Hough search for continuous gravitational waves using LIGO S4 data

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- Introduction
- The Hough transform
  - Overview of S2 analysis
  - Improvements for S4
  - Preliminary S4 results
- Future work

Sources of Continuous Gravitational Waves

(This talk shows results for isolated sources only.)

Search can detected any periodic source.

Upper limits are set on gravitational-wave amplitude, h_0, of rotating triaxial ellipsoid.

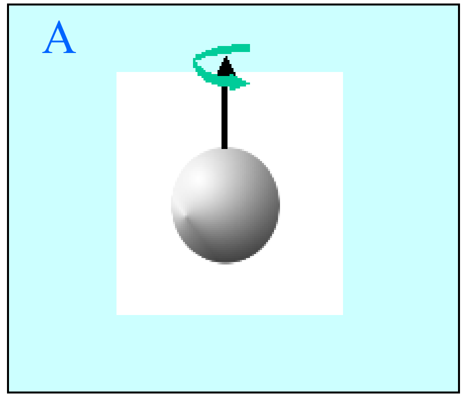
Credits:

A. image by Jolien Creighton; LIGO Lab Document G030163-03-Z.

B. image by M. Kramer; Press Release PR0003, University of Manchester - Jodrell Bank Observatory, 2 August 2000.

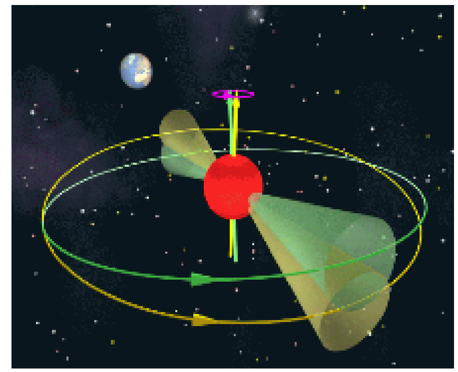
C. image by Dana Berry/NASA; NASA News Release posted July 2, 2003 on Spaceflight Now.

D. image from a simulation by Chad Hanna and Benjamin Owen; B. J. Owen's research page, Penn State University.

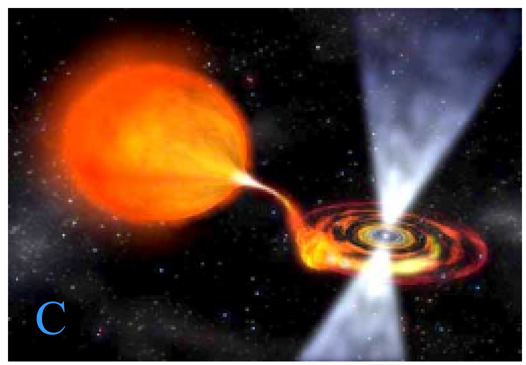


Mountain on neutron star

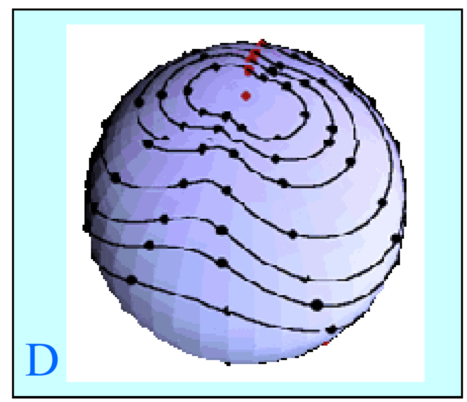
B



Precessing neutron star



Accreting neutron star



Oscillating neutron star

The expected waveform

- Expected waveform from an isolated spinning NS is sinusoidal with small spin-down:

$$h(t) = F_+(t,\psi)h_+(t) + F_\times(t,\psi)h_\times(t)$$
$$h_+ = A_+ \cos\Phi(t)$$
$$h_\times = A_\times \sin\Phi(t)$$

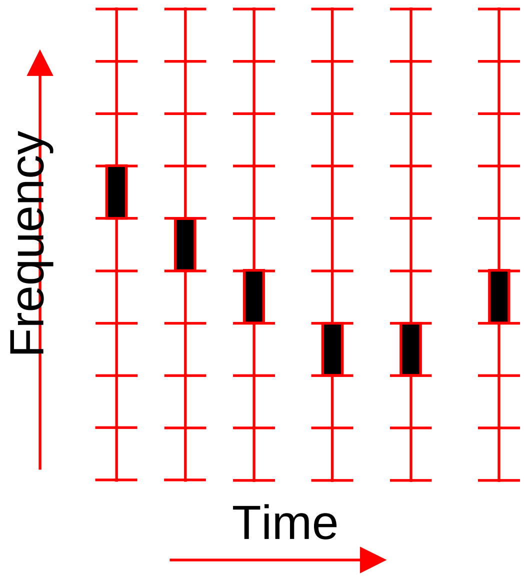
$$\Phi(t) = \phi_0 + 2\pi\sum_{n=0}^{\infty}\frac{f_{(n)}}{(n+1)!}(t-t_0)^{n+1}$$

- Doppler frequency modulation due to motion of Earth and amplitude modulation due to detector antenna pattern.
- For setting upper limits only, we assume the emission mechanism is due to deviations of the pulsar's shape from perfect axial symmetry, $f_{GW}=2f_r$

$$A_+ = \frac{1}{2}h_0(1+\cos^2\iota)$$
$$A_\times = h_0\cos\iota$$
$$h_0 = \frac{16\pi^2 G}{c^4}\frac{I_{zz}\varepsilon f_r^2}{d}$$

Incoherent power-sum methods

- Three methods have been developed to search for cumulative excess power from a hypothetical periodic gravitational wave signal by examining successive spectral estimates:



StackSlide

PowerFlux

Hough

They are all based on breaking up the data into segments, FFT each, producing Short (30 min) Fourier Transforms (SFTs) from h(t), as a coherent step (although other coherent integrations can be used if one increasing the length of the segments),

and then track the frequency drifts due to Doppler modulations and df/dt as the incoherent step.

- Other fully coherent methods:
  - Frequency domain match filtering/maximum likelihood estimation
  - Time domain Bayesian parameter estimation

Differences among the incoherent methods

What is exactly summed?

