



Thoughts on Thermal Compensation and Stable Recycling Cavity Design for Advanced LIGO

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Main Points

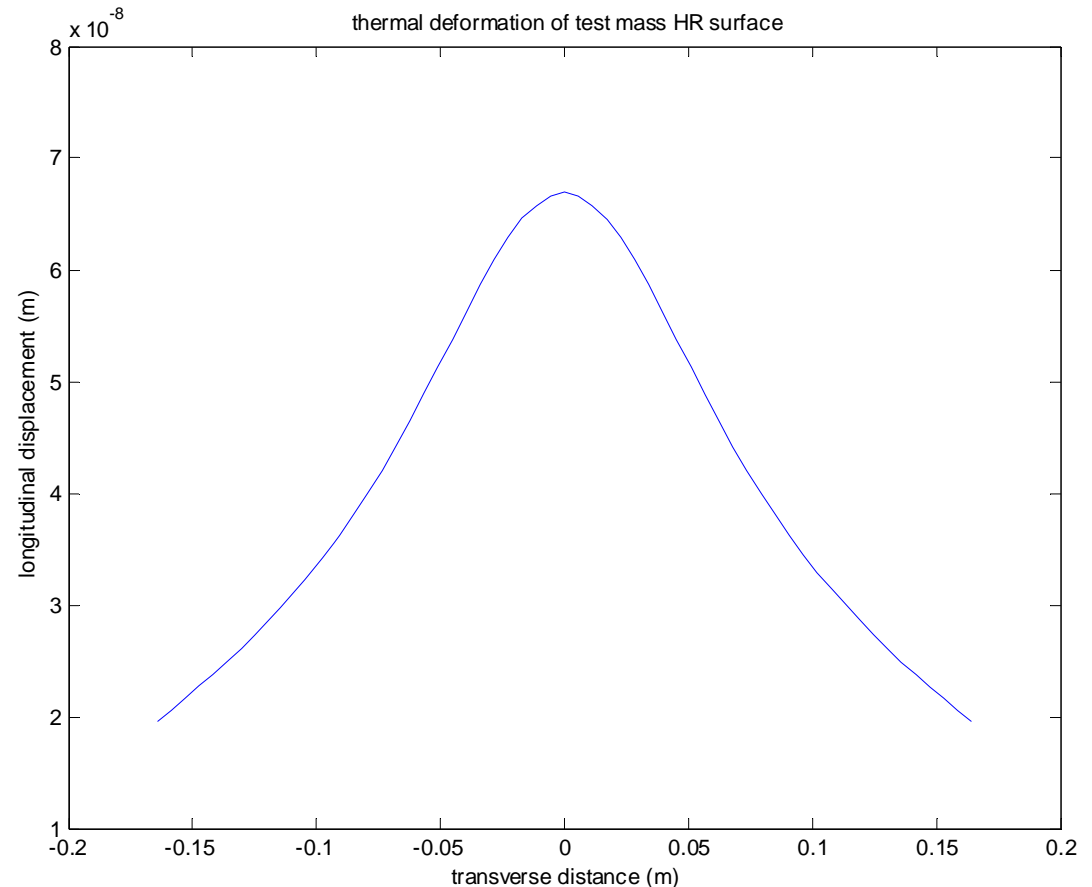
- Modes in thermally distorted arm cavities
 - » looks pretty good
- “Thermally invariant” stable recycling cavity design
 - » has its drawbacks but the GW sidebands sail through to output unharmed

Arm Cavity Modes

Previously we have assumed a thermal radius of curvature when calculating arm cavity modes.

