Limits on a

Stochastic Background of Gravitational Waves

For the stochastic analysis group

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Outline

i. Introduction

ii. Isotropic background search

iii. Anisotropic background (directional) search

iv. Other activities:
   i. Hanford 4km + 2km search
   ii. Hanford 4km + 2km high frequency (37.5kHz) search
   iii. ALLEGRO + Ligo Livingston search

Introduction

The Gravitational Wave background can be

- Isotropic (i.e. only small anisotropies)
  - Primordial origin (e.g. inflation)
  - Astrophysical origin, from many weak sources
- Anisotropic
  - Astrophysical origin, from fewer strong sources

Wave form unknown, but signal always present

- Analysis method:
  - Cross-correlation between two detectors

Isotropic Background of Gravitational Waves

- Energy density:

$$\rho_{GW} = \frac{c^2}{32\pi G} < \dot{h}_{ab} \, \dot{h}^{ab} >$$

- Characterized by log-frequency spectrum:

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}(f)}{d \ln f}$$

- Related to the strain power spectrum:

$$S(f) = \frac{3H_0^2}{10\pi^2} \frac{\Omega_{GW}(f)}{f^3}$$

- Strain scale:

$$h(f) = 6.3 \times 10^{-22} \sqrt{\Omega_{GW}(f)} \left(\frac{100 \text{ Hz}}{f}\right)^{3/2} \text{Hz}^{-1/2}$$

Detection Strategy, isotropic

- Cross-correlation estimator

$$Y = \int_{-T/2}^{+T/2} dt_1 \int_{-T/2}^{+T/2} dt_2 \; s_1(t_1) \; s_2(t_2) \; Q(t_2 - t_1)$$

$$Y = \int_{-\infty}^{+\infty} df \; \tilde{s}_1^*(f) \; \tilde{s}_2(f) \; \tilde{Q}(f)$$

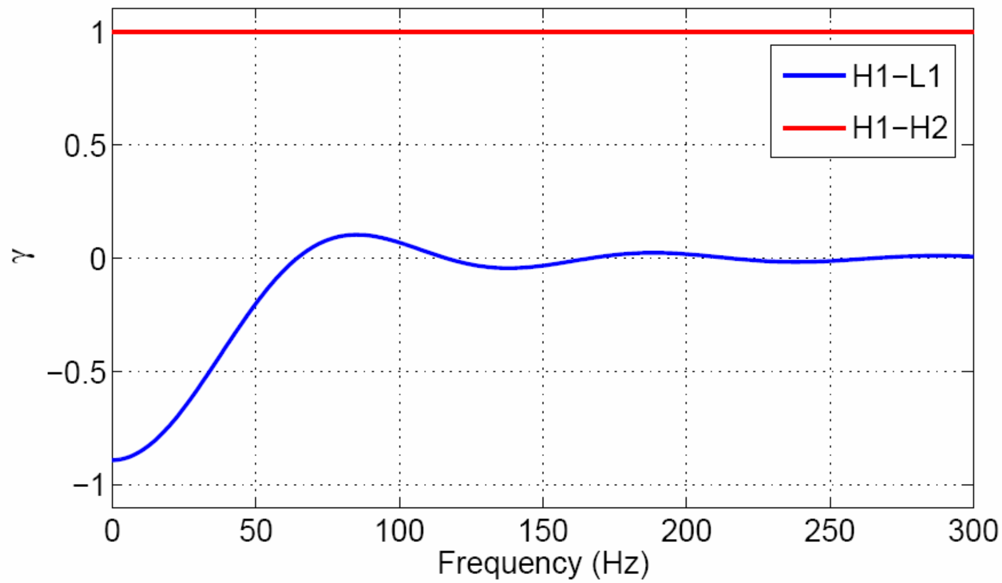
- Theoretical variance

$$\sigma_Y^2 \approx \frac{T}{2} \int_0^{+\infty} df \; P_1(f) \; P_2(f) \; | \; \tilde{Q}(f) \; |^2$$

