Coherent Bayesian analysis of inspiral signals

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Overview:

1. The Bayesian approach
2. MCMC methods
3. The inspiral signal
4. Priors
5. Example application

The Bayesian approach

- idea: **assign probabilities to parameters** θ
- pre-experimental knowledge: **prior probabilities / -distribution** $p(\theta)$
- data model: **likelihood** $p(y|\theta)$
- application of **Bayes' theorem** yields the **posterior distribution**

$$p(\theta|y) \;\propto\; p(\theta)\, p(y|\theta)$$

  conditional on the observed data y.
- posterior distribution combines the **information in the data** with the **prior information**

MCMC methods - what they do

- Problem -
  given: posterior distribution $p(\theta|y)$ (density, function of θ)
  wanted: mode(s), integrals,...

- what MCMC does:
  simulate random draws from (any) distribution, allowing to approximate any integral by sample statistic (e.g. means by averages etc.)

- Monte Carlo integration

MCMC methods - how they work

- **M**arkov **C**hain **M**onte **C**arlo
- random walk
- **Markov property**: each step in random walk only depends on previous
- **stationary distribution** is equal to the desired posterior $p(\theta|y)$
- most famous: **Metropolis- (Hastings-) sampler**
  especially convenient: normalising constant factors to $p(\theta|y)$ don't need to be known.

MCMC methods

- Metropolis-algorithm may also be seen as **optimisation algorithm**: improving steps always accepted, worsening steps sometimes ($\rightarrow$ *Simulated Annealing*)

- in fact: purpose often *both* **finding** mode(s) *and* **sampling** from them

The inspiral signal

- measurement: time series (signal + noise)
  at, say, 3 separate interferometers

- **signal**: chirp waveform; 2.5PN amplitude, 3.5PN phase[1,2]

- **9 parameters**: masses (m_1, m_2), coalescence time (t_c), coalescence phase (ϕ_0), luminosity distance (d_L), inclination angle (ι), sky location (δ, α) and polarisation (ψ)

[1] K.G. Arun et al.: *The 2.5PN gravitational wave polarizations from inspiralling compact binaries in circular orbits*, Class. Quantum Grav. 21, 3771 (2004).

[2] L. Blanchet et al.: *Gravitational-wave inspiral of compact binary systems to 7/2 post-Newtonian order*. Phys. Rev. D 65, 061501 (2002).

The signal at different interferometers

- **'local' parameters** at interferometer I:

$$\begin{array}{rcl} \text{sky location } (\delta,\, \alpha) & \rightarrow & \text{altitude } (\vartheta^{(I)}) \;/\; \text{azimuth } (\varphi^{(I)}) \\ \text{coalescence time } (t_c) & \rightarrow & \text{local coalescence time } (t_c^{(I)}) \\ \text{polarisation } (\psi) & \rightarrow & \text{local polarisation } (\psi^{(I)}) \end{array}$$

- **noise** assumed **gaussian**, **coloured**; interferometer-specific spectrum

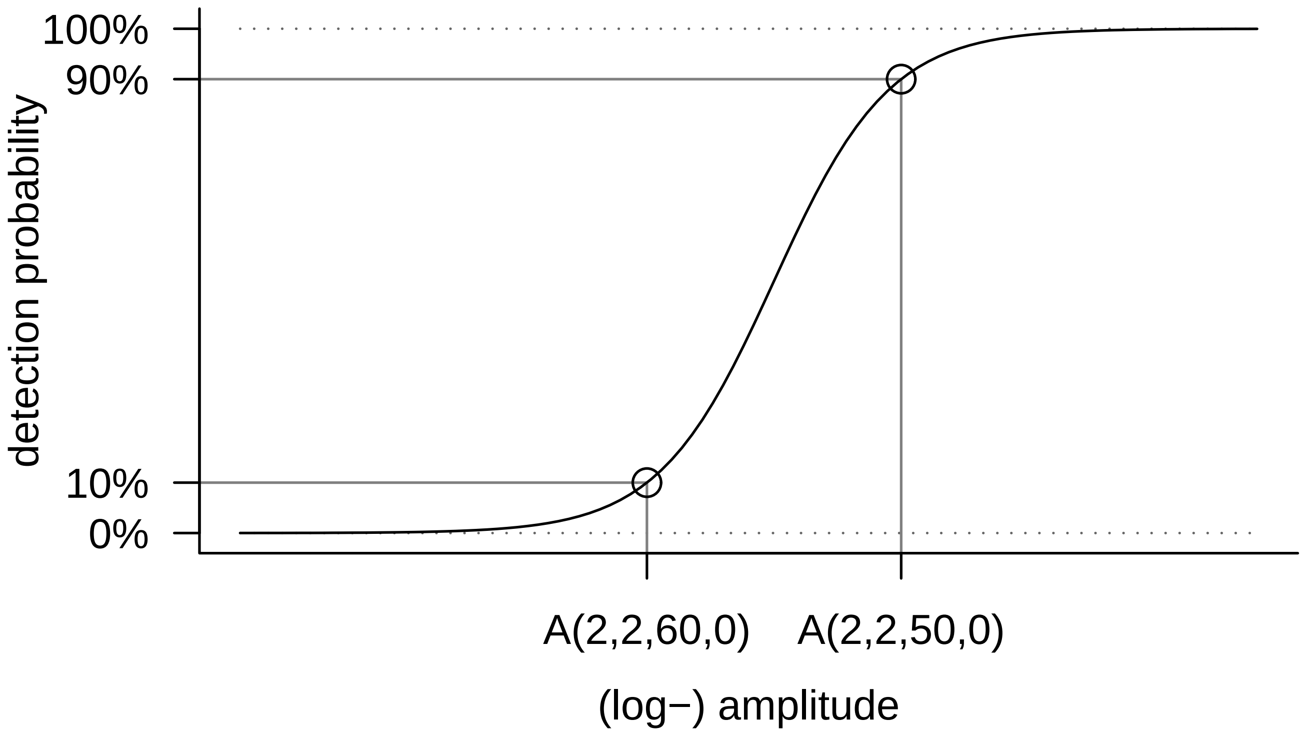
- **likelihood** computation based on Fourier transforms of data and signal

- noise **independent** between interferometers
  $\Rightarrow$ coherent network likelihood is **product** of individual ones

