An Evidence Based Search For Neutron Star Ringdowns

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Overview

Objective: Construct a (triggered) Bayesian search algorithm for neutron star ring-downs

- Neutron star ring-downs
- Bayesian model selection & evidence
- Application & analysis pipeline
- Preliminary sensitivity estimates
- Future work

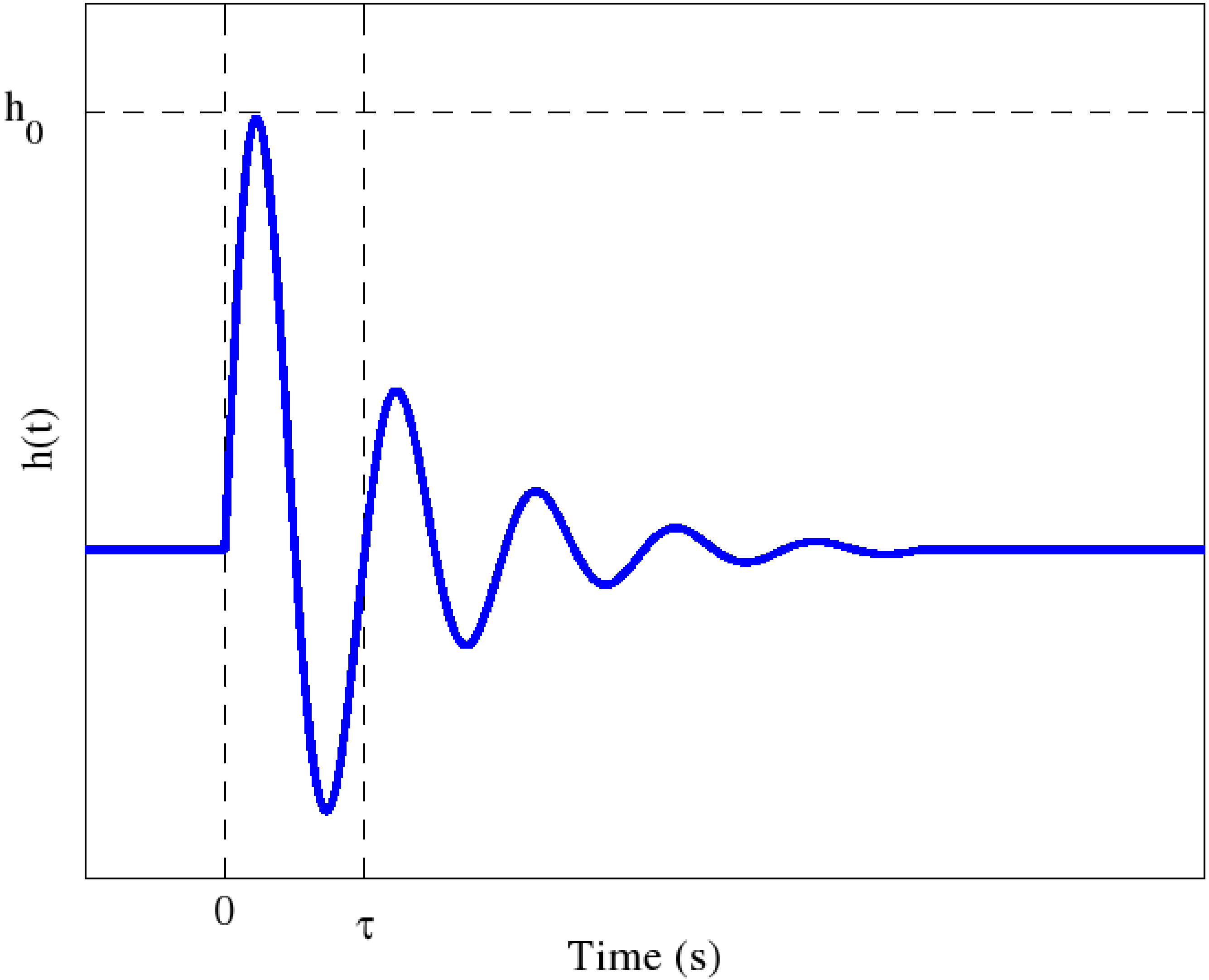
Neutron Star Ring-downs

Possible GW emission from neutron stars via quasi-normal mode (QNM) oscillations. QNMs may be excited by (e.g.):

- Birth of neutron star in core-collapse supernova
- **Soft gamma repeater (SGR) flares:**
  - highly magnetised NS, B-field stresses induce crustal cracking & excite QNMs, leading to GWs
  - Trigger: GRB observations (e.g., SGR1806-20 – GEO & LHO data)
- **Pulsar glitches**
  - Spin-down (and or de-coupling of crust/core, internal phase transition) induces crustal cracking due to relaxation of ellipticity: starquake.
  - Trigger: pulsar timing data

Neutron Star Ring-downs

$$h_0 \sim 5 \times 10^{-24} \left(\frac{E}{10^{-11} M_\odot c^2} \right)^{1/2} \left(\frac{\tau}{200\ ms} \right)^{-1/2} \left(\frac{\nu}{2\ kHz} \right) \left(\frac{D}{15\ kpc} \right)^{-1}$$



Bayesian Model Selection

- For competing models $\mathcal{M}_1, \mathcal{M}_2$ compute the **odds ratio** (ratio of posteriors probabilities) :

$$O_{12} = \frac{p(\ \mathcal{M}_1 \mid \{\mathcal{D}\},\ I)}{p(\ \mathcal{M}_2 \mid \{\mathcal{D}\},\ I)}$$

- Odds ratio consists of 2 terms:

$$O_{12} = \boxed{\frac{p(\ \mathcal{M}_1 \mid I)}{p(\ \mathcal{M}_2 \mid I)}} \times \boxed{\frac{p(\ \{\mathcal{D}\} \mid M_1,\ I)}{p(\ \{\mathcal{D}\} \mid M_2,\ I)}}$$

prior odds → ← **Bayes factor**

- $p(\{\mathcal{D}\} \mid \mathcal{M}_i,\ I)$ is the **evidence** for the model (likelihood, marginalised over some model parameters $\underline{\theta}$ and weighted by the prior) :

$$p(\{\mathcal{D}\} \mid \mathcal{M}_i,\ I) = \int_{\underline{\theta}} p(\ \underline{\theta} \mid \mathcal{M}_i,\ I) \times p(\{\mathcal{D}\} \mid \underline{\theta},\ \mathcal{M}_i,\ I) d\underline{\theta}$$

Applying Model Selection

$\mathcal{M}_1$: **Data contains a ring-down in Gaussian white noise**

- Likelihood function for a single datum $\mathcal{D}$, given an arbitrary signal power S & Gaussian noise σ is a non-central chi-squared distribution with non-centrality parameter S:

$$p\left(\mathcal{D} \mid S,\ \sigma,\ M\right) = \frac{1}{2\sigma^2} \exp\left[-\frac{\mathcal{D}+S}{2\sigma^2}\right] I_0\left(\frac{\sqrt{\mathcal{D}S}}{\sigma^2}\right)$$