- Optimal Filter

$$\tilde{Q}(f) = \frac{1}{N} \frac{\gamma(f)\Omega_t(f)}{f^3 P_1(f) P_2(f)}$$

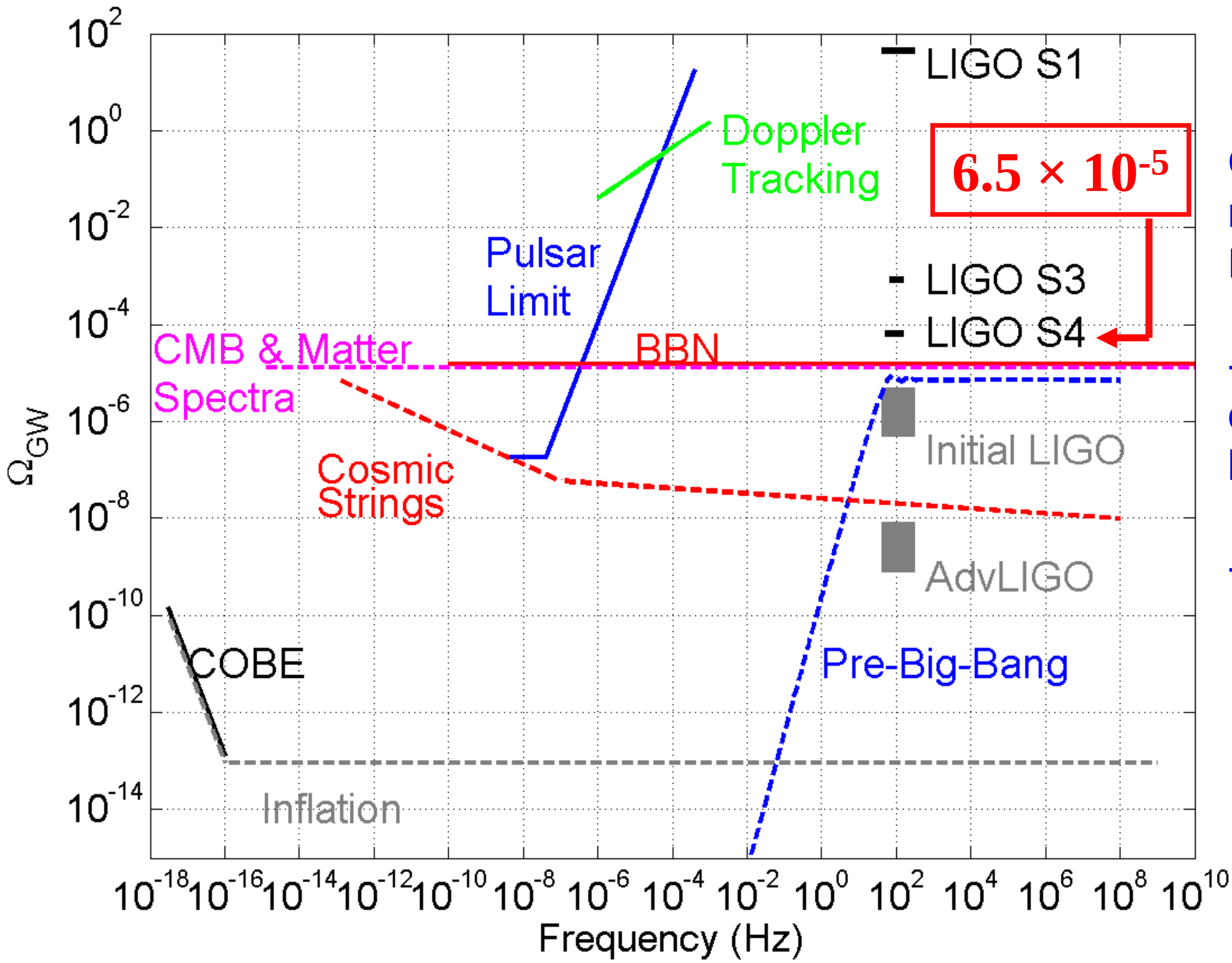
Overlap Reduction Function



For template: $\Omega_t(f) = \Omega_\alpha (f / 100Hz)^\alpha$

Choose *N* such that: $< Y >= \Omega_\alpha$

Landscape



Cosmic Strings models and Pre-Big-Bang models:

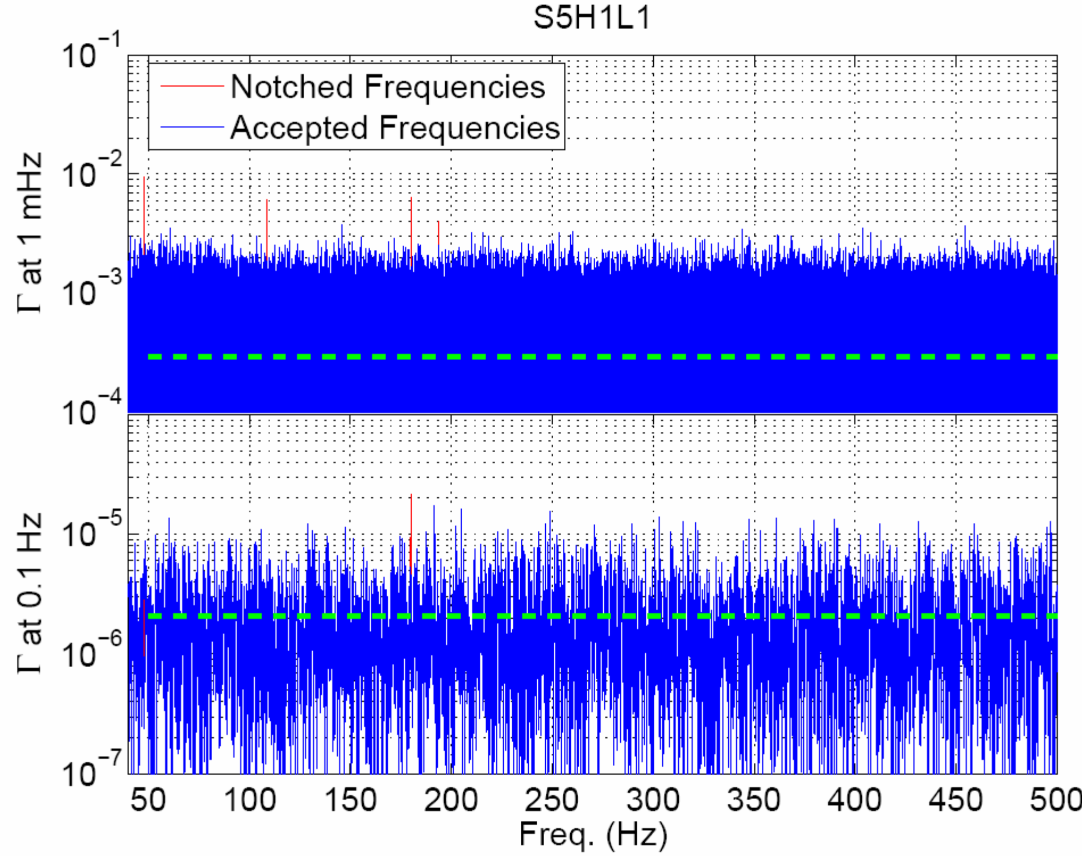
- can easily escape other experimental bounds

- accessible to LIGO.

S5 Status (1)

- Online analysis:
  - Quick feedback
  - No data-quality cuts, calibration not up to date...
- First pass at H1L1:
  - **Time-shift: defining cuts blindly.**
  - Calibration available:
    Nov 5, 2005 – Apr 3, 2006.
  - 32 Hz high-pass filter, in order to push analysis down to 40 Hz.
  - Several lines correlated between H1 and L1:
    - 48.0 Hz,
    - 108.9 Hz (simulated pulsar),
    - 179.9-180.1 Hz,
    - 193.7 Hz (simulated pulsar)

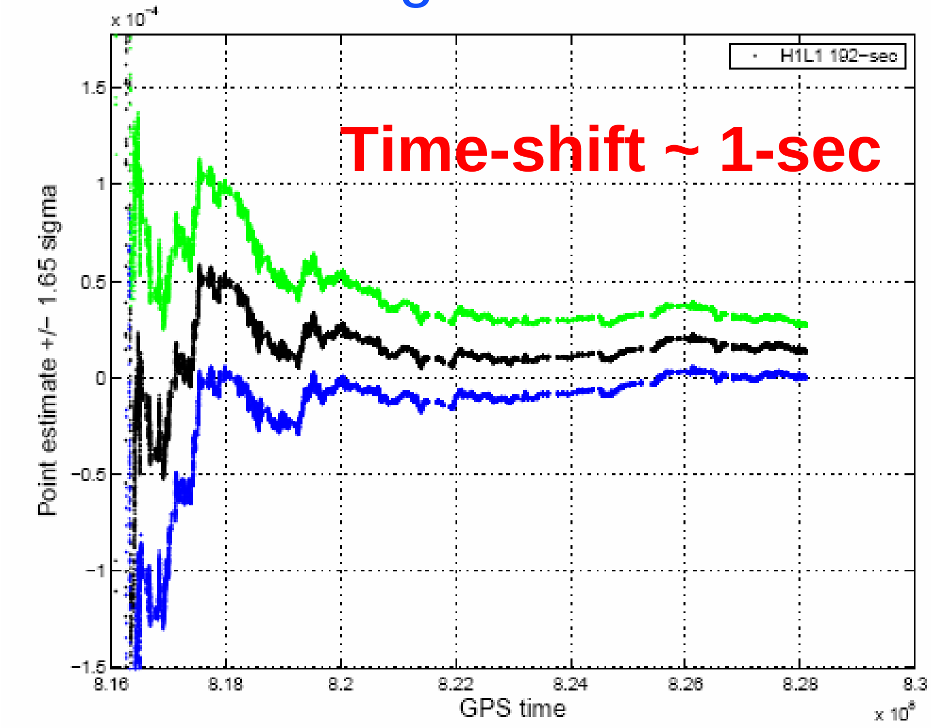
$$\mathrm{Coh} = \mathrm{CSD}^2 / \mathrm{PSD}_1 / \mathrm{PSD}_2$$