Prior information about parameters

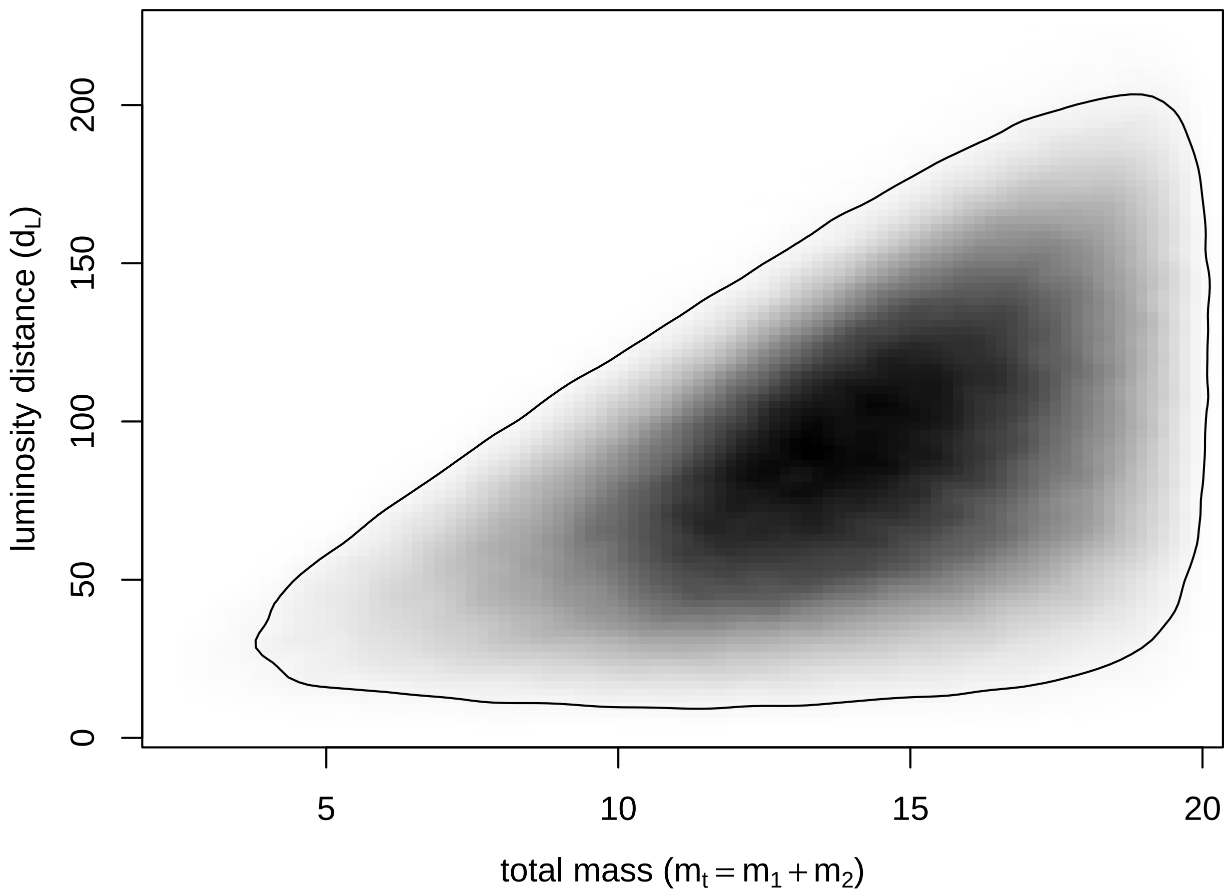
- different locations / orientations equally likely
- masses: uniform across $[1\,\mathrm{M}_\odot,\ 10\,\mathrm{M}_\odot]$
- events spread uniformly across space: $\mathrm{P}(d_L \leq x) \propto x^3$
- but: **certain SNR required** for detection
- cheap **SNR substitute**: signal **amplitude** $\mathcal{A}$
- primarily dependent on **masses**, **distance**, **inclination**: $\mathcal{A}(m_1, m_2, d_L, \iota)$

- introduce sigmoid function linking **amplitude** to **detection probability**[3]

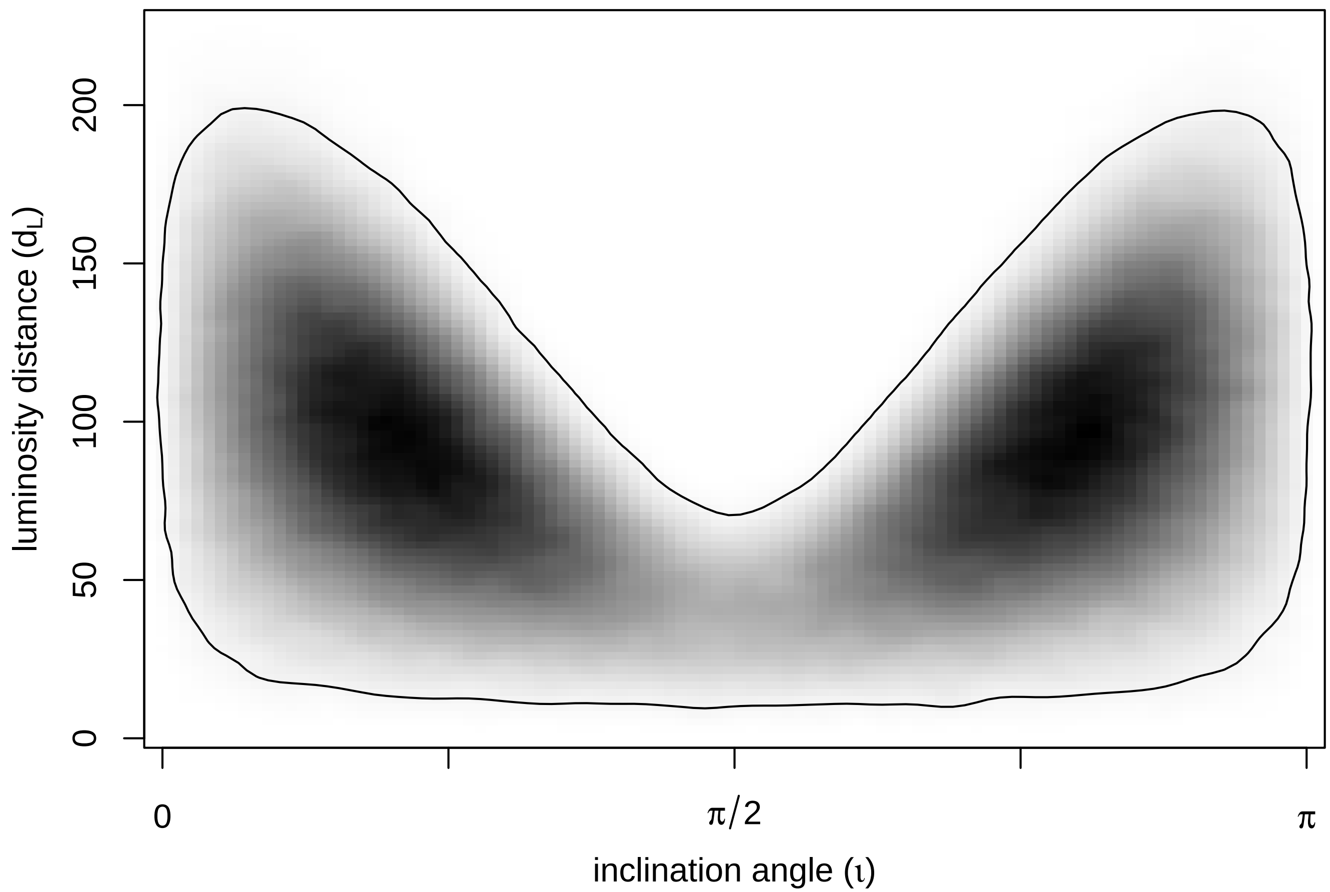


[3] R. Umstätter et al.: *Setting upper limits from LIGO on gravitational waves from SN1987a*. Poster presentation; also: paper in preparation.

