Detection and reconstruction of burst signals with networks of gravitational wave detectors

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LIGO Scientific Collaboration

Outline

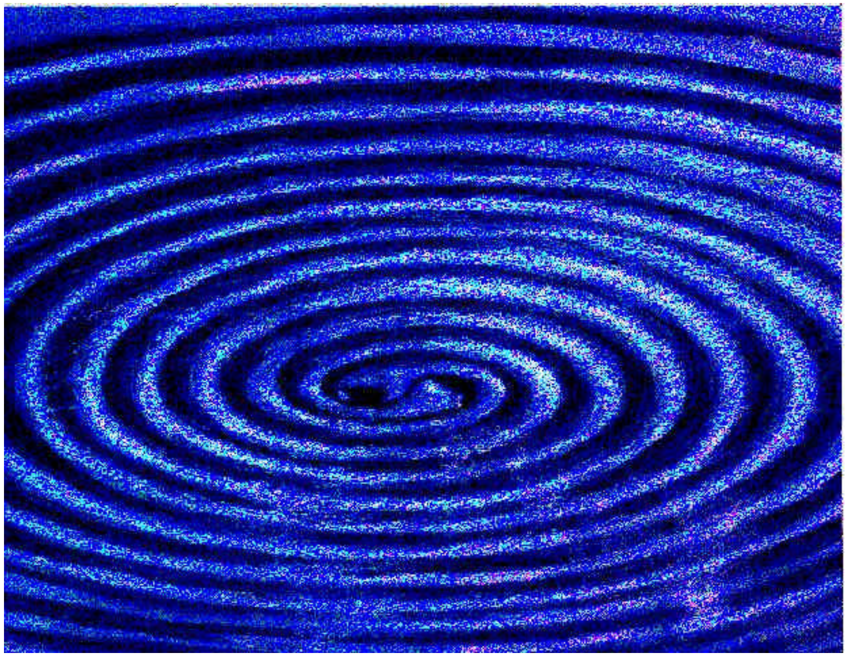
- Gravitational Waves
  - bursts
- Gravitational wave detectors
  - Detector response
  - Networks of GW detectors
- Detection of GW signals
  - Coincident methods
  - Coherent methods
- Coherent network analysis
  - Likelihood analysis
  - Constraint likelihood
- Reconstruction of GW signals
- Consistency tests for burst events
- Summary

gravitational waves

- time dependent gravitational fields come from the acceleration of masses and propagate away from their sources as a space-time warpage at the speed of light

- In the weak-field limit, linearize the equation in “transverse-traceless gauge”

$$\nabla^2 h - \frac{\partial^2 h}{c^2 \partial t^2} = 16\pi \frac{G_N}{c^4} T$$



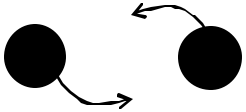
gravitational radiation
binary inspiral of compact objects

where $h_{\mu\nu}$ is a small perturbation of the space-time metric

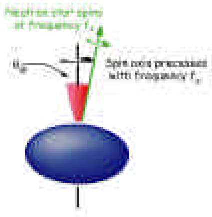
$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Gravitational waves

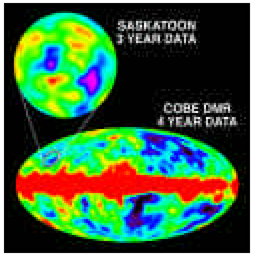
- **Perturbation of space-time metric predicted by GR**



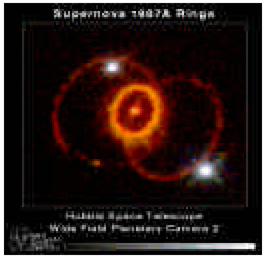
- **Compact binary inspiral: “chirps”**
  - **neutron stars / black holes**



- **Pulsars in our galaxy: “periodic”**
  - **GW from observed neutron stars**



- **Cosmological/astrophysical signals: “stochastic”**
  - **Early universe (like CMBR) or unresolved sources**



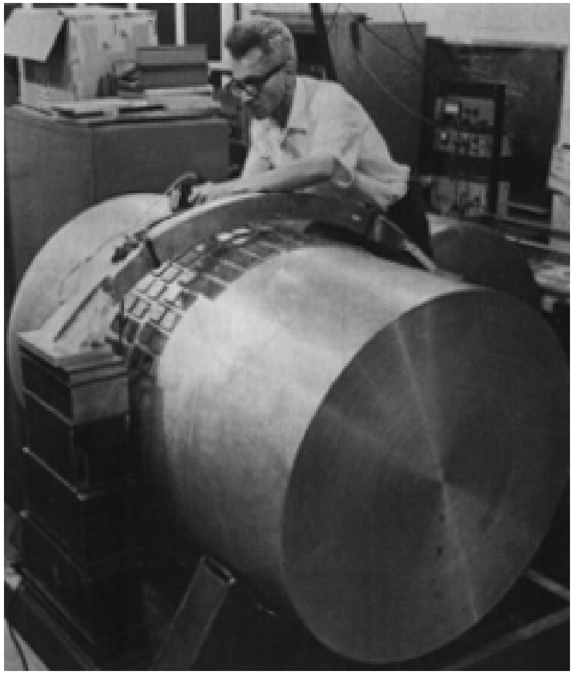
- **Supernovae / GRBs/ BH mergers/…: “bursts”**
  - **triggered – coincidence with GRB/neutrino detectors**
  - **un-triggered – coincidence of GW detectors**

Detectors

Bars

narrowband (~1Hz)

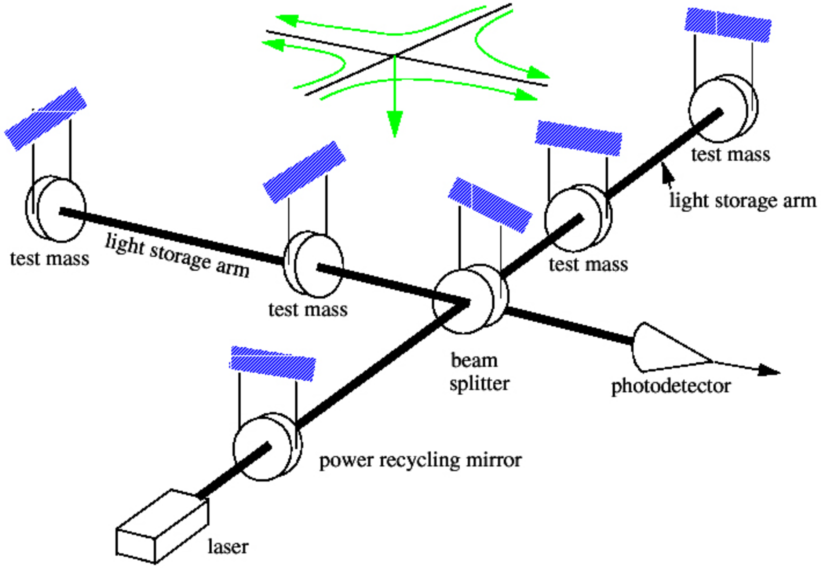
recent improvements (~10Hz)



ALLEGRO, AURIGA, EXPLORER, NAUTILUS, NIOBE, …

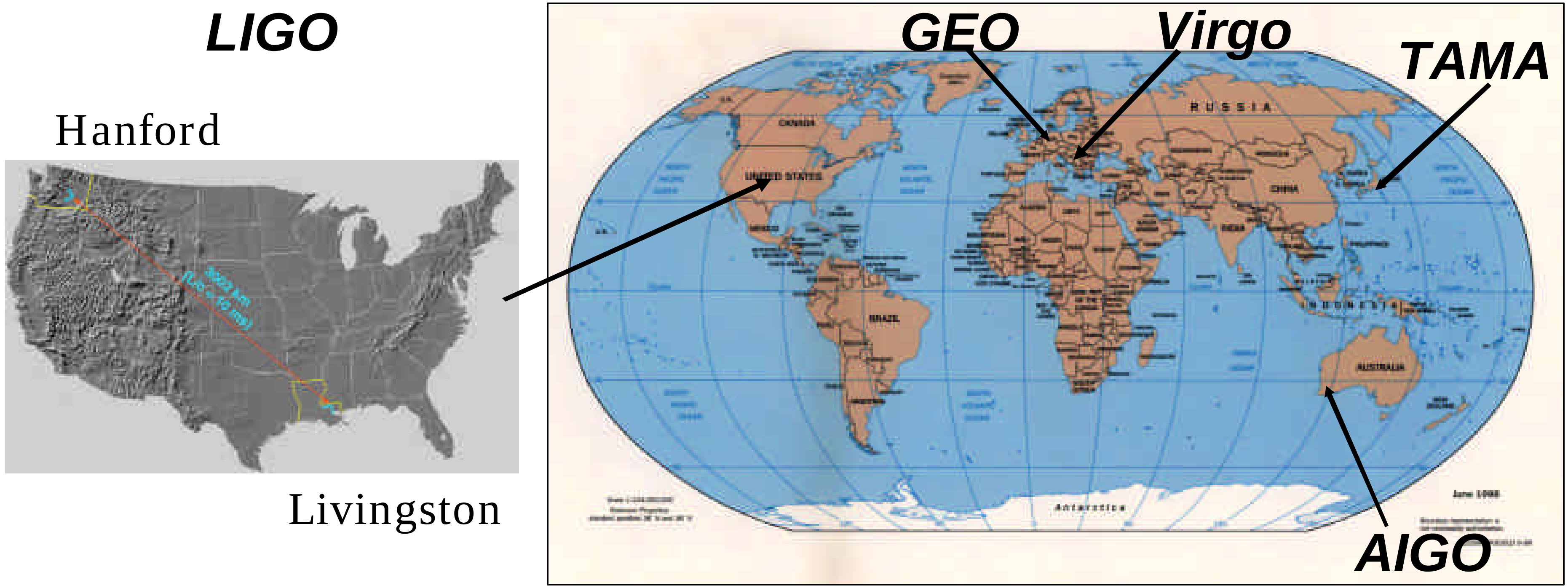
Interferometers

wideband (~10000 Hz)