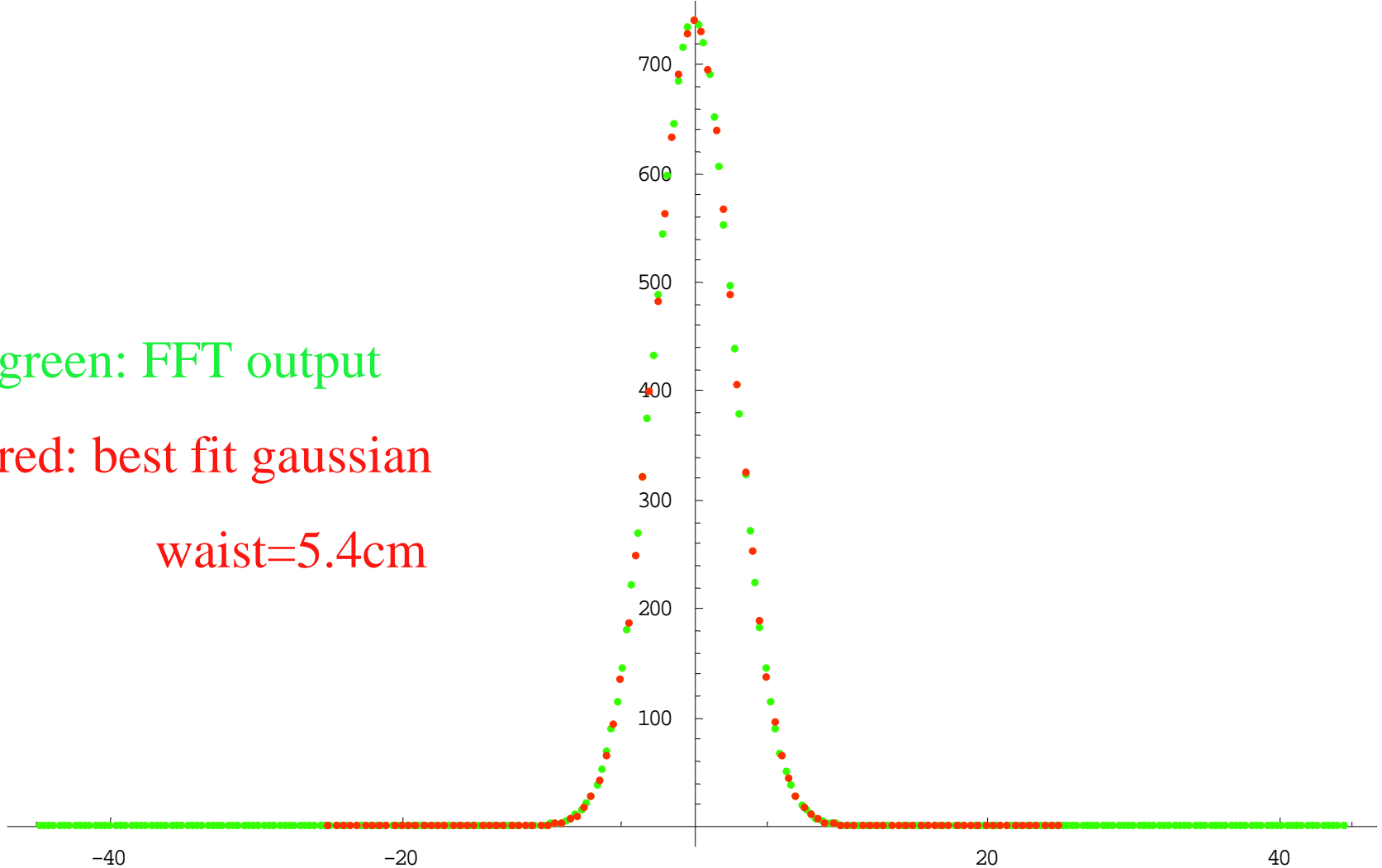
As the thermal model shows, the surface change is not a pure curvature.

Does this matter?