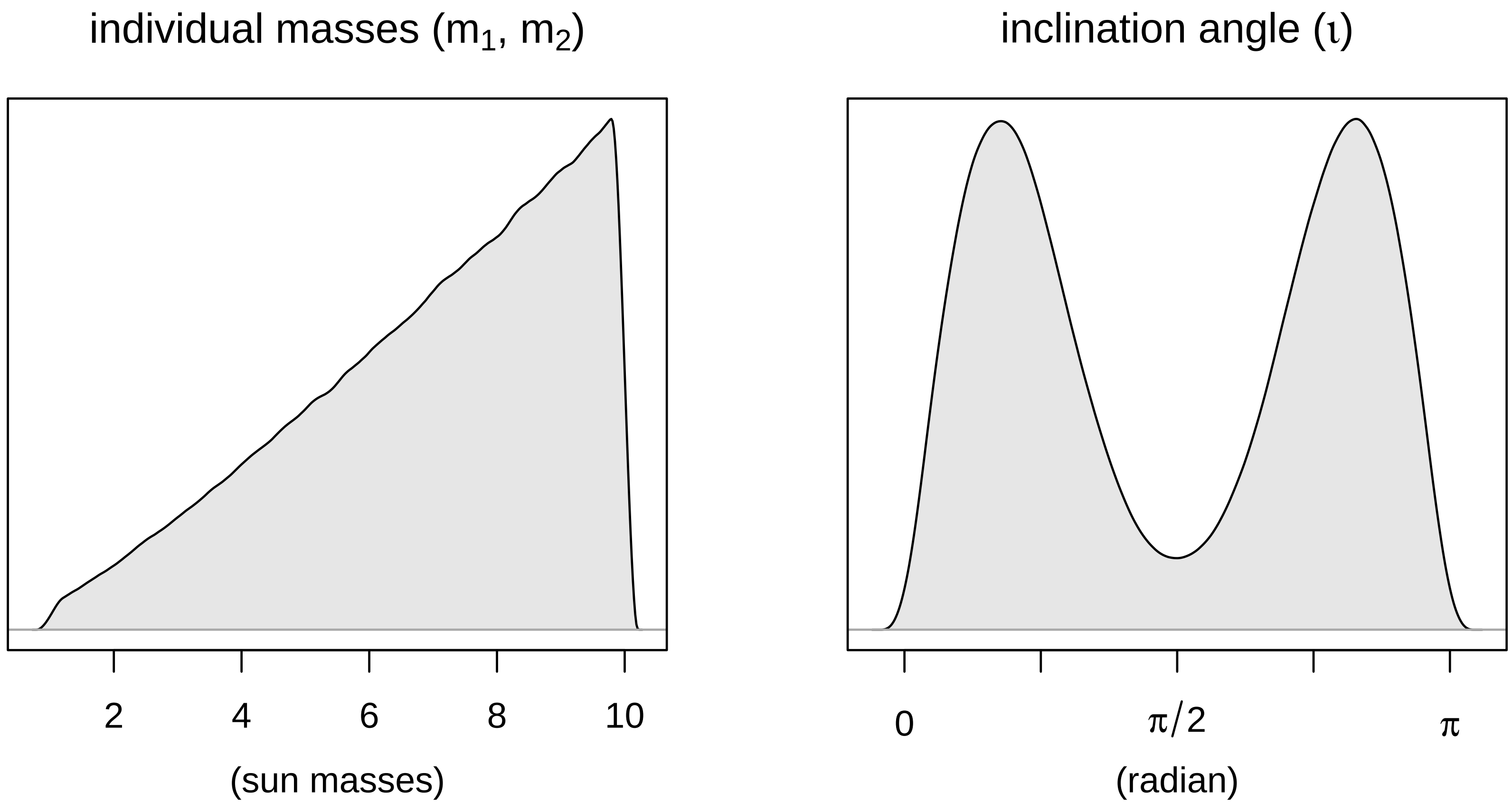
Resulting (marginal) prior density



Marginal prior density



Marginal prior densities



Prior

- prior ‘considers’ **Malmquist effect**
- more realistic settings once **detection pipeline** is set up

MCMC details

- **Reparametrisation**,
  most importantly: **chirp mass** m_c, **mass ratio** η

- **Parallel Tempering**[4]
  several *tempered* MCMC chains running in parallel
  sampling from $p(\theta|y)^{\frac{1}{T_i}}$ for 'temperatures' $1 = T_1 \leq T_2 \leq \ldots$

- **Evolutionary MCMC**[5]
  'recombination' steps between chains—motivated by Genetic algorithms

[4] W.R. Gilks et al.: *Markov chain Monte Carlo in practice* (Chapman & Hall / CRC, 1996).

[5] F. Liang, H.W. Wong: *Real-parameter Evolutionary Monte Carlo with applications to Bayesian mixture models*. J. Am. Statist. Assoc. 96, 653 (2001)

