Numerical Calculations of Diffraction Losses in Advanced Interferometric Gravitational Wave Detectors

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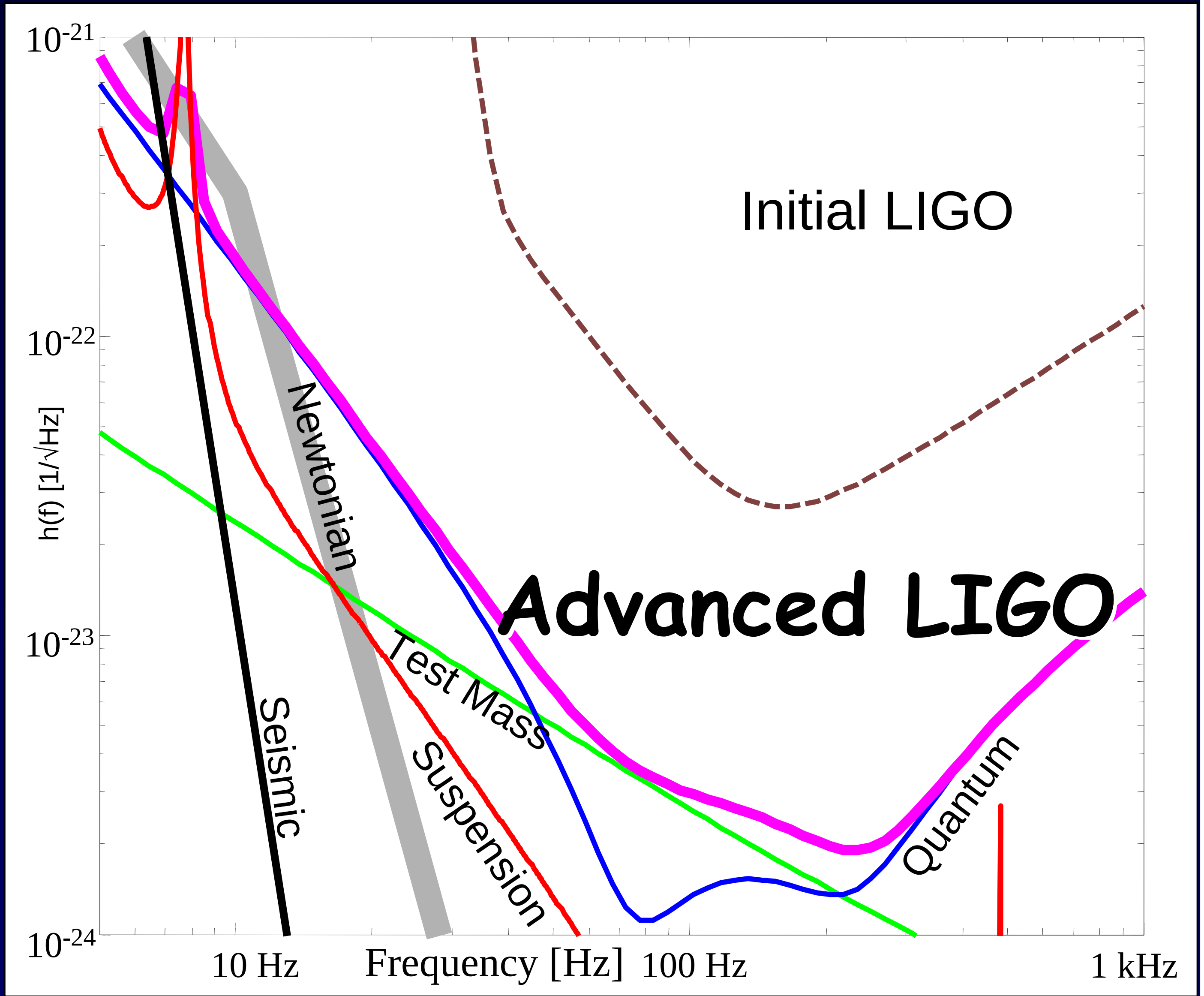
Introduction

Noise Budget

- Newtonian background
- Seismic ‘cutoff’
- Suspension thermal noise
- Test mass thermal noise
- Unified quantum noise



Considerations

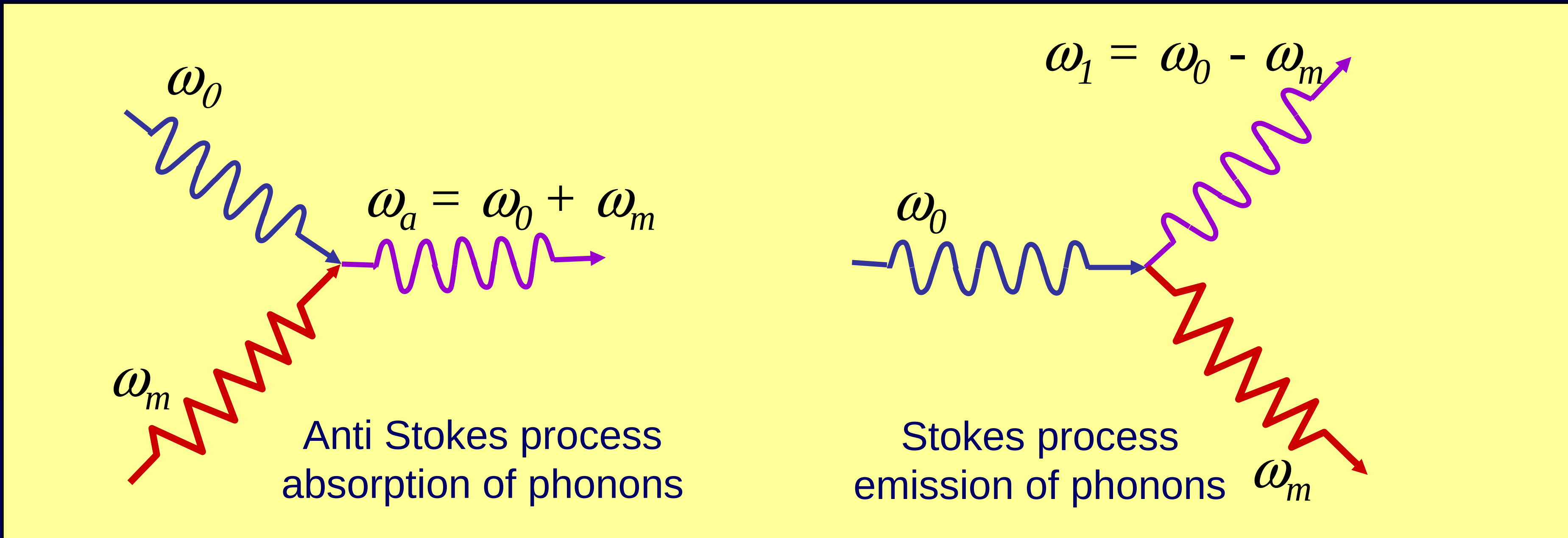
- Advanced gravitational wave detectors need high power laser.
- Downside: Thermal lensing, radiation pressure noise and the possibility of parametric instabilities.
- With finite size mirrors a fraction of the beam will fall outside the test mass.
- Diffraction losses depend on the relation between the spot size, the mode shape and the mirror size.

Parametric Instabilities

When energy densities get high things go unstable…

- Braginsky et al predicted parametric instabilities can happen in advanced detectors.
  – resonant scattering of photons with test mass phonons.
  – acoustic gain like a laser gain medium.

Photon - Phonon Scattering



Instabilities from photon-phonon scattering

- A test mass phonon can be **absorbed** by the photon, increasing the photon energy (damping);
- The photon can **emit** the phonon, decreasing the photon energy (potential acoustic instability).

Parametric Gain

Cavity Power

Mechanical Q

Stokes mode contribution

Anti-Stokes mode contribution

$$R \approx \frac{2PQ_m}{McL\omega_m^2}\left(\frac{Q_1\Lambda_1}{1+\Delta\omega_1^2/\delta_1^2} - \frac{Q_{1a}\Lambda_{1a}}{1+\Delta\omega_{1a}^2/\delta_{1a}^2}\right) > 1$$

$$\Delta\omega_{1(a)} = |\omega_0 - \omega_{1(a)}| - \omega_m$$

Λ - overlap factor

$$\delta_{1(a)} = \frac{\omega_{1(a)}}{2Q_{1(a)}}$$

Fundamental mode frequency

High order transverse mode frequency

Acoustic mode frequency

V. B. Braginsky, S.E. Strigin, S.P. Vyatchanin, *Phys. Lett. A*, 305, 111, (2002)

Instability Conditions

- High circulating power P
- High mechanical Q
- High optical mode Q

+

- Mode shapes overlap (High overlap factor Λ)
- Frequency coincidence – $\Delta\omega$ small

$R > 1 \Longrightarrow$ Instability

Diffraction Losses

Diffraction Losses

- Clipping approximation:

$$\mathcal{L}_{clip} = \int_a^\infty |U(r)|^2 \, 2\pi r \, dr$$

- Cavity Eigenvalues:

$$\gamma_{mn} U_{mn}(x, y) = \iint K(x, y, x_0, y_0) U_{mn}(x_0, y_0) \, dx_0 \, dy_0$$

$$\text{Power loss per round trip} = 1 - |\gamma_{mn}|^2$$

Diffraction Losses

- Geometry of the system determines whether the clipping underestimates or overestimate the diffraction losses.