- StackSlide – Normalized power (power divided by estimated noise)
  $\rightarrow$ Averaging gives expectation of 1.0 in absence of signal
- Hough – Weighted binary counts (0/1 = normalized power below/above SNR), with weighting based on antenna pattern and detector noise
- PowerFlux – Average strain power with weighting based on antenna pattern and detector noise
  $\rightarrow$ Signal estimator is direct excess strain noise
  (circular polarization and 4 linear polarization projections)

The Hough transform

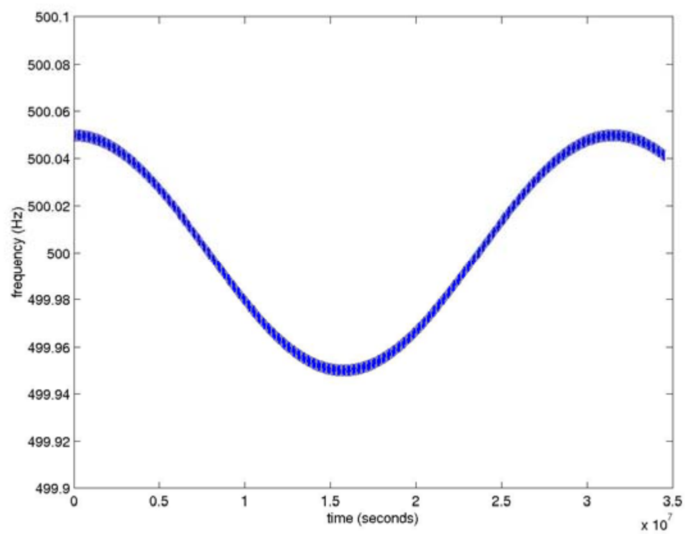
The *Hough transform* is a general method for pattern recognition that was developed and patented many years ago.

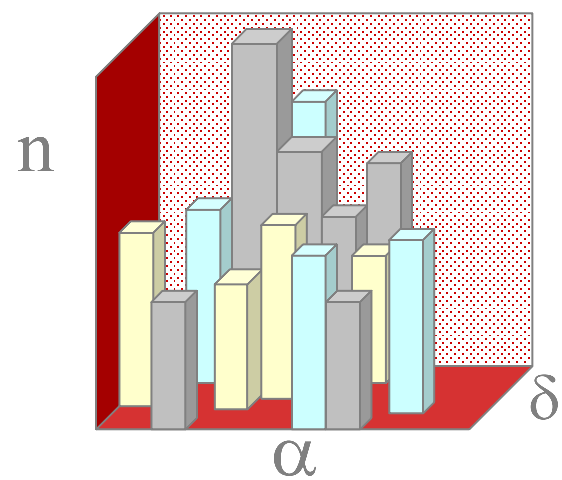
We use the *Hough transform* to find a pattern produced by the Doppler modulation & spin-down of a GW signal in the time-frequency plane of our data.

For isolated NS the expected pattern depends on: $\{\alpha, \delta, f_0, f_n\}$

$$f(t) - \hat{f}(t) = \hat{f}(t)\,\frac{\mathbf{v}(t)\cdot\mathbf{n}}{c}$$

$$\hat{f}(t) = f_0 + \dot{f}\,(t - t_0)$$





Review of S2 Hough analysis

- Start with 1800s SFTs for each detector
- Select frequency bins by setting a threshold on normalized power – gives time-frequency collection of 0s and 1s
- For N SFTs, the final number count for a given parameter space point is

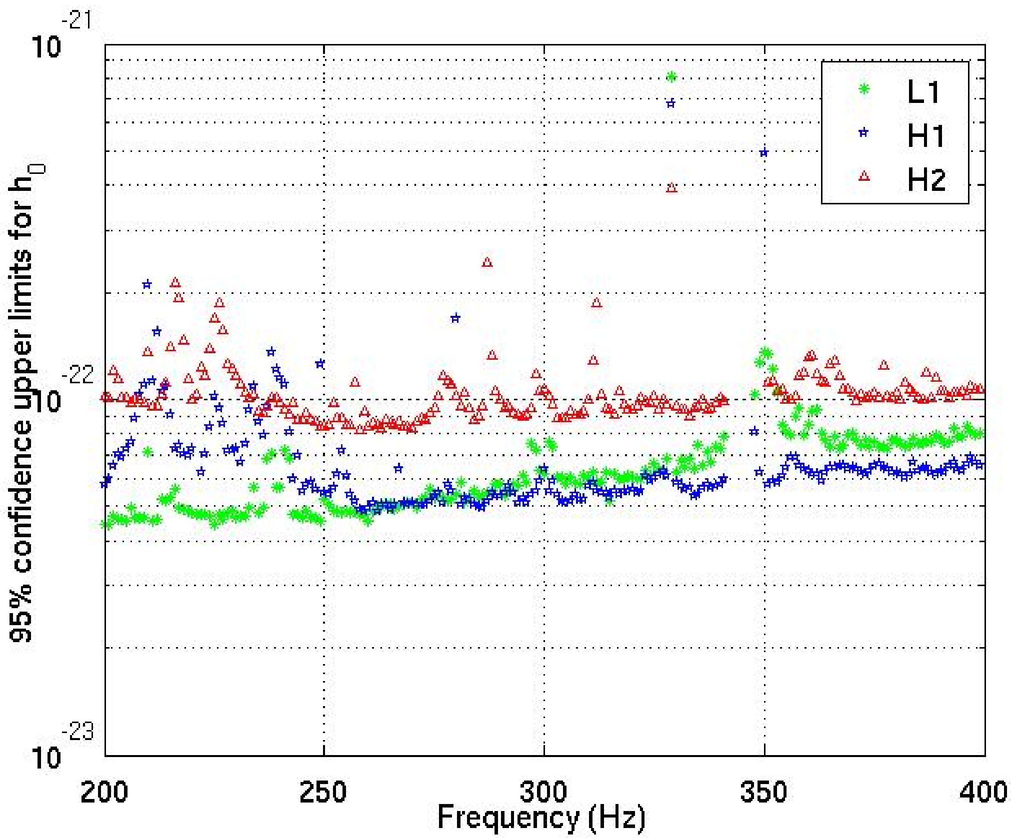
$$n = \sum_{i=1}^{N} n_i \quad \text{where} \quad n_i = \begin{cases} 0 \\ 1 \end{cases}$$

- Using 0s and 1s leads to gain in computational efficiency and it is more robust against large transient power artifacts

Hough S2: UL Summary

Feb.14-Apr.14,2003

- S2 analysis covered 200-400Hz, over the whole sky, and 11 values of the first spindown ($\Delta f = 5.55\times10^{-4}$ Hz, $\Delta f_1 = -1.1\times10^{-10}$ Hz s^{-1})
- Templates: Number of sky point templates scales like (frequency)^2
  - 1.5×10^5 sky locations @ 300 Hz
  - 1.9×10^9 @ 200-201 Hz
  - 7.5×10^9 @ 399-400 Hz
- Three IFOs analyzed separately
- No signal detected
- Upper limits obtained for each 1 Hz band by signal injections: Population-based frequentist limits on h_0 averaging over sky location and pulsar orientation



Detector	L1	H1	H2
Frequency (Hz)	200-201	259-260	258-259
$h_0^{95\%}$	**4.43×10^{-23}**	4.88×10^{-23}	8.32×10^{-23}