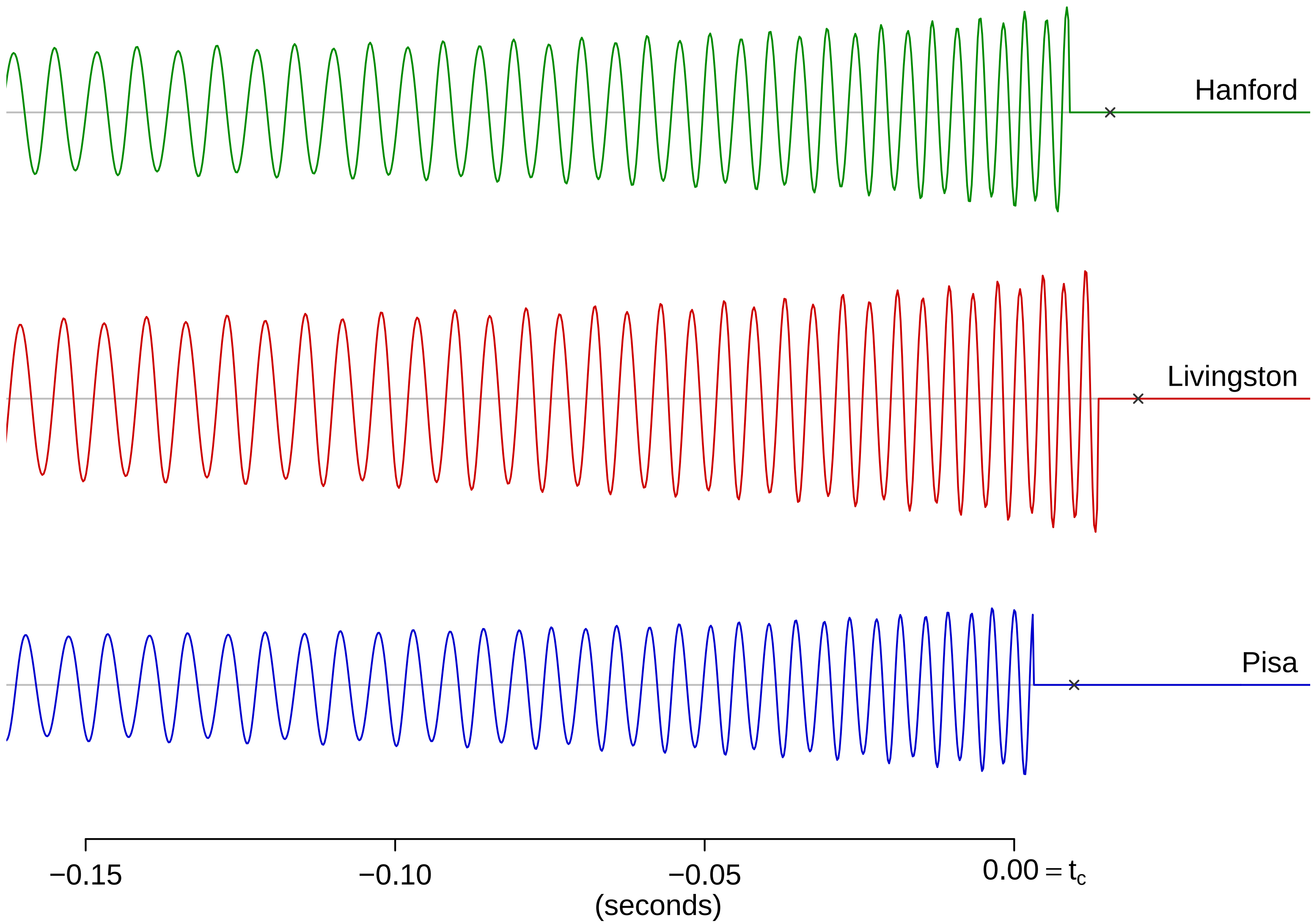
Example application

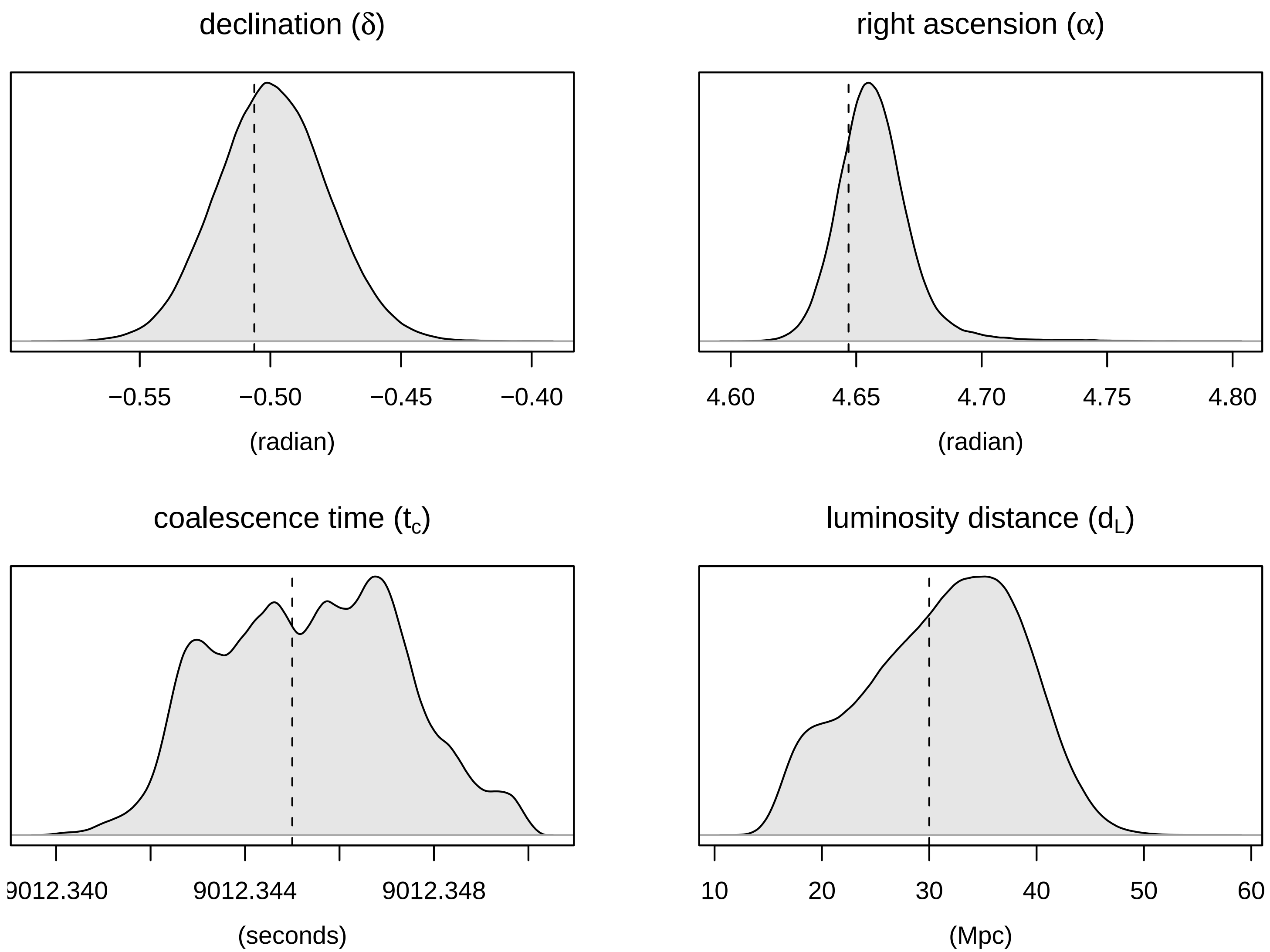
- simulated data:
  $2\,M_\odot$ - $5\,M_\odot$ inspiral at 30 Mpc distance
  measurements from 3 interferometers:

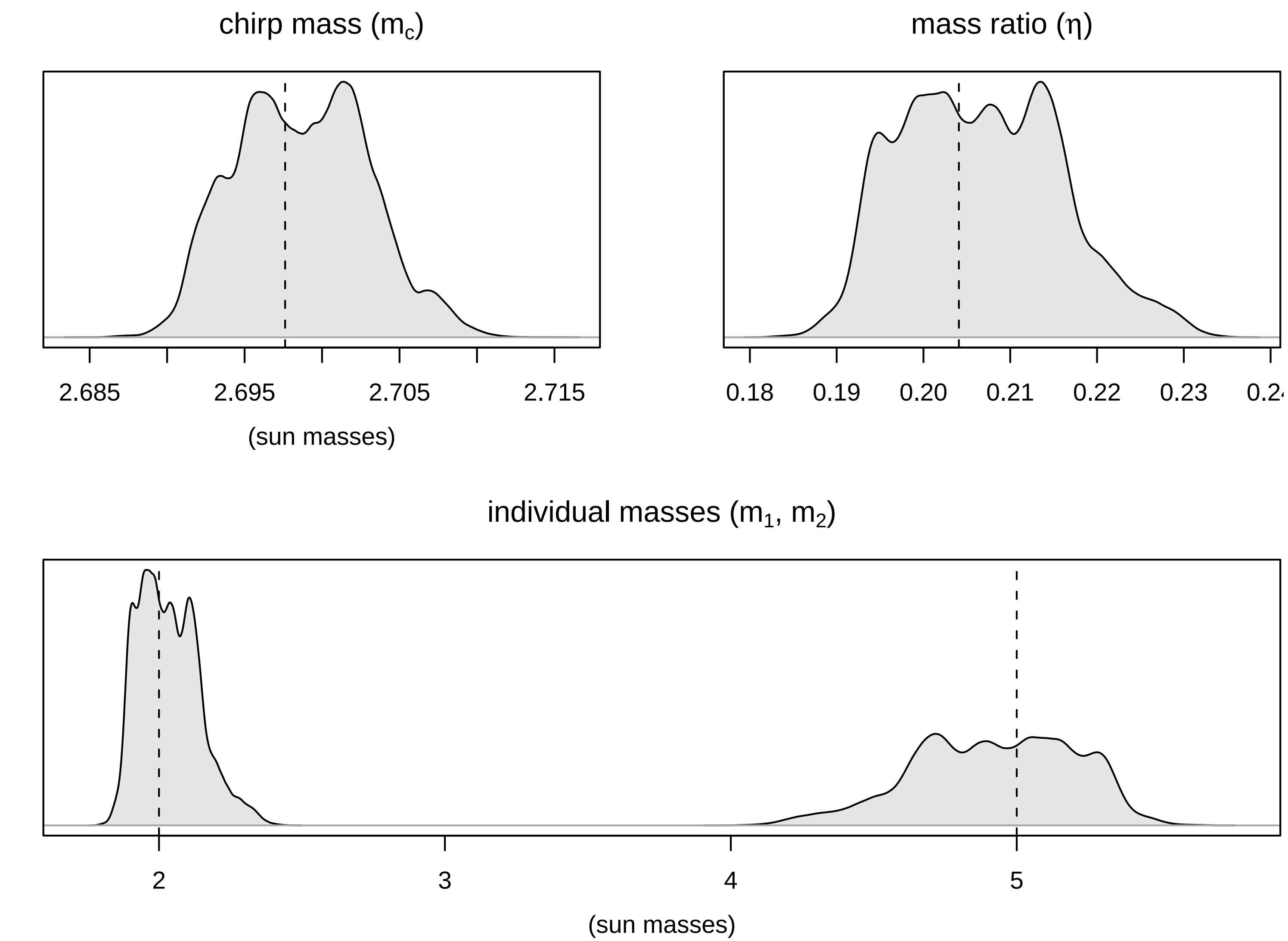
	SNR
LHO (Hanford)	8.4
LLO (Livingston)	10.9
Virgo (Pisa)	6.4
network	15.2

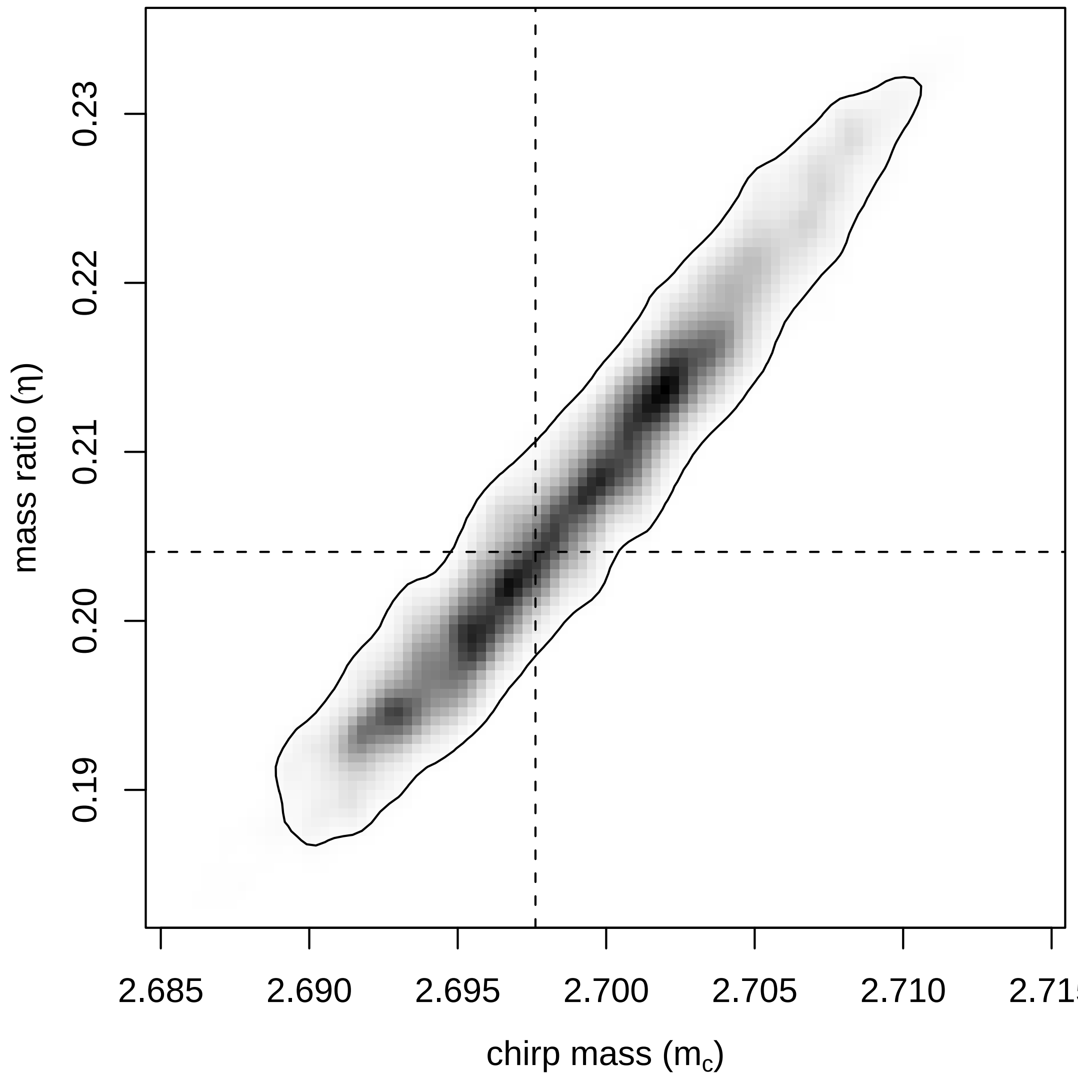
- **data**: 10 seconds (LHO/LLO), 20 seconds (Virgo) before coalescence, **noise** as expected at design sensitivities