  (R. E. Spero LIGO-T0920002-D and his later update in 2001)

- Fresnel number:

$$N = \frac{a^2}{\lambda L}$$

a: Mirror size radius
L: Cavity length
λ: laser wavelength

Diffraction Losses

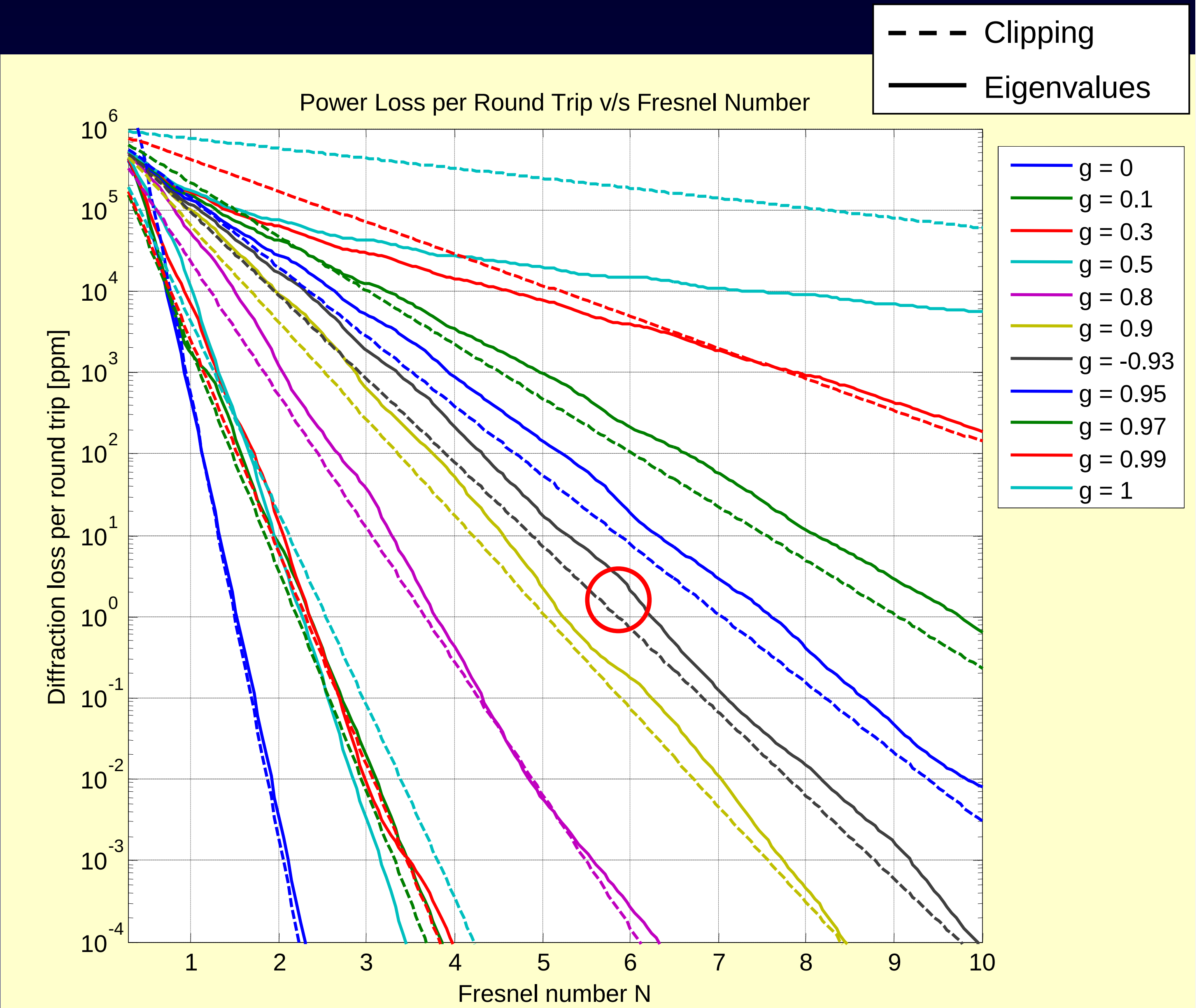
Mirror size:
a = 0.157 m

Cavity length:
L = 4000 m

Fresnel number:
N = 5.79

Mirror g-factor:
g = -0.927

Diffraction loss per round trip:
δ_{00} = 3.608 ppm



Diffraction Losses

- Cavity Finesse:

$$\mathcal{F} \cong \frac{2\pi}{T_i + D_i + L_i + T_e + D_e + L_e}$$

- Intensity at resonance:

$$I_{circ} \approx \frac{4(T_i + L_i)}{(T_i + D_i + L_i + T_e + D_e + L_i)^2} I_{inc}$$

Diffraction Losses

- Infinite mirror v/s R size mirror:

$$\sqrt{\frac{I_\infty}{I_R}} = \frac{T_i + D_i + L_i + T_e + D_e + L_e}{T_i + L_i + T_e + L_e}$$

- Total diffraction loss:

$$D_T = D_i + D_e = (T_i + L_i + T_e + L_e)\left(\sqrt{\frac{I_\infty}{I_R}} - 1\right)$$

Numerical Simulation

Gaussian Modes

- Hermite – Gaussian:

$$U_{m,n}(x,y,z)=\left(\frac{1}{2\pi}\right)^{\frac{1}{2}}\sqrt{\frac{\exp\{j(2m+2n+1)\psi(z)\}}{2^m 2^n m!n!\omega(z)^2}}\times$$

$$H_m\left(\frac{\sqrt{2}x}{\omega(z)}\right)H_n\left(\frac{\sqrt{2}y}{\omega(z)}\right)\exp\left\{-j2kz-jk\left(\frac{x^2+y^2}{2R(z)}\right)-\frac{x^2+y^2}{\omega(z)^2}\right\}$$

- Laguerre – Gaussian:

$$U_{l,m}(r,\phi,z)=\sqrt{\frac{4l!}{(1+\delta_{0,m})\pi(l+m)!}}\left(\frac{\exp\{j(2l+m+1)\psi(z)\}}{\omega(z)}\right)\cos(m\phi)\times$$

$$\left(\frac{\sqrt{2}r}{\omega(z)}\right)^m L_{l,m}\left(\frac{2r^2}{\omega(z)^2}\right)\exp\left\{-jkz-jk\frac{r^2}{2R(z)}-\frac{r^2}{\omega(z)^2}\right\}$$

Fast Fourier Transform

- Propagation matrix:

$$A(p,q,z_L) = \exp\left\{-jkz_L + j\pi\lambda\left(p^2+q^2\right)z_L\right\}$$

- The Fourier transform of a gaussian function is always a another gaussian transform of the same order.

$$\mathscr{F}\{U(x,y,z)\} = \iint U(x,y,z)e^{-jpx}e^{-jqy}\,dxdy$$

- The inverse Fourier transform is then written:

$$\mathscr{F}^{-1}\{U(p,q,z)\} = \left(\frac{1}{2\pi}\right)^2 \iint U(p,q,z)e^{jpx}e^{jqy}\,dpdq$$

