OPTIMIZED COATING STATUS

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Summary of Work Done

Coating Optimization in a Nutshell

Motivation: thermal (structural) noise in the coating is the limiting factor in setting LIGO sensitivity in key frequency band

Rationale: Standard quarter-wavelength (QWL) coatings are optimal only in the sense of providing the *minimal* number of layers for a prescribed transmittance.
They do *not* yield the lowest possible noise among all designs yielding the prescribed trasmittance, in view of the different noisiness of silica and tantala, viz.:

Coating noise PSD $\propto L_{SiO_2} + \gamma L_{Ta_2O_5}, \quad \gamma > 1$

physical length of silica/tantala

Summary of Work Done

Genetic Optimization Approach

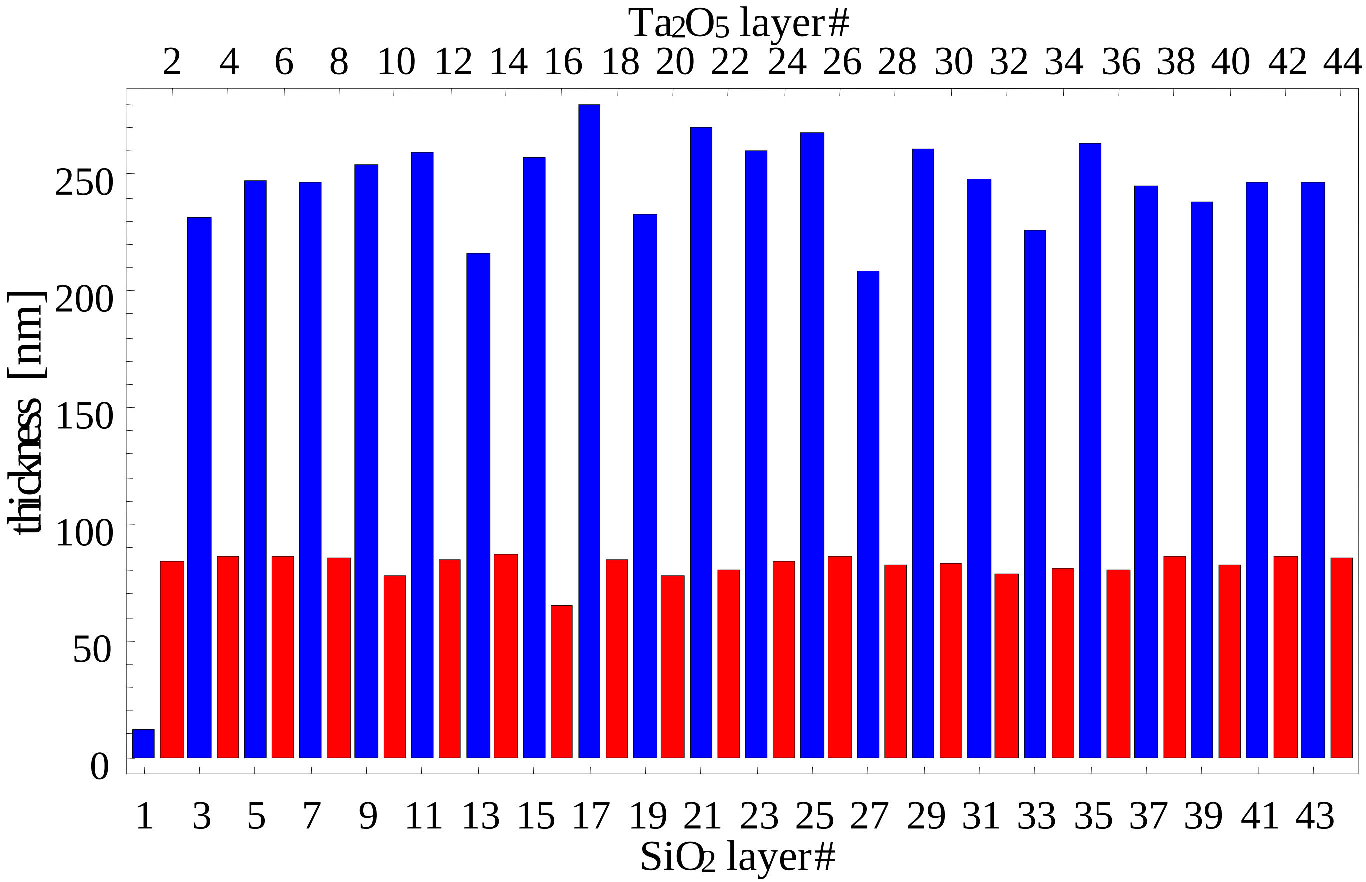
Motivation: Flexibility [more than two materials; multiple goals (e.g., reflectance, noise, optical losses) and constraints combining continuous (e.g., layers thicknesses) as well as discrete (e.g.,no. of layers) design optimization parameters;

Main Result(s): for 2-materials coatings, minimum-noise coatings w. prescribed reflectance are made of stacked non-QWL-doublets, except for the top and bottom layers.
Led to studying optimized stacked doublet (SD) designs, with or without end layer tweaking

Products/deliverables: genetic coating optimization code, based on public-comain PIKAIA engine (FORTRAN)

GA Engineered Prototype

Goal: $1-|\Gamma|^2 < 15$ ppm. $\gamma=10$. (after 10^5 generations)



GA Engineered Prototype, cont.d

	QWL-1	**Genetic**	QWL-2
N (cap included)	36	**44**	28
$1-\|\Gamma\|^2$ ppm	16.20	**14.91**	**235.46**
$L(Ta_2O_5)$ nm	**2359.43**	**1815.61**	1835.11
$L(SiO_2)$ nm	3479.98	**5217.4**	2747.35
L_{tot} nm	5839.41	**7033.01**	4582.46

Summary of Work Done, contd.

Linearization of Rigorous Coating Noise Formula

Motivation: Backing up naive formula used in genetic simulations

$$PSD = \Pi_0 (L_{SiO_2} + \gamma L_{Ta_2O_5})$$

understanding the underlying coating physics, finding a formula For the specific-loss-ratio γ based on first principles.

Main result(s): accurate linearized approximation (rel. accuracy of the order of 0.5%) obtained.

Products/deliverables: code computing linearized structural coating noise formula, symbolic & numeric form (MATHEMATICA)

Rigorous Coating PSD Model - Linearization

[G. Harry, LIGO-T040029-00-R, 2004]

k_B = Boltzmann constant
T = absolute temperature
f = frequency
w = beam half-width

$$S_{coat}(f) = \left(\frac{2k_B T}{\pi^{\frac{3}{2}} f}\right)\left(\frac{1-\sigma^2}{wY}\right)\phi_{\text{eff}}^{\text{coat}}$$

$$\begin{aligned}\phi_{\text{eff}}^{\text{coat}} &= \frac{d_{\text{coat}}}{\sqrt{\pi}w}\frac{1}{Y_\perp}\left\{\left[\frac{Y}{1-\sigma^2}-\frac{2\sigma_\perp^2 Y Y_\parallel}{Y_\perp(1-\sigma^2)(1-\sigma_\parallel)}\right]\phi_\perp+\right.\\ &+ \frac{Y_\parallel\sigma_\perp(1-2\sigma)}{(1-\sigma_\parallel)(1-\sigma)}(\phi_\parallel-\phi_\perp)+\\ &+ \left.\frac{Y_\parallel Y_\perp(1+\sigma)(1-2\sigma)^2}{Y(1-\sigma_\parallel^2)(1-\sigma)}\phi_\parallel\right\},\end{aligned}$$

Sporadic errors

$\boxed{\sigma_\perp^2}$ $\boxed{\sigma}$

Warning !!!
Formula propagated w. errors through technical Literature

$\sigma, \sigma_\perp, \sigma_\parallel$ = Poisson ratio
$Y, Y_\perp, Y_\parallel$ = Young modulus
$\phi, \phi_\perp, \phi_\parallel$ = Loss angle
(substrate, coating-normal, coating-parallel)

Coating PSD Model – Linearization, cont.d

[G. Harry, LIGO-T040029-00-R, 2004]