Frequentist upper limit

- Perform the Hough transform for a set of points in parameter space $\lambda=\{\alpha,\delta,f_0,f_i\} \in \mathbf{S}$, given the data:

$$\textbf{HT: S} \rightarrow \textbf{N}$$

$$\lambda \rightarrow n(\lambda)$$

- Determine the maximum number count n*

$$n^* = \max\,(n(\lambda)) : \lambda \in \mathbf{S}$$

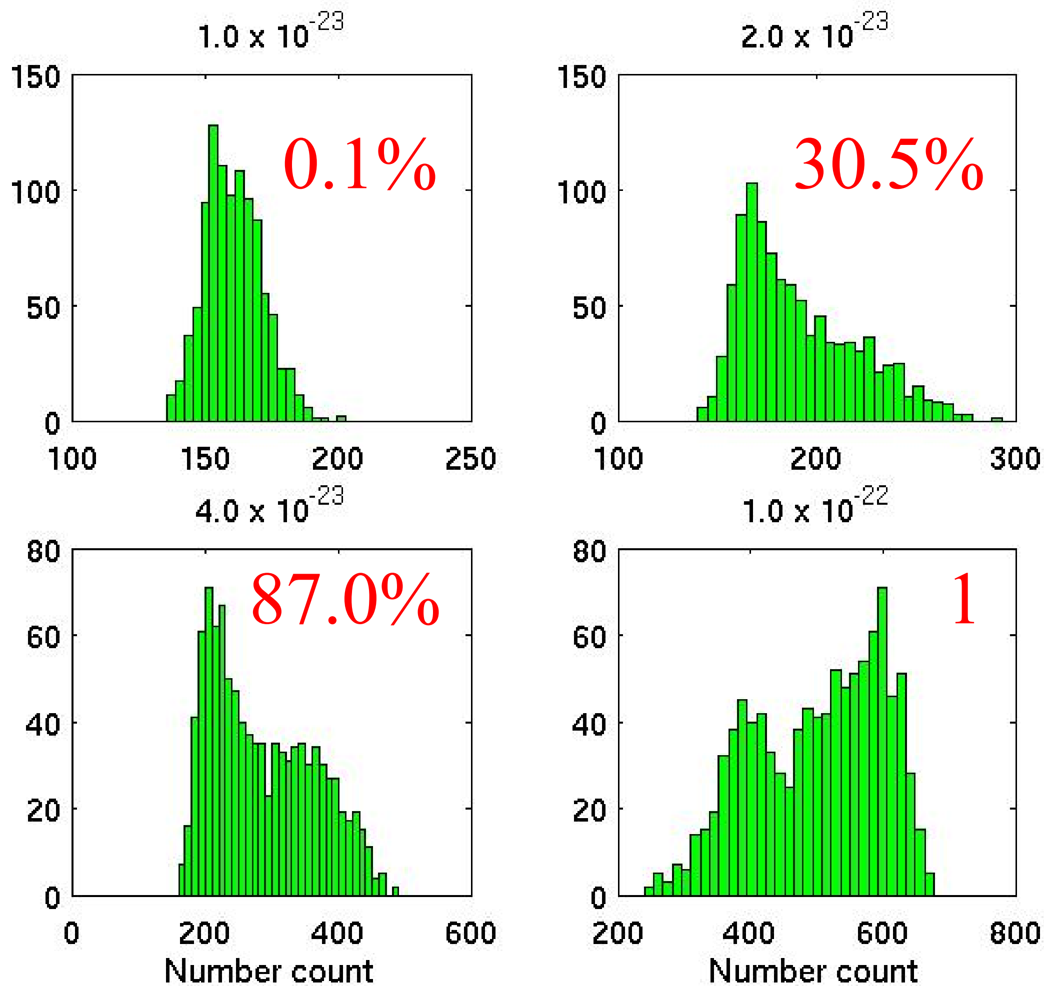
- Determine the probability distribution $p(n|h_0)$ for a range of h_0
- The 95% frequentist upper limit $h_0^{95\%}$ is the value such that for repeated trials with a signal $h_0 \geq h_0^{95\%}$, we would obtain $n \geq n^*$ more than 95% of the time

$$0.95 = \sum_{n=n^*}^{N} p\left(n|h_0^{95\%}\right)$$

Compute $p(n|h_0)$ via Monte Carlo signal injections

Number count distribution for signal injections in S2 data

L1: 200-201 Hz, n^* =202, 1000 injections



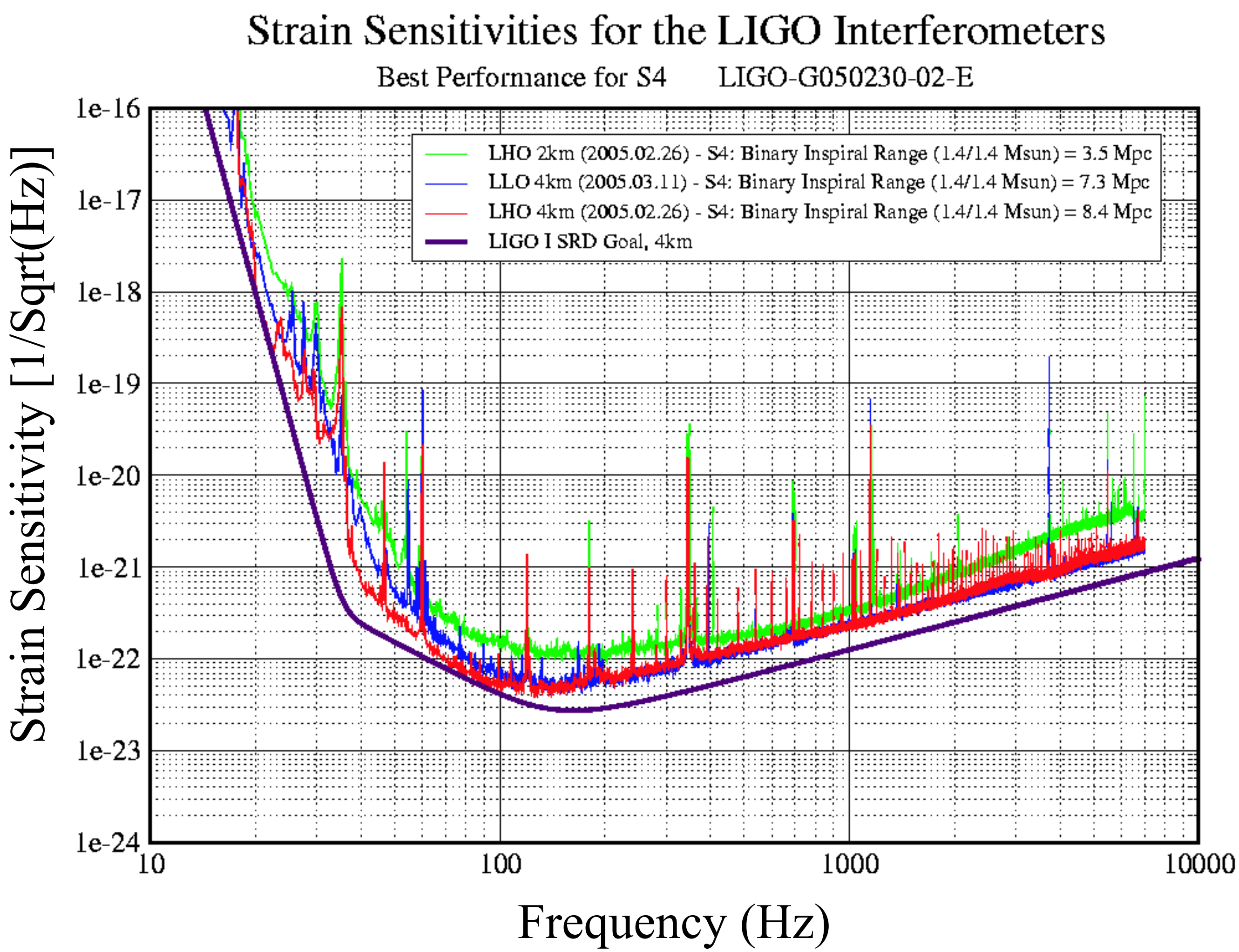
p(n|h₀) ideally binomial for a target search, but:

- Non stationarity in the noise
- Amplitude modulation of the signal for different SFTs
- Different sensitivity for different sky locations and pulsar orientations
- Random mismatch between signal & templates

'smear' out the binomial distributions

Fourth Science Run (S4) Sensitivity

(February 22, 2005 – March 23, 2005)



S5 is currently running at design sensitivity!

Improvements for S4

- The S2 Hough search has been modified to take into account that the SFTs have different noise floors and the signal amplitude changes in time – SNR changes across SFTs
- We use a *weighted Hough* to give more weight to SFTs having greater SNR. Weights are proportional to the beam pattern functions and inversely proportional to the SFT noise floor.
- Weighting method applied to Hough was initially suggested by C.Palomba and S.Frasca at GWDAW-2004 and it is similar to the one used by the PowerFlux method.
- Number count n is not an integer anymore

$$n = \sum_{i=1}^{N} w_i n_i \qquad \sum_{i=1}^{N} w_i = N$$

- Using the weights does not lead to any loss in computational efficiency or robustness
- It has also been generalized to the Multi-IFO case

Improvements for S4

- Nominal sensitivity for given FA and FD assuming a perfectly matched template averaged over sky, orientations and polarization angles:

$$h_0 = 5.34 \frac{S^{1/2}}{N^{1/4}} \sqrt{\frac{S_n}{T_{coh}}}$$

$$S = \mathrm{erfc}^{-1}(2\alpha_H) + \mathrm{erfc}^{-1}(2\beta_H)$$

- Improved Sensitivity: Assumes template is perfectly matched to signal, and average over all pulsar orientations and polarization angles (but not over sky-positions)

$$h_0 = 3.38 S^{1/2} \left(\frac{\|\vec{w}\|}{\vec{w} \cdot \vec{X}} \right)^{1/2} \sqrt{\frac{\langle S_n^{(i)} \rangle}{T_{coh}}}$$

$$X_i = \langle S_n^{(i)} \rangle \frac{\left(F_+^{(i)}\right)^2 + \left(F_\times^{(i)}\right)^2}{S_n^{(i)}}$$

- Optimal choice of weights is:

$$w_i \propto \frac{\left(F_+^{(i)}\right)^2 + \left(F_\times^{(i)}\right)^2}{S_n^{(i)}}$$

- Optimally weights should be calculated at same sky-location as signal

Improvements for S4

- Gain in sensitivity is large if standard deviation of SFT noise floors is large or if signal amplitude changes rapidly across SFTs
- Mean number count is unchanged due to normalization of weights:

$$\langle n \rangle = N\alpha = N\exp(-\rho_{th})$$

- Standard deviation always increases:

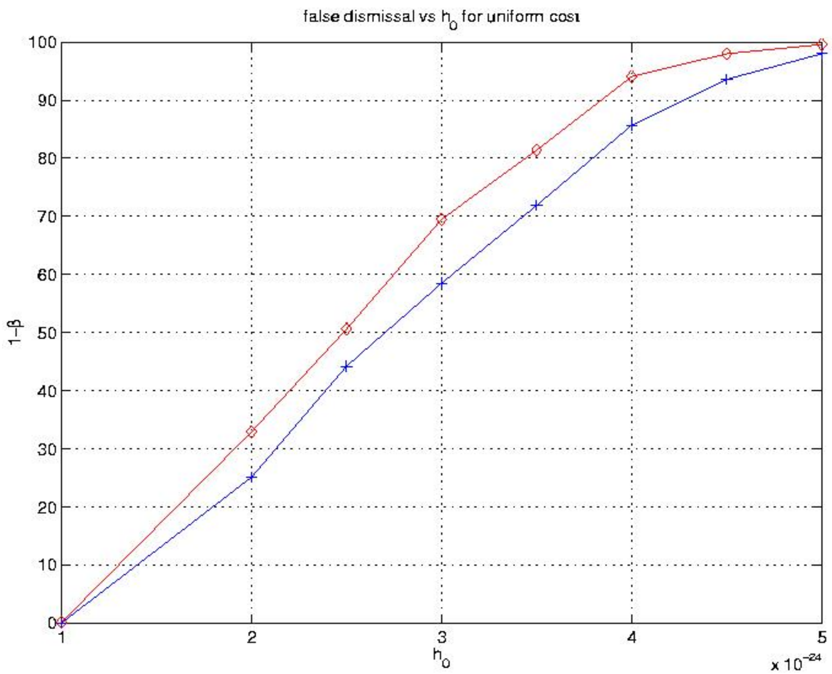
$$\sigma_n = \|\vec{w}\|\sqrt{\alpha(1-\alpha)}$$

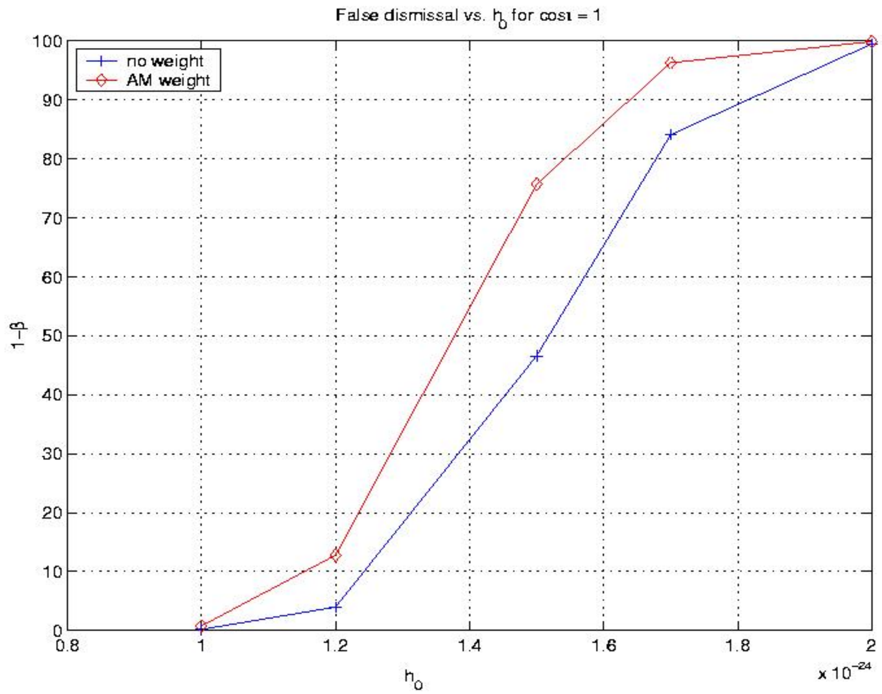
- Number count threshold for a given false alarm:

$$n_{th} = N\alpha + \sqrt{2\|\vec{w}\|^2\alpha(1-\alpha)}\ \mathrm{erfc}^{-1}(2\alpha_H)$$