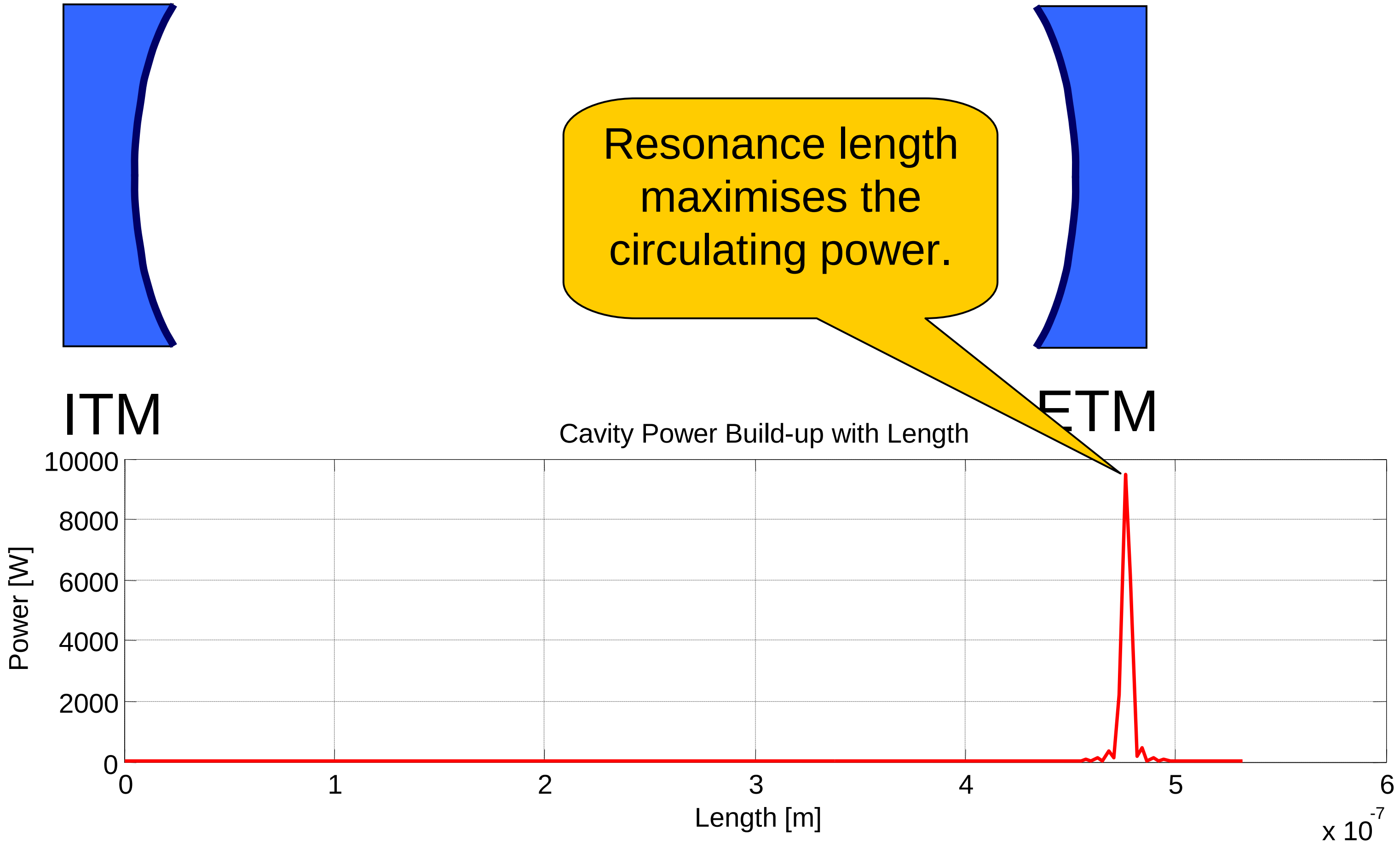
- If z_0 is the propagation starting point then the final field is:

$$U(x,y,z_L) = \mathscr{F}^{-1}\left\{\mathscr{F}\{U(x,y,z_0)\}\times A(p,q,z_L)\right\}$$

J. Y. Vinet et al, *J. Phys. I France,* **2,** 1287, (1992)

FFT Algorithm

- Separate codes developed in Matlab® at UWA and Caltech.
- Find resonance length: Move the ETM away from the ITM.
- Resonance length is calculated for a particular mode.

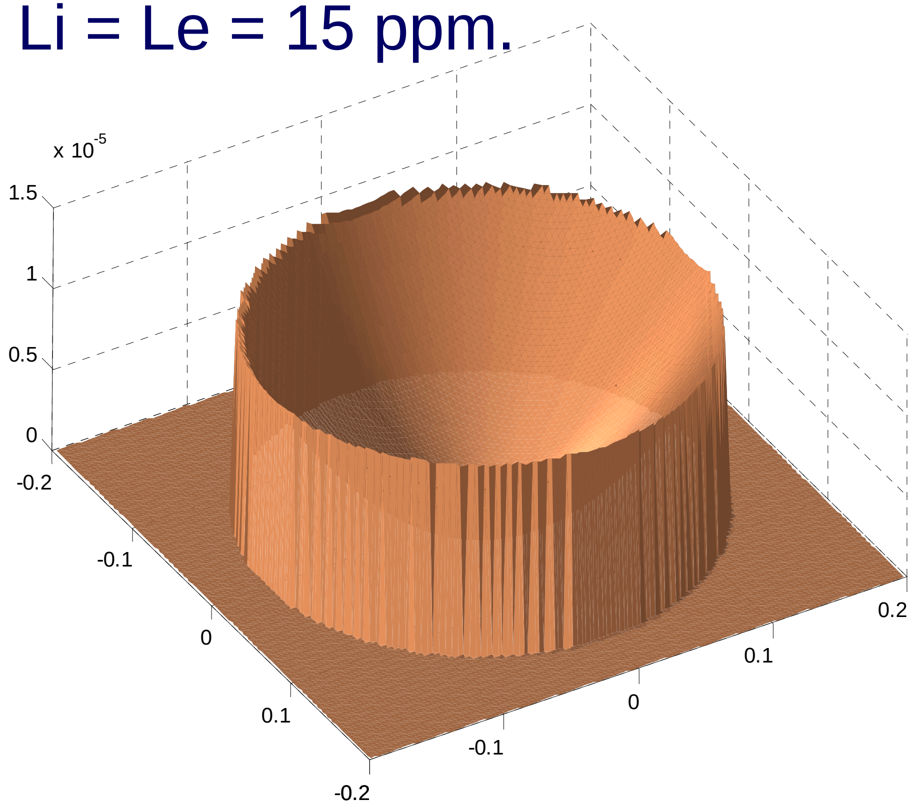


FFT Algorithm

- Start from the waist propagating towards the ETM.
- Bounce off the ETM towards the ITM.
- Part of the light is transmitted through the ITM.
- The rest bounces back towards the cavity waist.
- This is iterated until steady state power is reached.

Mirrors

- Standard Fused Silica mirrors.
- Smooth surface, but imperfections can be easily added.
- Mirror radius of curvature 2076 m.
- Ti = 5000 ppm, Te = 1 ppm, Li = Le = 15 ppm.
  (R. Lawrence, MIT *PhD Thesis*, 2003)

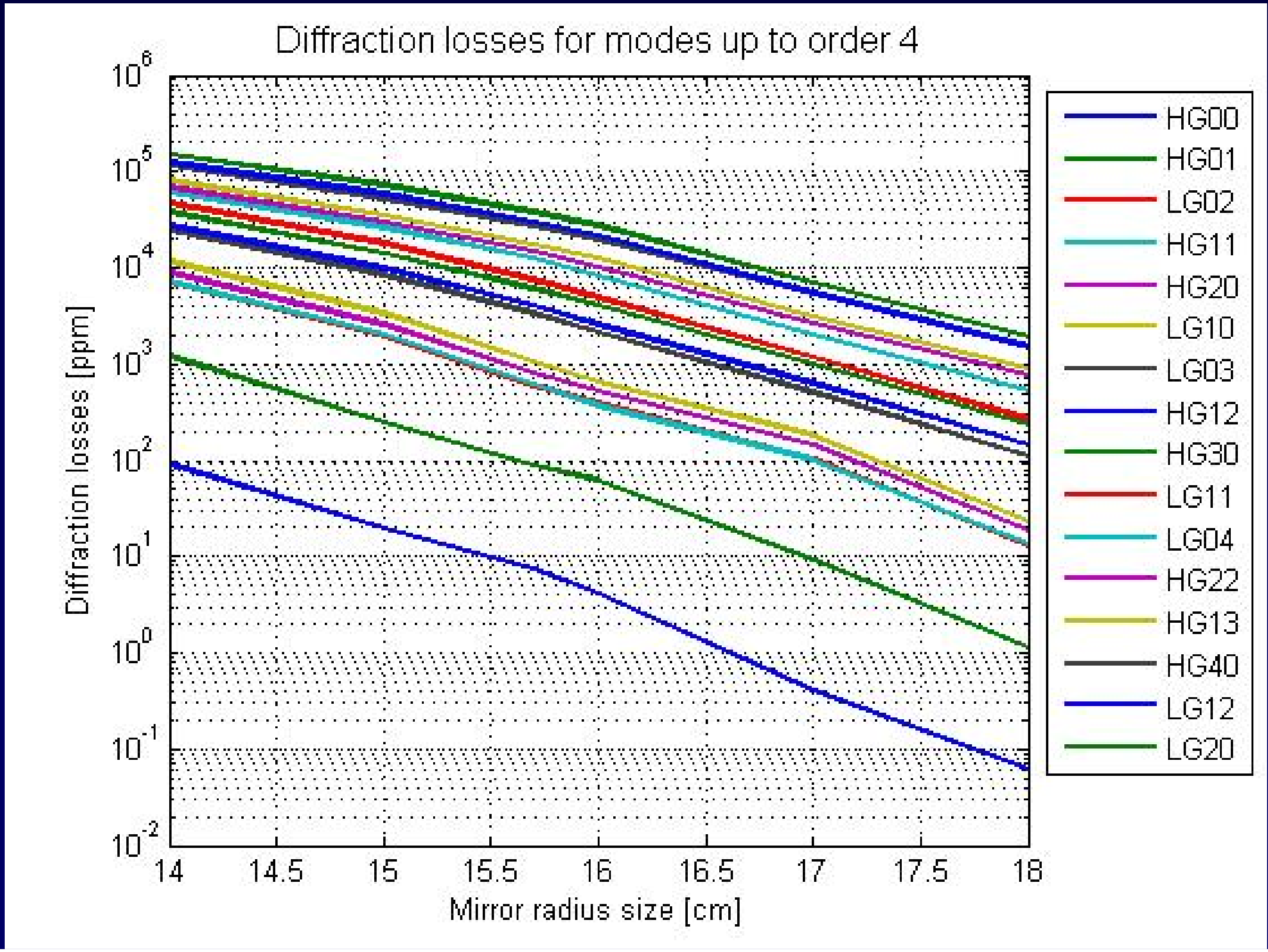


Results

A.- Diffraction Losses

Diffraction Losses

- Diffraction losses for different modes for varying mirror sizes.
- HG order number *(m+n)*, LG order number *(2l+m).*