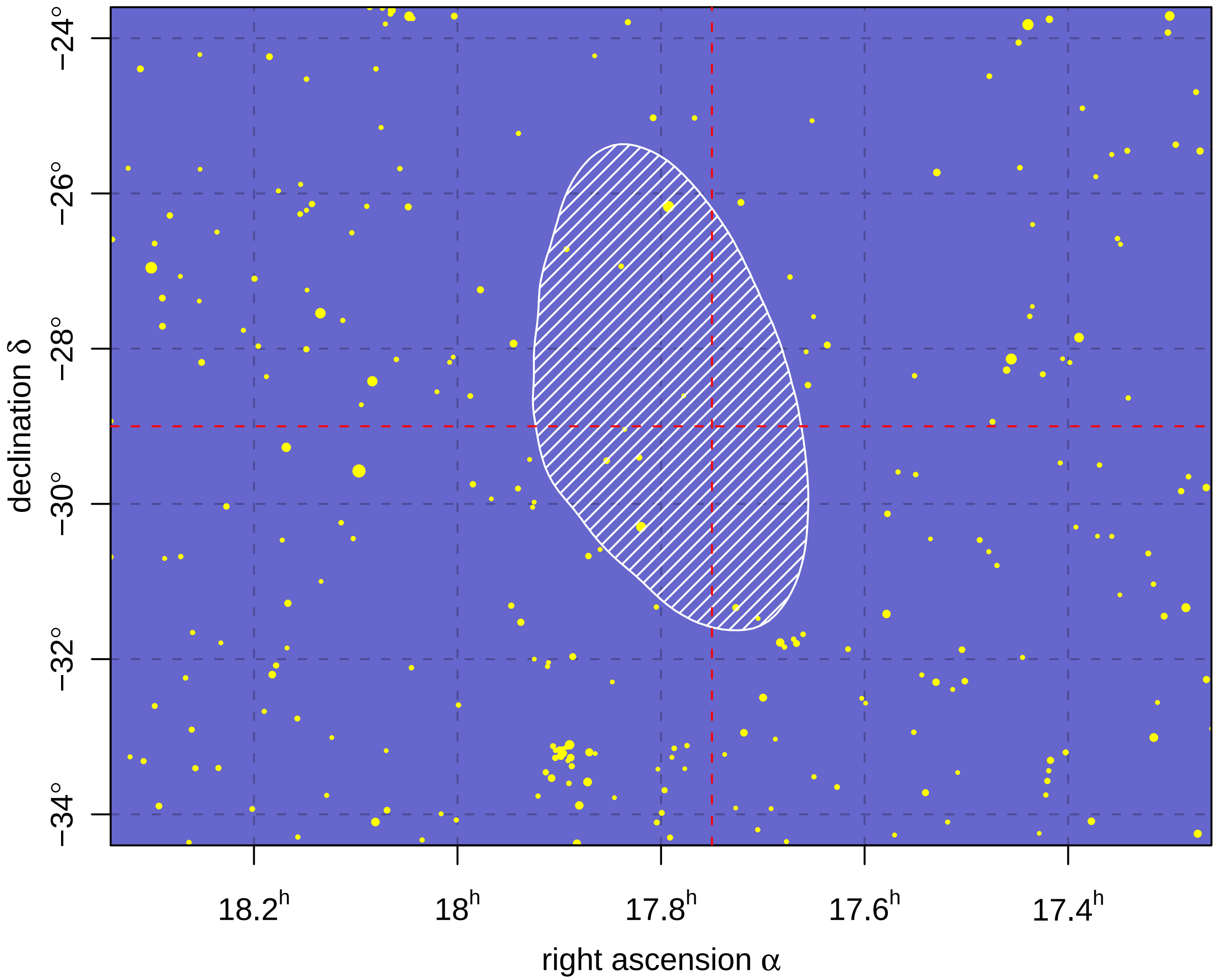
- computation **speed**: 1–2 likelihoods / second


Hanford
Livingston
Pisa
−0.15
−0.10
−0.05
0.00 = t_c
(seconds)


declination (δ)
−0.55
−0.50
−0.45
−0.40
(radian)
right ascension (α)
4.60
4.65
4.70
4.75
4.80
(radian)
coalescence time (t_c)
9012.340
9012.344
9012.348
(seconds)
luminosity distance (d_L)
10
20
30
40
50
60
(Mpc)


chirp mass (m_c)
2.685
2.695
2.705
2.715
(sun masses)
mass ratio (η)
0.18
0.19
0.20
0.21
0.22
0.23
individual masses (m_1, m_2)
2
3
4
5
(sun masses)


0.23
0.22
0.21
0.20
0.19
mass ratio (η)
2.685
2.690
2.695
2.700
2.705
2.710
2.715
chirp mass (mc)

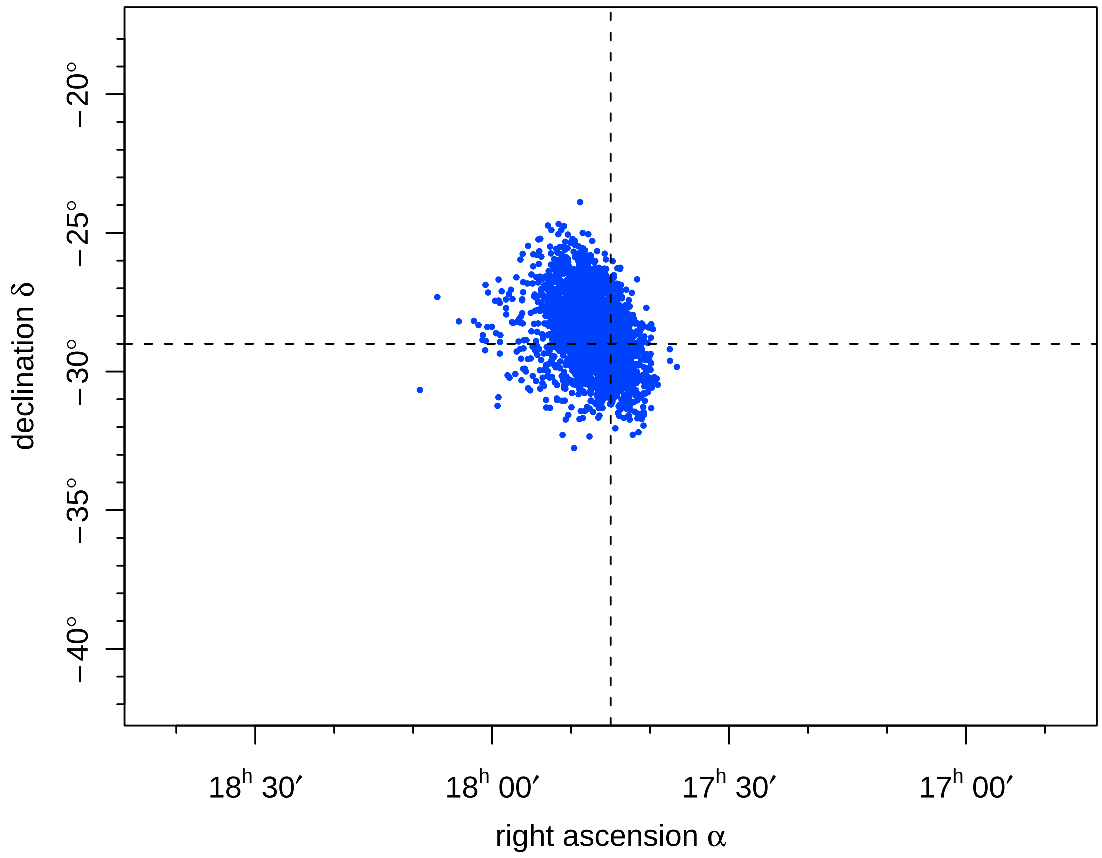


−24°
−26°
−28°
−30°
−32°
−34°
declination δ
18.2h
18h
17.8h
17.6h
17.4h
right ascension α