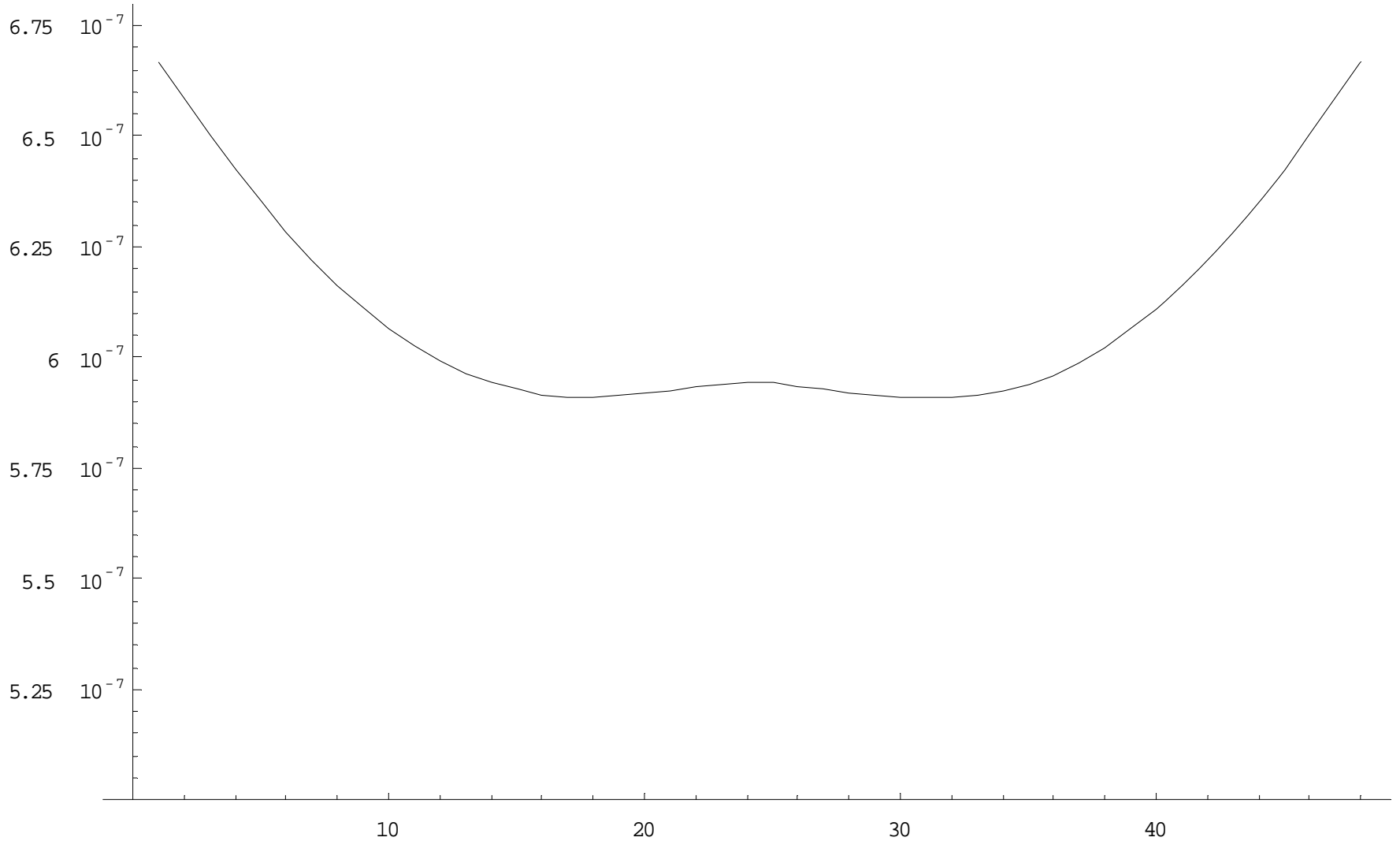
Thermally Distorted Arm Mode

green: FFT output
 red: best fit gaussian
 waist=5.4cm