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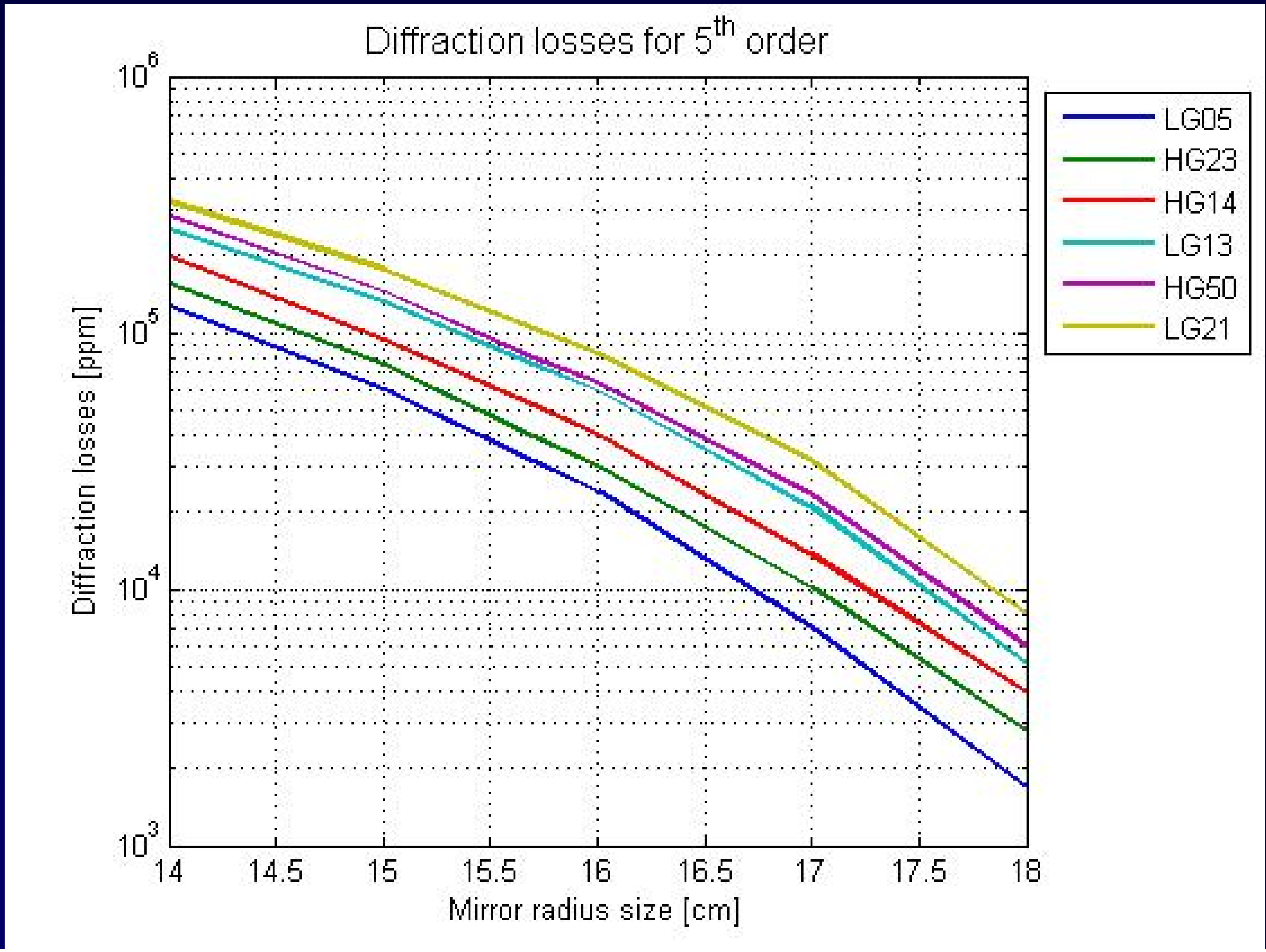

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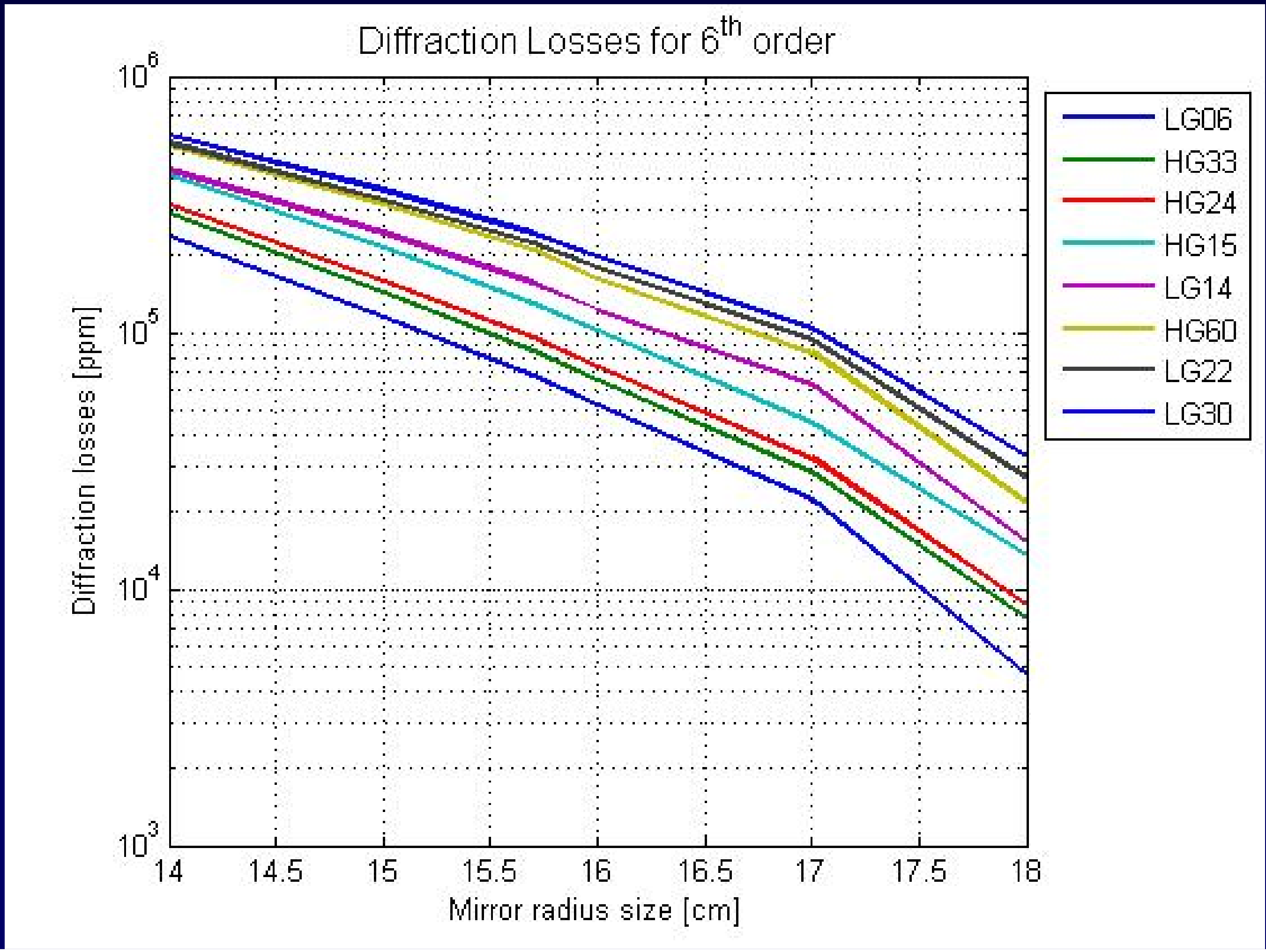