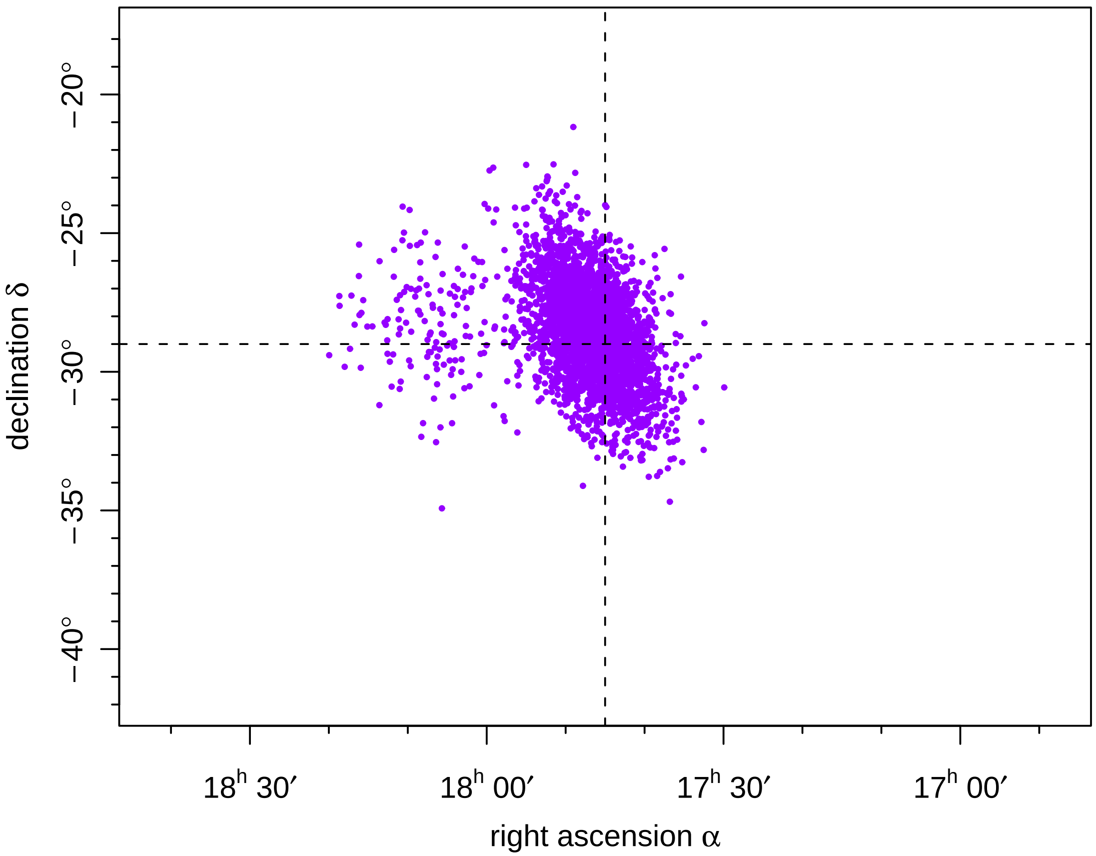
some posterior key figures

	mean	95% c.i.	true	unit
chirp mass (m_c)	2.699	(2.692, 2.707)	2.698	$M_\odot$
mass ratio (η)	0.207	(0.192, 0.225)	0.204	
coalescence time (t_c)	12.3455	(12.3421, 12.3490)	12.3450	s
luminosity distance (d_L)	31.4	(17.4, 43.5)	30.0	Mpc
inclination angle (ι)	0.726	(0.159, 1.456)	0.700	rad
declination (δ)	-0.498	(-0.539, -0.456)	-0.506	rad
right ascension (α)	4.657	(4.632, 4.688)	4.647	rad
coalescence phase (ϕ_0)			2.0	rad
polarisation (ψ)			1.0	rad