Improvement in detection efficiency

Signal injections in fake data, 250-260Hz, random sky-position and polarization angles. Number count threshold set for $\alpha_H = 10^{-10}$





- Improvement in sensitivity at 90% efficiency is roughly 10% in signal amplitude for a perfectly matched template and stationary noise. The gain depends on pulsar orientation
- Will be somewhat degraded when searching in a sky-patch because of a mismatch and also because we will use a single set of weights for the whole sky-patch (calculated at the center), but sensitivity can also improve in case of non-stationary noise.

The S4 Hough search

- As before, input data is a set of *N* 1800s SFTs (no demodulations)
- Weights allow us to use SFTs from all three IFOs together: 1004 SFTS from H1, 1063 from H2 and 899 from L1
- Search frequency band 50-1000Hz
- 1 spin-down parameter. Spindown range $[-2.2,0]\times10^{-9}$ Hz/s with a resolution of 2.2×10^{-10} Hz/s
- All sky search
- Sky is broken up into 92 patches 0.4 rad × 0.4 rad wide
- Line cleaning used to remove known narrow spectral lines
- All-sky upper limits set in 0.25 Hz bands
- Multi-IFO and single IFOs have been analyzed

Contribution of different IFOs

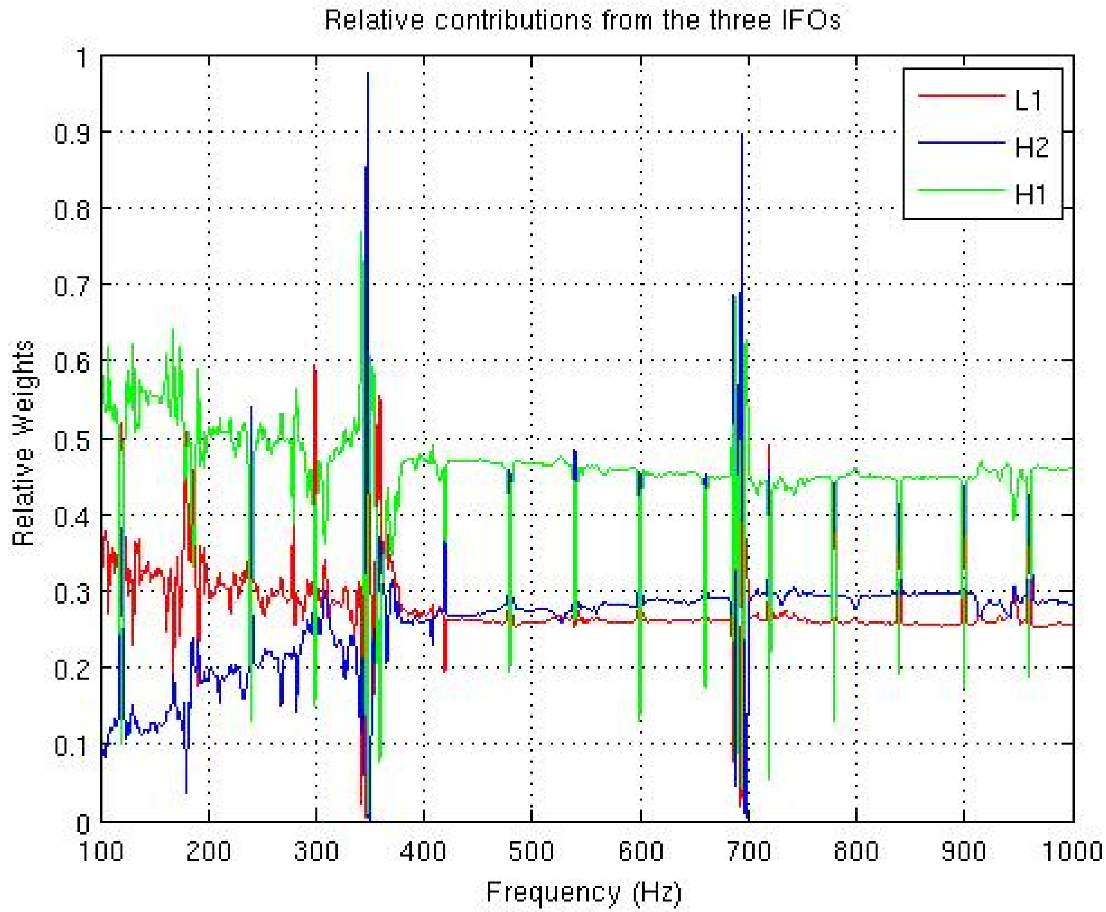
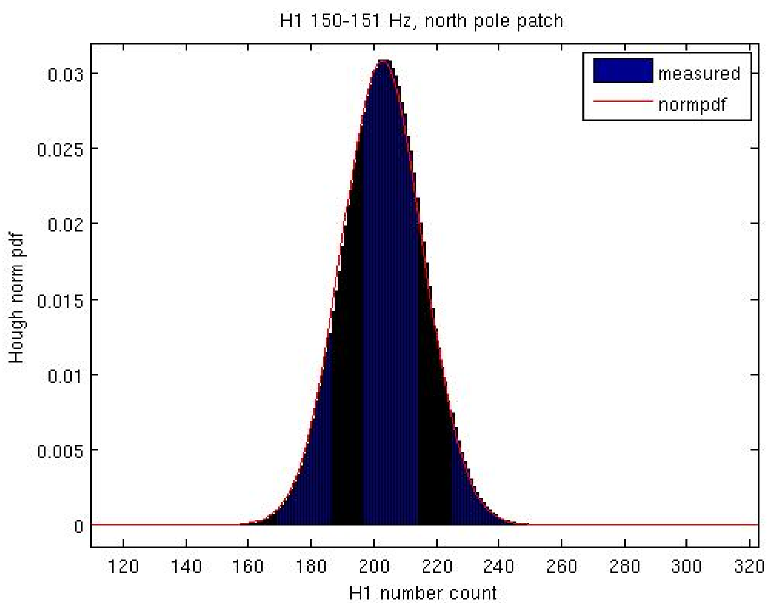