- where S is modelled with a Lorentzian line profile, parameterised by

$$\underline{\theta} = \{h_0,\ \nu_0,\ \tau\}$$

$\mathcal{M}_2$: **Data only contains Gaussian white noise**

- Know *a priori* that the `signal power' is zero – use the same likelihood function with a strong prior on S to get the central chi-squared distribution for the evidence

$$p\left(S \mid \mathcal{M}_2\right) = \delta\left(S\right)$$

Priors

Choice of priors for $\mathcal{M}_1$

- Assume parameters are independent so that:

$$p(h_0, \nu_0, \tau) = p(h_0)p(\nu_0)p(\tau)$$

$$e.g., \ p(h_0) = \frac{1}{max(h_0) - min(h_0)}$$

Parameter	Prior	Range[a]
amplitude, h_0	uniform	$0 \ - \ 5 \times 10^{-20}$
frequency, ν_0	uniform	$(1500 \ - \ 3000) \ Hz$
decay time, τ	uniform	$0.05 - 0.5 \ s$

[a] zero outside of this range

Applying Model Selection

Aim is to detect a known waveform in a stretch of noisy interferometer data with known properties:

- $p(\,\mathcal{M}_1 \mid \{\mathcal{D}\},\ I)$ - probability that the data contains a ring-down waveform and white noise
- $p(\,\mathcal{M}_2 \mid \{\mathcal{D}\},\ I)$ - probability that the data contains only white noise

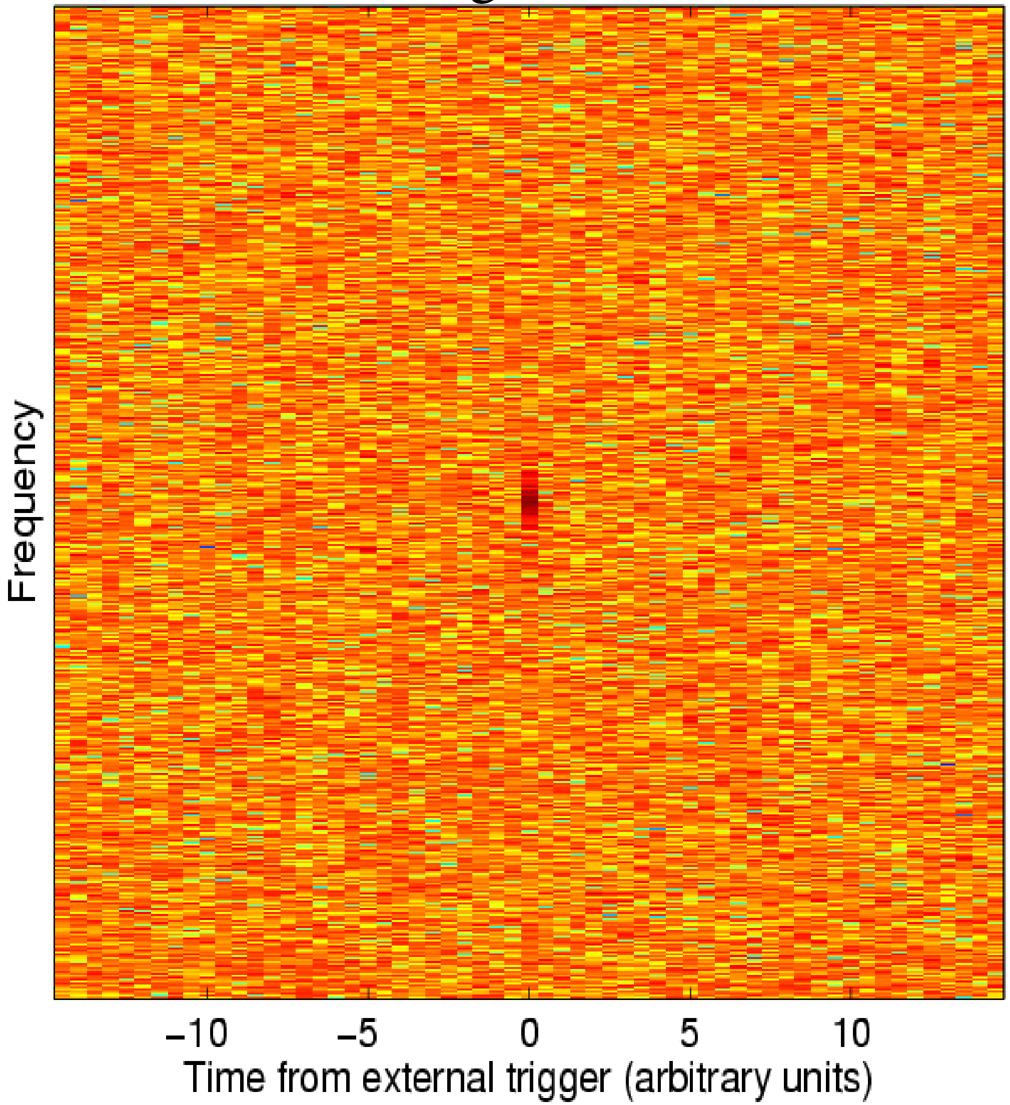
→ **Odds ratio O_{12} acts like a detection statistic for ring-downs versus white noise**

Analysis Pipeline

1. Construct spectrogram centered on external trigger (e.g., pulsar glitch)
2. Compute all possible $\mathcal{M}_1$ & $\mathcal{M}_2$ likelihoods for pixels & marginalise to get evidences in each time bin
3. Assume no prior model bias and compute odds ratio:
4. $$O_{12} = \frac{p(\ \{\mathcal{D}\}\ |\ \mathcal{M}_1,\ I)}{p(\ \{\mathcal{D}\}\ |\ \mathcal{M}_2,\ I)}$$
5. Finally, identify events with:

$$O_{12} > O_{thresh}$$

illustrative example spectrogram with ringdown:



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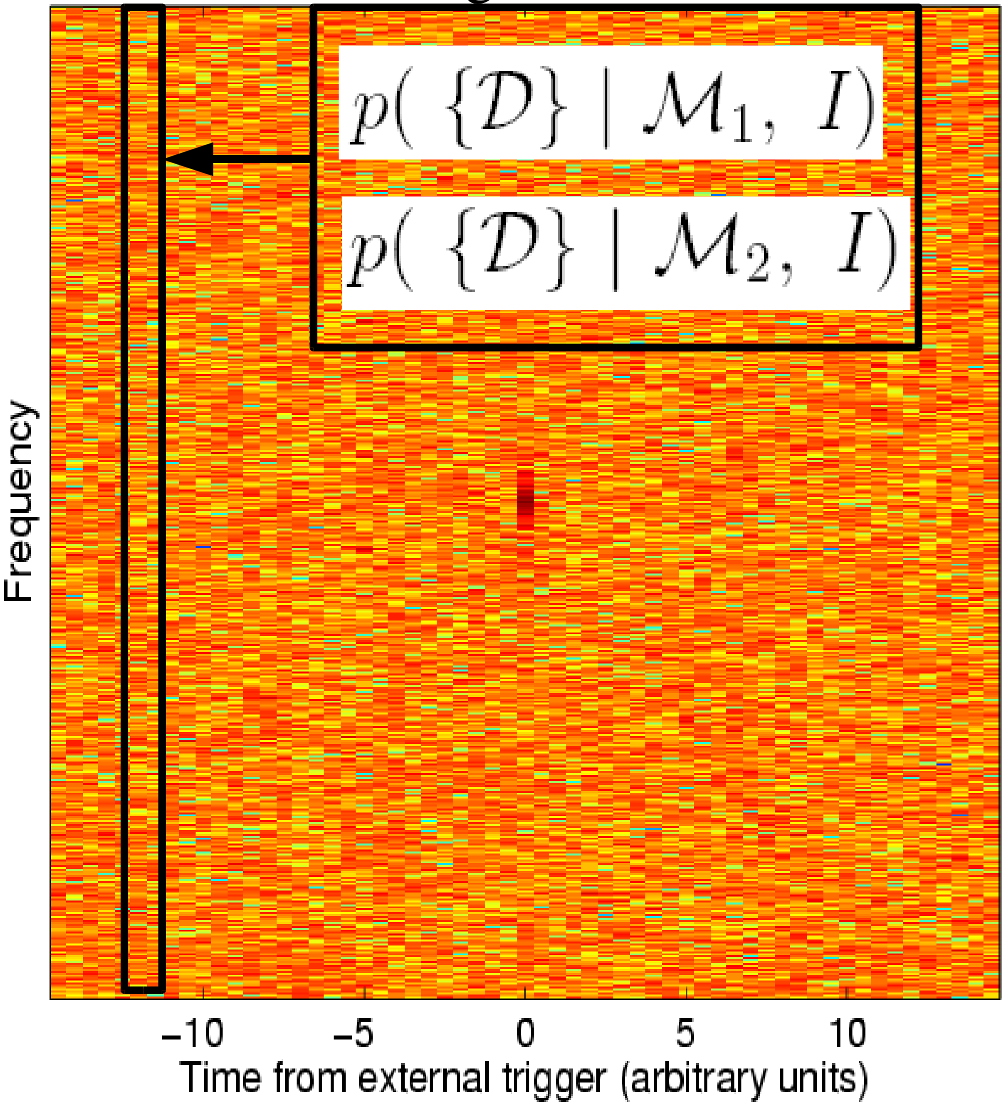