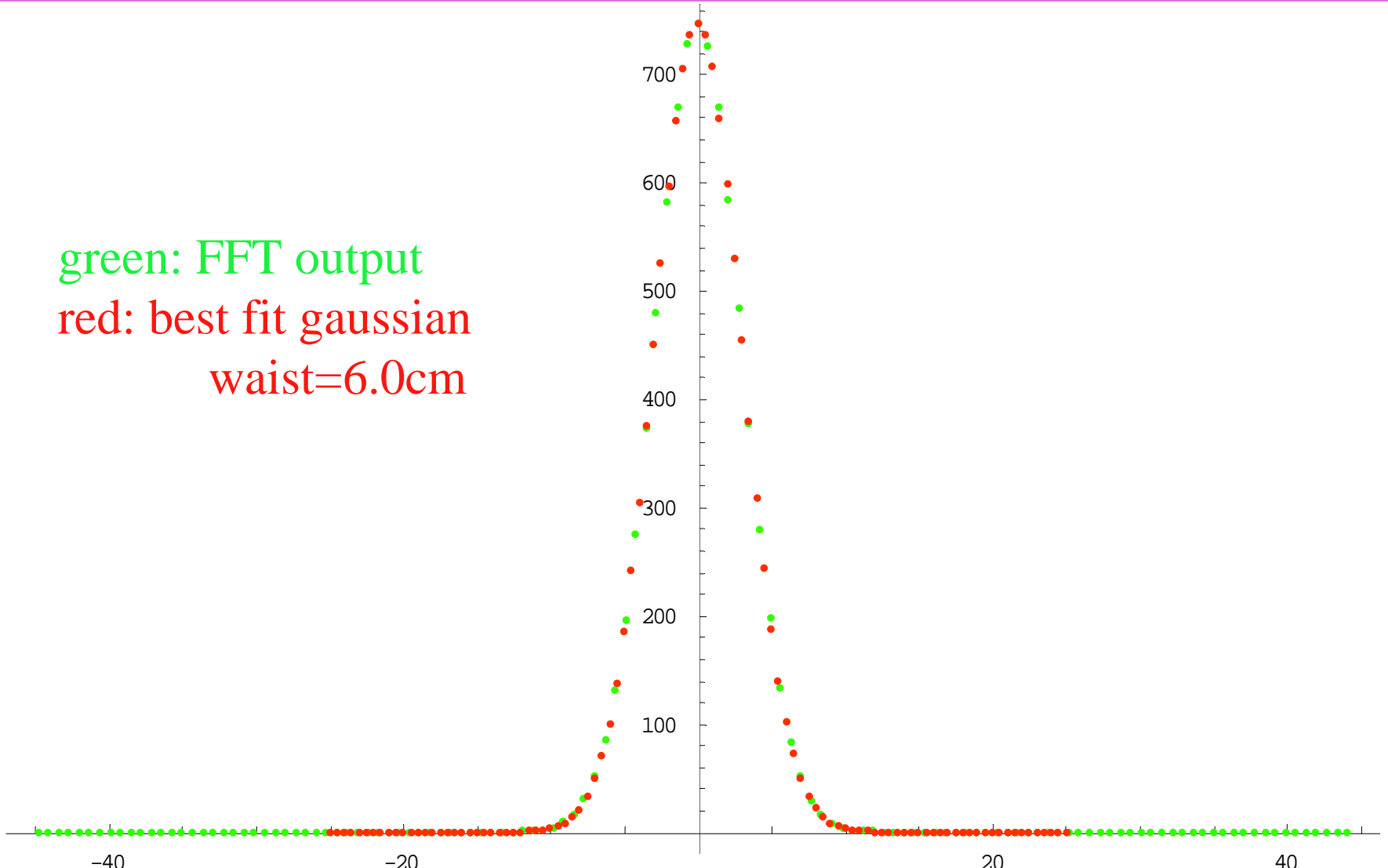


Deformation of Ring-Compensated Mirror

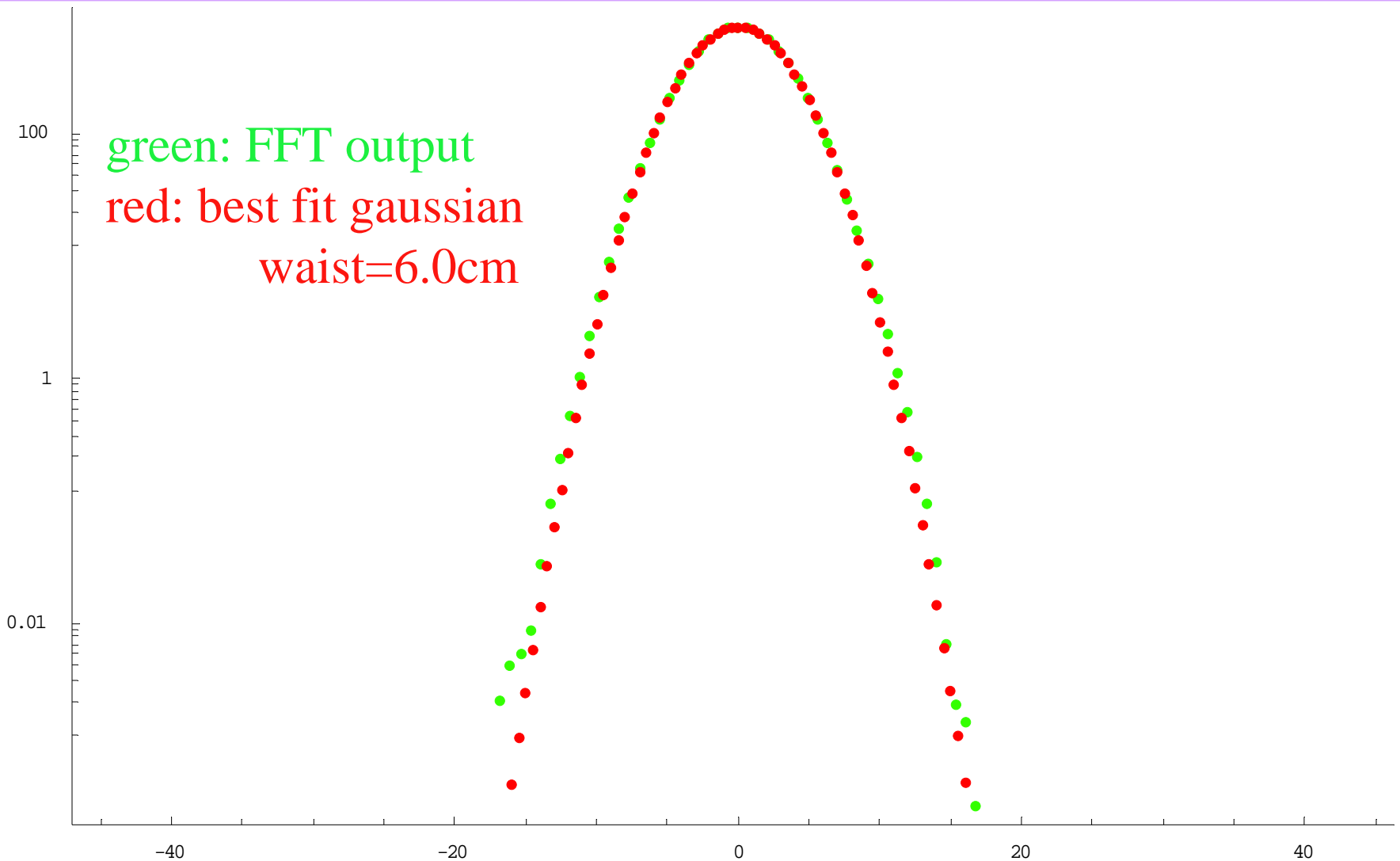