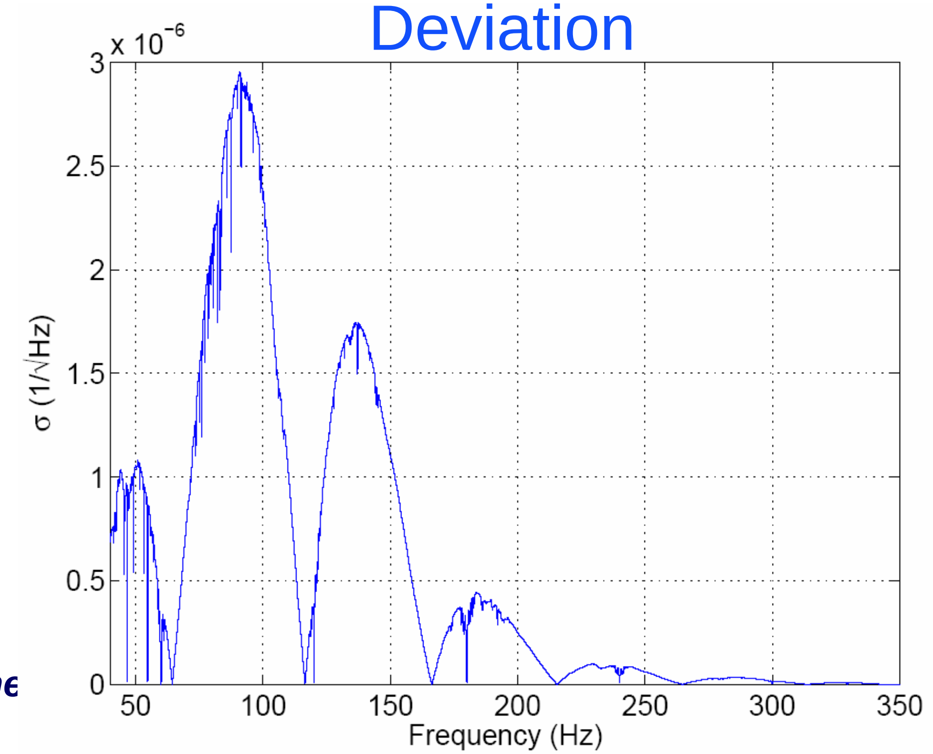
S5 Status (2)

- Flat spectrum, H_0 = 72 km/s/Mpc: $\sigma_\Omega = 1.67 \times 10^{-5}$
  - 2.5× better than S4, but still weaker than the BBN bound (~1.1×10^{-5} in our frequency band).

Running Point Estimate



Theoretical Standard Deviation



Directional search motivation

- Stochastic GW Background due to Astrophysical Sources?
  - Not isotropic if dominated by nearby sources
  - ➔ Do a ***Directional Stochastic Search***

- Source position information from
  - Signal time delay between different sites (sidereal time dependent)
  - Sidereal variation of the single detector acceptance

➔ Time-Shift and Cross-Correlate!