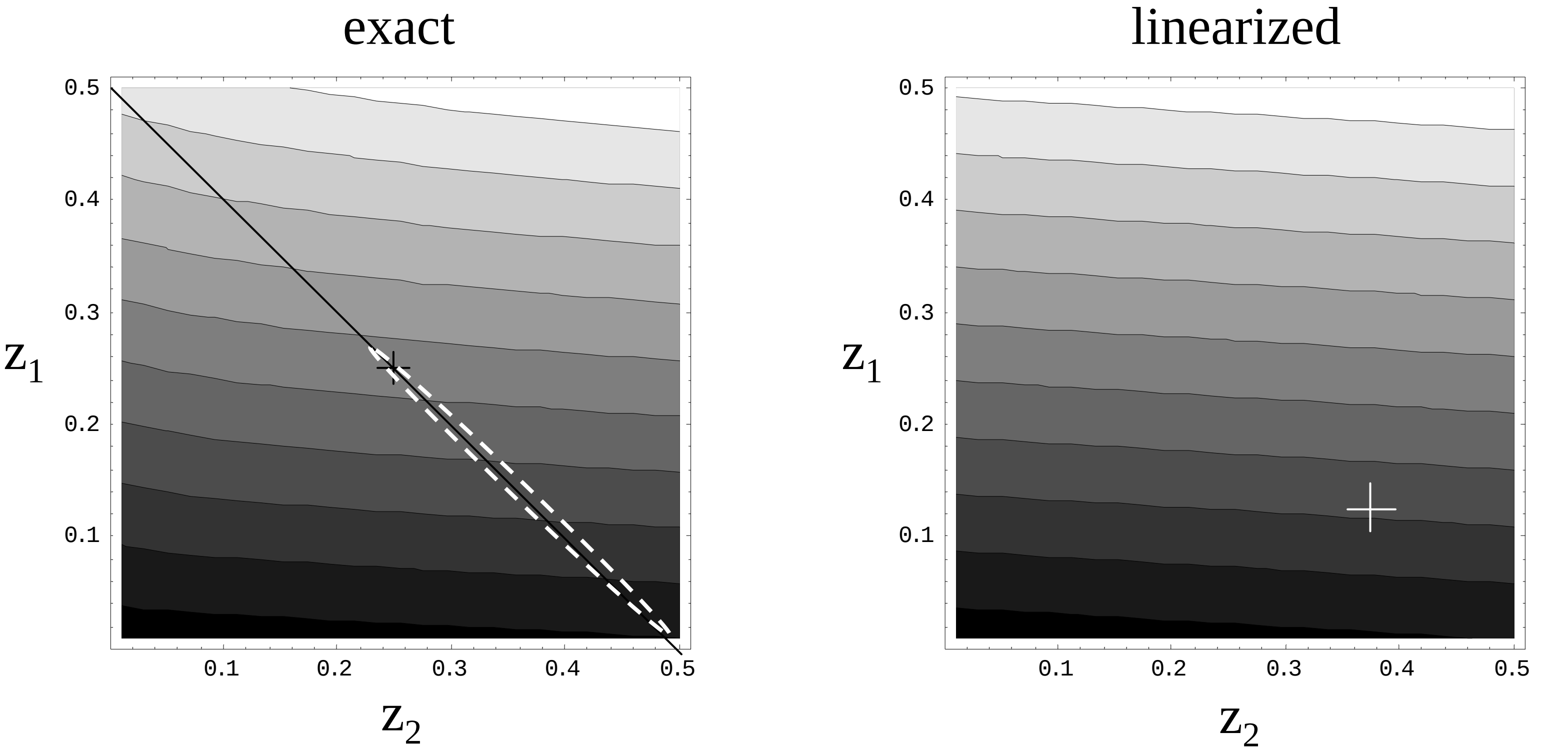
$$\begin{aligned}
Y_\perp &= \frac{1+d_2/d_1}{Y_1^{-1}+(d_2/d_1)Y_2^{-1}}, \quad Y_\parallel = \frac{Y_1+(d_2/d_1)Y_2}{1+d_2/d_1}, \\
\phi_\perp &= Y_\perp \frac{\phi_1 Y_1^{-1}+(d_2/d_1)\phi_2 Y_2^{-1}}{1+d_2/d1}, \quad \phi_\parallel = Y_\parallel^{-1}\frac{Y_1\phi_1+(d_2/d_1)Y_2\phi_2}{1+d_2/d_1} \\
\sigma_\perp &= \frac{\sigma_1 Y_1+(d_2/d_1)\sigma_2 Y_2}{Y_1+(d_2/d_1)Y_2},
\end{aligned}$$

defines implicitly $\sigma_\parallel$

$$\frac{\sigma_1 Y_1}{(1+\sigma_1)(1-2\sigma_1)}+\frac{(d_2/d_1)\sigma_2 Y_2}{(1+\sigma_2)(1-2\sigma_2)} = -\frac{Y_\parallel(\sigma_\perp^2 Y_\parallel+\sigma_\parallel Y_\perp)(1+d_2/d_1)}{(\sigma_\parallel+1)[2\sigma_\perp^2 Y_\parallel-(1-\sigma_\parallel)Y_\perp]}$$

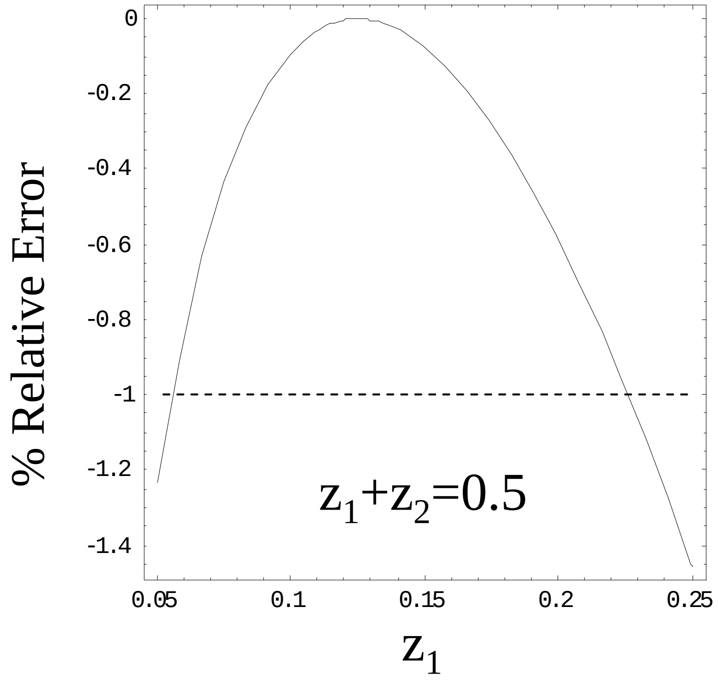
Quadratic equation. Elementary solution in analytic form.
Has one and only one positive (acceptable) root.

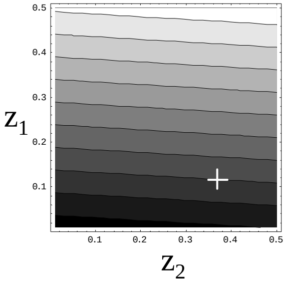
Coating PSD Model – Linearization, cont.d



Single doublet noise contour plots vs. $z_1 = n_{Ta_2O_5} \Delta_{Ta_2O_5} / \lambda_0$ **and** $z_2 = n_{SiO_2} \Delta_{SiO_2} / \lambda_0$.
Left panel: exact. Region of interest for optimization highlighted.
Right panel: first-order truncated Taylor-McLaurin expansion with initial point midway optimization range, $z_1 = 1/8$, $z_2 = 3/8$ **(white-cross marker)**.

Coating PSD Model – Linearization, cont.d





Single doublet noise. Percent relative error between exact and first-order truncated Taylor - McLaurin (linearized) expansion w. initial point $z_1 = 1/8$, $z_2 = 3/8$ **as a function of** z_1 **in the range** $0 < z_1 < 1/4$, $z_2 = 1/2 - z_1$**.**

Summary of Work Done, contd.