LIGO, VIRGO, GEO, TAMA, AIGO, …

Network of GW detectors



LIGO

Hanford

Livingston

GEO

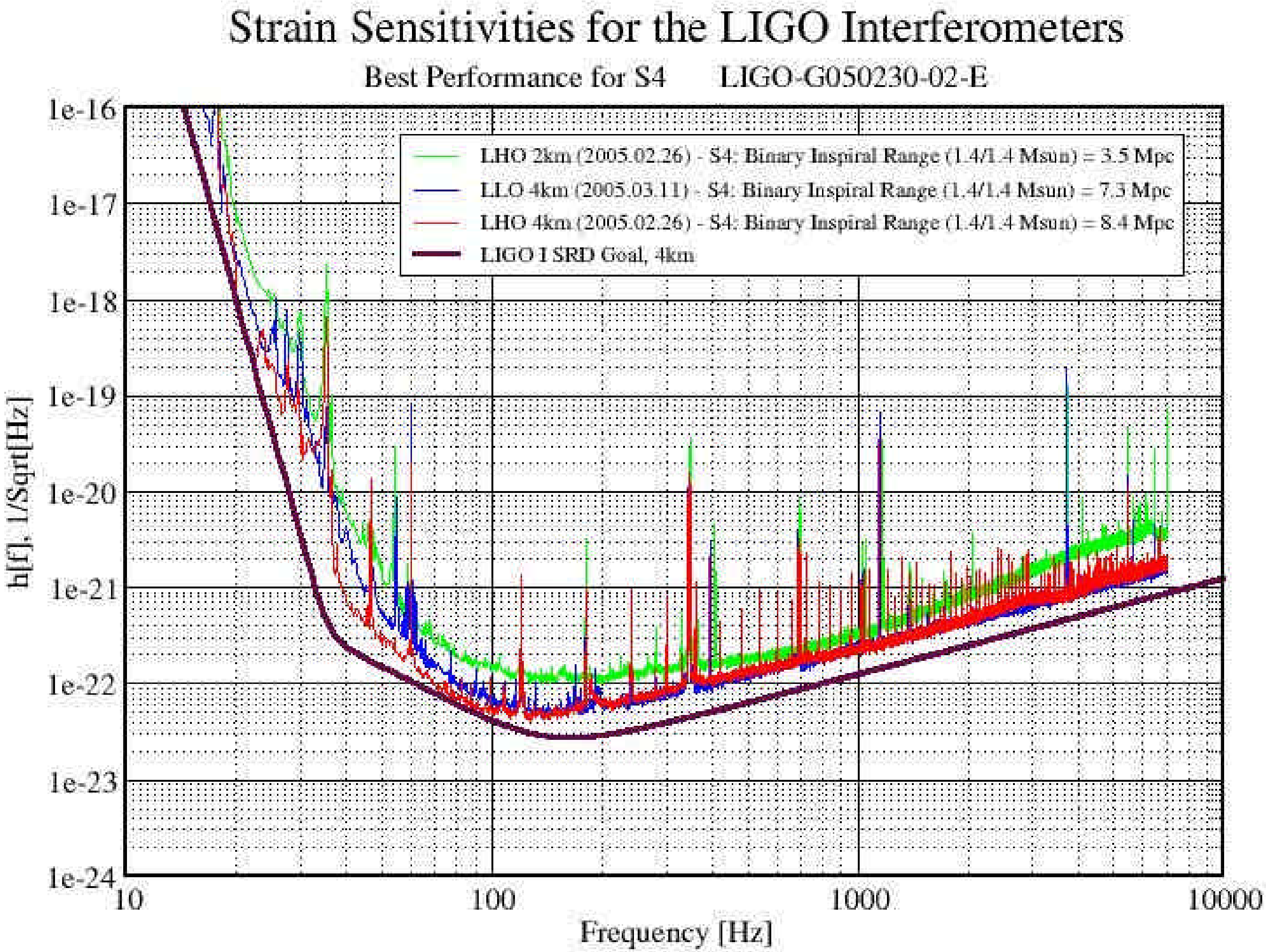
Virgo

TAMA

AIGO

- **Detection confidence – unlike instrumental/environmental artifacts, GW signal are coincident in the detectors**
- **Reconstruction of GW waveforms and direction to the source, which is not possible with a single GW detector.**
- **How to combine individual measurements?**

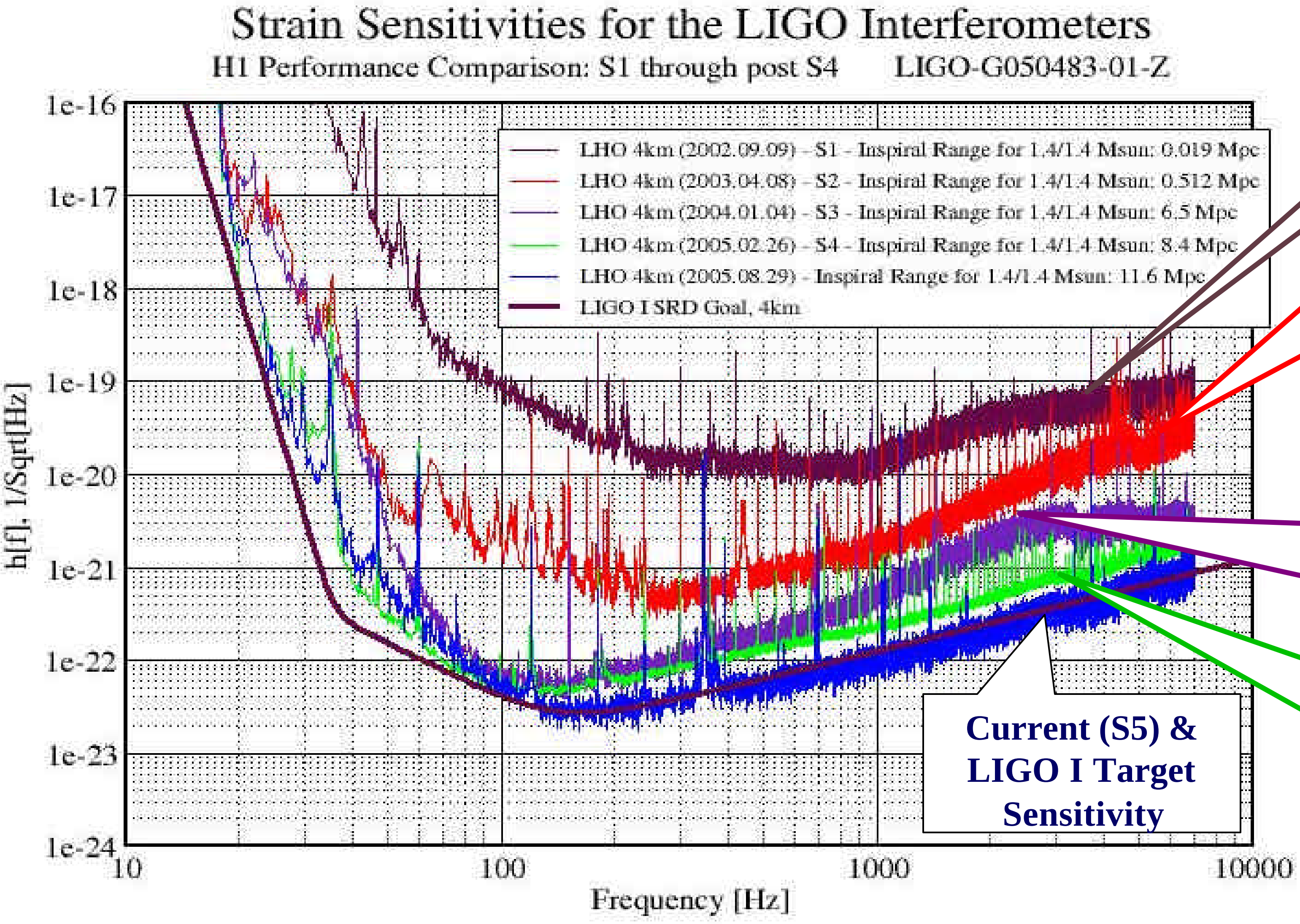
LIGO Sensitivity



LIGO achieved design sensitivity in S5 run which is currently in progress

LIGO runs

Science Runs



Current (S5) & LIGO I Target Sensitivity

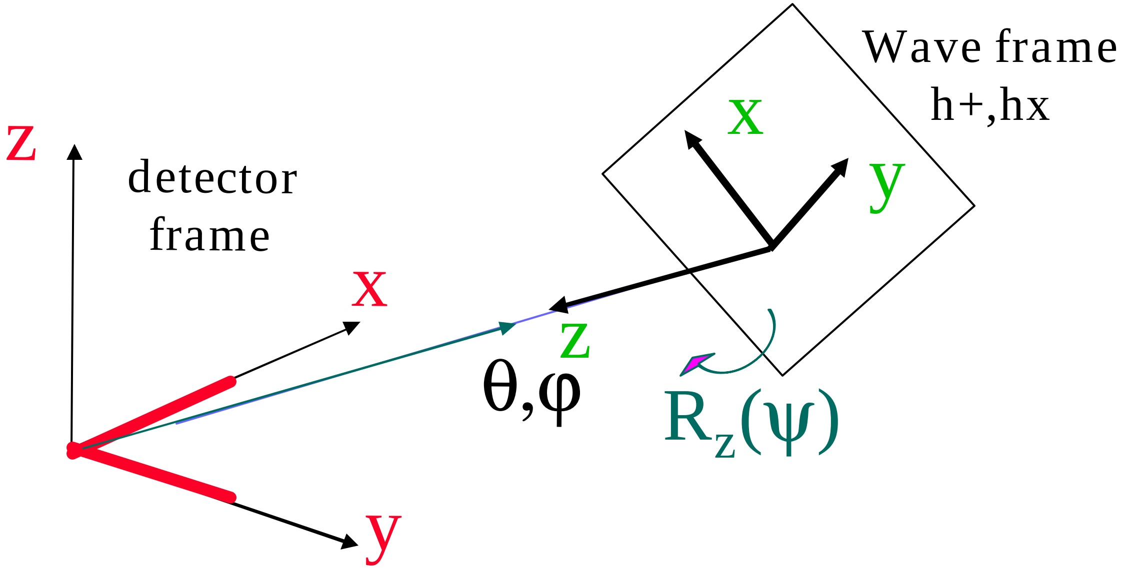
S1 run
Sep 02
(17 days)
(3x 96h)