Detection Strategy, point source

- Cross-correlation estimator

$$Y = \int_{-T/2}^{+T/2} dt_1 \int_{-T/2}^{+T/2} dt_2 \; s_1(t_1)\; s_2(t_2)\; Q(t_2 - t_1)$$

$$Y = \int_{-\infty}^{+\infty} df \; \tilde{s}_1^*(f)\; \tilde{s}_2(f)\; \tilde{Q}(f)$$

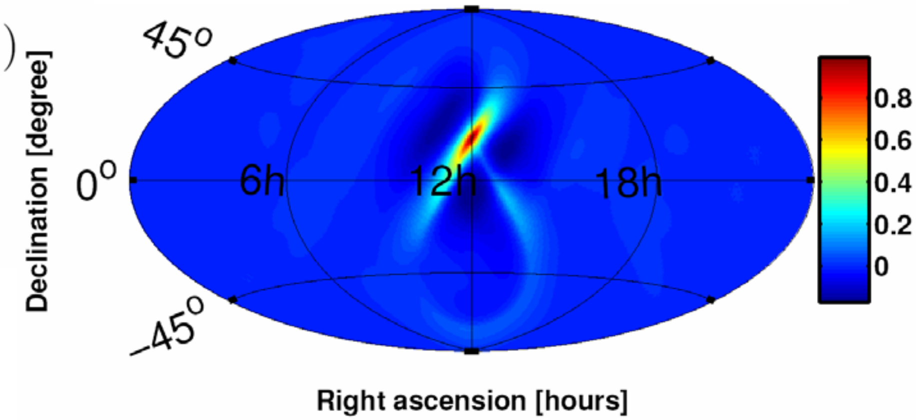
- Theoretical variance

$$\sigma_Y^2 \approx \frac{T}{2} \int_0^{+\infty} df \; P_1(f)\; P_2(f)\; |\, \tilde{Q}(f)\, |^2$$

- Optimal Filter

$$\tilde{Q}(t,f) = \frac{1}{N} \frac{\gamma_{\text{point}}(t,f) H(f)}{P_1(f) P_2(f)}$$