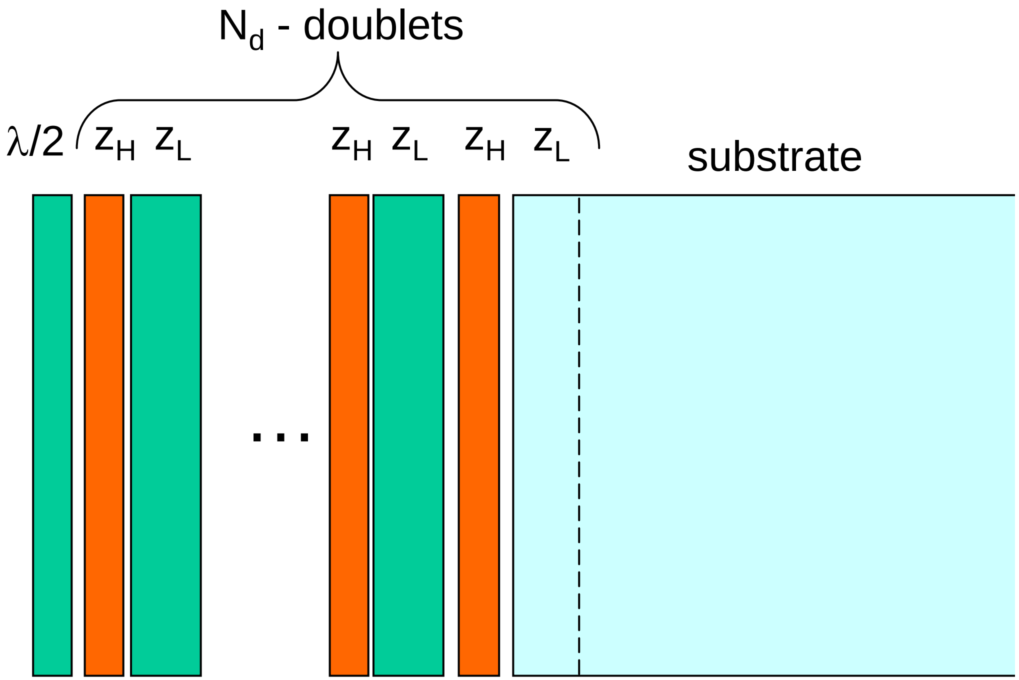
SD (TSD) Optimization Approach

Motivation: suggested by blind genetic optimization using only two refractive materials.

Main result(s):<u>Tradeoff curves</u>, coating reflectance vs. noise PSD, for fixed no.of doublets; <u>Optimization curves</u> (coating noise PSD vs. number of doublets, at prescribed reflectance); <u>Design tables</u>, yielding the corresponding layer thicknesses.
Plain SD, end-layer-tweaked SD, and naive ($z_1+z_2=1/2$) syntheses investigated.

Products/deliverables: codes for computing the above curves tables (MATHEMATICA)

Stacked-Doublet Coating (w. λ/2 silica cap)



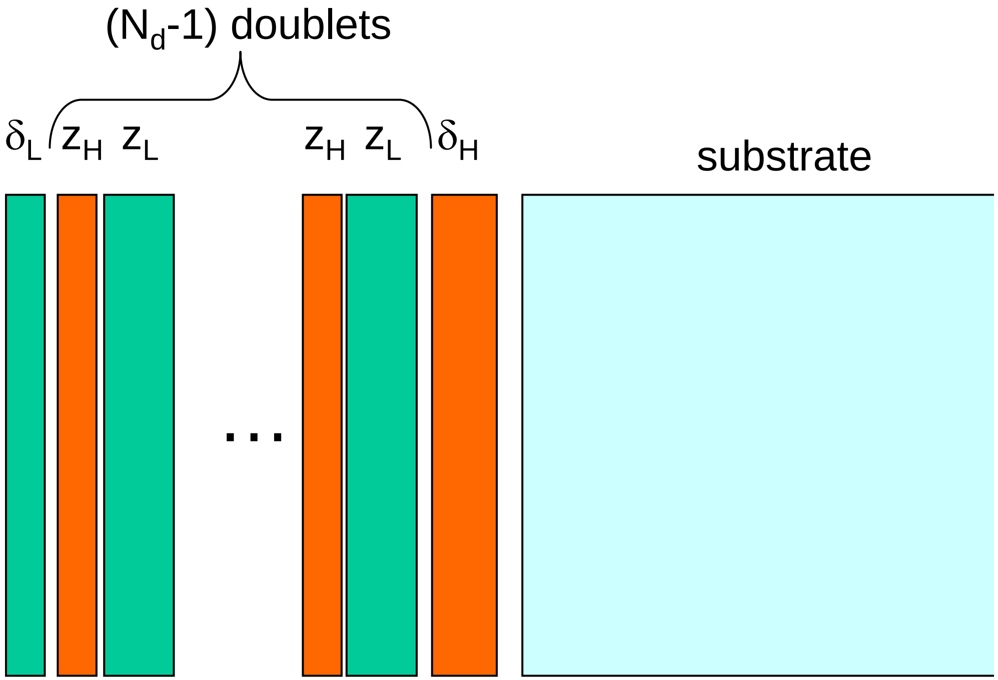
Noise thickness:

$$L_n = \lambda_0 \left(\frac{1}{2n_L} + (N_d - 1)\frac{\overline{z}_L}{n_L} + N_d \frac{\overline{z}_H}{n_H} \right)$$

where:

$$\overline{z}_{L,H} = \frac{z_{L,H}}{\lambda_{L,H}} = \frac{z_{L,H} n_{L,H}}{\lambda_0}$$

End-tweaked Stacked-Doublet Coating Geometry



Noise thickness:

$$L_n = \lambda_0 \left(\frac{\bar{\delta}_L}{n_L} + (N_d - 1)\left(\frac{\bar{z}_L}{n_L} + \frac{\bar{z}_H}{n_H} \right) + \frac{\bar{\delta}_H}{n_H} \right)$$

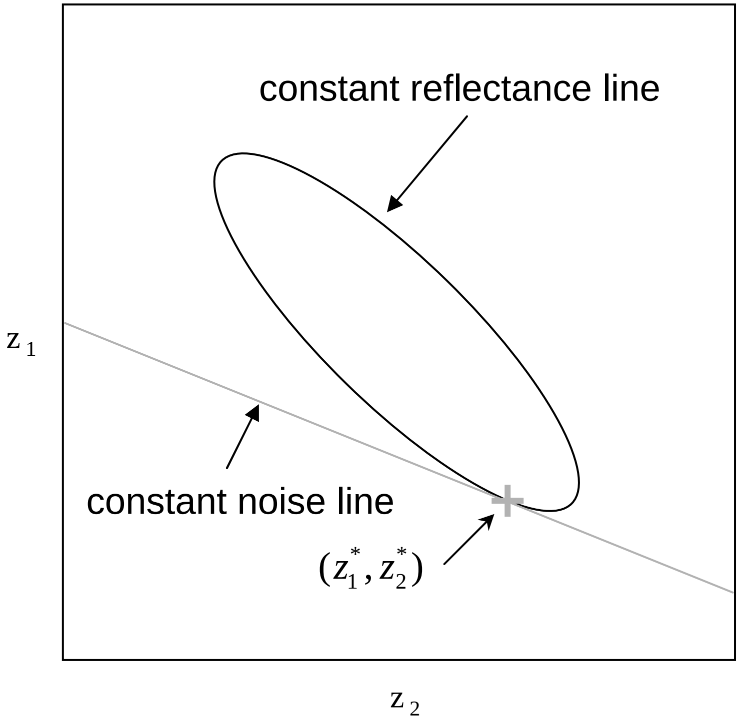
where:

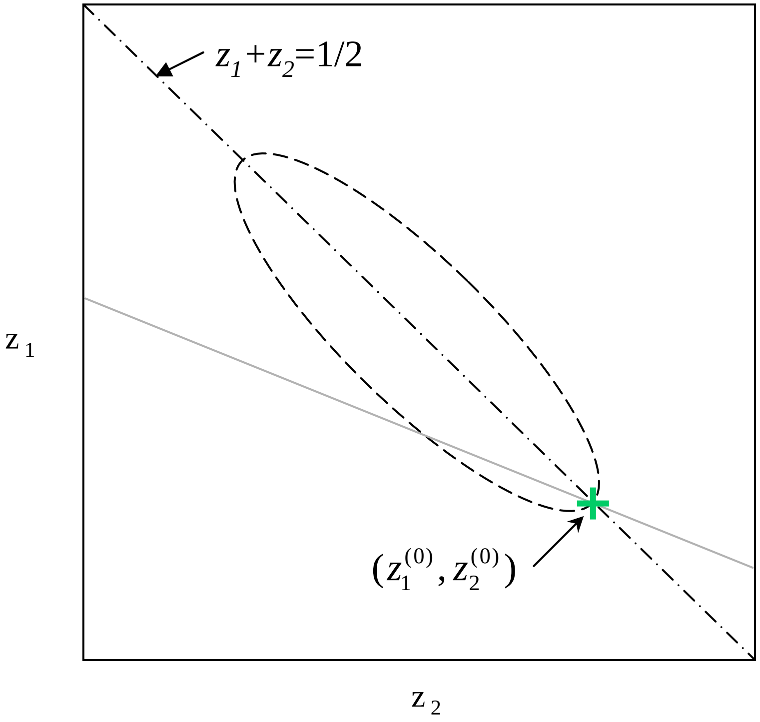
$$\bar{z}_{L,H} = \frac{z_{L,H}}{\lambda_{L,H}} = \frac{z_{L,H}\, n_{L,H}}{\lambda_0},$$

$$\bar{\delta}_{L,H} = \frac{\delta_{L,H}}{\lambda_{L,H}} = \frac{\delta_{L,H}\, n_{L,H}}{\lambda_0}$$

SD Optimization: Exact (left) vs Approximate (right)

…finding the lowest-noise doublet w. prescribed reflectance

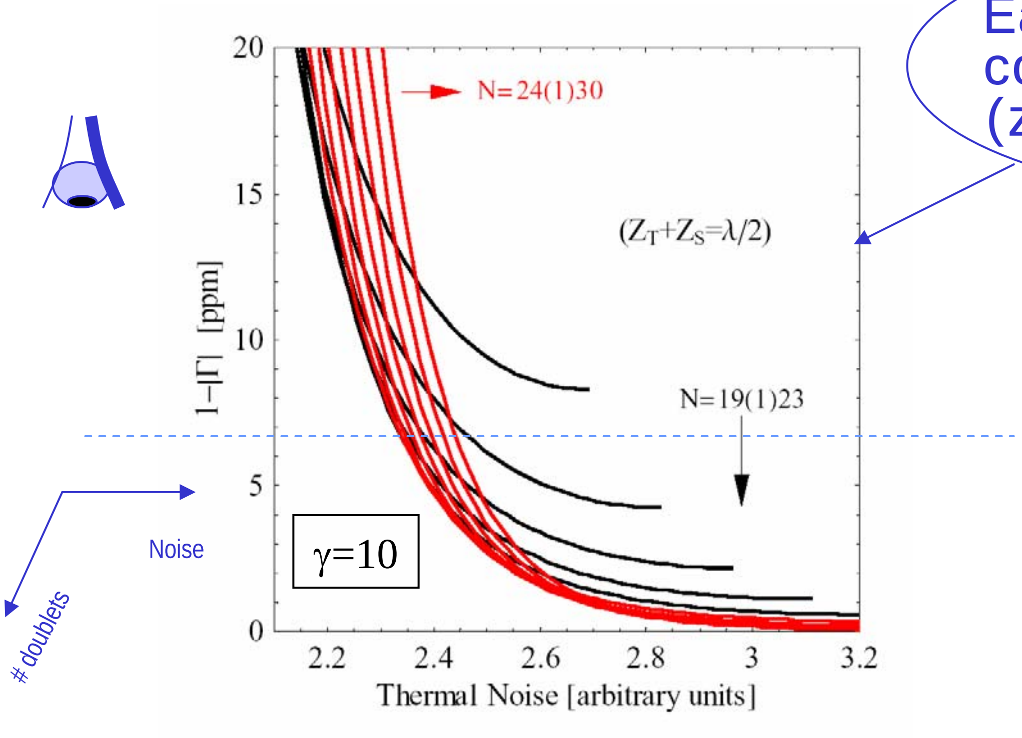




constant reflectance contours squeeze along $z_1+z_2=1/2$ line corresponding to naive (Bragg) synthesis.

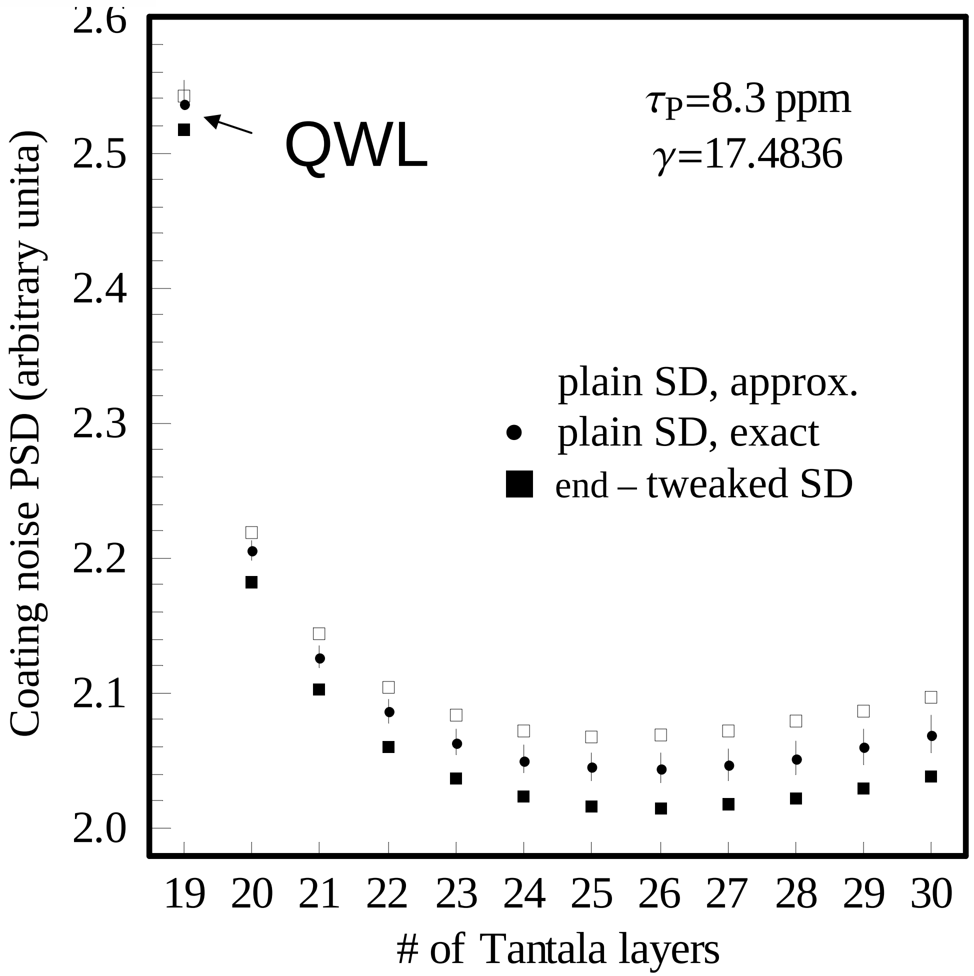
SD Coating Tradeoff Curves

Each point on any curve corresponds to a specific (z_1,z_2) value.