S2 run
Feb 03
Apr 03
(59 days)
3x 318h

S3 run
Nov 03
Jan 04
(70 days)
3x 265h

S4 run
Feb 05
Mar 05
(30 days)
3x 403h

GW signal and detector response



- **Direction to the source θ,φ and polarization angle Ψ define relative orientation of the detector and wave frames.**
- **two GW polarizations:** $\vec{h} = \left(h_+(t), h_\times(t)\right)$
- **Antenna patterns:** $\vec{F} = \left(F_+(q,j), F_\times(q,j)\right)$
- **Detector response:** $x = F_+ h_+ + F_\times h_\times = \vec{F} \cdot \vec{h}$

Detector response

- **complex GW waveform**

$$u(t) = h_{+}(t) + ih_{x}(t)$$

- **Antenna pattern (assume polarization angle Y=0)**

$$A(q,j) = \frac{1}{2}\left[F_{+}(q,j) + iF_{x}(q,j)\right]$$

- **Detector response (~ - complex conjugate)**

$$x = u\tilde{A} + \tilde{u}A$$

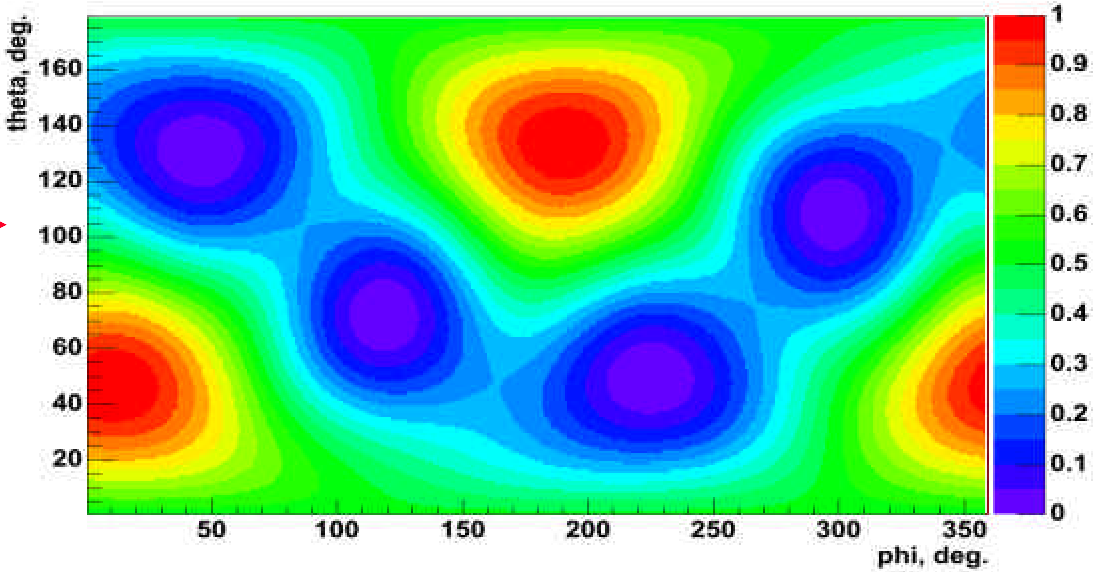
- **Observable parameters are $R_Z(Y)$ invariant**

$$u \rightarrow ue^{i2\Psi} \qquad A \rightarrow Ae^{i2\Psi}$$

Detector Antenna Patterns

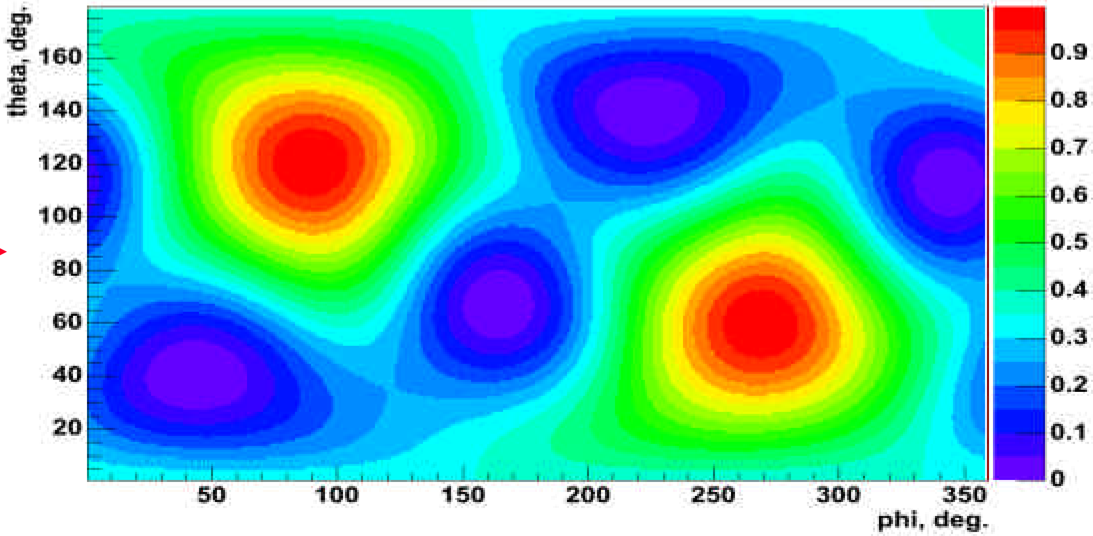
$$A = \frac{1}{2}\left(F_+ + iF_x\right)$$

- $|A|^2$ **for L1** →



- **Several misaligned detectors increase coverage of the sky**

- $|A|^2$ **for Virgo** →



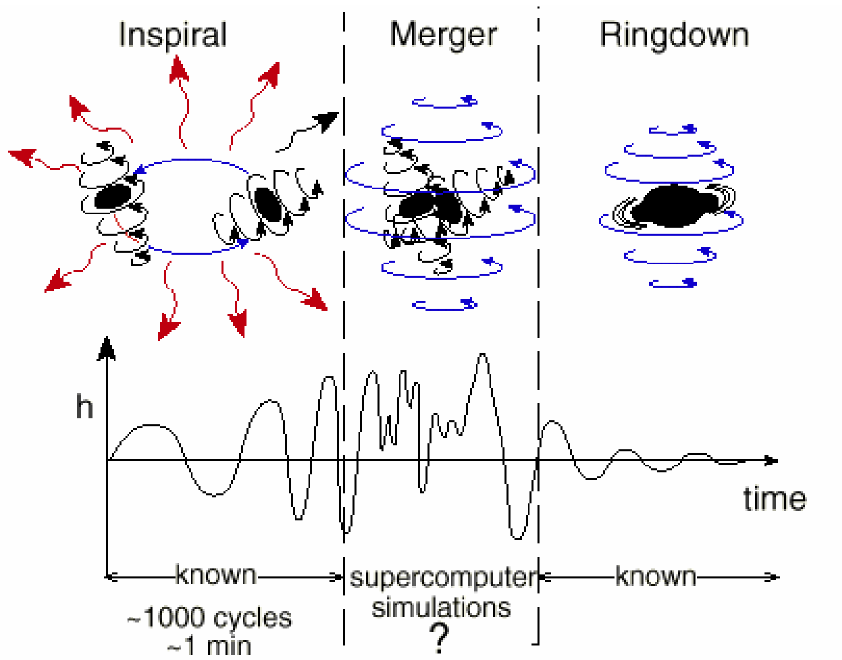
Gravitational Waves from Bursts Sources

- **Any short transient of gravitational radiation (< few sec).**
- **Astrophysically motivated**
  - **Un-modeled signals -- Gamma Ray Bursts, …**
  - **Poorly modeled -- supernova, inspiral mergers,..**
  - **Modeled – cosmic string cusps**
- **In most cases matched filters will not work**
- **Characterize un-modeled bursts by**
  - **characteristic frequency *fc***
  - **duration (dt) & bandwidth (df) & TF volume (dt X df)**
  - **strain amplitude h_{rss}**

$$h^2{}_{rss} = \int_{-\infty}^{+\infty} \left[h_+^2(t) + h_\times^2(t) \right] dt$$

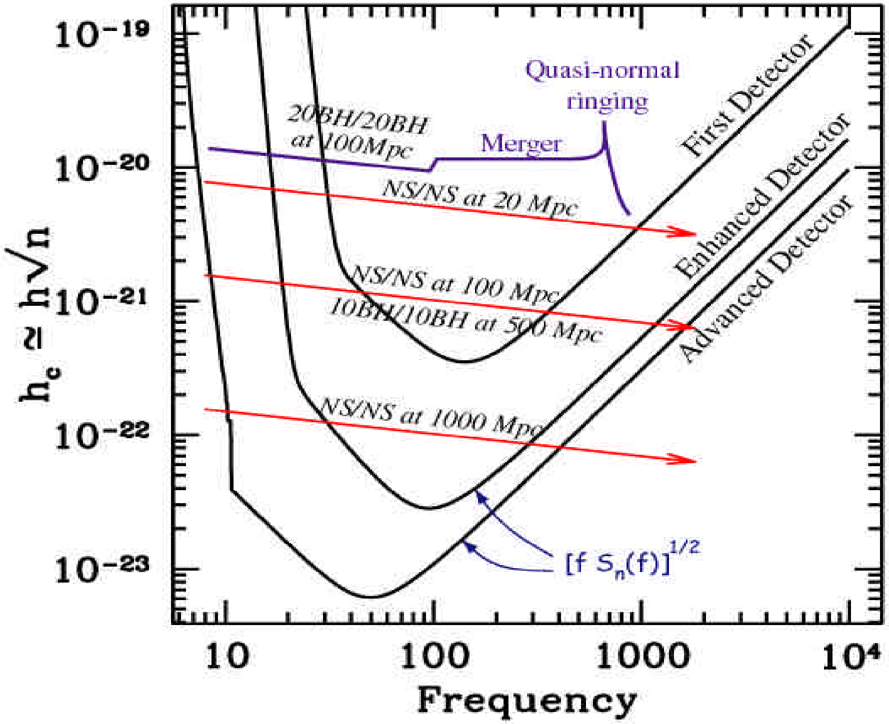
Inspiral Mergers

Compact binary mergers



Sensitivity of LIGO to coalescing binaries

K.Thorne



- **massive BH-BH objects can be detected via merger and ring-down**
- **One of the most promising source to be detected with LIGO**
- **Recent progress in NR (see C.Lousto's talk) will help to extract information about BH-BH dynamic when mergers are detected.**