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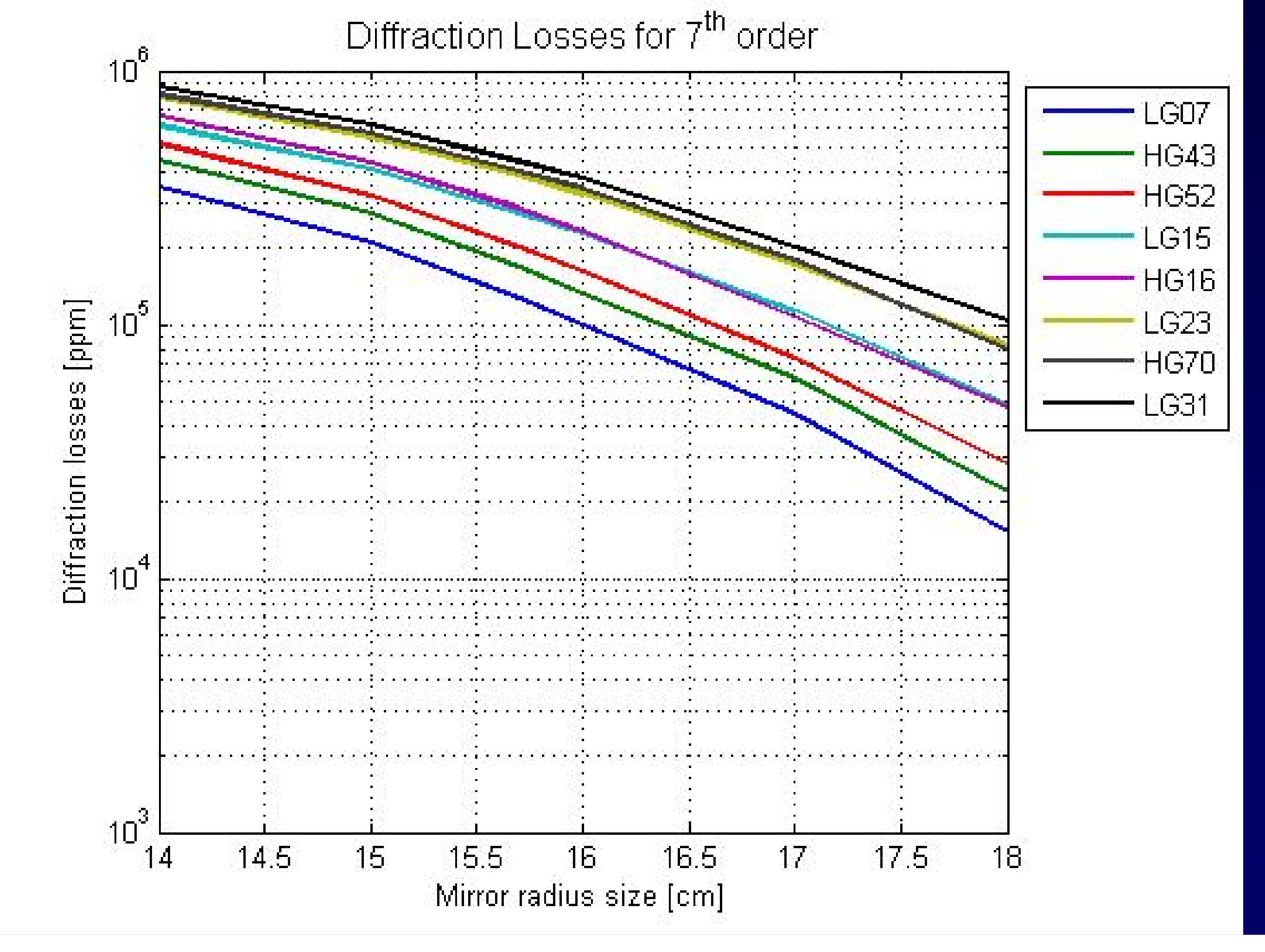
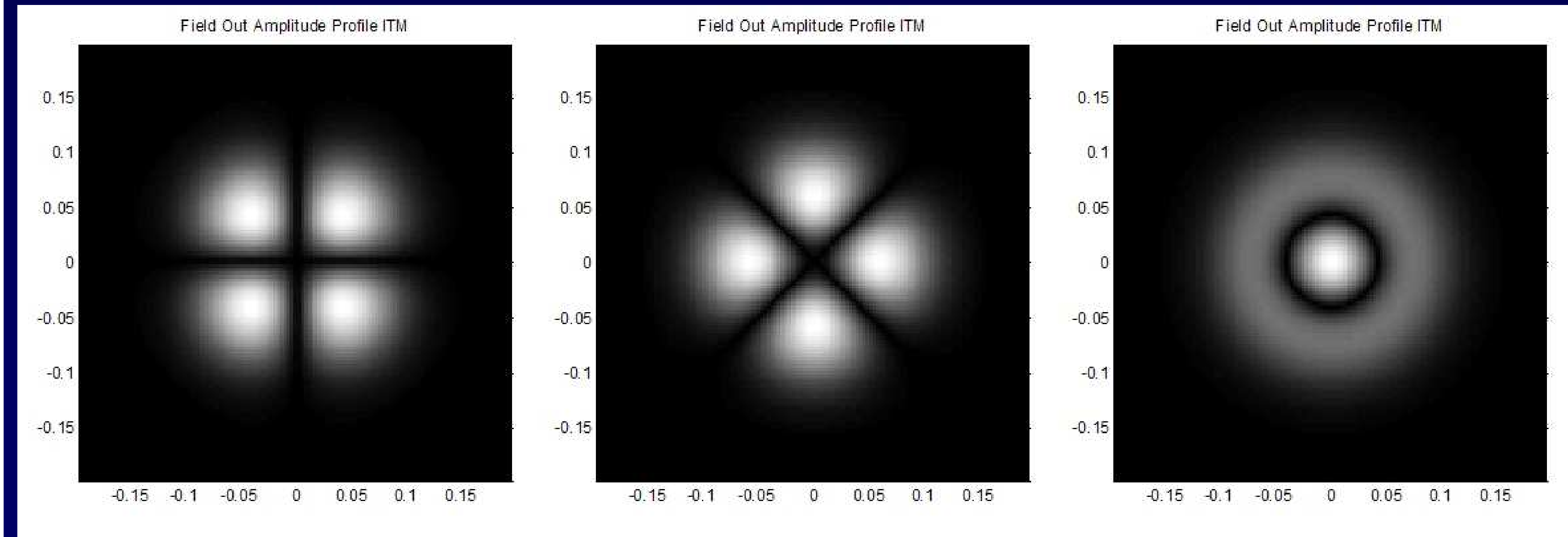


Diffraction Losses

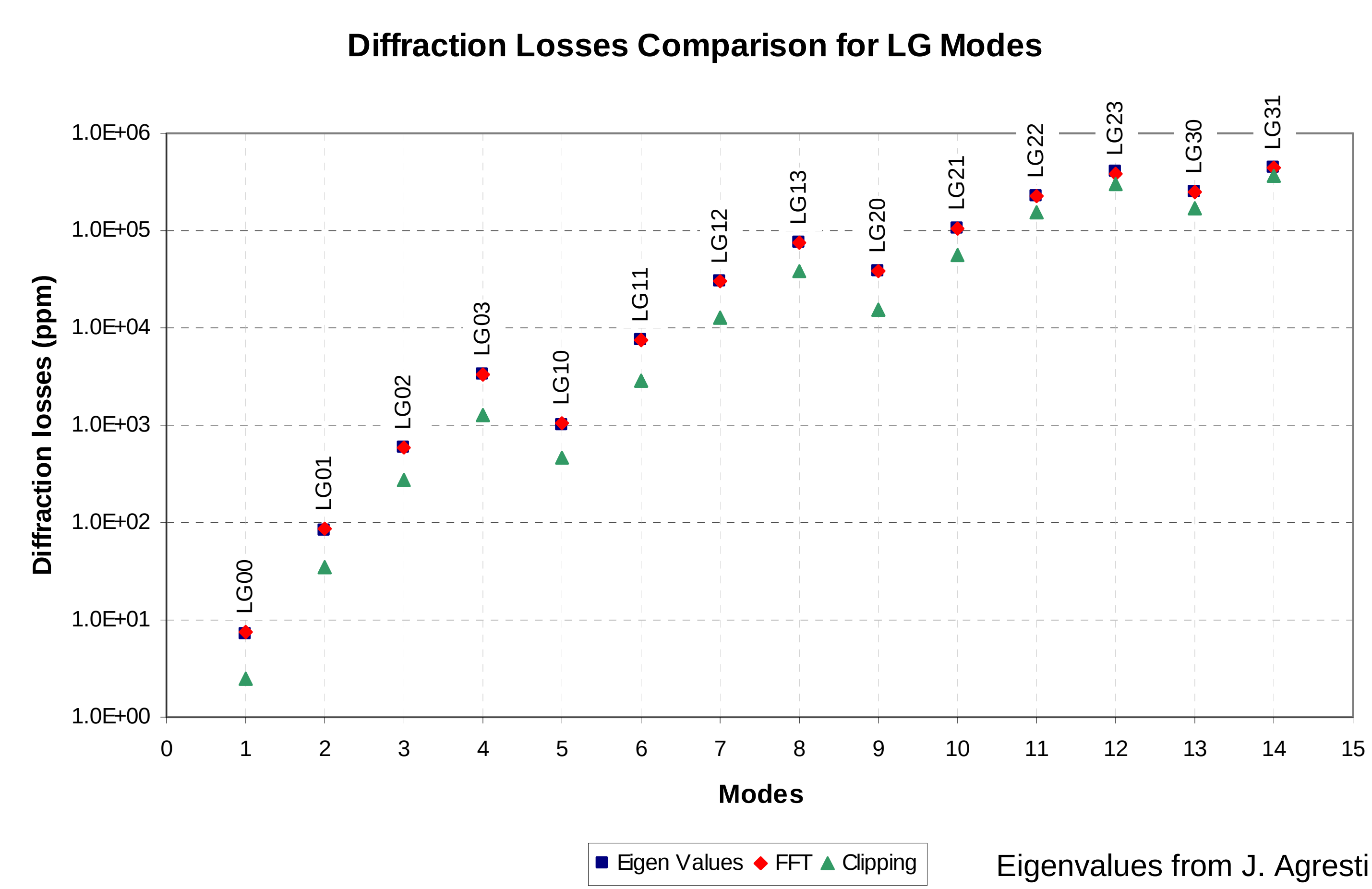
- Similar diffraction losses between HG11 and LG02.
- LG02 corresponds to HG11 twisted by 45º (or vice versa).
- For comparison we show LG10.