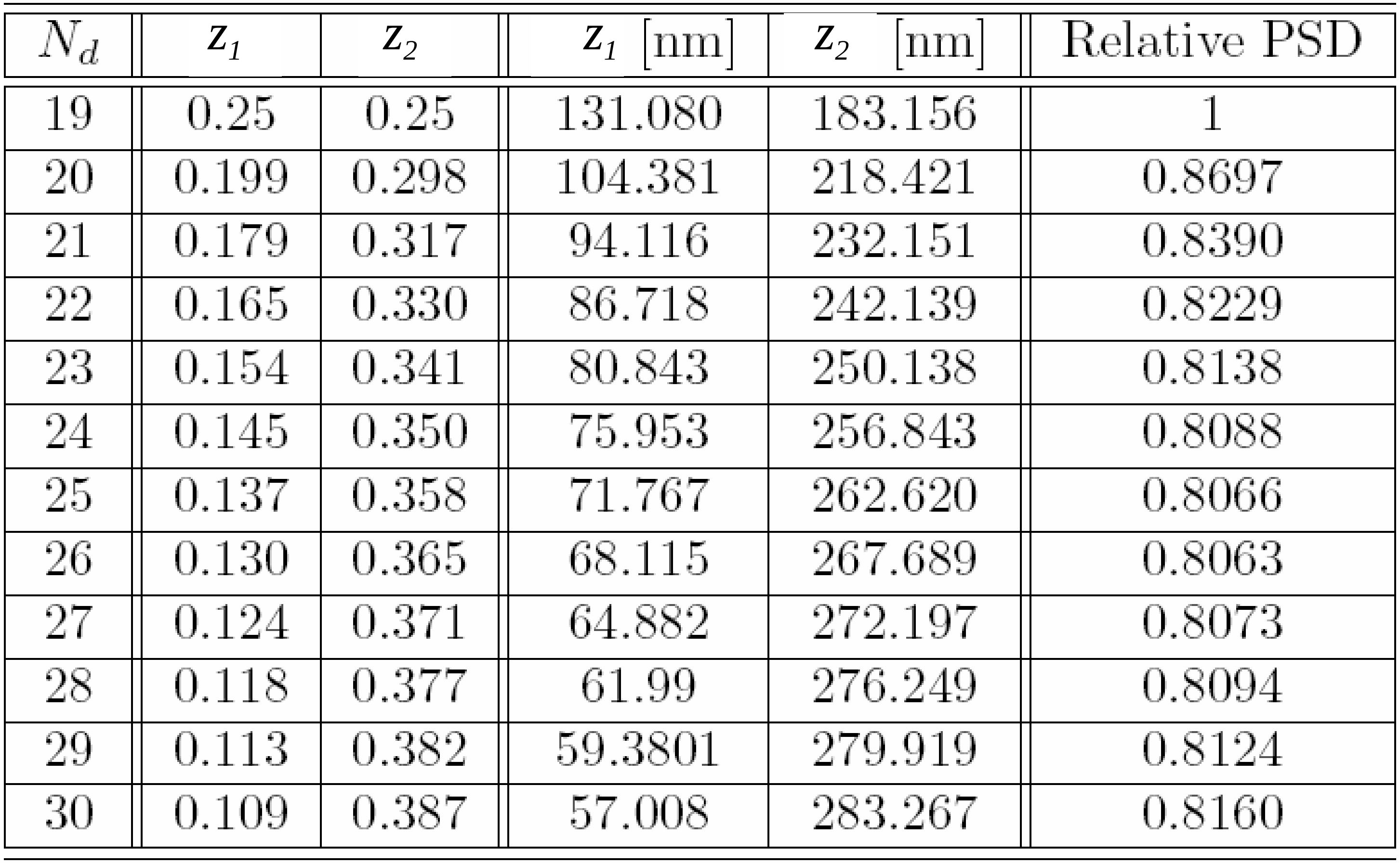
8.3 ppm

Stacked Doublet (SD) Designs with $\tau_P = 8.3\,ppm$



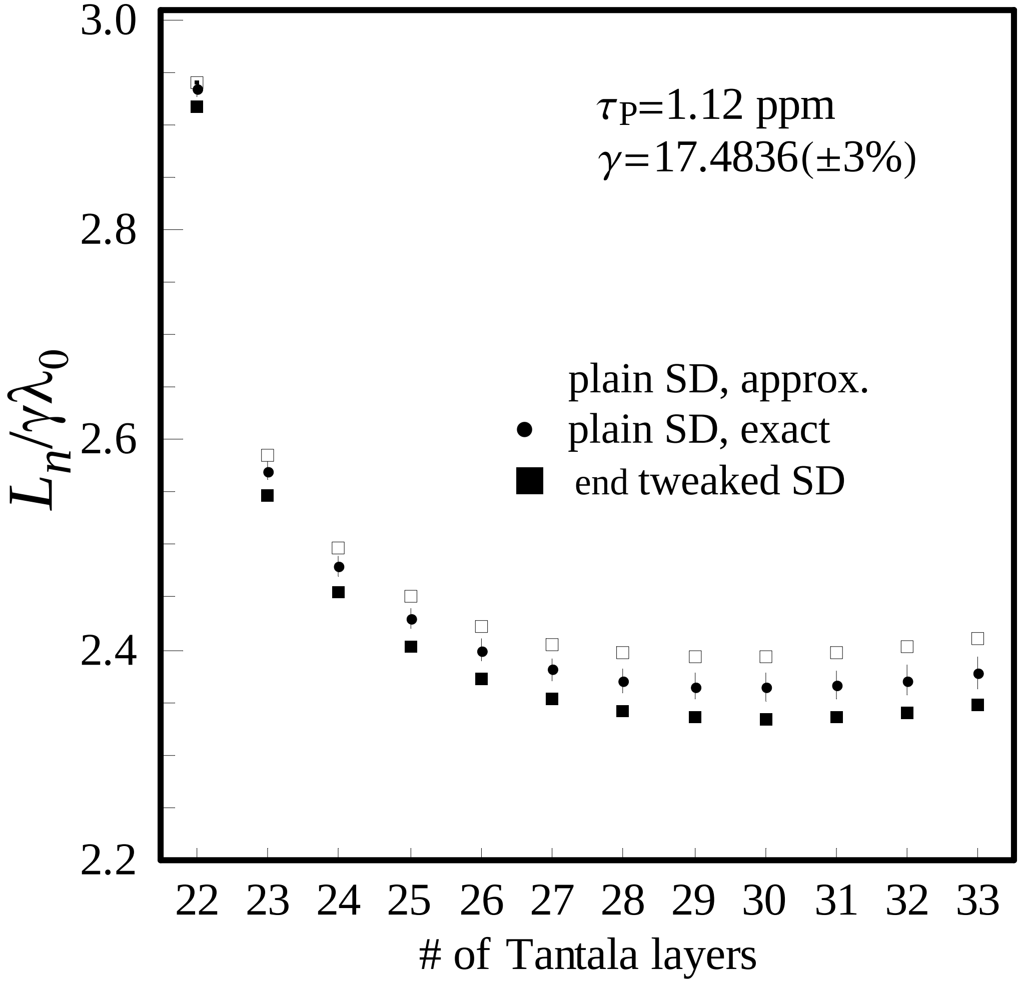
~10% uncertainties in material parameters may blur the difference between end-tweaked, plain-exact and approximate syntheses…