Objectives of Burst Analysis

- In most cases matched filters do not work
  - need robust model independent detection algorithms
- Combine measurements from several detectors
  - handle arbitrary number of co-aligned and misaligned detectors
  - confident detection, elimination of instrumental/environmental artifacts
  - reconstruction of source coordinates
  - reconstruction of GW waveforms
- Detection methods should account for
  - variability of the detector responses as function of source coordinates
  - differences in the strain sensitivity of detectors
- Extraction of source parameters
  - confront measured waveforms with source models

Coincidence methods

- Apply (usually excess power) filter to a single detector stream and record instances of time (triggers) when the data is inconsistent with the noise model
  - ➢LSC: ExceessPower, WaveBurst, Q-transform, BlockNormal, KleineWelle, …
  - ➢Virgo: PowerFilter, ALF, EGC,…
- Reduce FA rate by coincidence of triggers in some time window

$$R_{coincidence} = \Delta T \times R_1 \times R_2$$

- Coincidence methods are successfully used in LIGO burst searches, very convenient tool for detector studies, however
  - ➢ sensitivity may be limited by least sensitive detector
  - ➢ do not reconstruct waveforms and source coordinates
  - ➢ depend on selection of a coincidence window DT
  - ➢ do not test “common origin” of waveforms in different detectors

Consistency Test of coincident events

- **Are triggers detected in different detectors consistent?**
- **Pearson's correlation between two detector data streams: r-statistic,** **Cadonati, CQG 22 S1159 (2005)**
  - **can test a consistency of waveforms in the detectors, works for co-aligned or closely aligned detectors**
  - **effective tool for FA reduction, successfully used in LIGO burst searches**
- **Null stream: Schutz et al, CQG 22 S1321 (2005)**
  - **construct linear combination of data streams where GW signal is cancelled out. Reject triggers if residual is not consistent with the noise**
  - **most straightforward is a null stream for co-aligned detectors: P.Ajith et al, CQG 23 S741-S749 (2006)**

$$N(t)=x_1(t)-x_2(t+t\ )$$

- **Both methods can significantly reduce false alarm, but they mainly work for co-aligned detectors and do not address the GW reconstruction.**

LIGO burst searches

- **use WaveBurst algorithm**

  (Klimenko et al, CQG 21, S181 (2004))

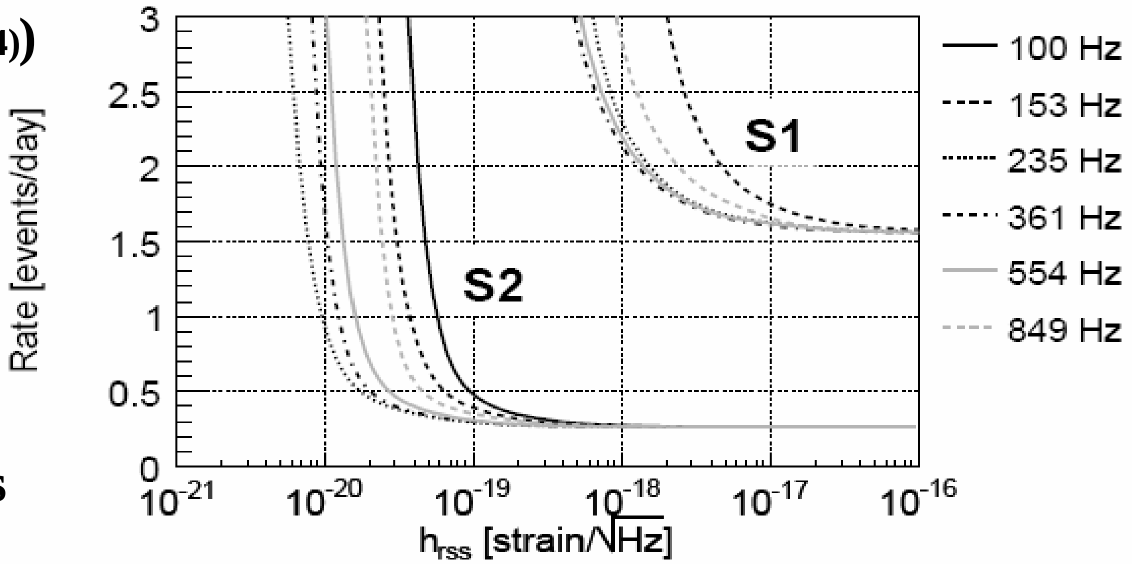
  to generate triggers reconstructed in wavelet (time-frequency) domain

- **use CorrPower algorithm**

  (Cadonati et al, CQG 21, S181 (2004))

  for consistency test of triggers

Abbot et al, PRD **69**, 102001 (2004)
Abbot et al, PRD **72**, 062001 (2005)



- **Set rate vs strength upper limit on generic GW bursts**
- **S2: set limit on rate <0.26 events/day at 90% conf. level**
- **S4: significant improvement in sensitivity (x10), to be published soon**
- **S5: significant increase of life time (x10), analysis in progress**

Coherent network analysis

Combine data, not triggers; solve inverse problem of GW detection

- **Guersel,Tinto, PRD 40 v12,1989**
  - **reconstruction of GW signal for a network of three misaligned detectors**
- **Likelihood analysis: Flanagan, Hughes, PRD57 4577 (1998)**
  - **likelihood analysis for a network of misaligned detectors**
- **Two detector paradox: Mohanty et al, CQG 21 S1831 (2004)**
  - **state a problem within likelihood analysis**
- **Constraint likelihood: Klimenko et al, PRD 72, 122002 (2005)**
  - **address problem of ill-conditioned network response matrix**
  - **first introduction of likelihood constraints/regulators**
- **Penalized likelihood: Mohanty et al, CQG 23 4799 (2006).**
  - **likelihood regulator based on signal variability**
- **Maximum entropy: Summerscales at al, to be published**
  - **likelihood regulator based on maximum entropy**
- **Rank deficiency of network matrix: Rakhmanov, CQG 23 S673 (2006)**
  - **likelihood based in Tickhonov regularization**
- **GW signal consistency: Chatterji et al, PRD 74 082005(2006)**
  - **address problem of discrimination of instrumental/environmental bursts**

Likelihood Analysis for Bursts

Flanagan, Hughes, PRD57 4577 (1998)

$$\Lambda(x|u) = \max_u \left(\frac{P(x|u)}{P(x|0)} \right) \qquad L(x|u) = -\ln[\Lambda(x|u)]$$

- **For Gaussian noise with variance s ^{2} and templates u(Q), where Q is a parameter set. *x_k* – detector outputs. x_k – detector response**

$$L(x|\Theta) = \sum_i \sum_k \frac{1}{2s_k^2} \left[x_k^2[i] - \left(x_k[i] - x_k[i,\Theta]\right)^2 \right]$$

- **For unknown GW signal treat every sample of *u(h+,hx)[i]* as an independent variable→ find solution from variation of *L***

$$L(x|\Theta) \Rightarrow L(x|u)$$

- **"Template search" in the limit of a large number of parameters**

Variation of the likelihood functional

- **Likelihood functional (time index *i* is omitted)** $u[i] = h_+[i] + ih_x[i]$

$$L(x|u) = \sum_i \left(u\tilde{X} + \tilde{u}X\right) - \frac{1}{2}\sum_i \left(2u\tilde{u}p + u^2\tilde{q} + \tilde{u}^2 q\right)$$

- **Network data vector** $X[i] = \sum_k \frac{x_k[i]A_k}{s_k^2}$

k – detector index

- **Network antenna patterns**

$$p = \sum_k \frac{A_k \tilde{A}_k}{s_k^2}, \quad q = \sum_k \frac{A_k^2}{s_k^2},$$

- **find solutions for *u* by *variation of L(x|u)***

$$\frac{dL}{du} = 0, \ \frac{dL}{d\tilde{u}} = 0 \rightarrow \begin{array}{l} X = pu + q\tilde{u} \\ \tilde{X} = p\tilde{u} + \tilde{q}u \end{array} \longrightarrow u_o[i] = \frac{pX[i] - q\tilde{X}[i]}{p^2 - q\tilde{q}}$$

Maximum Likelihood Ratio

- **Replace *u* in *L*(*x* | *u*) with the solution u_o**

$$L_{MLR} = \frac{1}{2}\sum_i \left(u_o \tilde{X} + \tilde{u}_o X \right) \approx \sum_k \frac{1}{2s_k^2} \left\langle x_k^2 \right\rangle$$

  - **L_{MLR} is a projection of u_o on *X***
  - **$2L_{MLR}$ is the *network* SNR**

- **Coherent/incoherent energy**

$$2L = \sum_{i,j} \left\langle x_i(t) x_j(t + t_{ij}) \right\rangle C_{ij} = E_{i=j} + E_{i \neq j}$$

  - **$<x_i x_j>$ - inner product of data vectors x_i and x_j.**
  - **$t_{ij}(q,f)$ is a time delay between detectors *i&j***
  - **diagonal terms – power, off-diagonal terms – correlation**

- **Likelihood analysis is very elegant and consistent approach for burst detection and reconstruction, but..**

Two detector paradox

Mohanty et al, CQG 21 S1831 (2004)

- **for simplicity assume unit noise variance**
- **aligned detectors (identical detector responses** x **):**

$$L = \sum_i x[i]\left(x_1[i] + x_2[i] - x[i]\right) \;\Rightarrow\; x = \frac{x_1 + x_2}{2}$$

$$L_A = \tfrac{1}{4}\left[\langle x_1, x_1\rangle + \langle x_2, x_2\rangle + 2\langle x_1, x_2\rangle\right]$$

power

cross-correlation

  - **If separated → $\mathbf{L_A}$ has directional sensitivity (circle on the sky) because correlation term depends on** q **and** f**.**
- **misaligned detectors:**
  - **solution for GW waveform:** $x_1 = x_1, \quad x_2 = x_2$

$$L_M = \tfrac{1}{2}\left[\langle x_1, x_1\rangle + \langle x_2, x_2\rangle\right]$$

- **Likelihood method does not work for two misaligned detectors**

  No directional sensitivity even if detectors are infinitesimally misaligned!