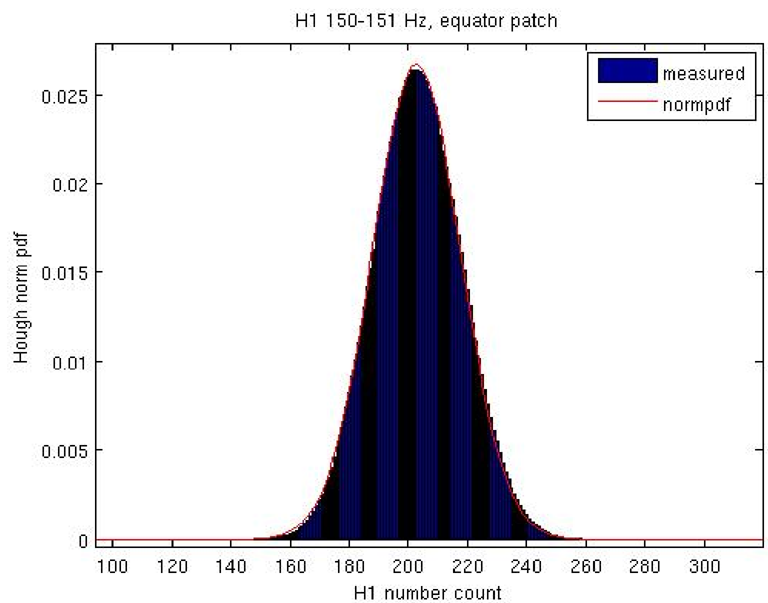
Figure plots the relative noise weights from H1, H2 and L1

$$\frac{\sum_{IFO} w_i}{\sum_{All} w_i}$$

Histogram of Hough number counts for the H1 detector

Histogram of the Hough number count compared to a Gaussian distribution for the H1 detector (1004 SFTs) in the frequency band 150-151 Hz.
Number of templates analyzed in each sky patch $\sim 11 \times 10^6$





$\langle n \rangle = 202.7$

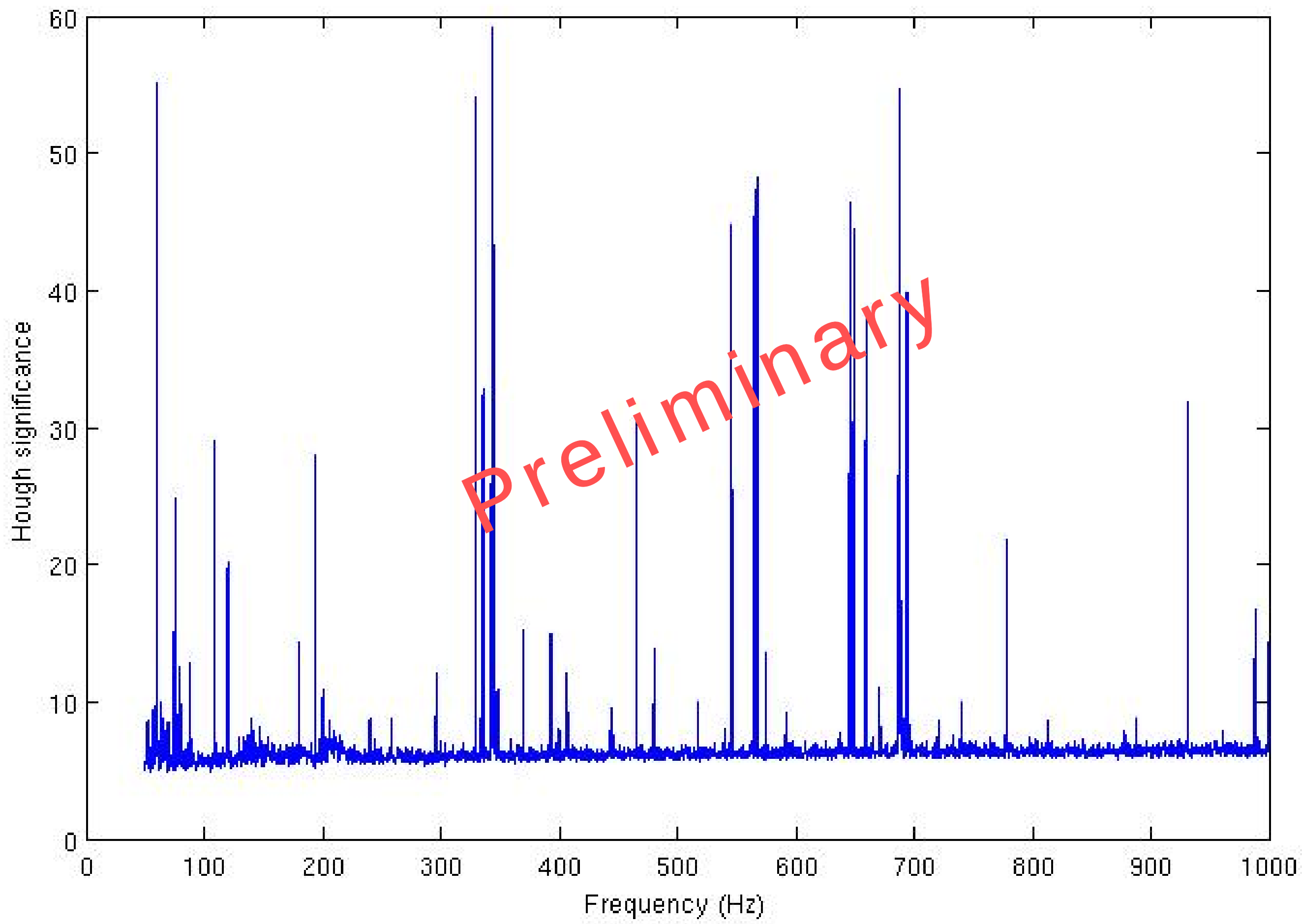
$\sigma = 12.94$ (obtained from the weights)

