

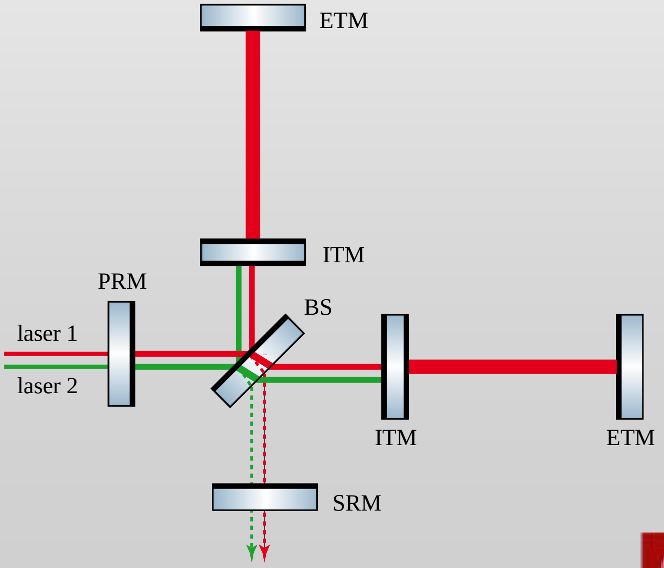
Equations of Motion (1)

$$\hat{x}_{\rm ITM} = -R_{xx}(\Omega)\left(\hat{F}^{(1)}(\Omega) + R^{(1)}_{FF}(\Omega)\,(\hat{x}_{\rm ETM} - \hat{x}_{\rm ITM})\right)$$
$$+R_{xx}(\Omega)\left(\hat{F}^{(2)}(\Omega) - R^{(2)}_{FF}(\Omega)\,(\hat{x}_{\rm ITM} + \sqrt{2}\,\hat{x}_{\rm BS})\right) + \hat{\xi}^{\rm cl}_{\rm ITM}$$
$$\hat{x}_{\rm ETM} = R_{xx}(\Omega)\left(\hat{F}^{(1)}(\Omega) + R^{(1)}_{FF}(\Omega)\,(\hat{x}_{\rm ETM} - \hat{x}_{\rm ITM})\right) + L\,h + \hat{\xi}^{\rm cl}_{\rm ETM}$$
$$\hat{x}_{\rm BS} = R^{\rm BS}_{xx}(\Omega)\left(\hat{F}^{(2)}(\Omega) + R^{(2)}_{FF}(\Omega)\,(\hat{x}_{\rm ITM} + \sqrt{2}\,\hat{x}_{\rm BS})\right)$$
$$+R^{\rm BS}_{xx}(\Omega)\hat{F}_{\rm BP}(\Omega) + \hat{\xi}^{\rm cl}_{\rm BS}$$

BS motion is

- only important in the local meter
- negligible for the large scale interferometer due to arm cavities
- also driven by bright port laser input fields

[Harms et. al. 2002]

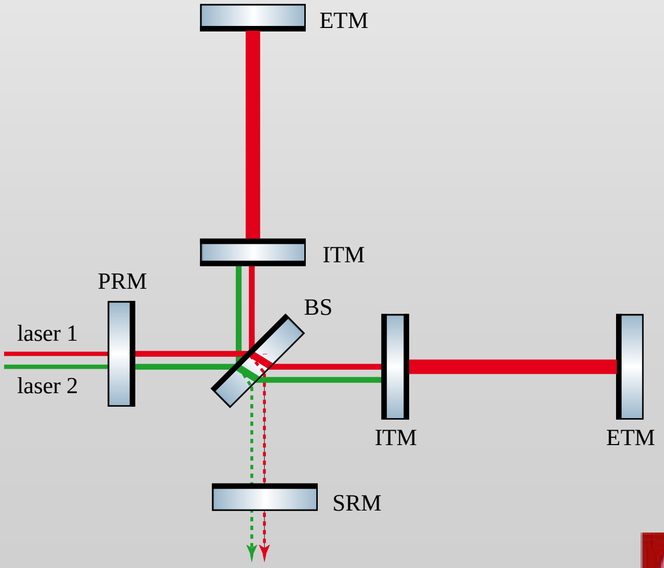


Equations of Motion (2)