Network response matrix

- **Solution for GW waveforms satisfies the equations:**

$$X = pu + q\tilde{u}$$

$$\Big\downarrow$$

Klimenko et al, PRD 72, 122002 (2005)

$$\begin{bmatrix} \mathrm{Re}(X) \\ \mathrm{Im}(X) \end{bmatrix} = \begin{bmatrix} p + \mathrm{Re}(q) & \mathrm{Im}(q) \\ \mathrm{Im}(q) & p - \mathrm{Re}(q) \end{bmatrix} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix} = \mathrm{M_R} \begin{bmatrix} h_+ \\ h_\times \end{bmatrix}$$

- **Network response matrix $\mathrm{M_R}$ takes diagonal form in the wave frame where Im(*q*)=0 (Dominant Polarization frame)**

$$M_R = \begin{bmatrix} p+|q| & 0 \\ 0 & p-|q| \end{bmatrix} = g \begin{bmatrix} 1 & 0 \\ 0 & e \end{bmatrix}$$

$$q = \sum_k \frac{A_k^2}{s_k^2}$$

  - ***g*** **– network sensitivity factor**
  - ***e*** **– network alignment factor**

- **Network has ill-conditioned matrix if *e*<<1**

Detection of two GW components

- **h_1 & h_2 - solutions for GW polarizations in the DP frame**
- **For aligned detectors *e* =0 for any** q **and** f
- **For misaligned detectors *e* can be <<1 for significant area in the sky**

- **total network SNR**

$$2L \approx 2g\left(\left\langle h_1^2\right\rangle + e\left\langle h_2^2\right\rangle\right) = SNR_{tot}$$

$\left\langle h_1^2\right\rangle, \left\langle h_2^2\right\rangle$ **-sum-square energies of GW components**

- **if *e*=0 only component h_1 can be measured**
- **Even for networks with several misaligned detectors the measurement of the second component not always possible**

Network alignment factor

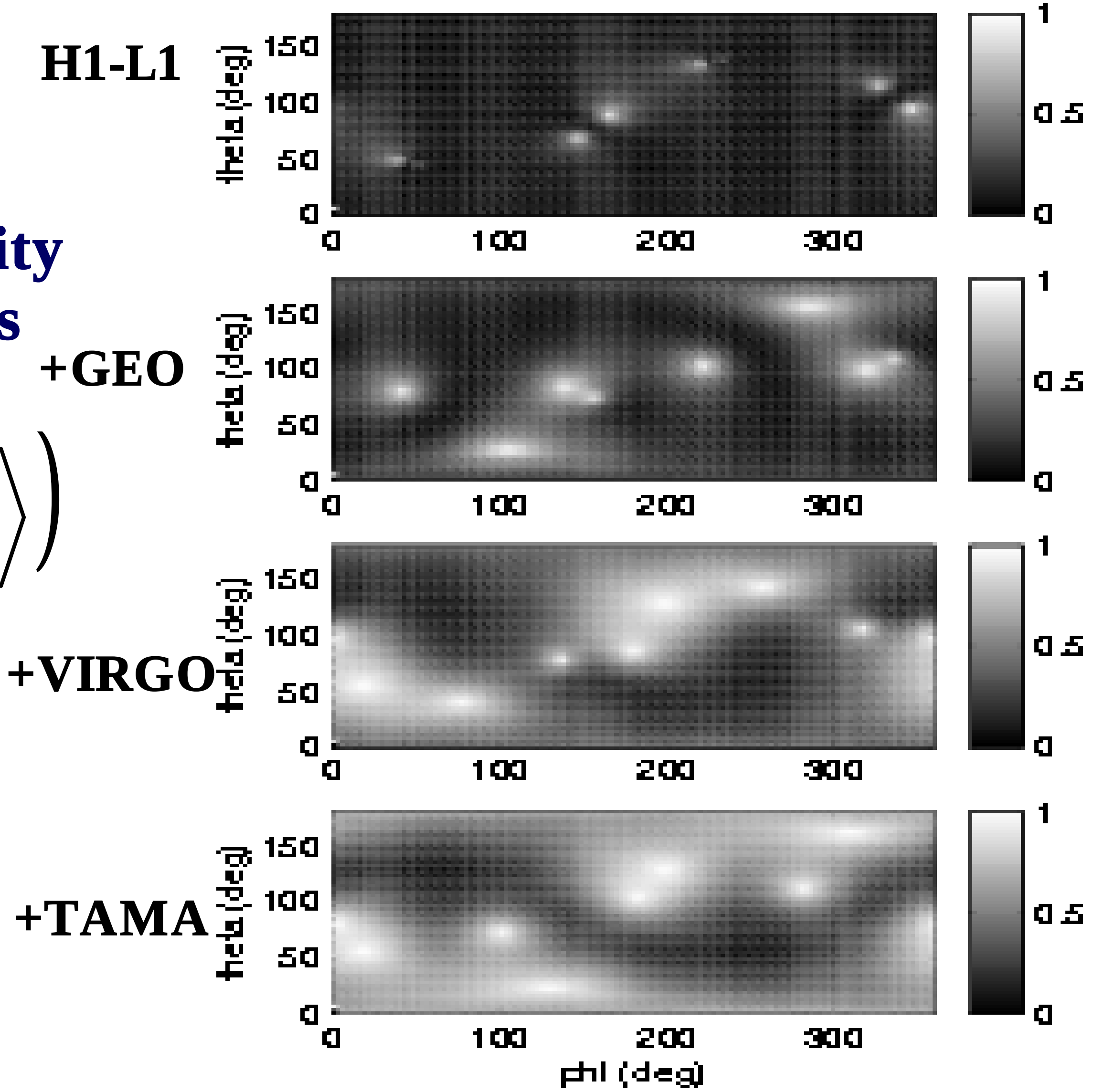
$$e = p - |q| \, / \, p + |q|$$

For aligned network e=0

e shows relative sensitivity to two GW components

$$L \propto \left(\left\langle h_1^2 \right\rangle + e \left\langle h_2^2 \right\rangle \right)$$

to be detected with the same SNR h_2 should be 1/e times stronger then h_1



Network sensitivity factor

$$g = p + |q|$$

$$SNR = 2g\left(\left\langle h_1^2 \right\rangle + e\left\langle h_2^2 \right\rangle\right)$$

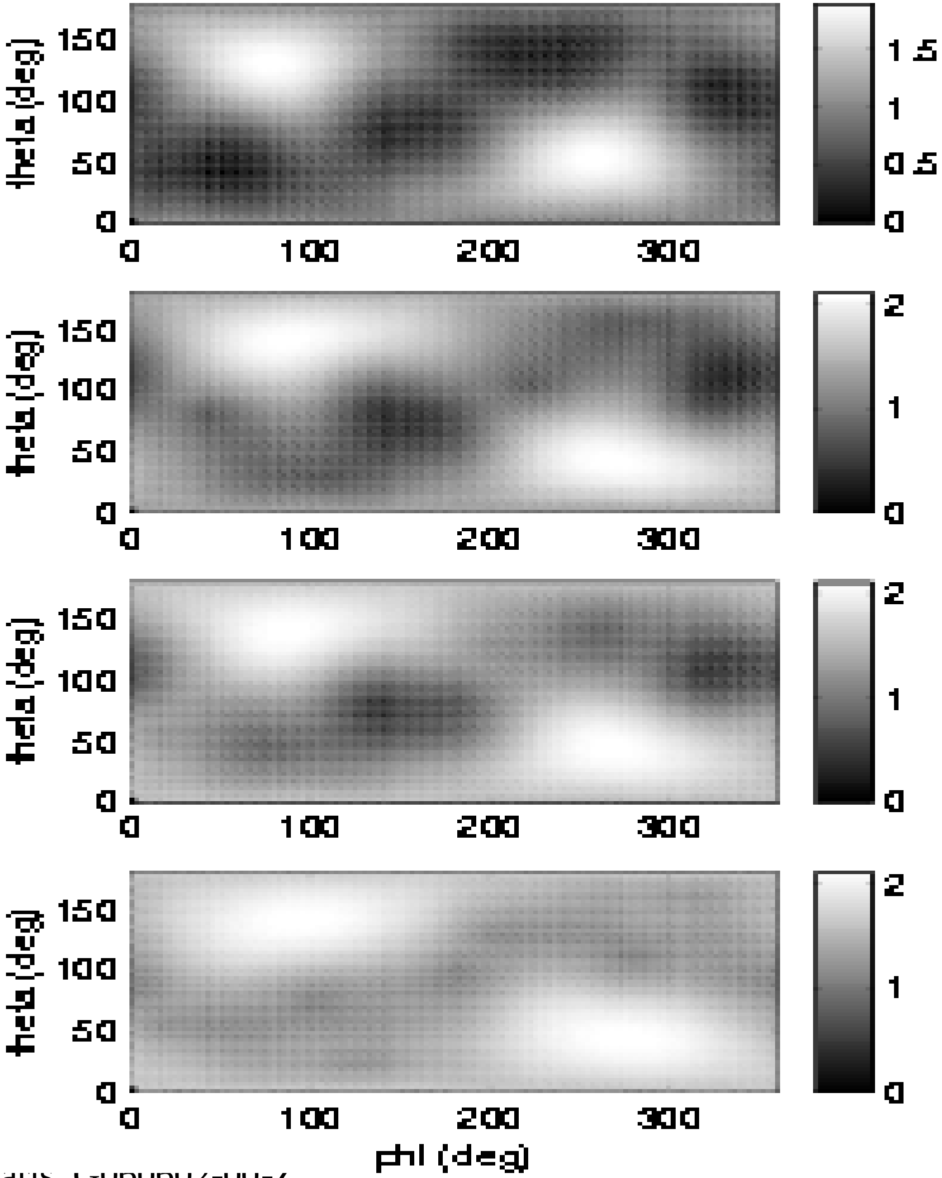
need several detectors for more uniform sky coverage