SD Coating Design Table γ =17.4836, $\tau_P = 8.3\,ppm$

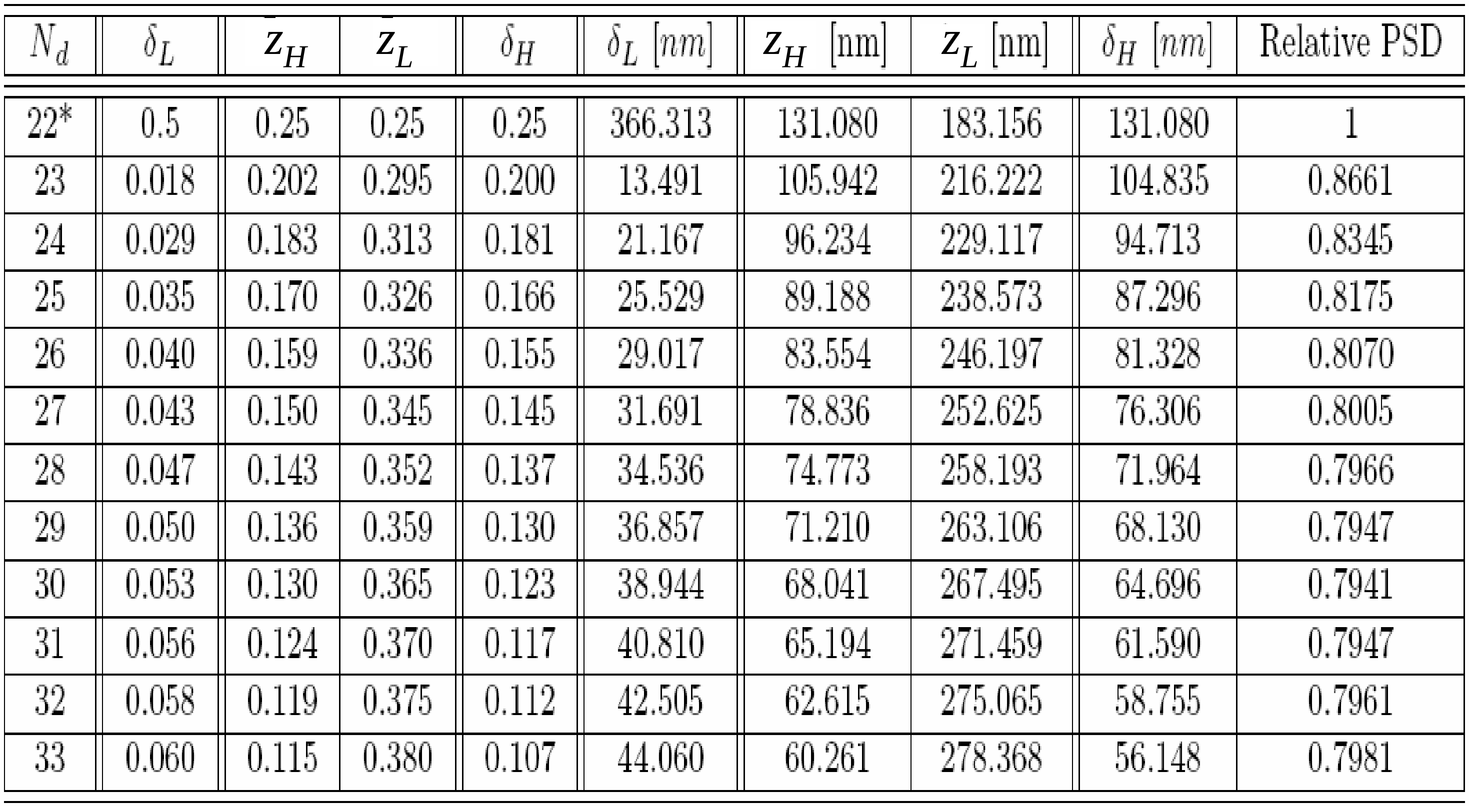


N_d	z_1	z_2	z_1 [nm]	z_2 [nm]	Relative PSD
19	0.25	0.25	131.080	183.156	1
20	0.199	0.298	104.381	218.421	0.8697
21	0.179	0.317	94.116	232.151	0.8390
22	0.165	0.330	86.718	242.139	0.8229
23	0.154	0.341	80.843	250.138	0.8138
24	0.145	0.350	75.953	256.843	0.8088
25	0.137	0.358	71.767	262.620	0.8066
26	0.130	0.365	68.115	267.689	0.8063
27	0.124	0.371	64.882	272.197	0.8073
28	0.118	0.377	61.99	276.249	0.8094
29	0.113	0.382	59.3801	279.919	0.8124
30	0.109	0.387	57.008	283.267	0.8160

Stacked Doublet (SD) Designs with $\tau_P = 1.12\,ppm$



ETSD Coating Design Table γ =17.4836, $\tau_P = 1.12\,ppm$



N_d	δ_L	$\bar{z}_H$	$\bar{z}_L$	δ_H	δ_L [nm]	z_H [nm]	z_L [nm]	δ_H [nm]	Relative PSD
22*	0.5	0.25	0.25	0.25	366.313	131.080	183.156	131.080	1
23	0.018	0.202	0.295	0.200	13.491	105.942	216.222	104.835	0.8661
24	0.029	0.183	0.313	0.181	21.167	96.234	229.117	94.713	0.8345
25	0.035	0.170	0.326	0.166	25.529	89.188	238.573	87.296	0.8175
26	0.040	0.159	0.336	0.155	29.017	83.554	246.197	81.328	0.8070
27	0.043	0.150	0.345	0.145	31.691	78.836	252.625	76.306	0.8005
28	0.047	0.143	0.352	0.137	34.536	74.773	258.193	71.964	0.7966
29	0.050	0.136	0.359	0.130	36.857	71.210	263.106	68.130	0.7947
30	0.053	0.130	0.365	0.123	38.944	68.041	267.495	64.696	0.7941
31	0.056	0.124	0.370	0.117	40.810	65.194	271.459	61.590	0.7947
32	0.058	0.119	0.375	0.112	42.505	62.615	275.065	58.755	0.7961
33	0.060	0.115	0.380	0.107	44.060	60.261	278.368	56.148	0.7981

Summary of Work Done, contd.

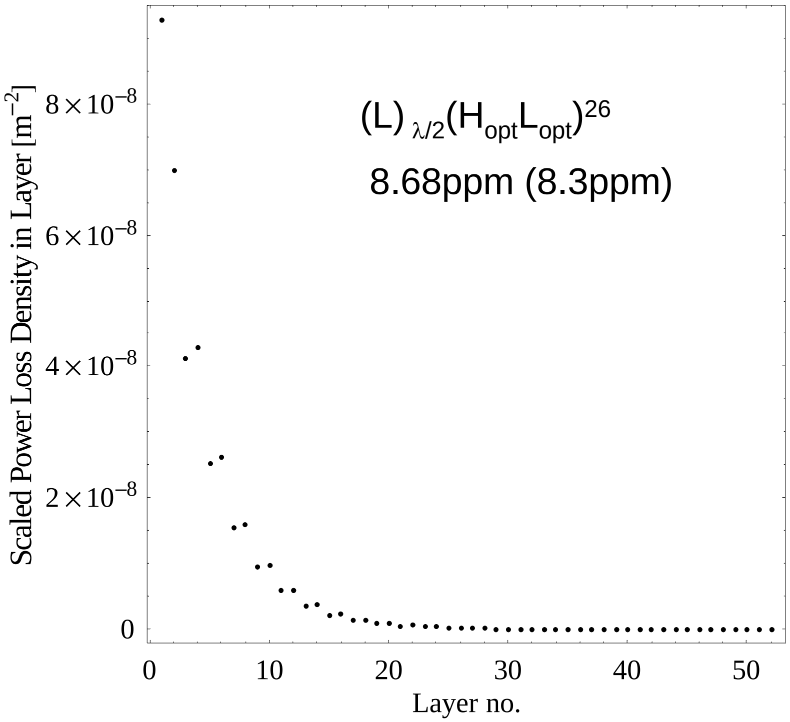
Coating Characterization Codes

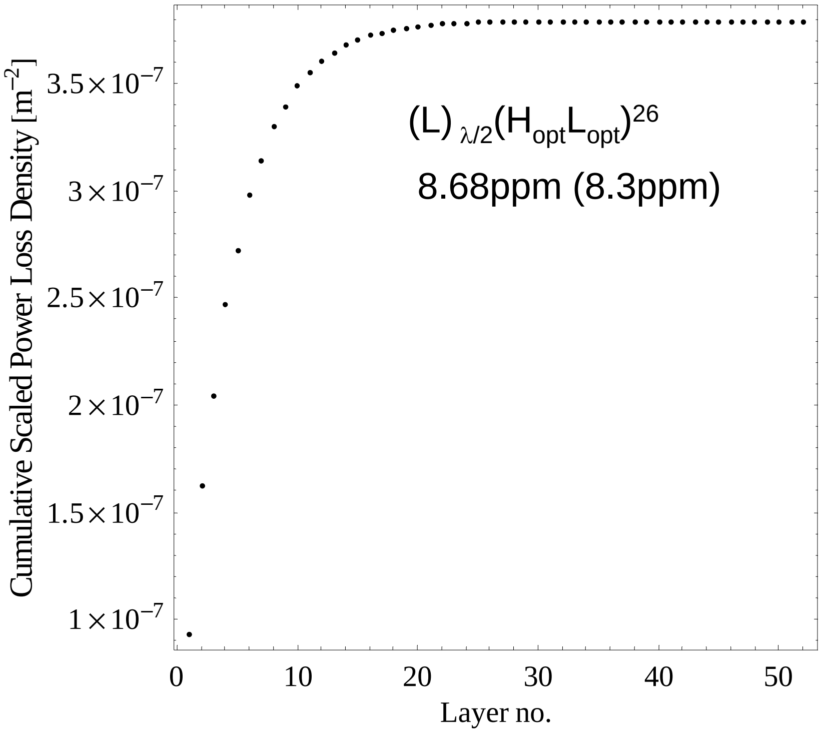
Motivation: Characterizing the optimized coatings.

Main result(s): Spatial distribution and amount of <u>optical losses</u>; <u>frequency response</u>; <u>angular response</u> (TE and TM incidence); <u>Robustness analysis</u> (distribution of reflectance on populations of realizations differing by the addition of random (uniform or gaussian) thickness errors.