$\langle n \rangle = 202.7$

$\sigma = 14.96$

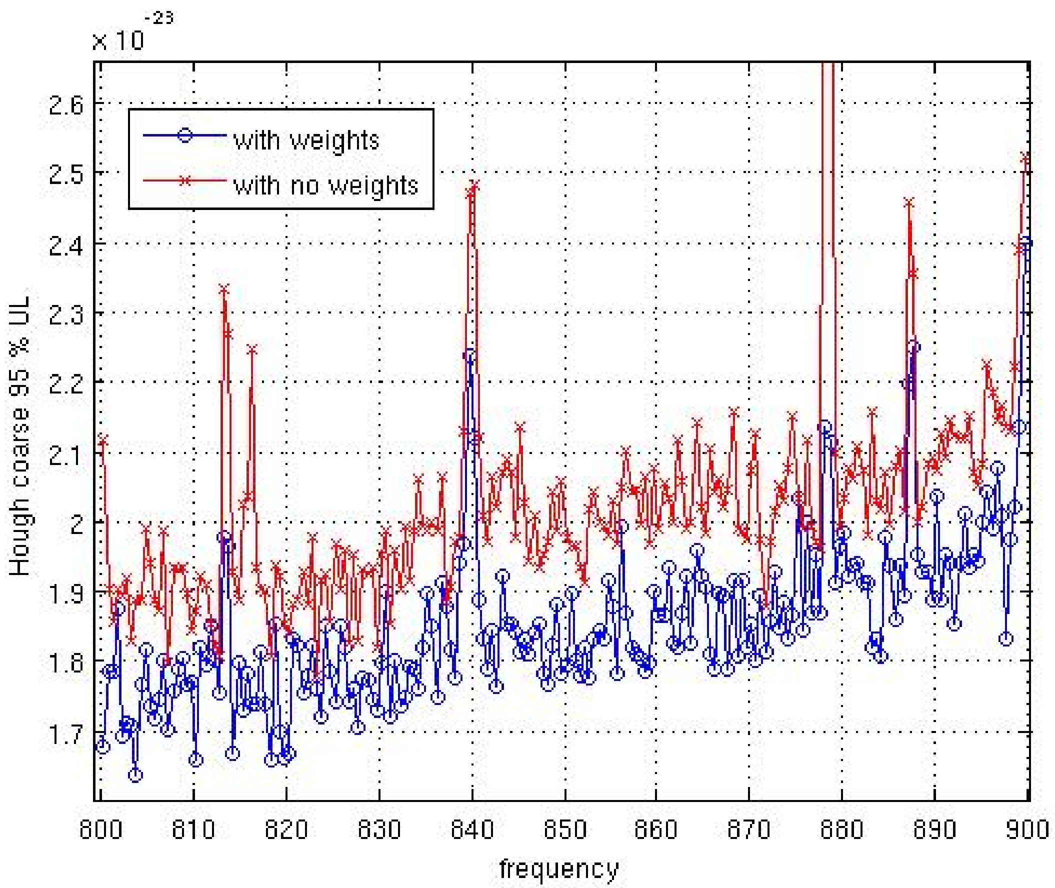
Loudest events for every 0.25 Hz

Multi interferometer case 50-1000 Hz



Significance defined as $s=(n_{max}-\langle n\rangle)/\sigma$

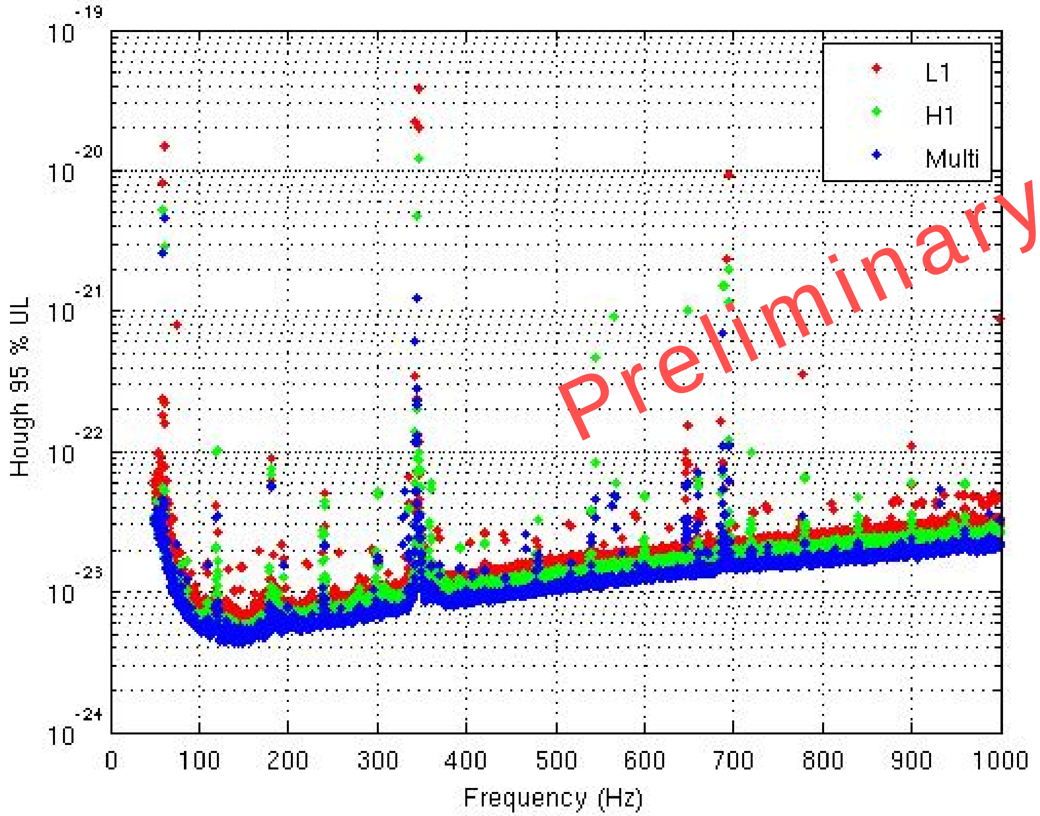
Improvements due to the weights



Comparison of the All-sky 95% upper limits obtained by Monte-Carlo injections for the multi-IFO case.

The average improvement by using weights in this band is 9.25% for the multi-IFO case, but only ~6% for the single IFO