HG11 LG02 LG10



Diffraction Losses Comparison

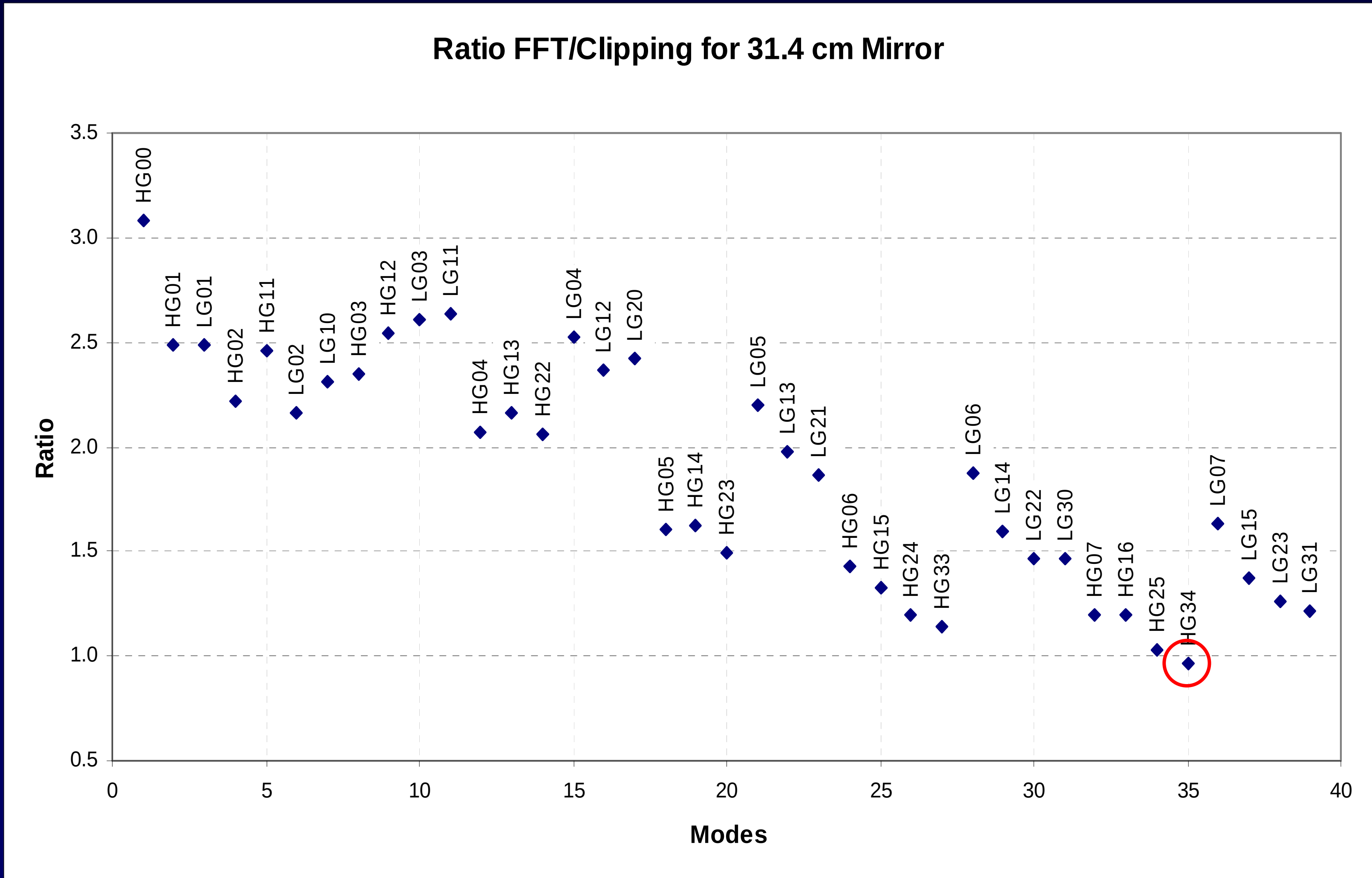
- Eigenvalues are very similar to the FFT simulation results.
- Difference between Clipping and FFT grows smaller with higher order modes.



Eigenvalues from J. Agresti

Clipping Approx v/s FFT Simulation

- Ratio between FFT simulation over clipping approximation.

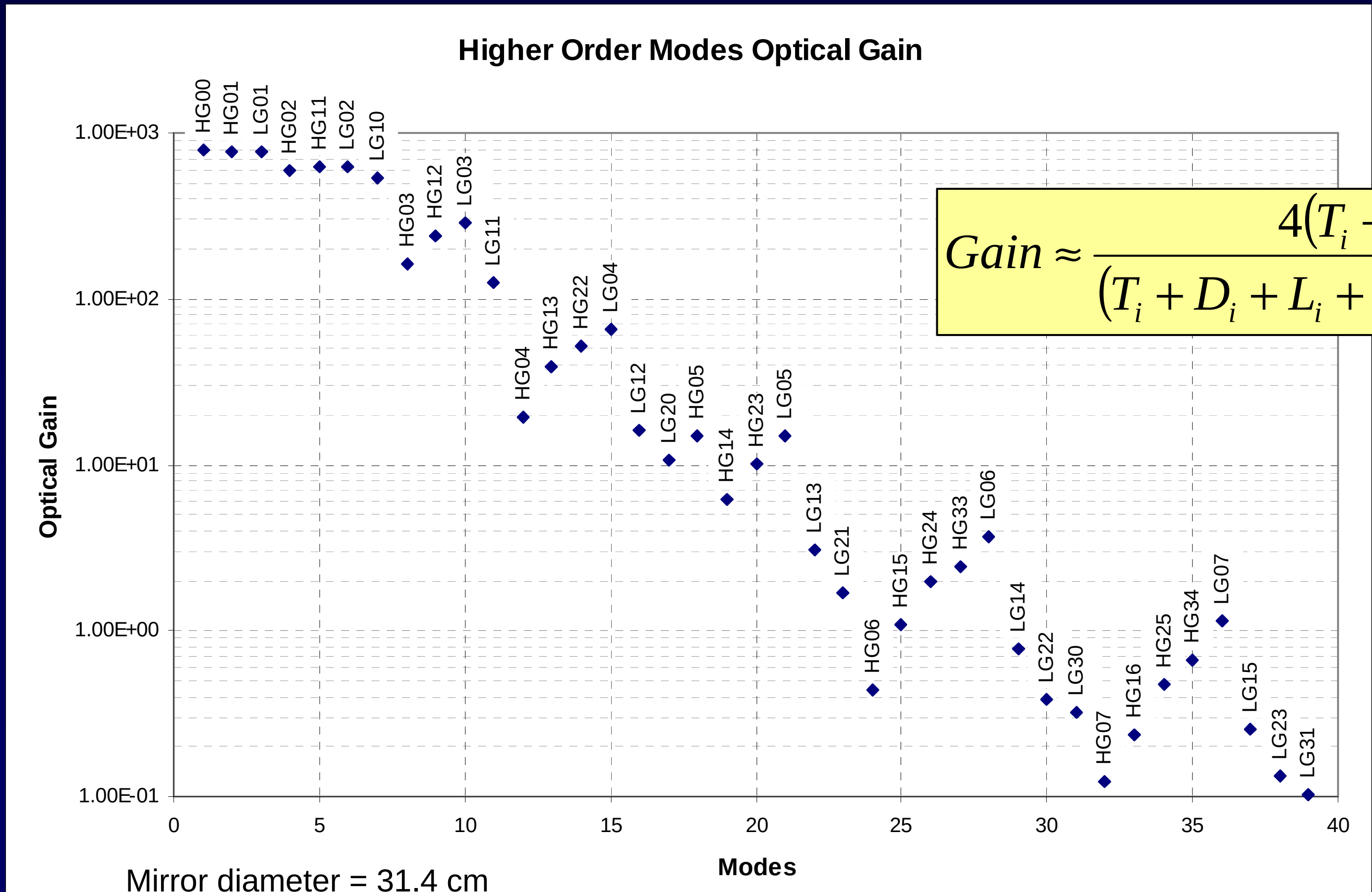


Results

A.- Diffraction Losses
B.- Optical Gain

Optical Gain

- Higher order modes have reduced optical gain.
- Optical gain also depends on the energy distribution of the mode.