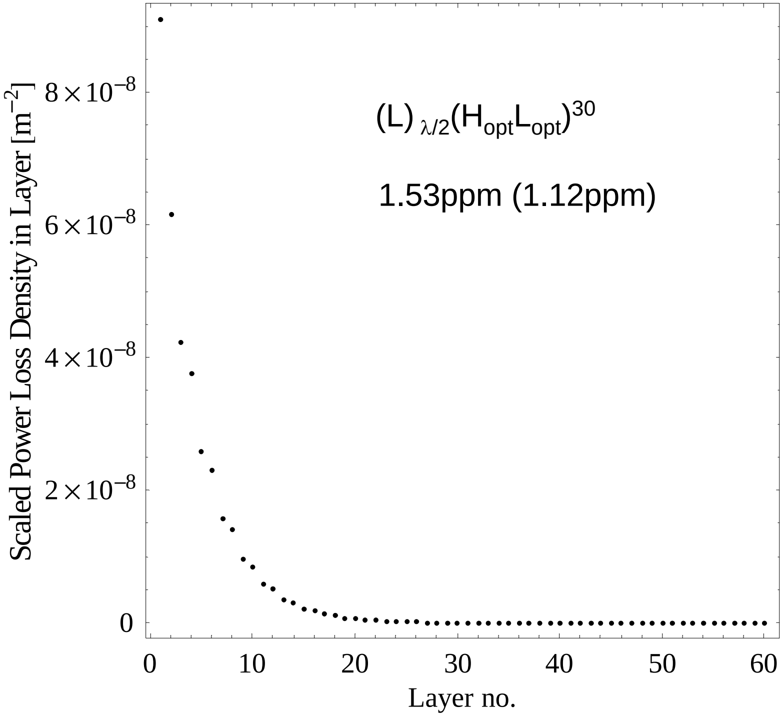
Products/deliverables: codes for computing the above curves tables (MATHEMATICA)

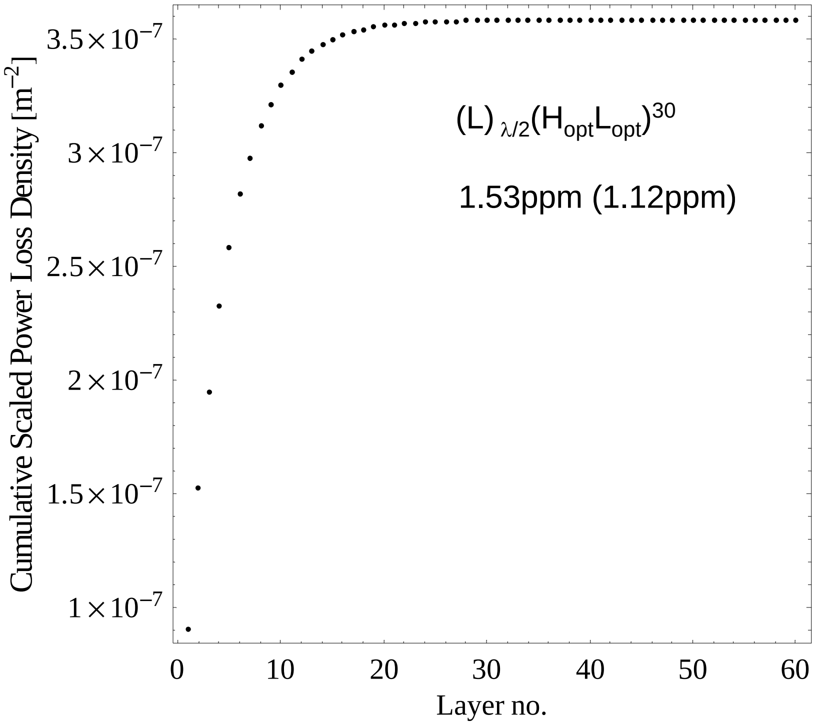
Optical Losses, I





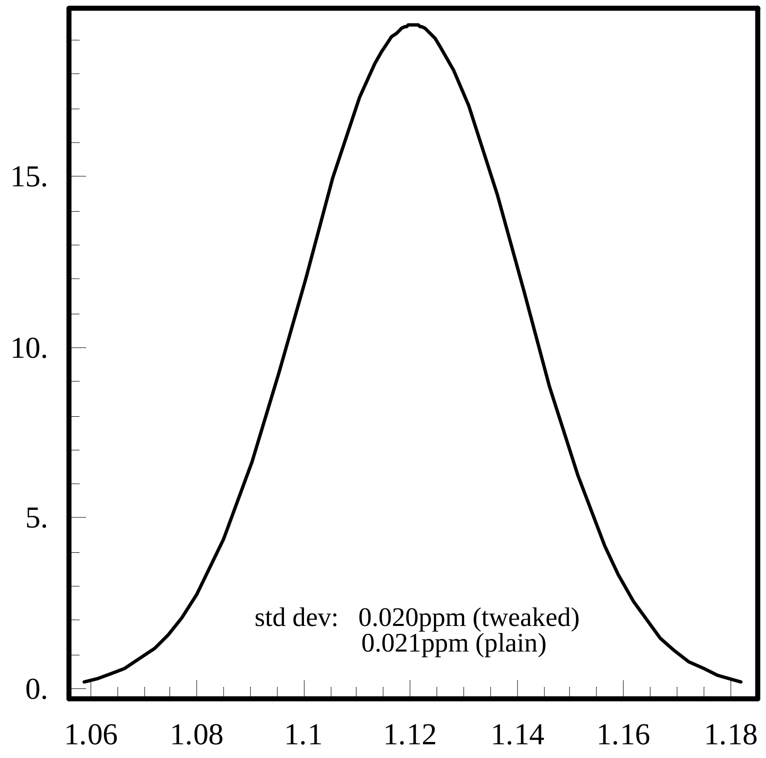
Optical Losses, II





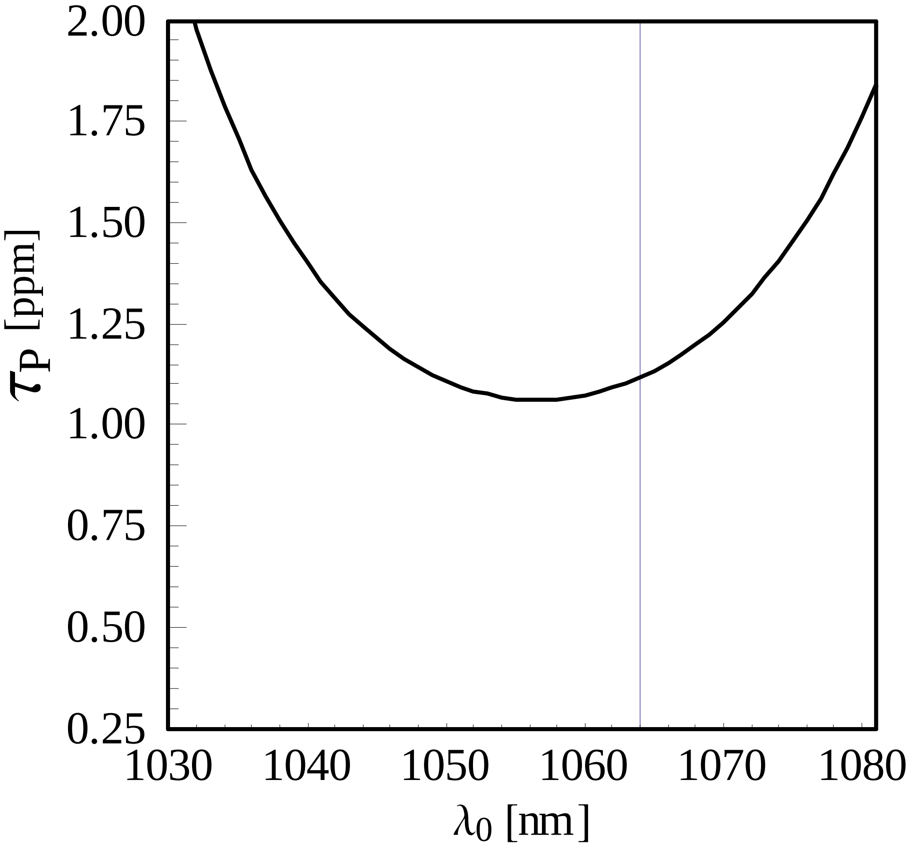
ETSD Coating Characterization, $\tau_P = 1.12\,ppm$