H1-L1

+GEO

+VIRGO

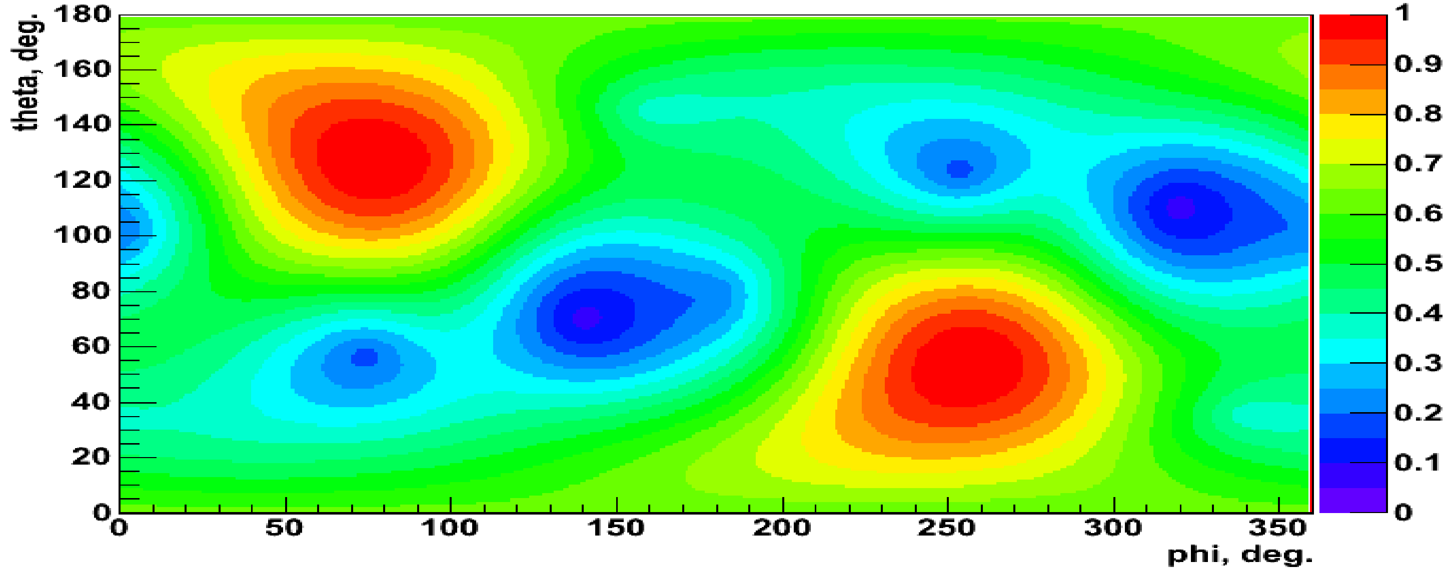
+TAMA



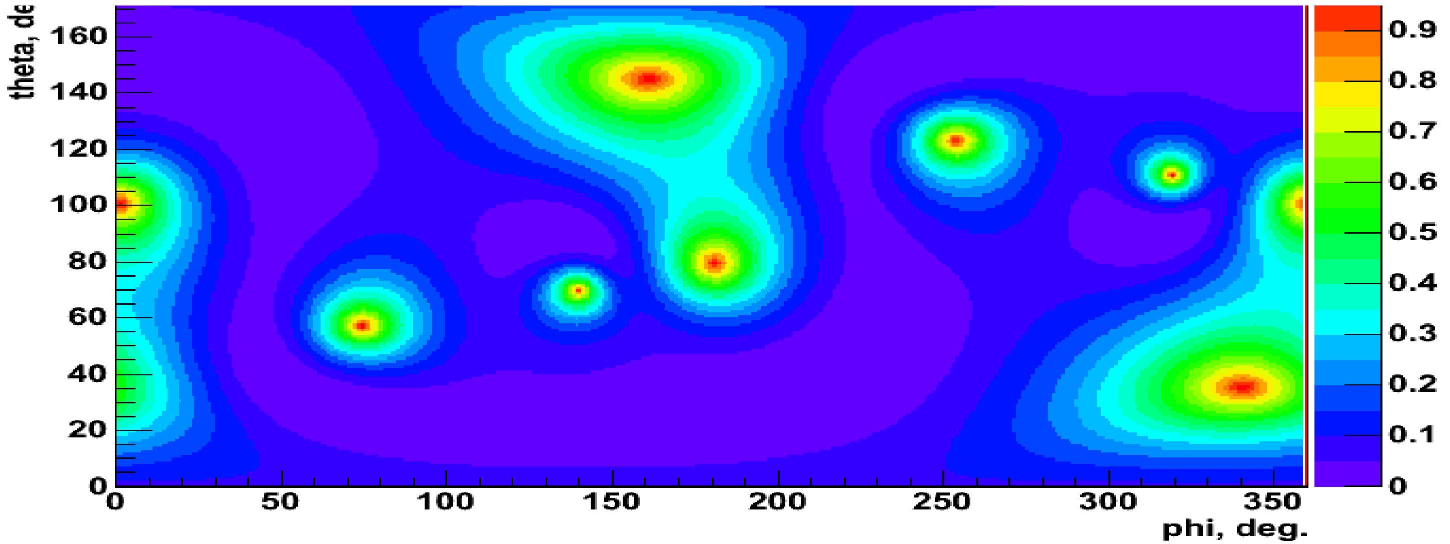
L1/H1/V1 network

- **Significant fraction of the sky has ill-conditioned network matrix**

g



e



Virtual Detectors

- **Any network can be described as two virtual detectors**

γ - phase of q

detector	output	noise variance	network SNR
VD_1	$\mathrm{Re}(X[\mathrm{t}]\mathrm{e}^{-\gamma/2})$	g	$g\langle h_1^2\rangle$
VD_2	$\mathrm{Im}(X[\mathrm{t}]\mathrm{e}^{-\gamma/2})$	eg	$eg\langle h_2^2\rangle$

- **In many cases only VD_1 is available for measurements. VD_2 does not contribute much if** e**<<1:**

$$eg\overline{\langle h_1^2\rangle} << g\overline{\langle h_1^2\rangle}$$

$\overline{<>}$ - average over source population

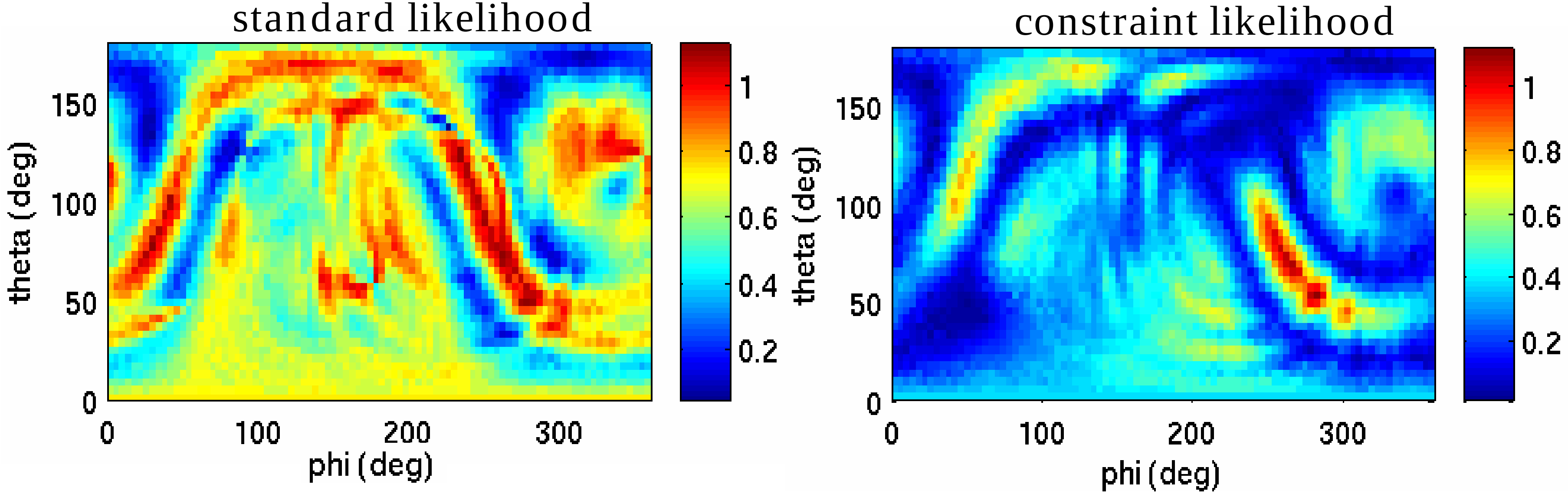
- **Solution: put constraints on measurement of the h_2 waveform.**
  - **remove un-physical solutions produced by noise**
  - **may sacrifice small fraction of GW signals but**
  - **enhance detection efficiency for the rest of sources**

Likelihood Constraints/Regulators

- **regulators – source-model independent constraints**
- **Soft regulator - weight the second component according to the network alignment factor (PRD 72, 122002 (2005))**