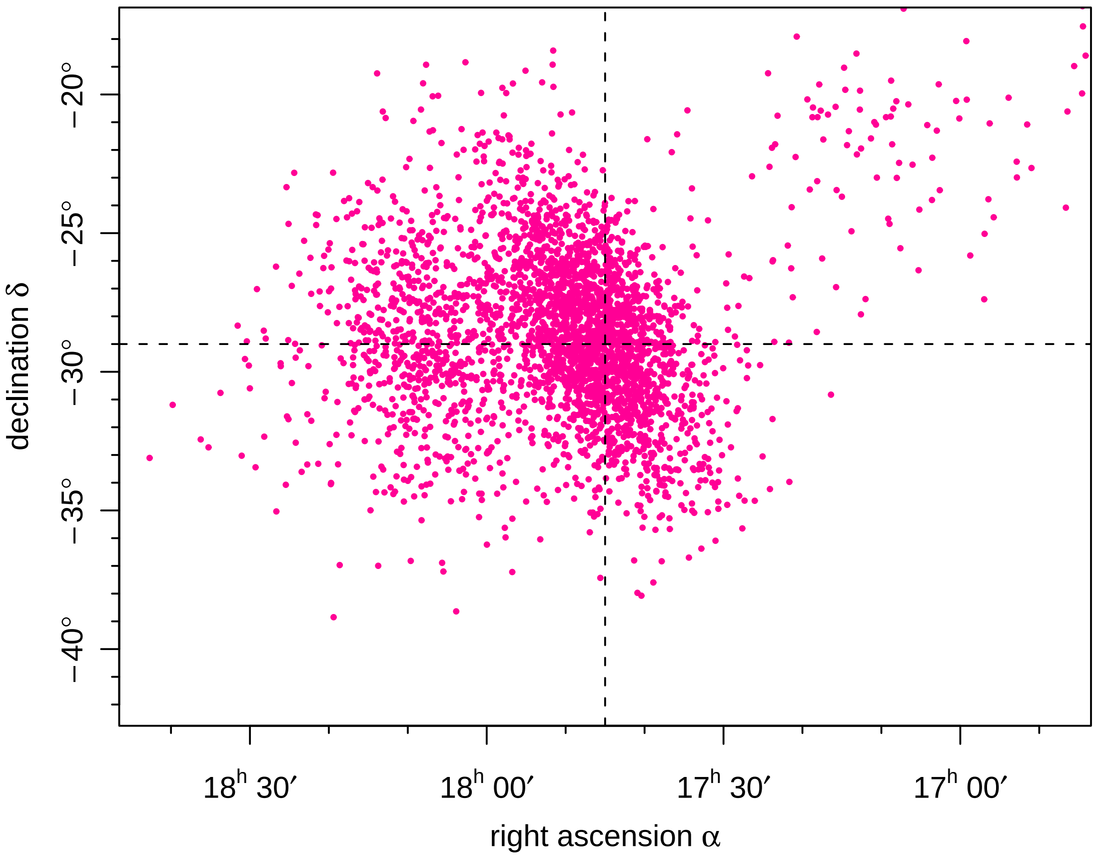
MCMC chain 1 — temperature = 1



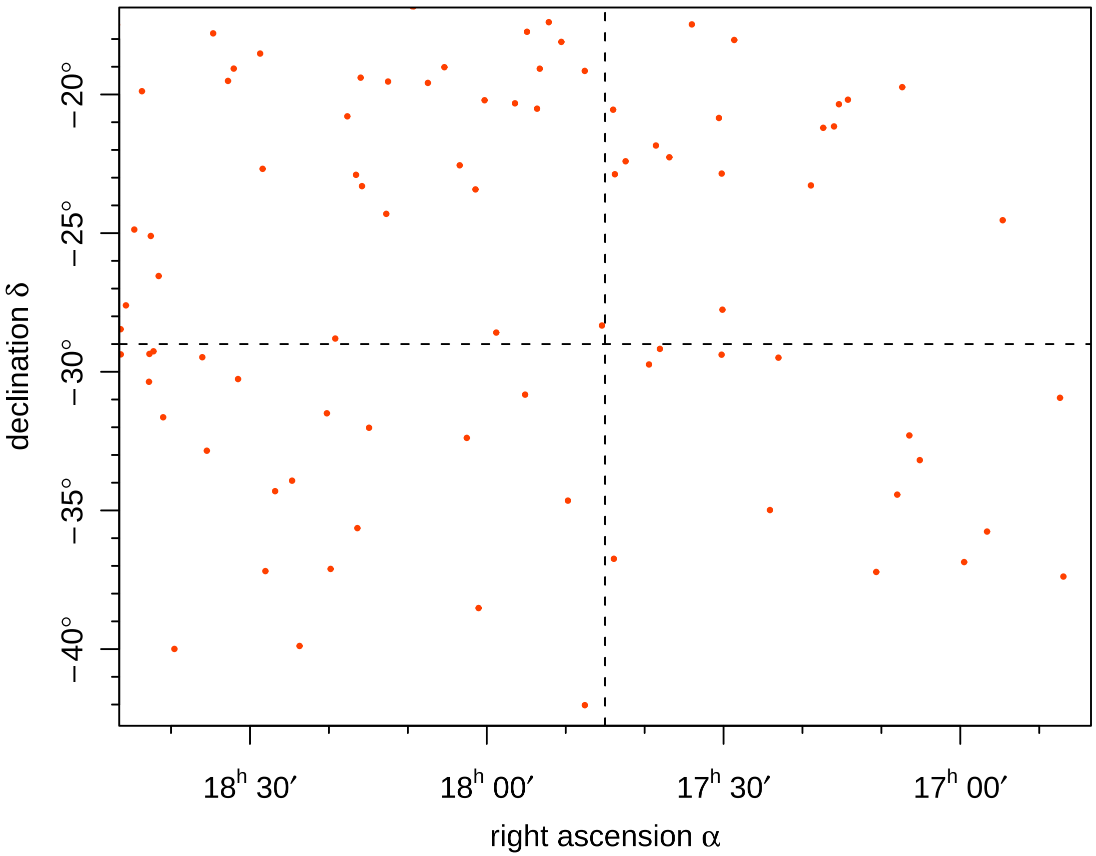
MCMC chain 2 — temperature = 2



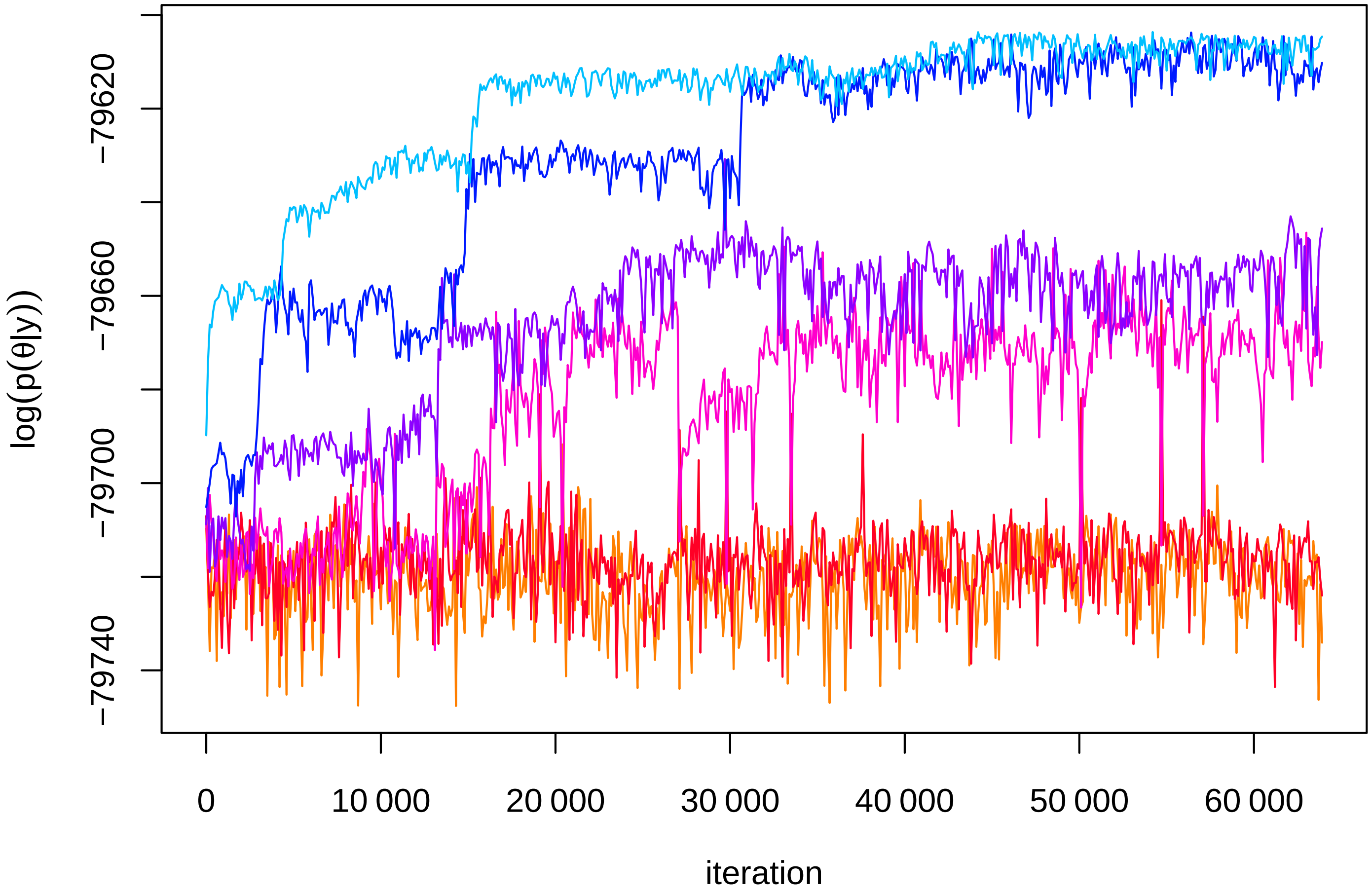
MCMC chain 3 — temperature = 4



MCMC chain 4 — temperature = 8



Six tempered chains 'in action'



Outlook

- incorporation into a 'loose net' detection pipeline for large mass ratio inspirals
- use information supplied by detection pipeline
- further parameters, e.g. spin effects