$$
\begin{aligned}
\hat{x}_{\rm ITM} &= -R_{xx}(\Omega)\ \left(\hat{F}^{(1)}(\Omega) + R^{(1)}_{FF}(\Omega)\ (\hat{x}_{\rm ETM} - \hat{x}_{\rm ITM})\right) \\
&\quad + R_{xx}(\Omega)\left(\hat{F}^{(2)}(\Omega) - R^{(2)}_{FF}(\Omega)\ (\hat{x}_{\rm ITM} + \sqrt{2}\ \hat{x}_{\rm BS})\right) + \hat{\xi}^{\rm cl}_{\rm ITM} \\
\hat{x}_{\rm ETM} &= R_{xx}(\Omega)\ \left(\hat{F}^{(1)}(\Omega) + R^{(1)}_{FF}(\Omega)\ (\hat{x}_{\rm ETM} - \hat{x}_{\rm ITM})\right) + L\ h + \hat{\xi}^{\rm cl}_{\rm ETM} \\
\hat{x}_{\rm BS} &= R^{\rm BS}_{xx}(\Omega)\ \left(\hat{F}^{(2)}(\Omega) + R^{(2)}_{FF}(\Omega)\ (\hat{x}_{\rm ITM} + \sqrt{2}\ \hat{x}_{\rm BS})\right) \\
&\quad + R^{\rm BS}_{xx}(\Omega)\hat{F}_{\rm BP}(\Omega) + \hat{\xi}^{\rm cl}_{\rm BS}
\end{aligned}
$$

Classical noise

- modeled by operators ξ^{cl}_{i}, i 5 {ITM,ETM,BS}
- contributions are uncorrelated
- each mirror is subject to fourth of the spectrum generally expected for whole differential mode



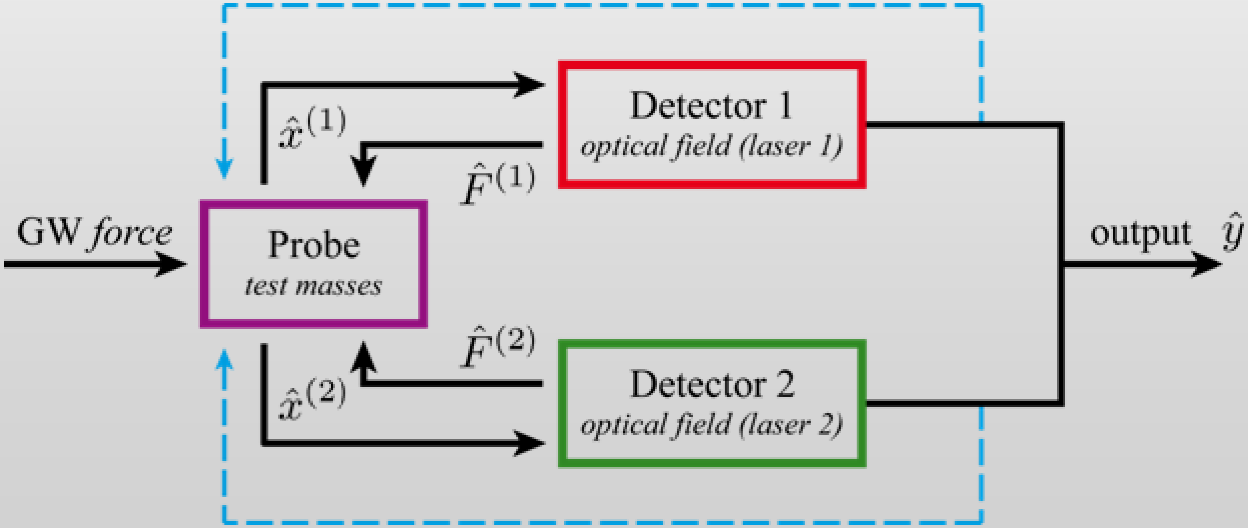
Implementation

- RF sidebands (1% of carrier light) already probe ITM in planed Advanced LIGO
- For our scheme a second laser beam is needed with the same power as the first laser (125 W).
- Polarized light or different frequencies can be used
- Frequency of second laser must be chosen such that it does not resonate in the arm cavities
- Circumvent shot-noise-limited sensitivity of large scale interferometer since sensitivity in the detection band cannot be lower than shot-noise level of local measurement of the ITMs in the control bandwidth

Control Filter

- Instability induced by optical spring requires feedback control
- In general local meter and large scale interferometer could be detuned and therefore show instabilities
- It can be shown that an appropriate control feedback leaves noise spectral density unchanged

$$\hat{\mathcal{C}}^{(1)} = \mathbf{K}_{\mathcal{C}}^{(1)} \hat{y}^{(1)}$$



$$\hat{\mathcal{C}}^{(2)} = \mathbf{K}_{\mathcal{C}}^{(2)} \hat{y}^{(2)}$$