Thermally Compensated Arm Mode

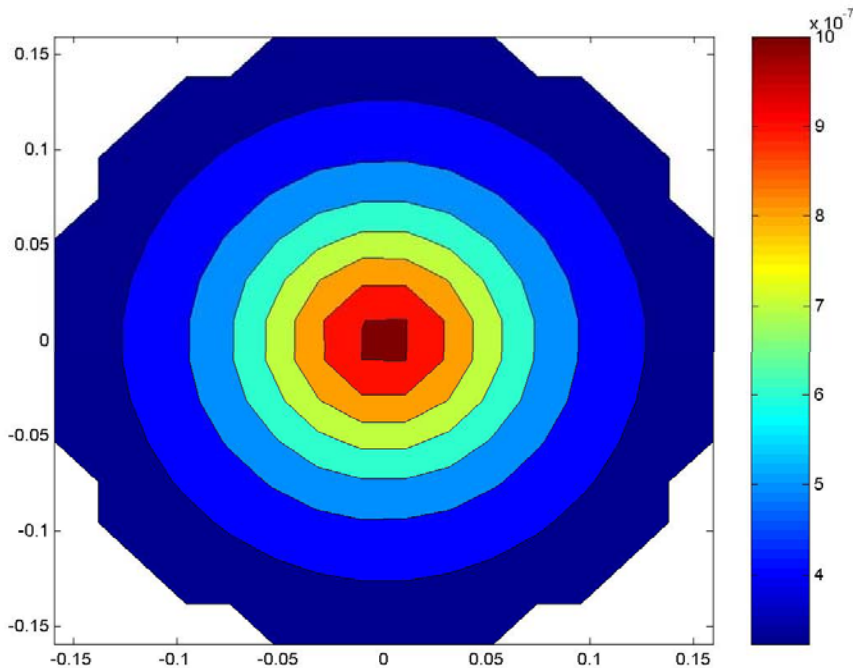
green: FFT output
 red: best fit gaussian
 waist=6.0cm