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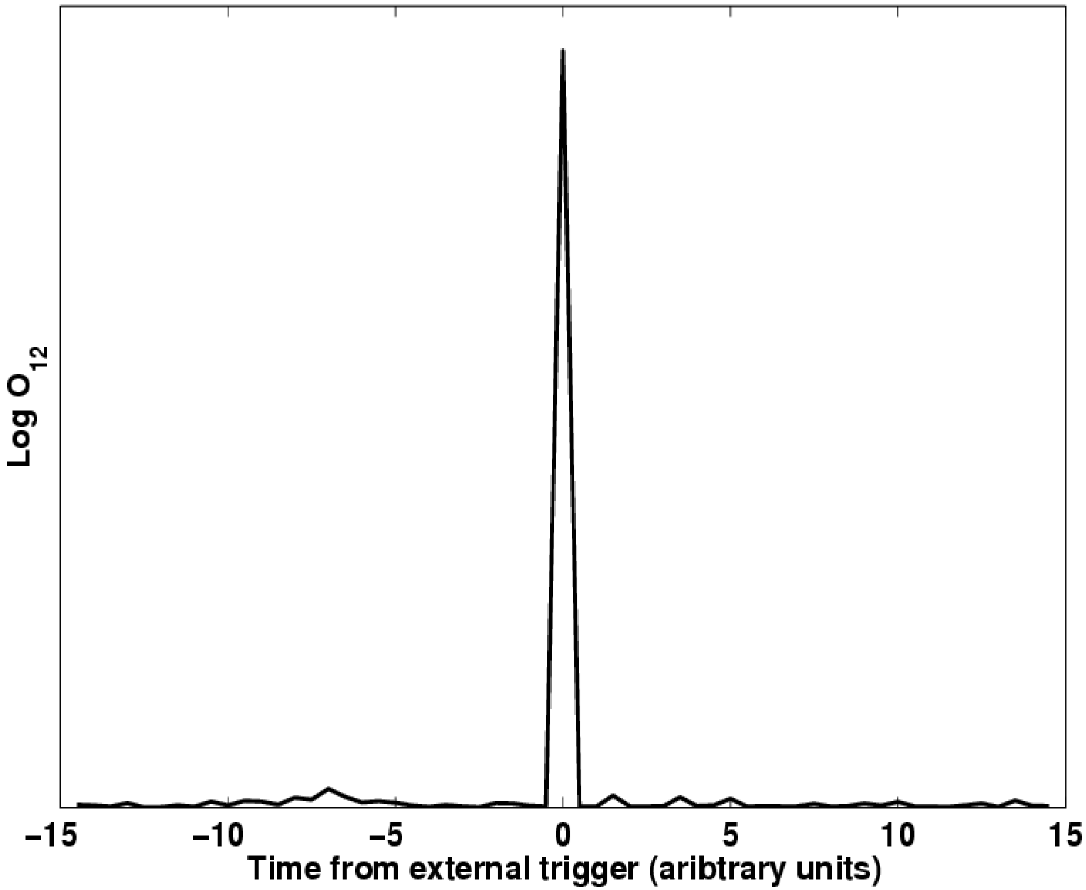
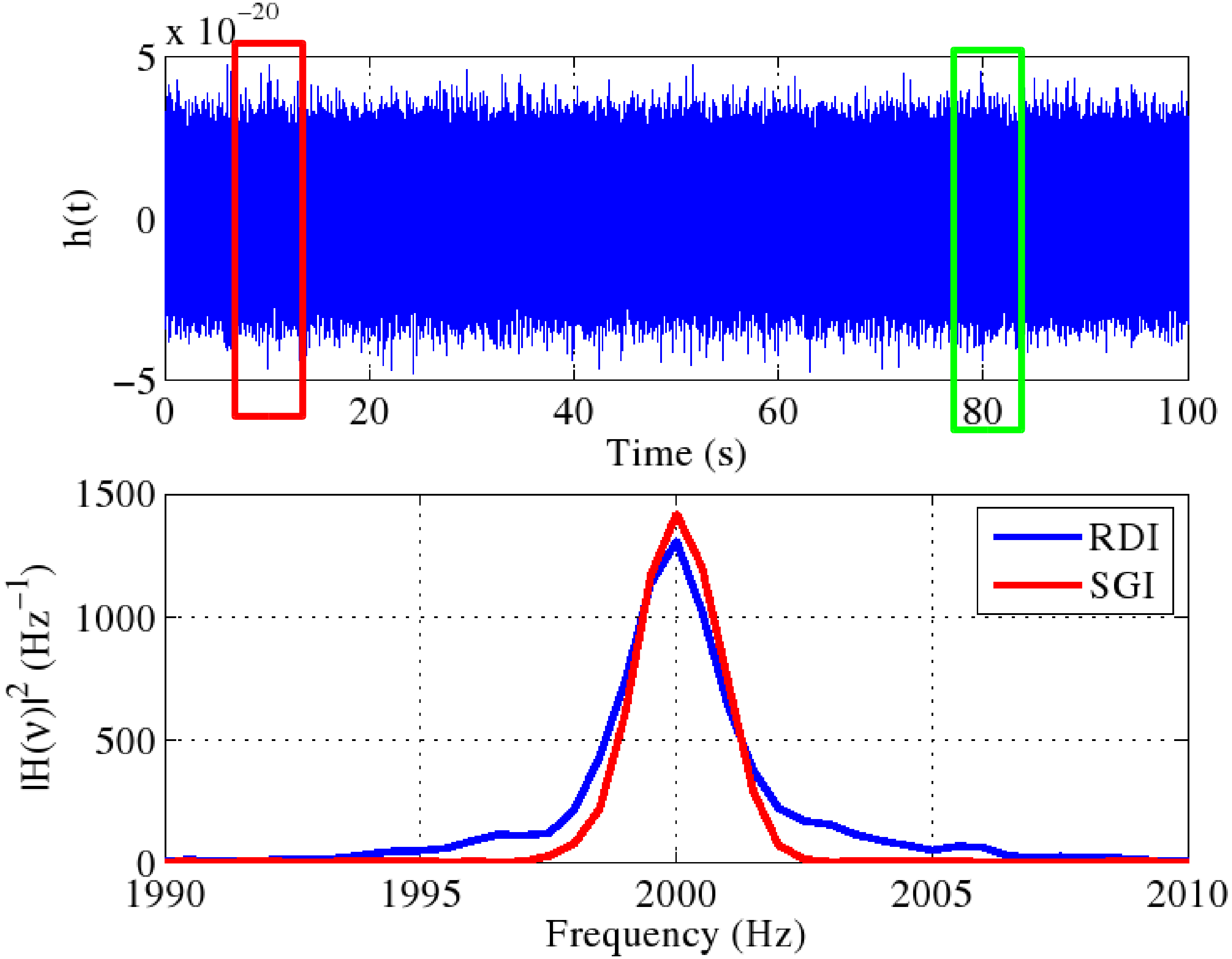
log odds from previous example:



An Example

- Compare response to target (ring-down - RD) waveform and an unwanted glitch (sine-Gaussian - SG)



RD

SG

Inject 1 ring-down and 1 sine-Gaussian of roughly equal SNR into synthetic Gaussian white noise

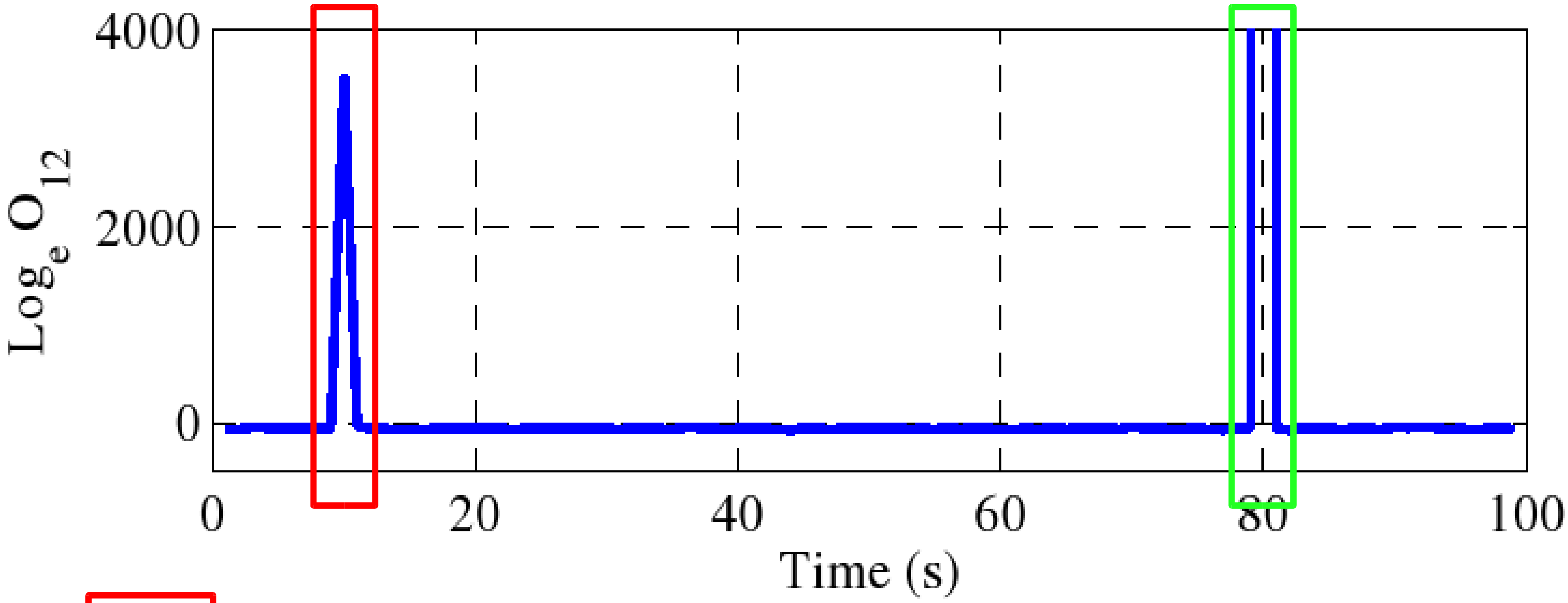
$$\rho_{RD} \approx \rho_{SG} \approx 40$$

$$\rho = \frac{h_{rss}}{ASD_{\text{noise}}}$$

$$h_{rss} = \sqrt{\int_{-\infty}^{+\infty} |h(t)|^2 dt}$$

Example Output

Output from odds algorithm:



- RD: ring-down is detected with odds well above that of background
- SG: sine-Gaussian is **also** detected

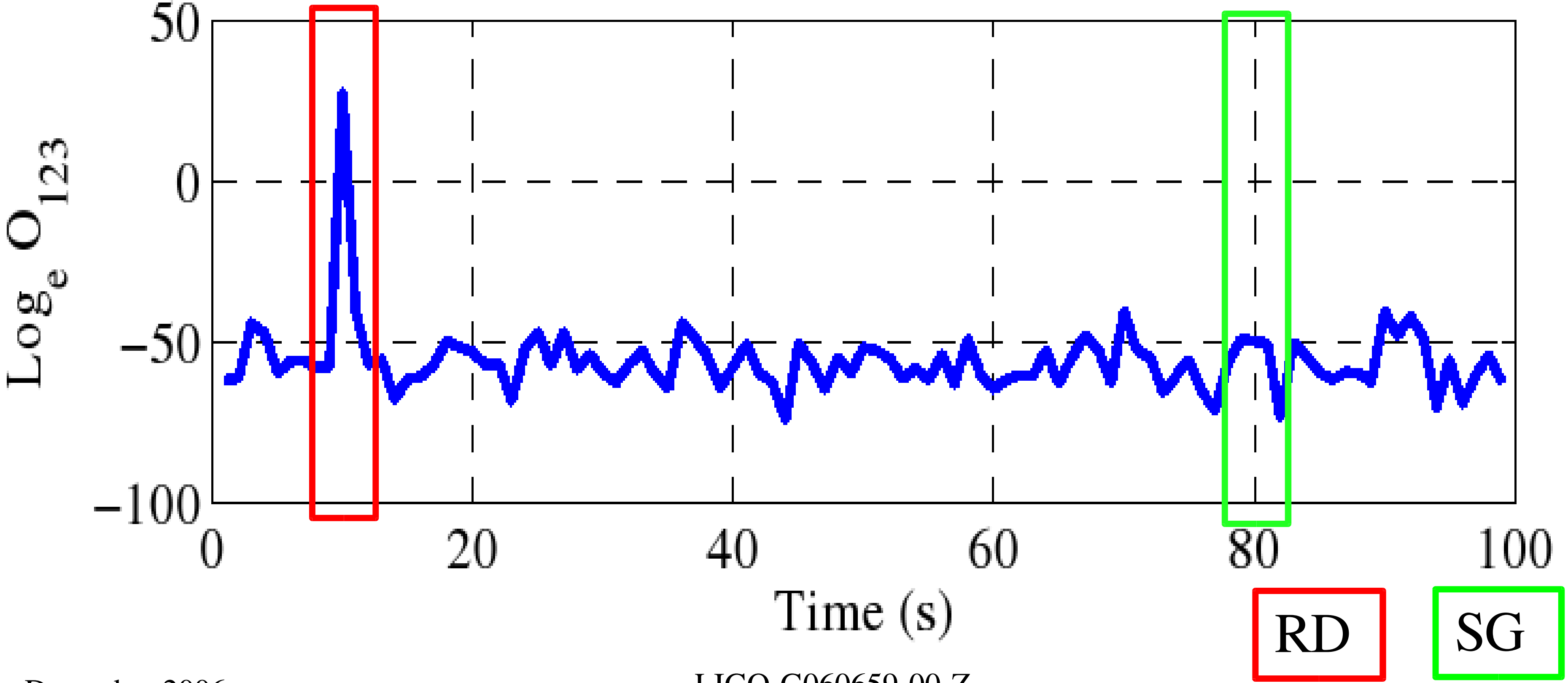
In fact, for the sine-Gaussian: $O_{12} \rightarrow +\infty \quad \text{as} \quad p(M_2|D,I) \rightarrow 0$