$$L_{soft} = L_1(h_1) + e\ L_2(h_2)$$

Simulated BH-BH merger (Lazarus) in L1/H1/G1 network

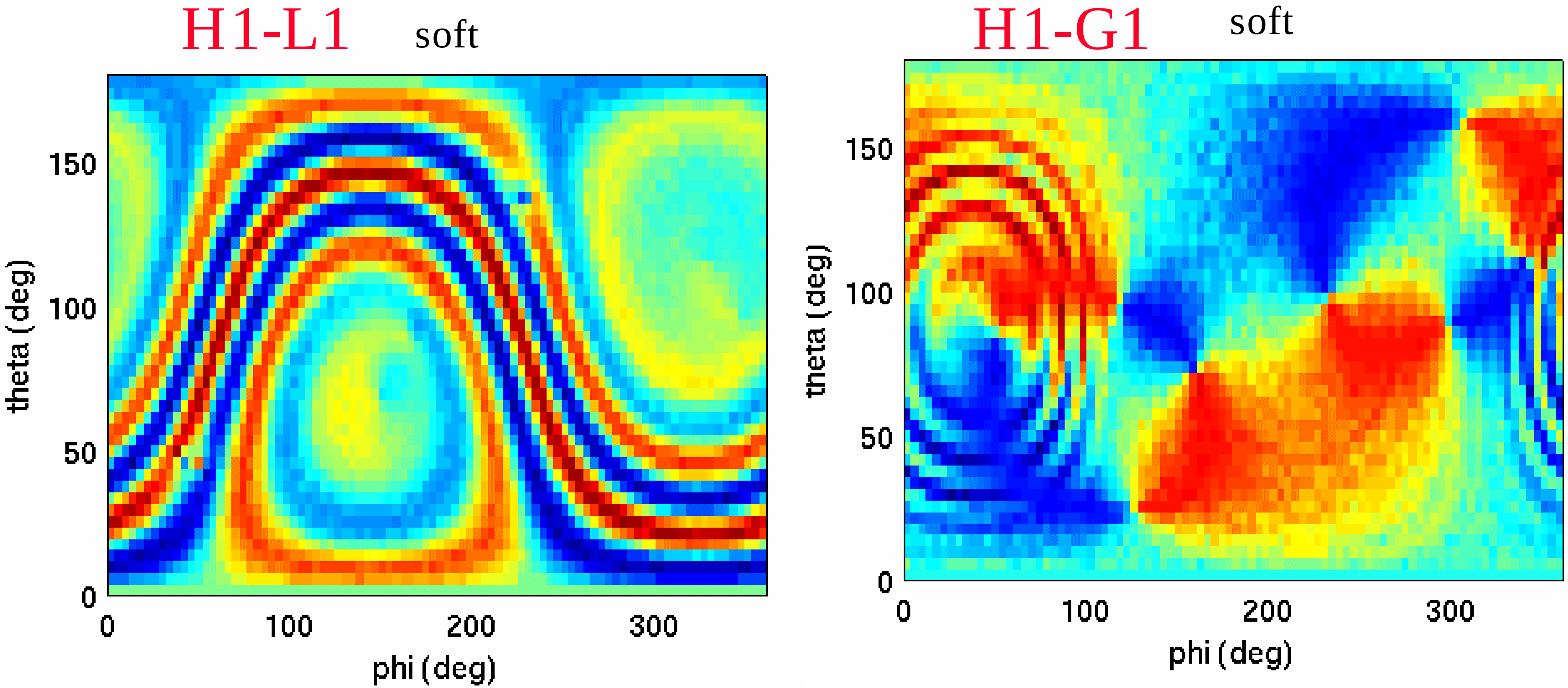

standard likelihood

constraint likelihood

- **various constraints are possible: hard, soft, entropy, Tickhonov regulators, etc..**

Two detector sky maps

Constrained likelihood gives directional information in case of two detectors

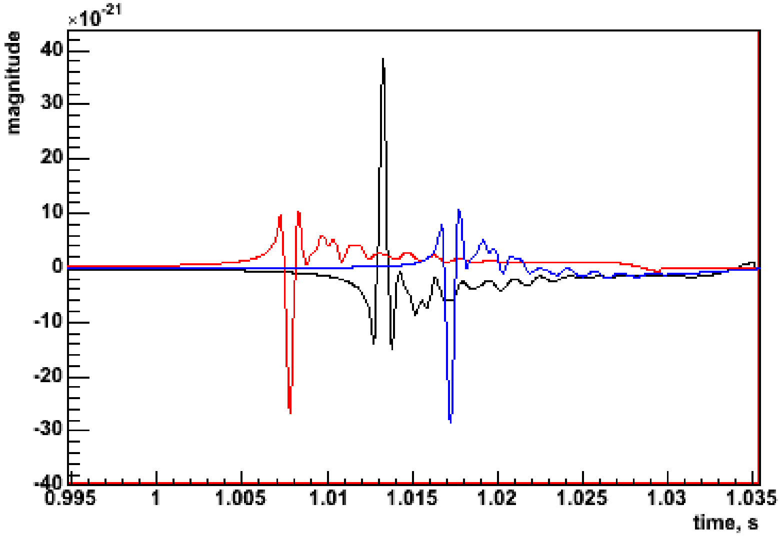


reproduce solution for aligned detectors

Source at θ=120, ϕ=80

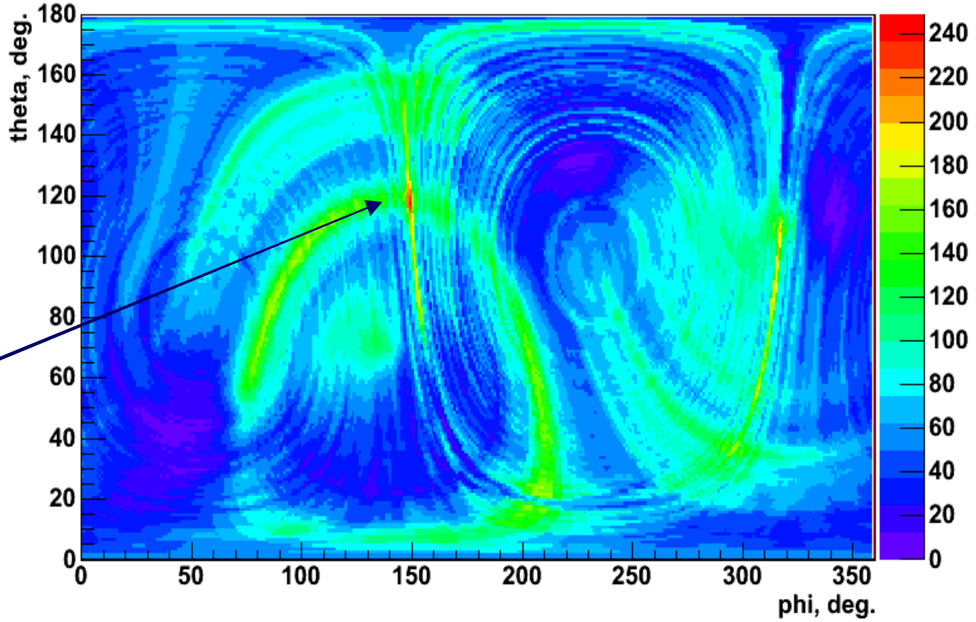
Reconstruction of source coordinates

simulated DFM-A1B2G1 waveform at $\theta=119$, $\phi=149$,
L1 / H1 / V1



simulated noise, average SNR=160 per detector

Likelihood sky map
Signal detected at
$\theta=118$, $\phi=149$



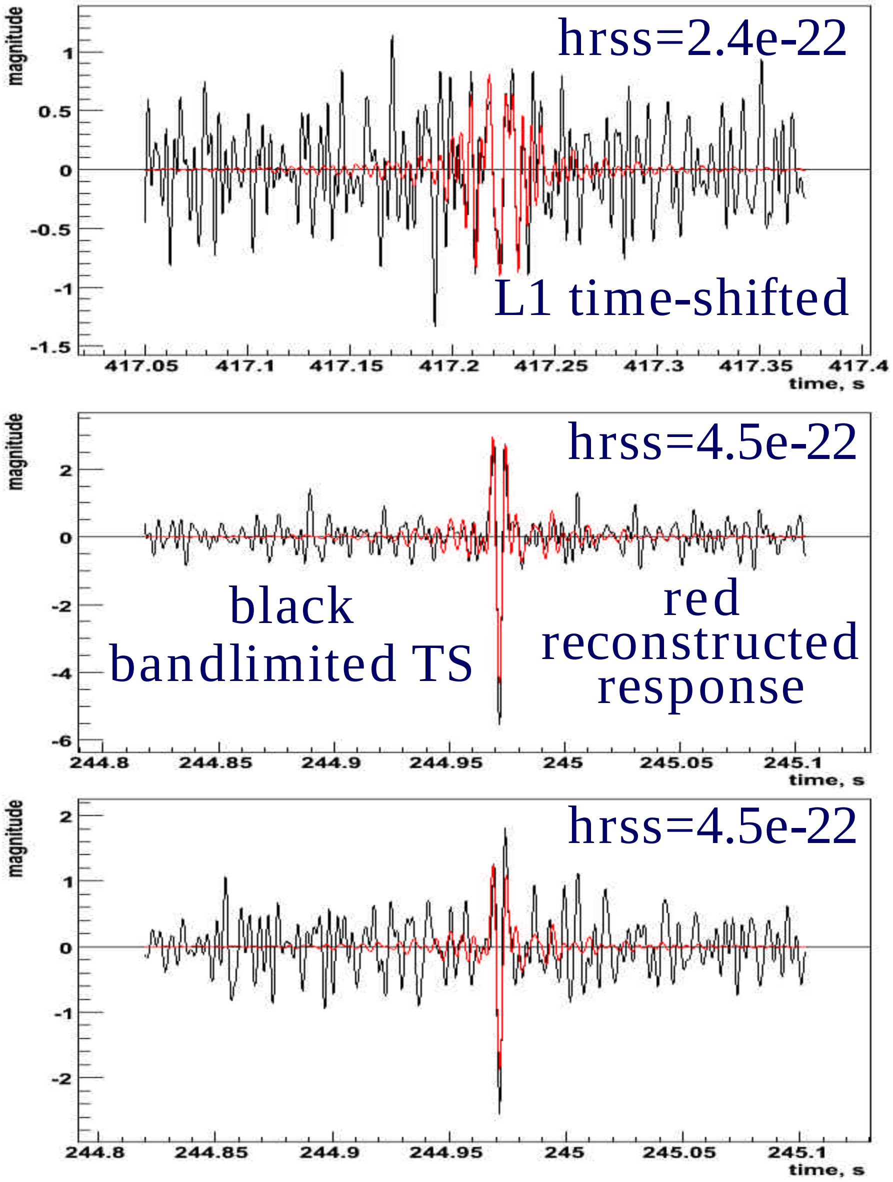
Reconstruction of burst waveforms

- If GW signal is detected, two polarizations and detector responses can be reconstructed and confronted with source models for extraction of the source parameters

- Figures show an example of LIGO glitch reconstructed with the coherent WaveBurst event display (A.Mercer et al.)

  → powerful tool for consistency test of coherent triggers.