$$Gain \approx \frac{4(T_i + L_i)}{(T_i + D_i + L_i + T_e + D_e + L_i)^2}$$

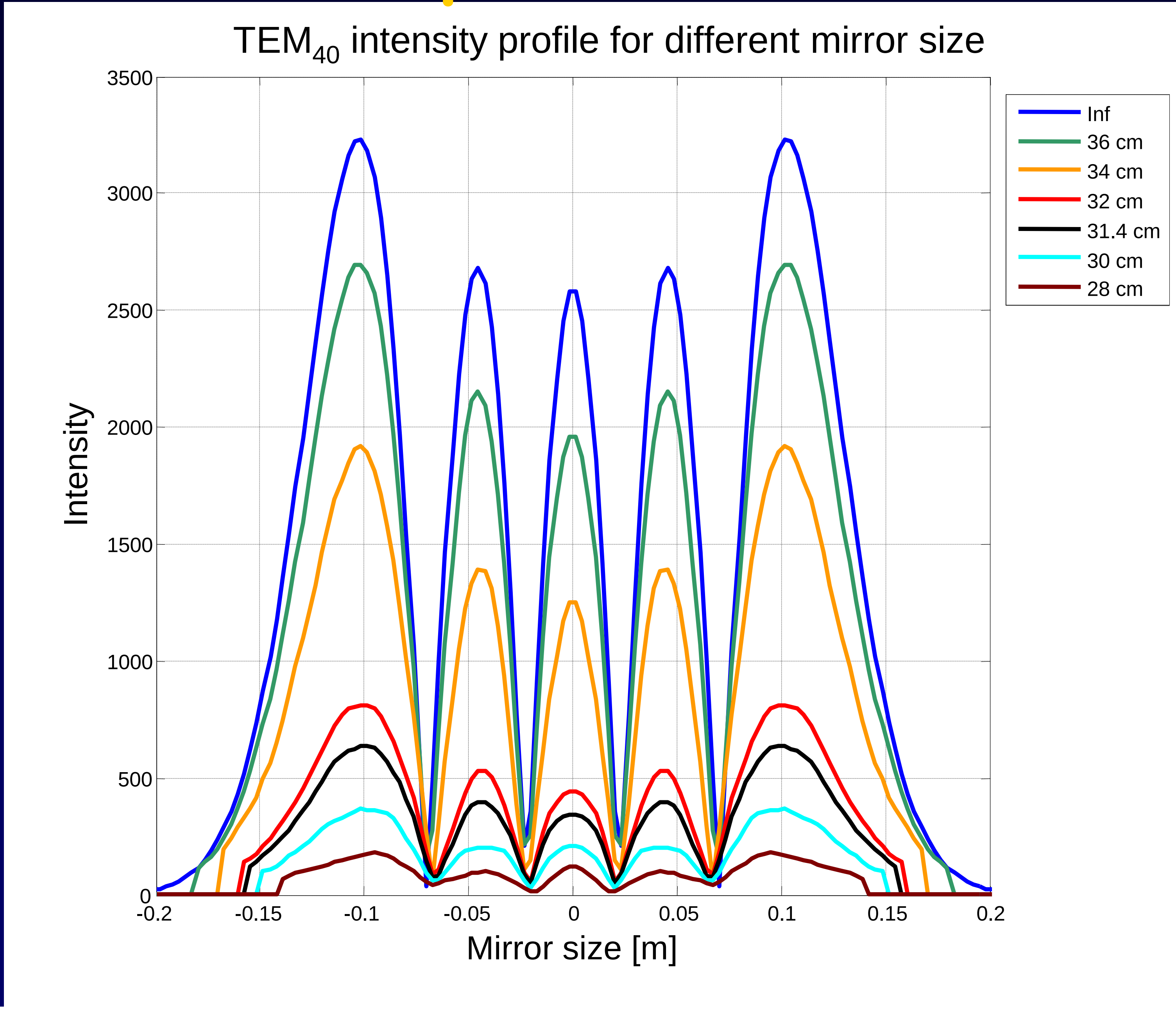
Mirror diameter = 31.4 cm

Optical Gain

- Bigger mirror size smaller diffraction losses.
- Bigger mirror size higher optical gain.
- Diffraction losses and optical gain for mode HG40.

Mirror Diameter	Diffraction Losses	Optical Gain
(cm)	(ppm)	
28	116022	1.37
30	52437	6.07
31.4	27092	19.44
32	19256	34.01
34	5315	187.40
36	1506	469.50

Optical Gain



Results

A.- Diffraction Losses
B.- Optical Gain
C.- Mode Frequency

Higher Mode Frequency

- Frequency shift for higher order mode:

$$\frac{\upsilon_0}{\pi}(m+n)\arccos\left(\sqrt{g_1 g_2}\right)$$ for HG modes,

$$\frac{\upsilon_0}{\pi}(2l+m)\arccos\left(\sqrt{g_1 g_2}\right)$$ for LG modes.

Higher Mode Frequency

- Infinite size mirrors agree with theoretical values.
- For modes of order 7 Δf = 32.151 kHz.
- Frequency given by: $\Delta f = \frac{\Delta l}{L} f_{YAG}$
- Infinite size mirrors:

Mode	Δfreq [kHz]
HG70	32.151
HG16	32.151
HG52	32.151
HG43	32.151
LG07	32.151

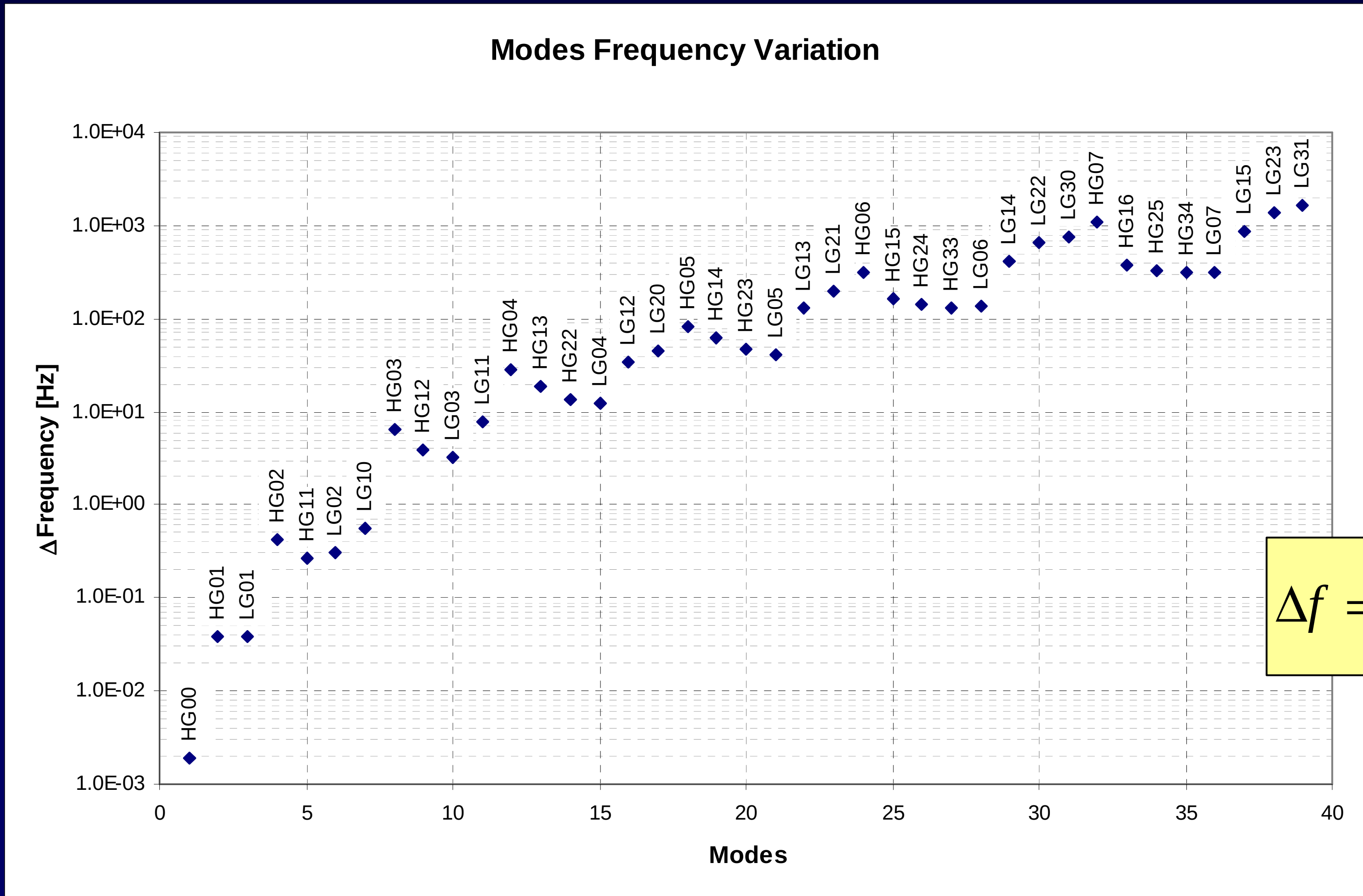
Higher Mode Frequency

- Frequency shift from theoretical value for modes of order 7.