Example Output

Solution - consider an alternative 'glitch' hypothesis

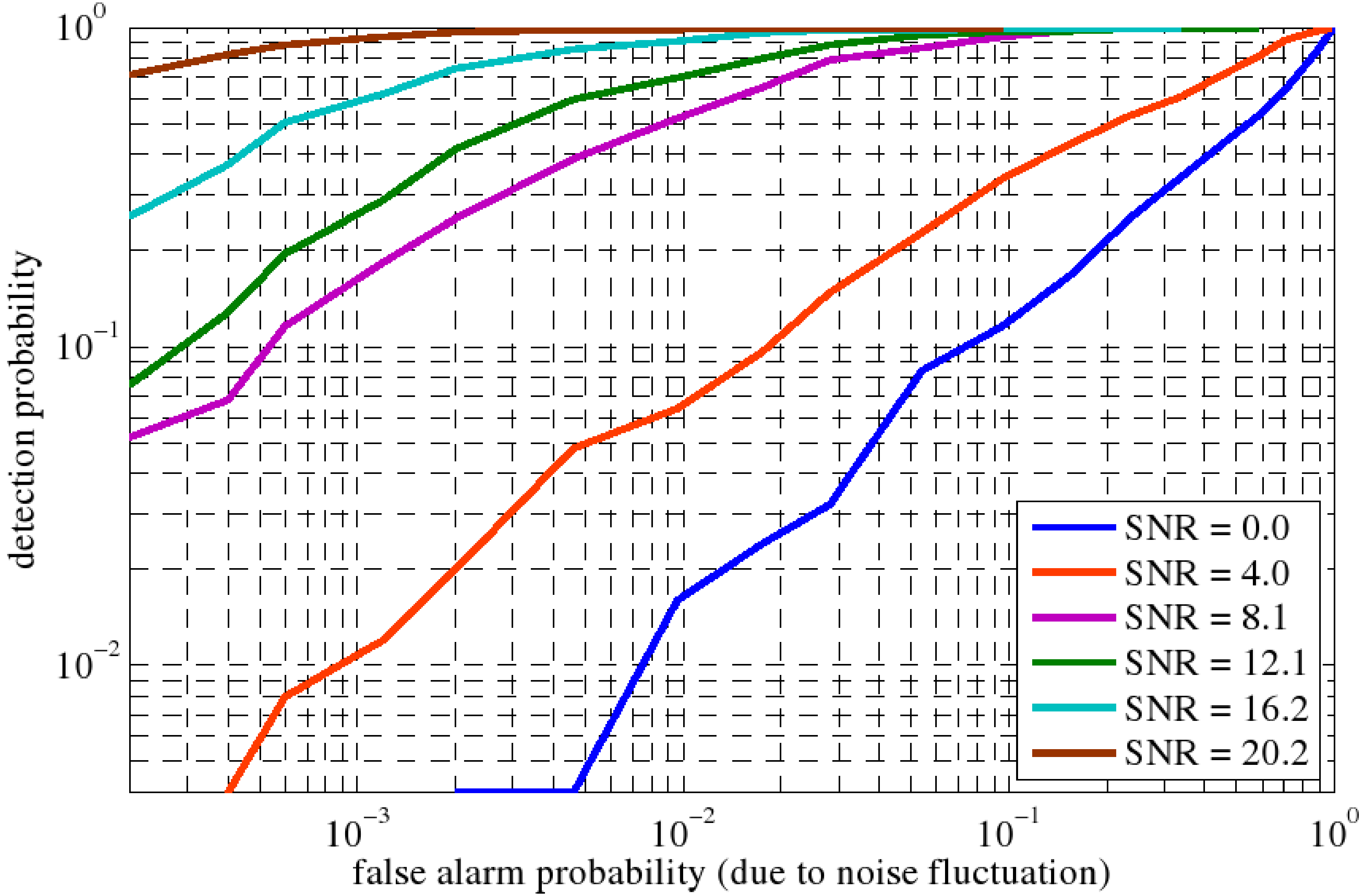
$\mathcal{M}_3$: data contains a sine-Gaussian in Gaussian white noise

$$O_{123} = \frac{p(\mathcal{M}_1 \mid \mathcal{D},\ I)}{p(\mathcal{M}_2 \mid \mathcal{D},\ I) + p(\mathcal{M}_3 \mid \mathcal{D},\ I)}$$



Performance Estimate

Receiver operating characteristics



Future Plans

Short-term:

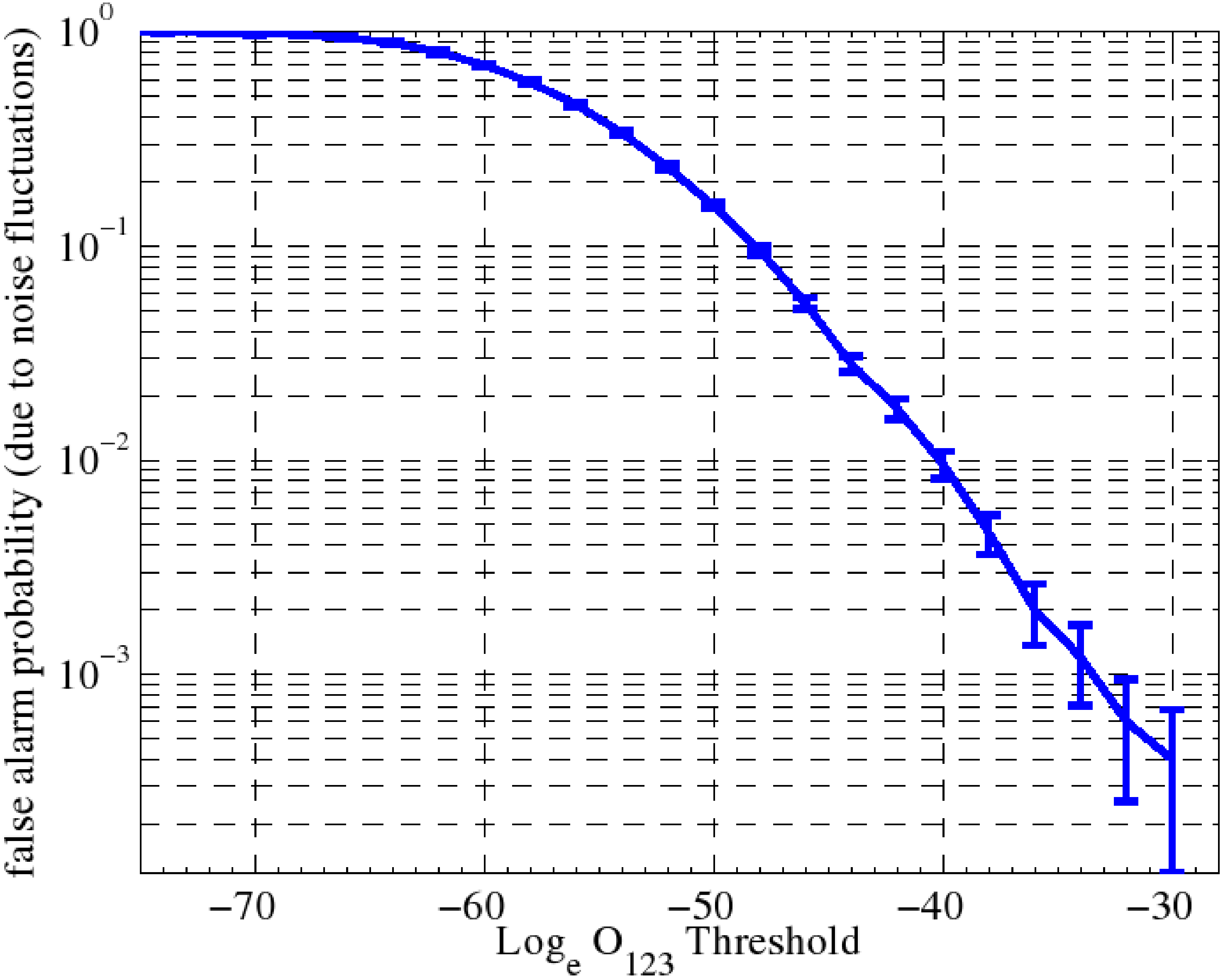
- Finish writing up methodology (J. Clark et al. in preparation)
- Run code on GEO & LIGO data from around SGR1806-20 – need to know what happens with *real* data... (have data)

Long-term:

- Upper limits on SGR1806-20 based on posterior probabilities and/or search sensitivity
- Look at other sources (pulsar glitches, GRB ring-downs)
- Potentially have a framework for multi-detector analysis from joint probabilities between detectors

end

Sensitivity Estimates



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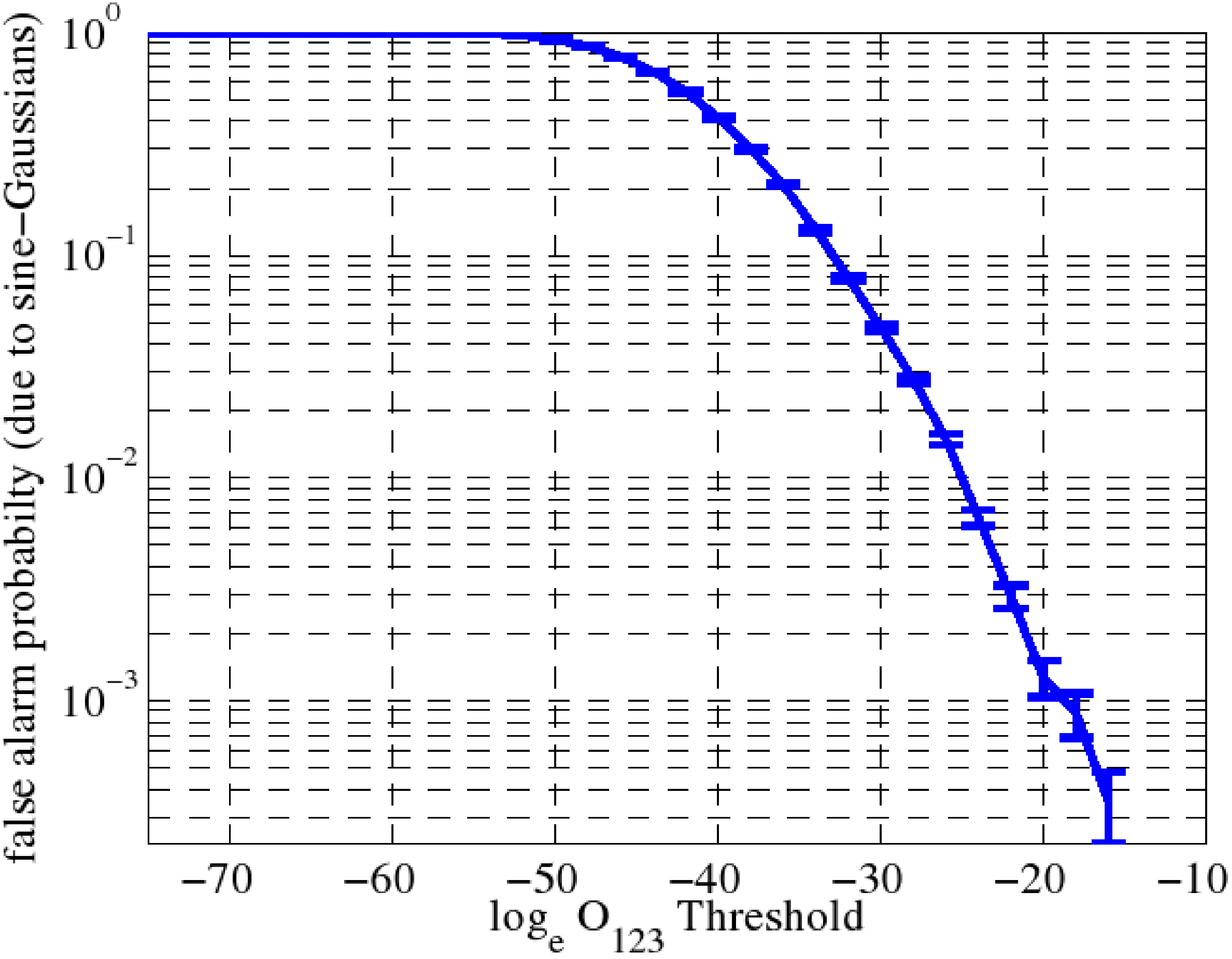