Preliminary Upper Limits



Best UL

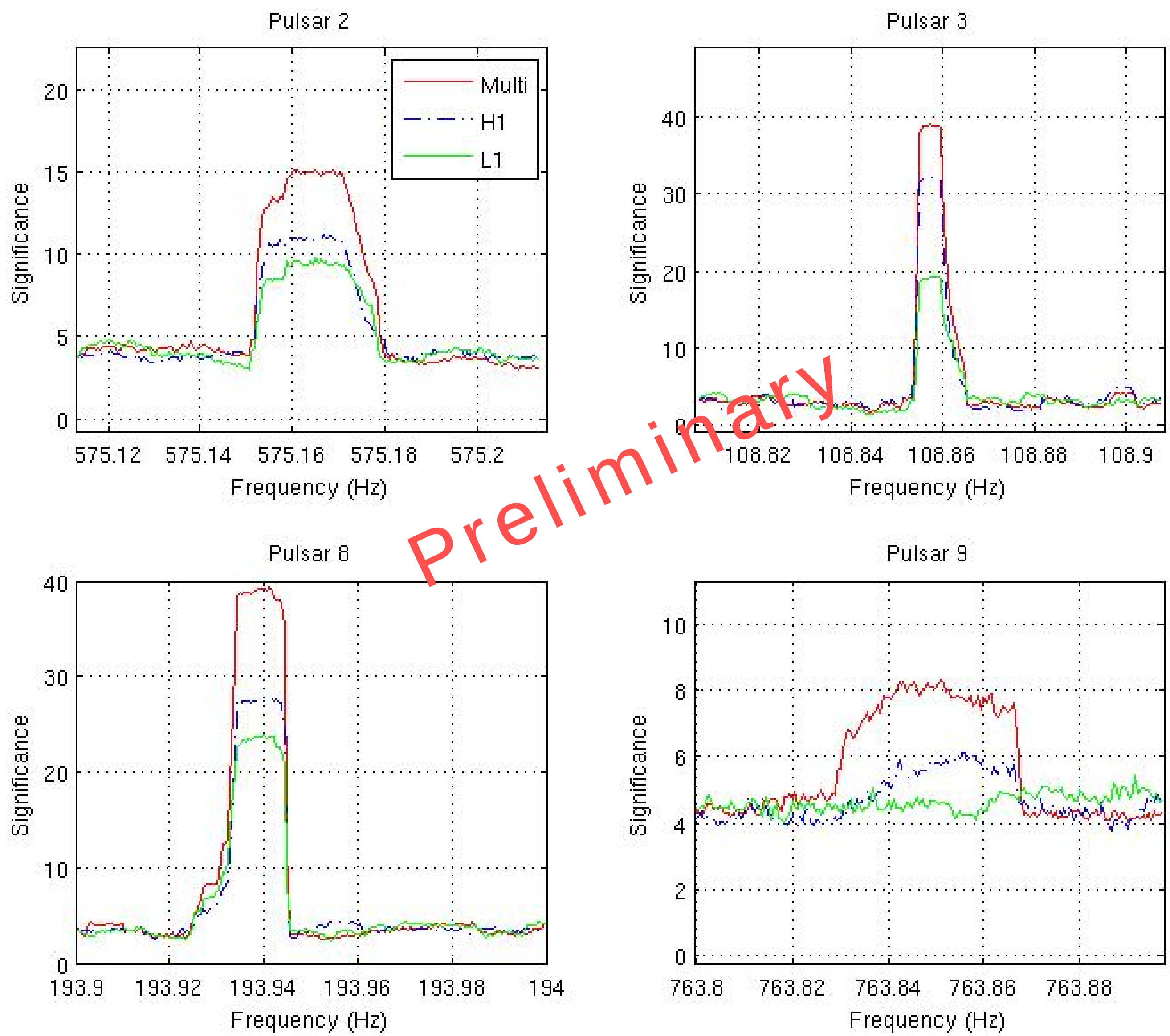
for L1: 5.9×10^{-24}
for H1: 5.0×10^{-24}
for Multi: 4.3×10^{-24}

It turns out that UL can be fitted by

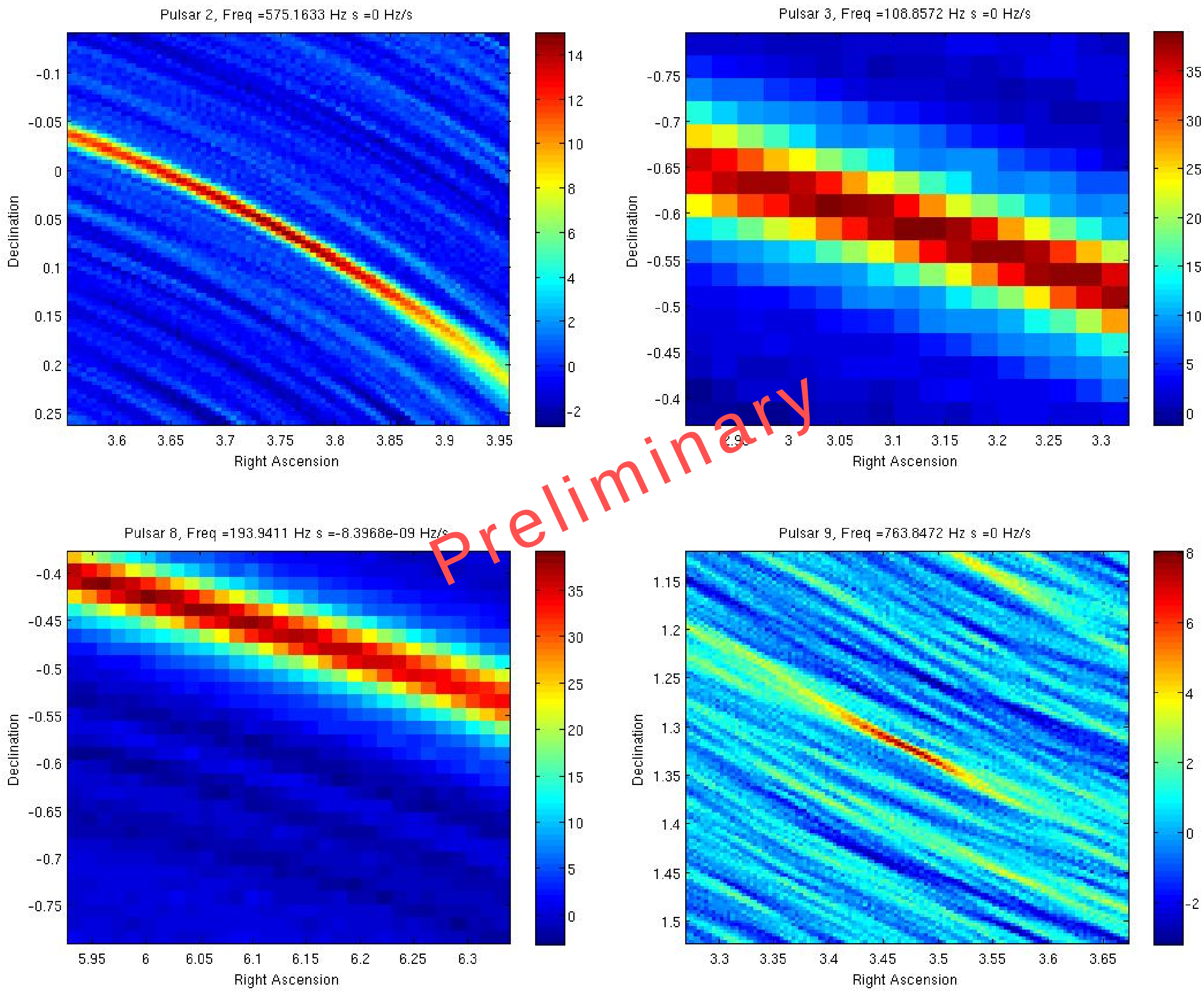
$$h_0^{95\%} = \frac{C}{\left(\sum_{i=1}^{N}\frac{1}{\left(S_n^{(i)}\right)^2}\right)^{1/4}}\sqrt{\frac{S^{1/2}}{T_{coh}}}, \qquad S = \frac{\max(significance)}{\sqrt{2}} + \mathrm{erfc}^{-1}(0.1)$$

with C=11.0± 0.5

Analysis of Hardware Injections



Analysis of Hardware Injections



Conclusions & Future work

- Results are interesting, but much better results are on the way!
- Improvements for S5:
  - S5: ~2x better sensitivity, 12x or more data
  - Increase the time of the coherent step
  - Hough on F-statistic segments from multiple IFOs
  - Ongoing development of Hierarchical pipeline that combines coherent and semi-coherent searches