H1 / H2 coincident magnetic glitch



Coherent statistics

- **Likelihood: estimator of network SNR → detection statistic**

$$L = \sum_i \sum_k \frac{1}{2s_k^2}\left[x_k^2[i] - \left(x_k[i] - x_k[i]\right)^2\right]$$

$$2L = E - N$$

detected (signal) energy

total energy

noise (null) energy

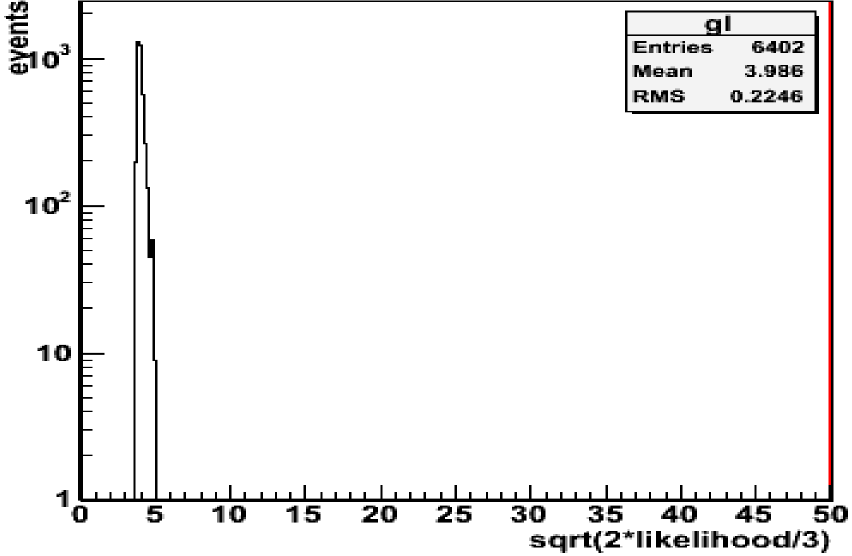
- **Individual statistics L_k, E_k, N_k for each detector are also available**
- **Likelihood matrix**

$$2L = \sum_{i,j} \langle x_i x_j \rangle C_{ij} = E_{i=j} + E_{i \neq j}$$

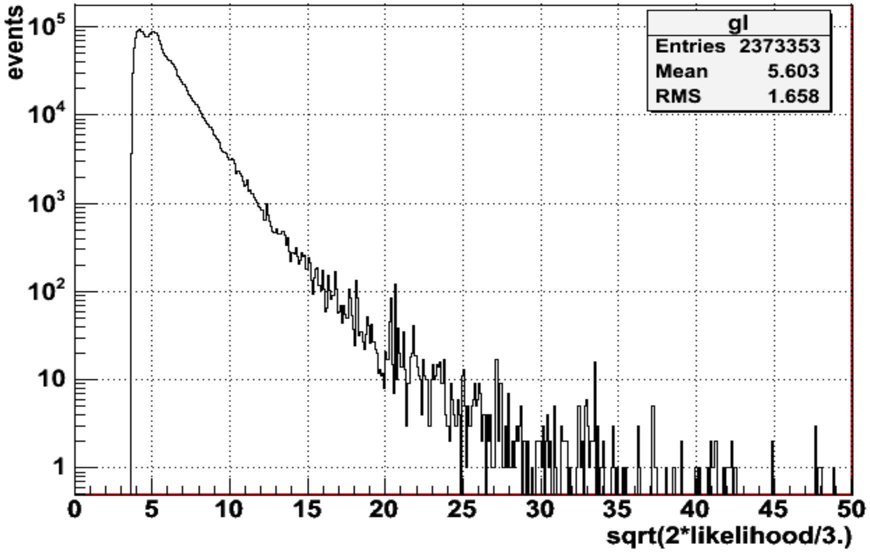
incoherent

coherent

Consistency test of network triggers



Ligo-Virgo simulated data



S4 data

- Likelihood statistic is designed to separate non-stationary bursts from stationary Gaussian noise
- Real data is dominated by glitches
- The coherent statistics is a powerful tool to reject glitches
- Consistency test for LIGO and LIGO-GEO data based on
  - ➢ reconstructed burst energy in individual detectors
  - ➢ network correlation

Coincidence between detectors

- Coherent triggers are coincident in time (and frequency) by construction → no coincidence window
- Define coincidence between detectors by applying threshold at reconstructed energy

$$E_i = \left\langle x_i^2 \right\rangle - N_i$$

  - E_i – reconstructed energy in i-th detector
  - N_i – detector null (noise) energy
- Optimal coincidence schemes can be selected after trigger production
  - strict: $E_{H1}+E_{H2}+E_{L1}>E_T$
  - double OR: $E_{H1}+E_{H2}>E_T$ & $E_{H1}+E_{L1}>E_T$ & $E_{H2}+E_{L1}>E_T$
  - loose: $E_{H1}+E_{H2}+E_{L1}>E_T$ (same as $2L>E_T$)

correlation of misaligned detectors

- **Pearson's statistic**

$$r = \frac{E_{ij}}{\sqrt{E_{ii}E_{jj}}} \rightarrow \frac{\langle x_i x_j \rangle}{\sqrt{\langle x_i^2 \rangle \langle x_j^2 \rangle}}$$

any detectors two aligned detectors

- **Cauchy's statistic**

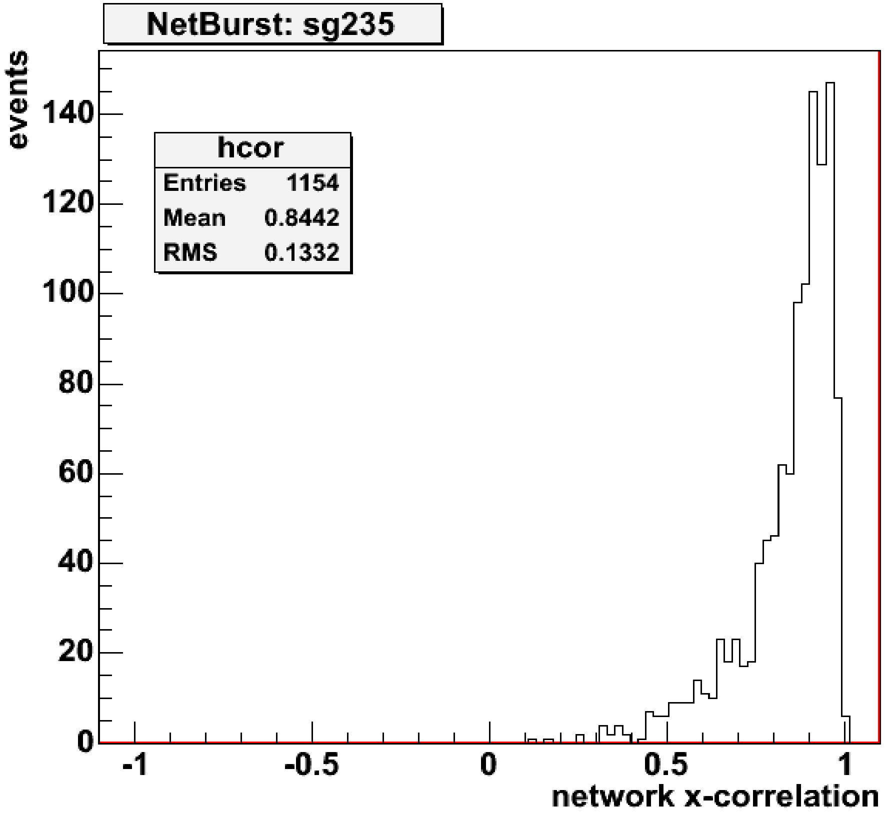
$$c = \frac{E_{ij}}{E - E_{ii} - E_{jj}} \rightarrow \frac{2\langle x_i x_j \rangle}{\langle x_i^2 \rangle + \langle x_j^2 \rangle}$$

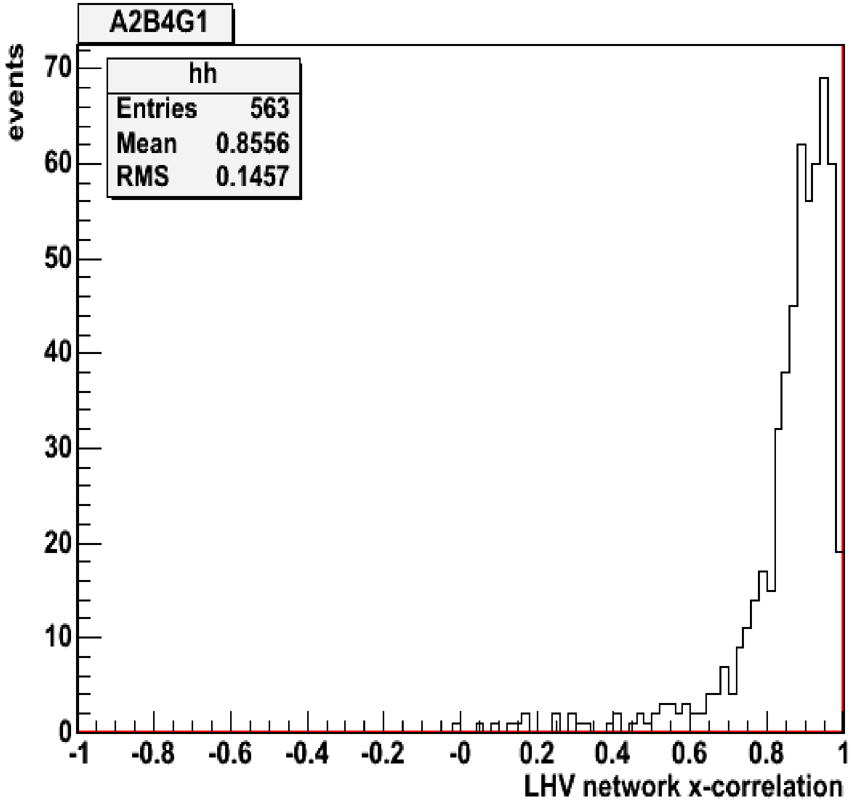
  - **network correlation coefficient**

$$C_{net} = \frac{\sum_{i \neq j} E_{ij}}{E - \sum E_{ii}} = \frac{E_{coherent}}{N_{ull} + E_{coherent}}$$

Network correlation

- **Distribution of the network correlation coefficient for simulated bursts injected into LIGO-Virgo simulated data (project 1b)**





Status of network analysis

- **LSC burst group concentrates on development of several aspects of network analysis**
  - **application of constraint likelihood method to non-hierarchical all sky searches, which are very CPU intensive. Coherent WaveBurst pipeline has been implemented and successfully used for analysis of simulated LIGO-Virgo data, S4 and S5 data sets.**
  - **follow up network analysis of triggers generated by power filters (hierarchical searches)**
  - **application of network analysis to externally triggered searches**
  - **development of network algorithms for rejection of instrumental/environmental glitches**
  - **reconstruction of waveforms and extraction of source parameters.**

Summary

- **Several GW detectors are now operating around the world forming a network**
- **Coherent network analysis addresses problems of detection and reconstruction of GW signals with detector networks**
- **Likelihood methods provide a universal framework for burst searches with arbitrary networks of GW detectors**
  - **likelihood ratio statistic is used for detection**
  - **GW waveforms can be reconstructed from the data**
  - **location of sources in the sky can be measured**
  - **consistency test of events in different detectors**
- **Constraints significantly improve the performance of coherent algorithms**
- **Coherent algorithms are started to be used for burst searches**