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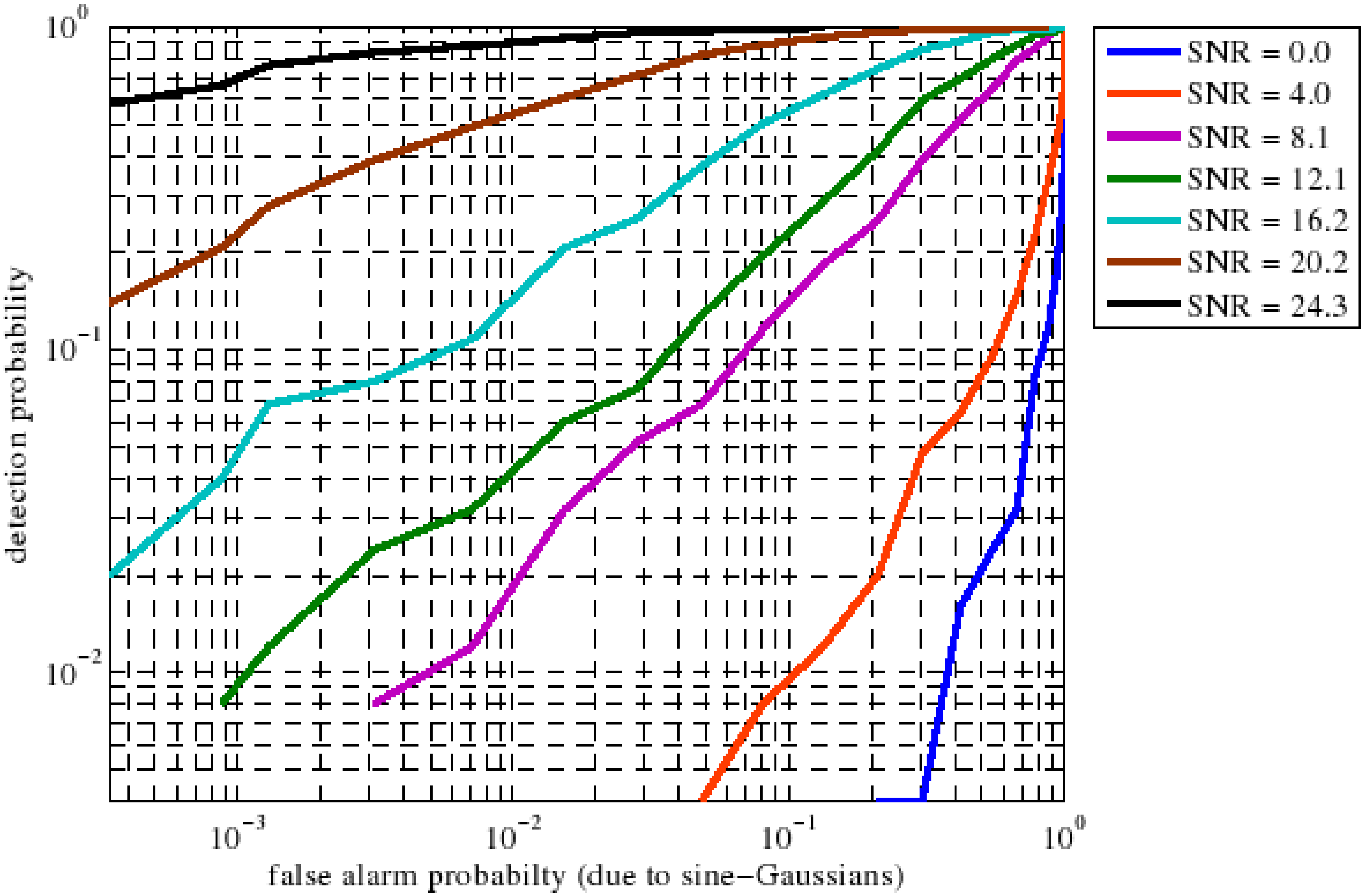

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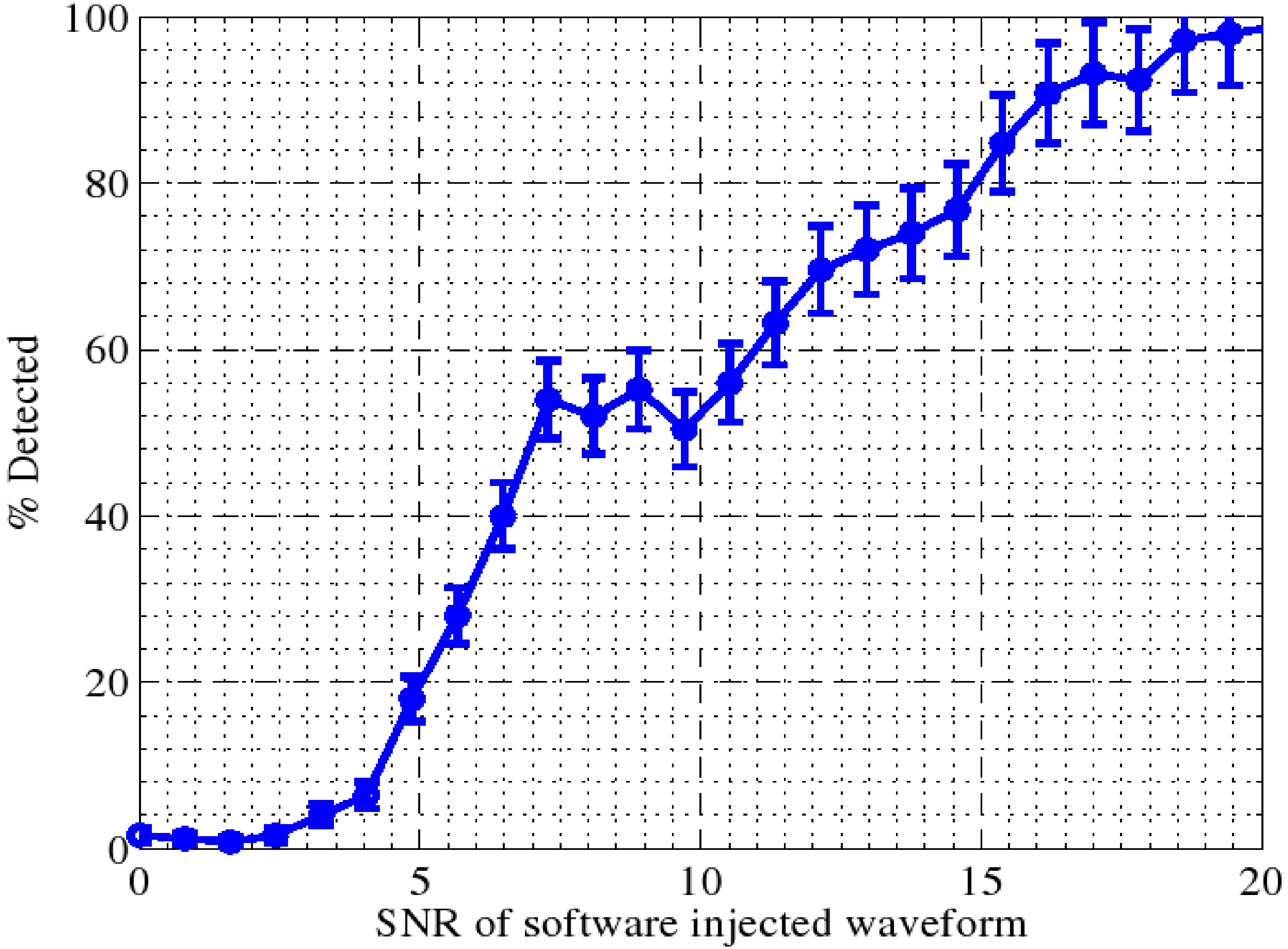