Design robustness check

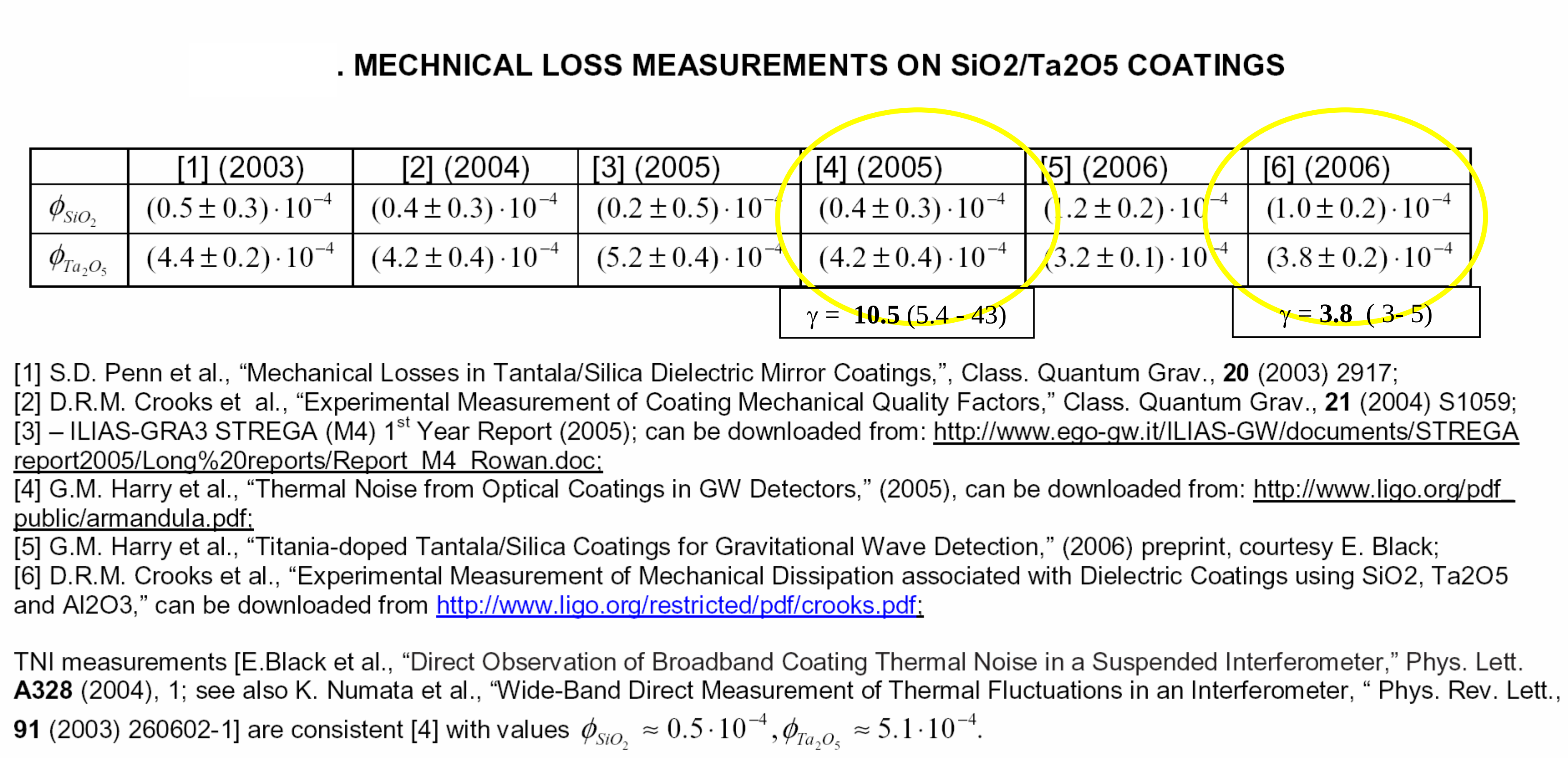


Transmittance [ppm] distribution over 10^4 realizations featuring random uniform layer thickness errors ~ ±1nm .

Frequency response



Loss Angle Measurements 2003-2006



. MECHNICAL LOSS MEASUREMENTS ON SiO2/Ta2O5 COATINGS

	[1] (2003)	[2] (2004)	[3] (2005)	[4] (2005)	[5] (2006)	[6] (2006)
ϕ_{SiO_2}	$(0.5 \pm 0.3) \cdot 10^{-4}$	$(0.4 \pm 0.3) \cdot 10^{-4}$	$(0.2 \pm 0.5) \cdot 10^{-4}$	$(0.4 \pm 0.3) \cdot 10^{-4}$	$(1.2 \pm 0.2) \cdot 10^{-4}$	$(1.0 \pm 0.2) \cdot 10^{-4}$
$\phi_{Ta_2O_5}$	$(4.4 \pm 0.2) \cdot 10^{-4}$	$(4.2 \pm 0.4) \cdot 10^{-4}$	$(5.2 \pm 0.4) \cdot 10^{-4}$	$(4.2 \pm 0.4) \cdot 10^{-4}$	$(3.2 \pm 0.1) \cdot 10^{-4}$	$(3.8 \pm 0.2) \cdot 10^{-4}$
				$\gamma =$ **10.5** (5.4 - 43)		$\gamma =$ **3.8** (3- 5)

[1] S.D. Penn et al., "Mechanical Losses in Tantala/Silica Dielectric Mirror Coatings,", Class. Quantum Grav., **20** (2003) 2917;
[2] D.R.M. Crooks et al., "Experimental Measurement of Coating Mechanical Quality Factors," Class. Quantum Grav., **21** (2004) S1059;
[3] – ILIAS-GRA3 STREGA (M4) 1st Year Report (2005); can be downloaded from: http://www.ego-gw.it/ILIAS-GW/documents/STREGA report2005/Long%20reports/Report_M4_Rowan.doc;
[4] G.M. Harry et al., "Thermal Noise from Optical Coatings in GW Detectors," (2005), can be downloaded from: http://www.ligo.org/pdf_public/armandula.pdf;
[5] G.M. Harry et al., "Titania-doped Tantala/Silica Coatings for Gravitational Wave Detection," (2006) preprint, courtesy E. Black;
[6] D.R.M. Crooks et al., "Experimental Measurement of Mechanical Dissipation associated with Dielectric Coatings using SiO2, Ta2O5 and Al2O3," can be downloaded from http://www.ligo.org/restricted/pdf/crooks.pdf;

TNI measurements [E.Black et al., "Direct Observation of Broadband Coating Thermal Noise in a Suspended Interferometer," Phys. Lett. **A328** (2004), 1; see also K. Numata et al., "Wide-Band Direct Measurement of Thermal Fluctuations in an Interferometer, " Phys. Rev. Lett., **91** (2003) 260602-1] are consistent [4] with values $\phi_{SiO_2} \approx 0.5 \cdot 10^{-4}, \phi_{Ta_2O_5} \approx 5.1 \cdot 10^{-4}$.

Silica figures changed by a factor 2 during last year

Loss Angle : Crucial Issues

•Reduction in the available loss-angle ratio, due to different (better) modeling of the measurement process by a factor ≈2 between 2005 and 2006;

•Further reduction expected from using low-noise Ti-doped tantala.

•Coating-thickness optimization makes sense if and only if loss angle ratio related quantity γ sensibly different from unity.

•LMA results [J.M. Mackowski] for Ti-doped Tantala (formula 5, old formula 2) suggest a value $\gamma \approx 6 \pm 5\%$ (would translate into $\gamma \approx 15$ for plain tantala).

More measurements needed for meaningful prototyping.

Loss Angle Uncertainty-Related Issues

- In deciding the design criterion to adopt, loss-angle uncertainties should be likely treated differently, depending on their nature, e.g.,

  -*measurement-related* uncertainties in ϕ should be likely assumed as being *Poisson distributed with peak at the minimum*, and the minimum value should be accordingly used in the synthesis;

  -*technology-related (process*) uncertainties in ϕ should likely be assumed as *Gaussian distributed around the average value*, and the average or least-favourable value should be used for synthesis.

We need better knowledge of material properties in order to produce a final prototype design