Point Spread Function

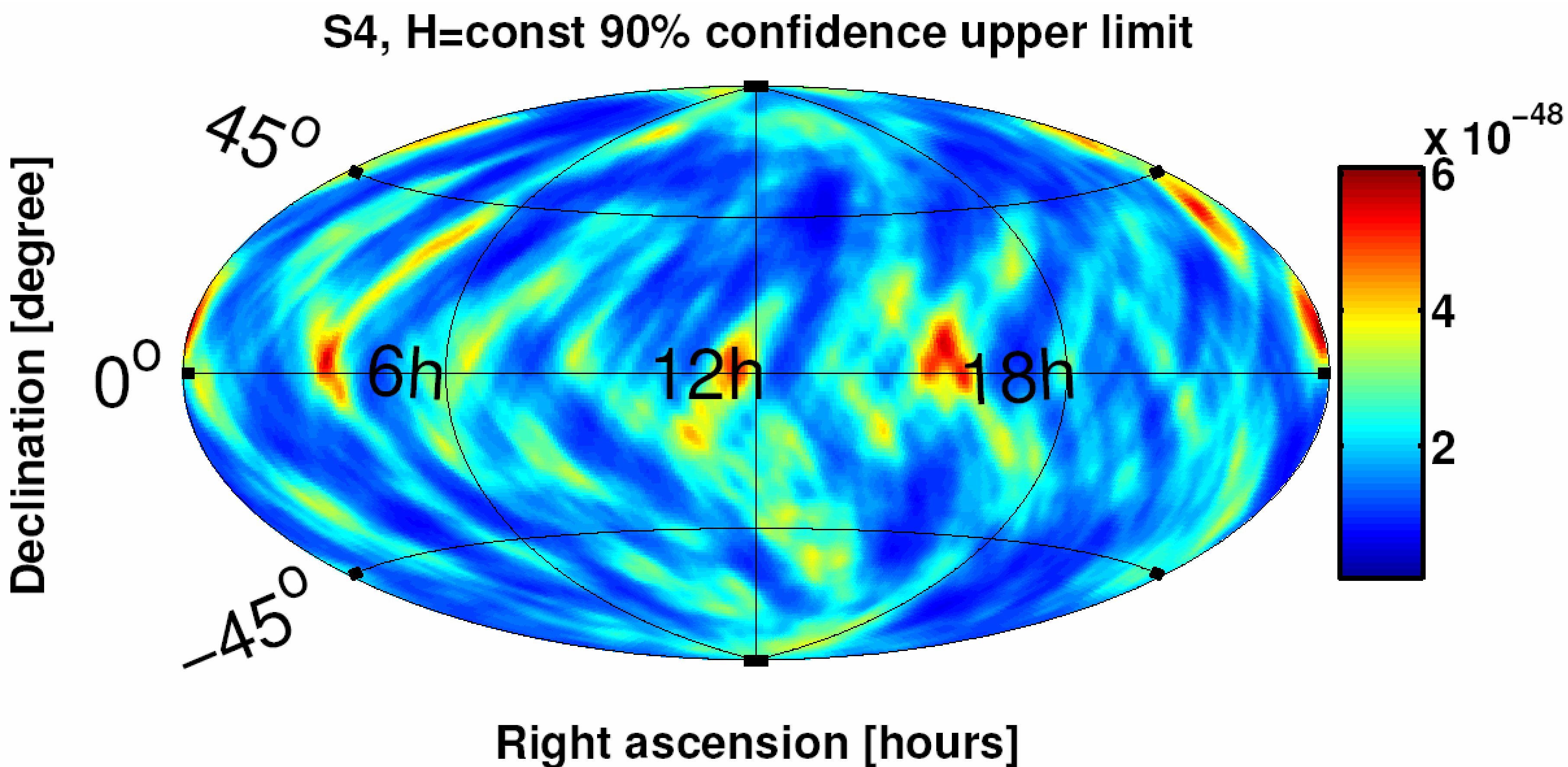


$$\gamma_{\text{point}}(t,f) = \sum_{A=+,\times} e^{i2\pi f \hat{\Omega} \frac{\Delta\vec{x}(t)}{c}} F_{1,t}^{A}(\Omega) F_{2,t}^{A}(\Omega)$$

Strain Power: $H(f) = H_\beta (f / 100Hz)^\beta$

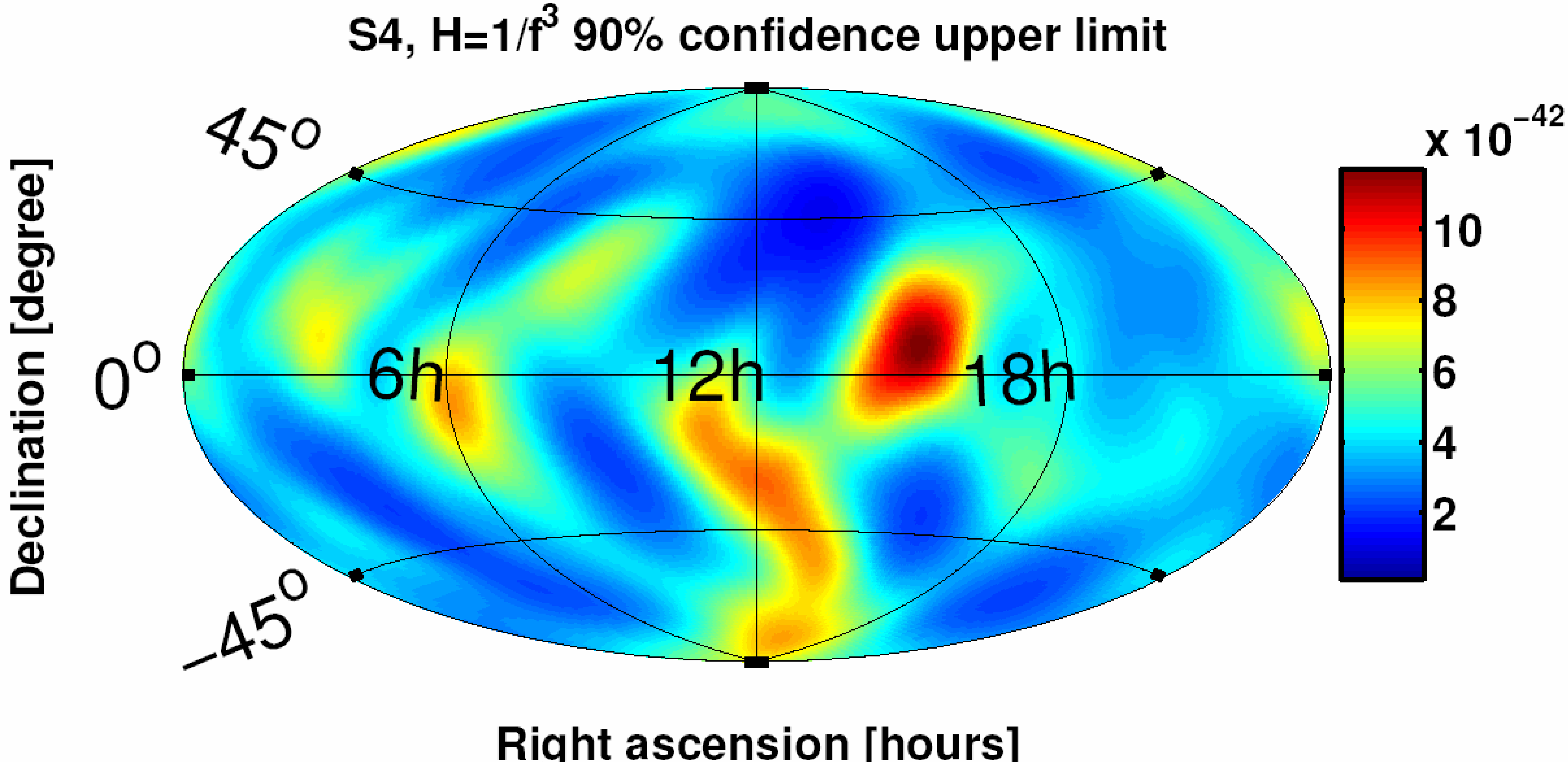
Choose *N* such that: $\langle Y \rangle = H_\beta$