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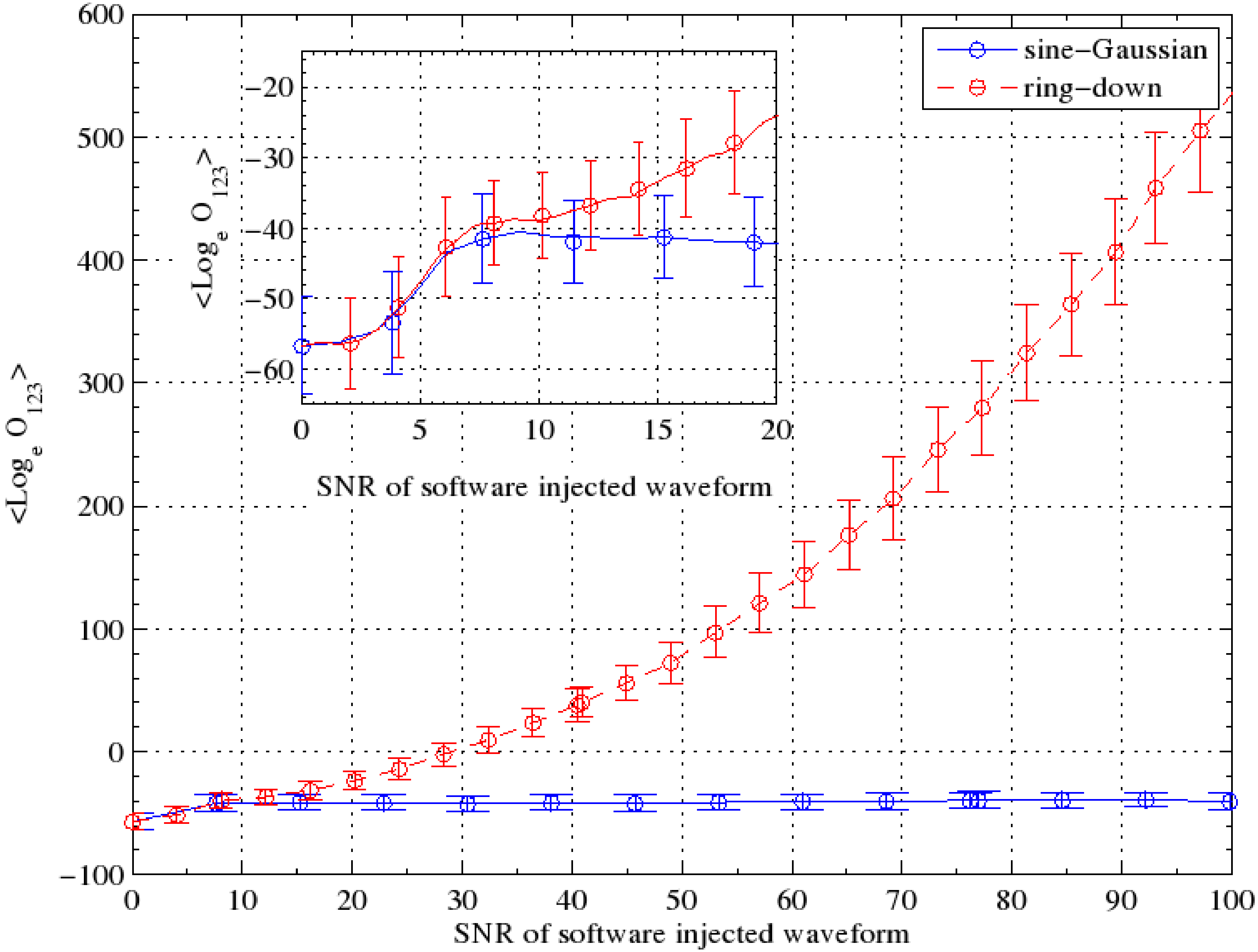
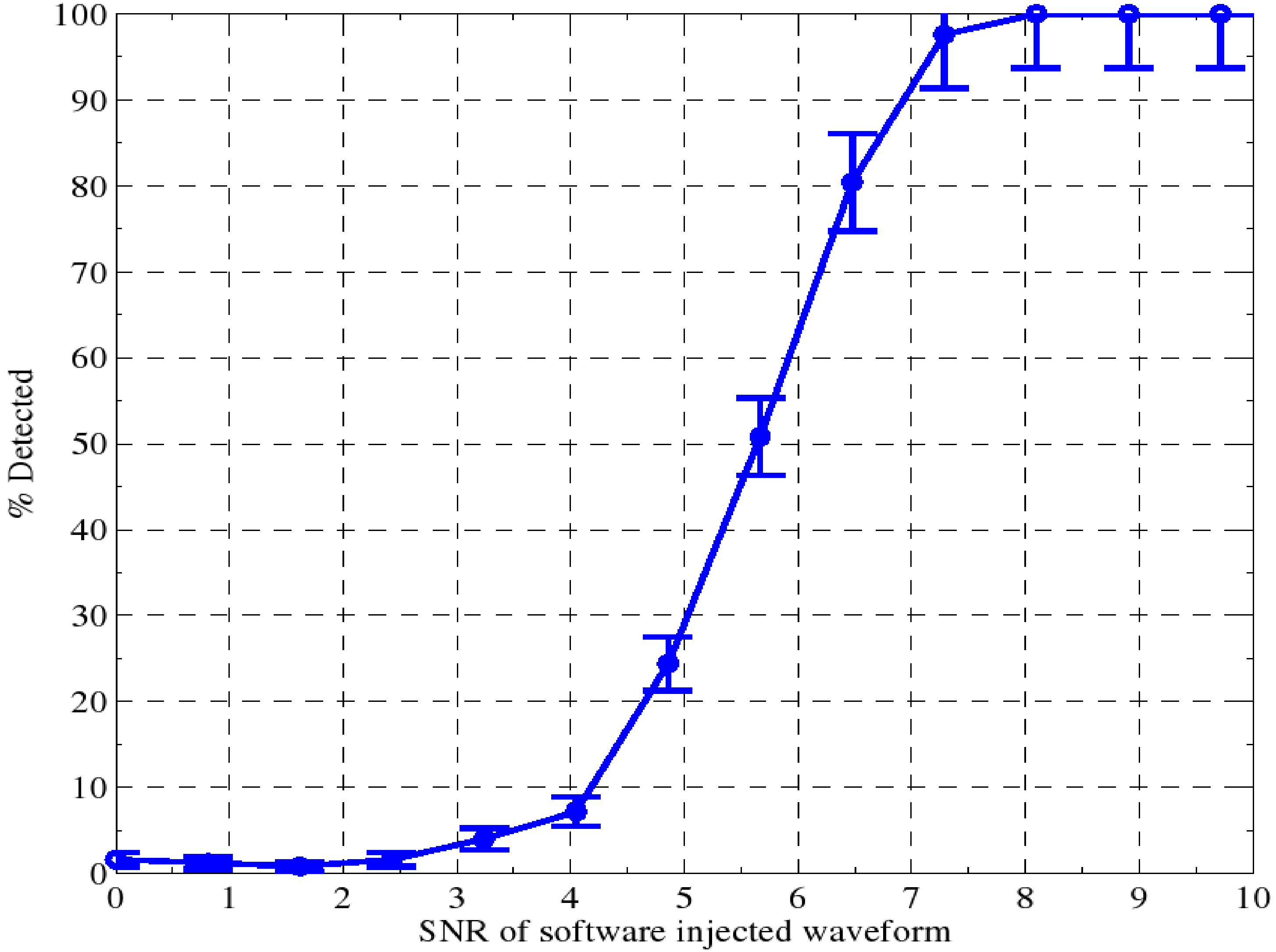


Sensitivity Estimates

Using original odds ratio, O_{12} :



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