Mirror Diameter	Δfreq HG70	Δfreq HG61	Δfreq HG52	Δfreq HG43	Δfreq LG07	Δfreq LG15	Δfreq LG23	Δfreq LG31
[cm]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]
28	3466	1658	1345	1277	1251	2715	3735	4213
30	2144	799	686	659	649	1584	2324	2698
31.4	1091	381	325	316	310	875	1401	1688
32	897	282	248	237	233	688	1127	1371
34	317	103	70	64	61	215	400	515
36	73	59	34	25	21	76	145	189

Frequency Variation

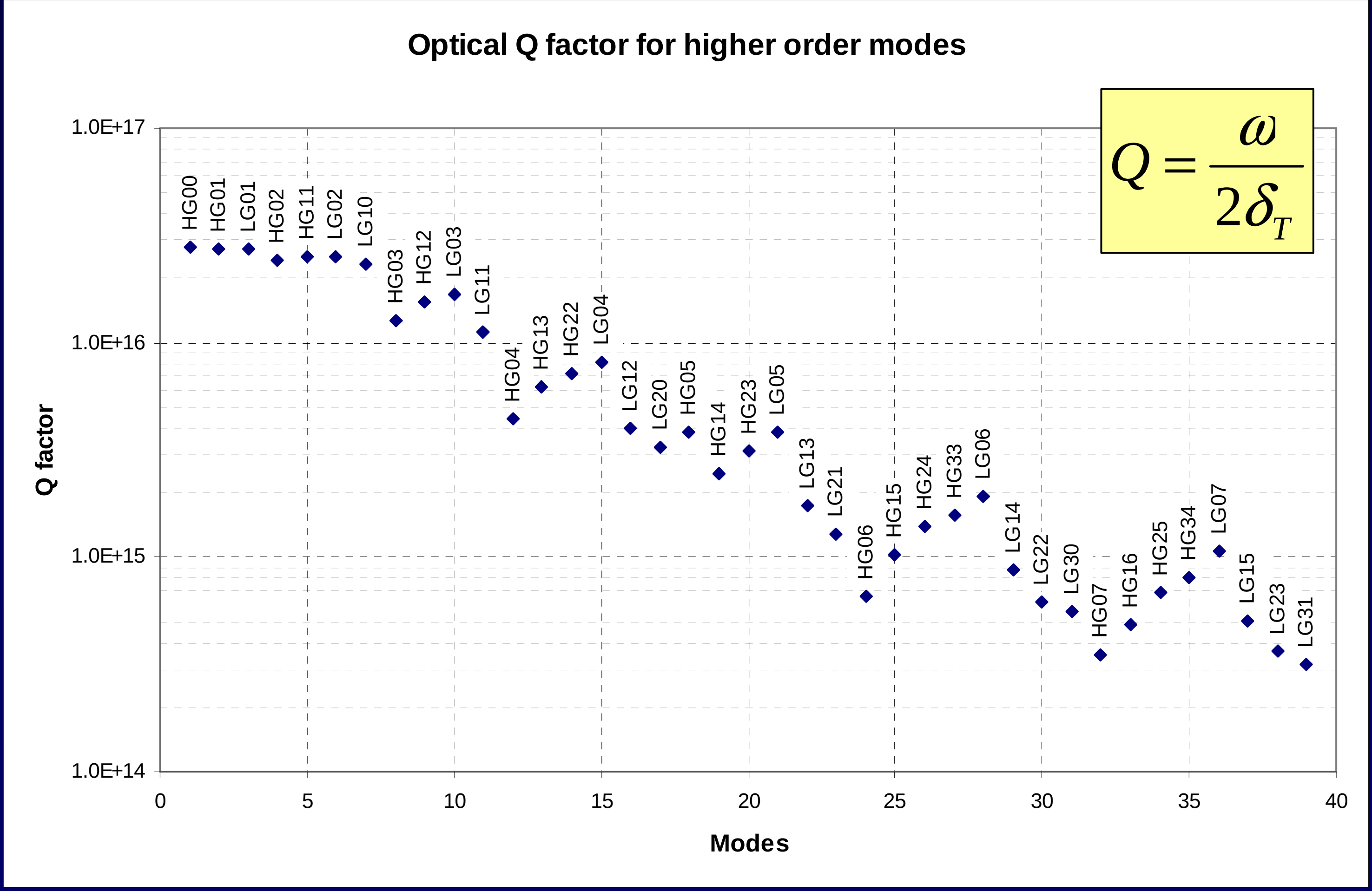
- Frequency variation for higher order modes.
- Mirror size = 31.4 cm.



$$\Delta f = \frac{\Delta l}{L} f_{YAG}$$

Optical *Q* factor

- Q variation for higher order modes.
- Mirror size = 31.4 cm.



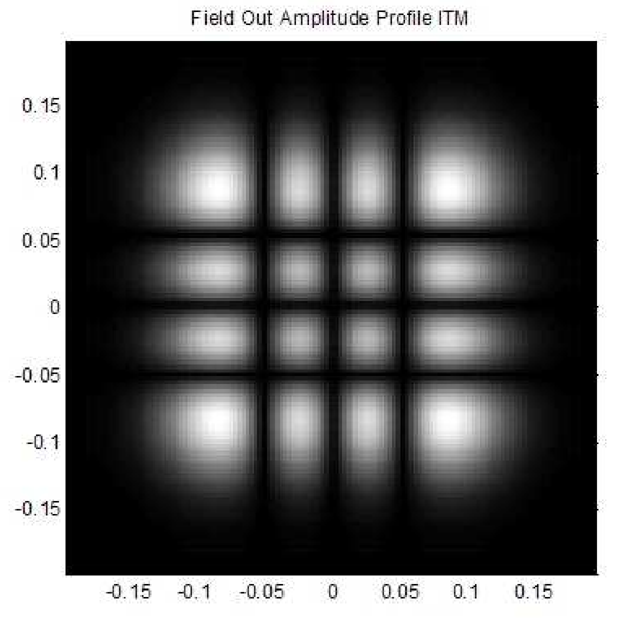
Results

A.- Diffraction Losses
B.- Optical Gain
C.- Mode Frequency
D.- Mode Shape

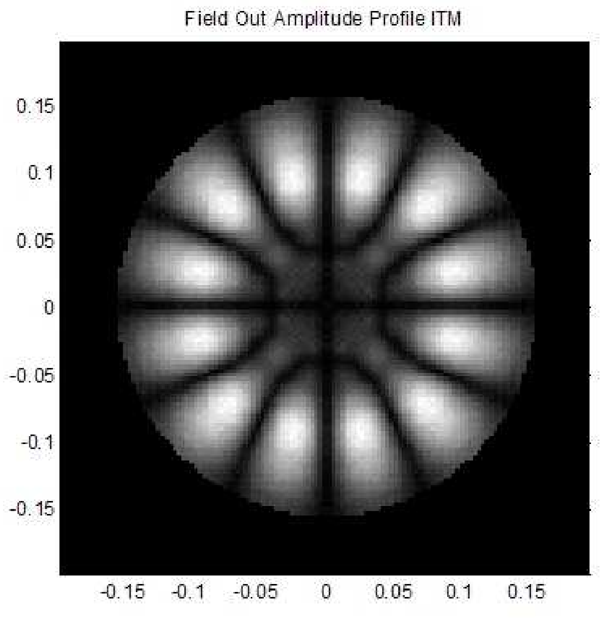
Mode Shape

- Resonant mode inside the cavity depends on the mirror size.
- Infinite size mirrors contains the “nominal” mode.
- Finite mirror: Is it still mode HG33?
- Similar to LG06 but twisted by 30º.

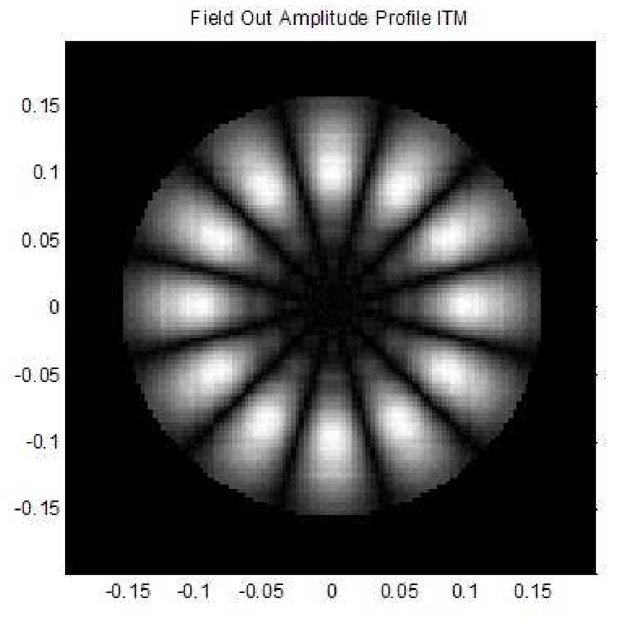
Mode HG33
Infinite size mirror



Mode HG33 Mirror
diameter = 31.4 cm



Mode LG06 Mirror
diameter = 31.4 cm



Conclusions

Conclusions

- Diffraction losses of higher order modes depend on the size of the mirrors.
- In general clipping approximation underestimates the diffraction losses.
- Predicted mode frequencies are offset from the infinite mirror case by up to a few kHz.
- Frequency shift will also affect the optical Q factor of the higher order modes.
- Finite size mirrors significantly alters the mode shape of higher order modes affecting the overlapping parameter.
- Diffraction losses, frequency shift, optical Q factor and mode shape are needed to calculate the parametric gain R.