S4 Upper Limit map , H(f)=const



$$H_{90\%} = (0.85 - 6.1) \times 10^{-48}\, Hz^{-1}$$

S4 Upper Limit map , $H(f) \sim f^{-3}$



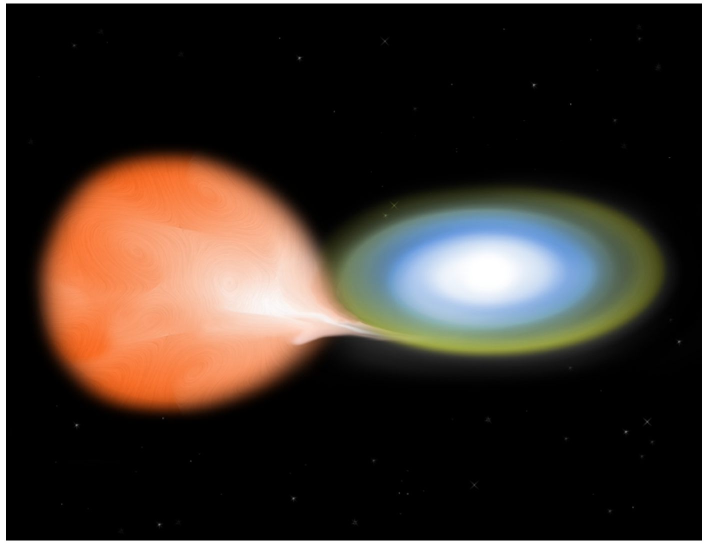
$$H_{90\%} = (0.12 - 1.2) \times 10^{-47}\, Hz^{-1} \left(100Hz / f\right)^{3}$$

Application: Low-Mass X-ray Binary (LMXB)

- Accretion driven pulsars
  - Spun up to 300Hz < f_{spin} < 730Hz
  - Torque balanced by GW?
    - Likely for B<< 10^{11} Gauss
- From torque balance:

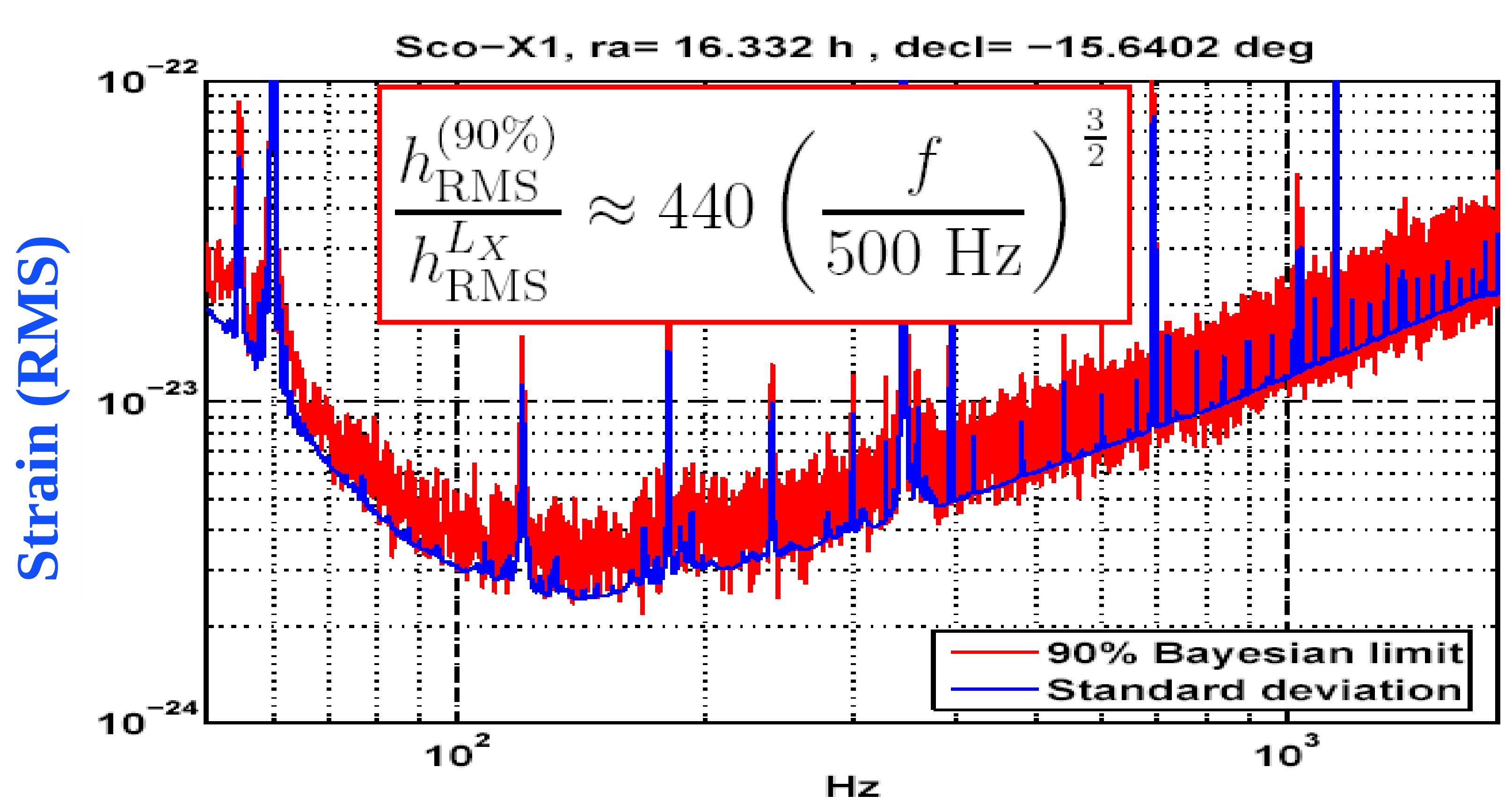
$$L_{GW} \approx \frac{f_{spin}}{f_{Kepler}} L_X$$

- Sco-X1:
  - Is brightest X-ray source in sky
  - Low magnetic field (~10^7 Gauss)
  - Spin frequency unknown



(Artist's impression: NASA)

Frequency dependent Strain Upper Limit Sco-X1



Other activities

- Deconvolution of maps
  - Use maximum likelihood estimator to reduce PSF
- Hanford 4km & 2km analysis
  - Environmental sources that explain excess coherence identified
- High frequency (37.5kHz) analysis
  - Exploit interferometer sensitivity at the 1st free spectral range
- Livingston – ALLEGRO (GW bar at LSU) analysis
  - has decent sensitivity around 915Hz (Strain ~ 10^{-21} $Hz^{-1/2}$)
  - 40km from LIGO Livingston (Overlap Reduction Function close to 1)
- LIGO-VIRGO
  - Working on code compatibility for future collaboration

Background material

- Method paper
  - Allen, Romano, PRD 59 102001 (1999) http://prola.aps.org/abstract/PRD/v59/i10/e102001
- Most recent paper:
  - Isotropic upper limit, S4, submitted to in ApJ: http://arxiv.org/abs/astro-ph/0608606
- Thesis
  - Directional search, Stefan Ballmer, MIT, (Ph.D).: http://ligo.mit.edu/~sballmer/thesis.pdf
  - H1-H2 low frequency search Nickolas Fotopoulos, MIT, (M.S.): http://web.mit.edu/~nvf/www/thesis_as_accepted.pdf

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End