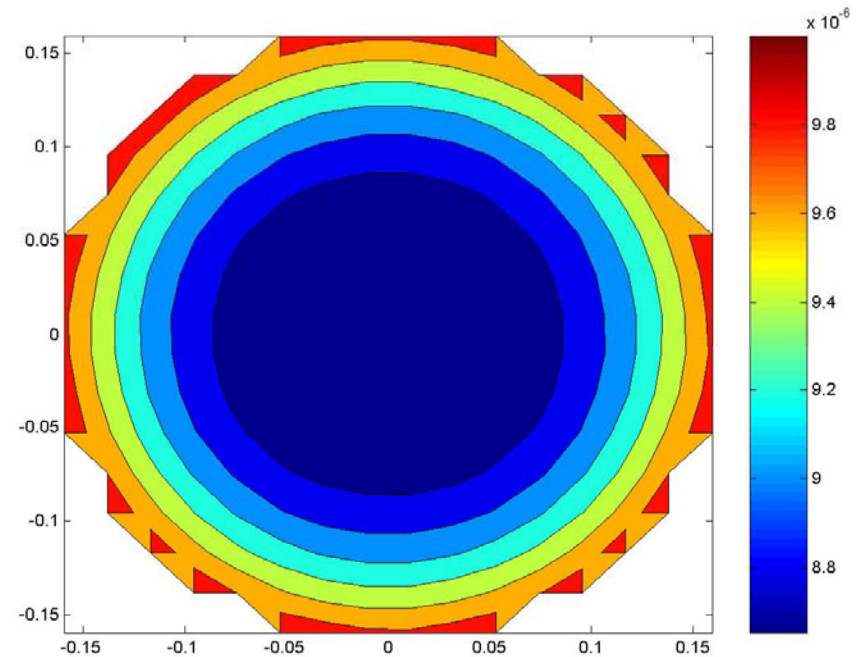
Thermally Compensated Arm Mode



Thermal Lensing in the Substrate



uncompensated:
4% mode overlap



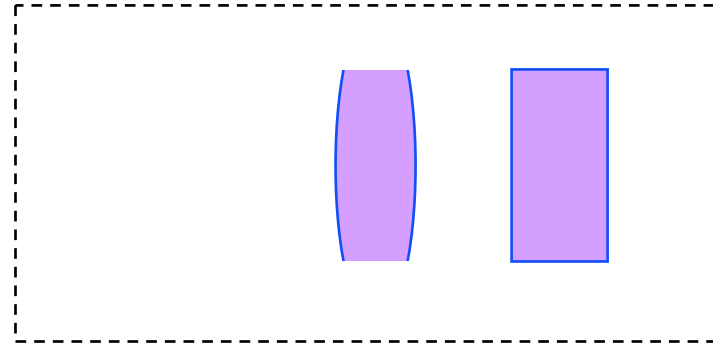
compensated:
72% mode overlap

'Thermally Invariant' Stable Recycling Cavity

ITM (could be distorted, focal length f)



reference plane



rest of cavity

$$\begin{array}{|c|c|} \hline 1 & 0 \\ \hline -1/f & 1 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline A & B \\ \hline C & D \\ \hline \end{array}$$

total cavity matrix:

$$\begin{array}{|c|c|} \hline A & B \\ \hline -A/f+C & -B/f+D \\ \hline \end{array}$$

Cavity Math

Stability criterion: $\left| \frac{A - B/f + D}{2} \right| \leq 1$

Waist size at reference plane: $w_{ref} = \sqrt{\frac{\lambda}{\pi}} \frac{\sqrt{|B|}}{\left(1 - \left(\frac{A - B/f + D}{2}\right)^2\right)^{1/4}}$

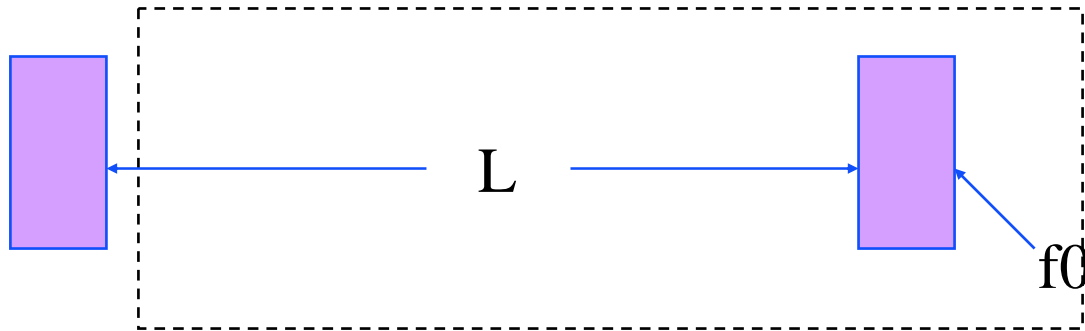
Waist size does not vary with f ($\frac{dw_{ref}}{df} = 0$) if: $A - B/f + D = 0$

Note: cavity stability is guaranteed by this condition.

Waist size is then $w_{ref} = \sqrt{\frac{\lambda}{\pi}} \sqrt{|B|} = 6cm \rightarrow B = \pm 10629m$

Apply this to a simple cavity...

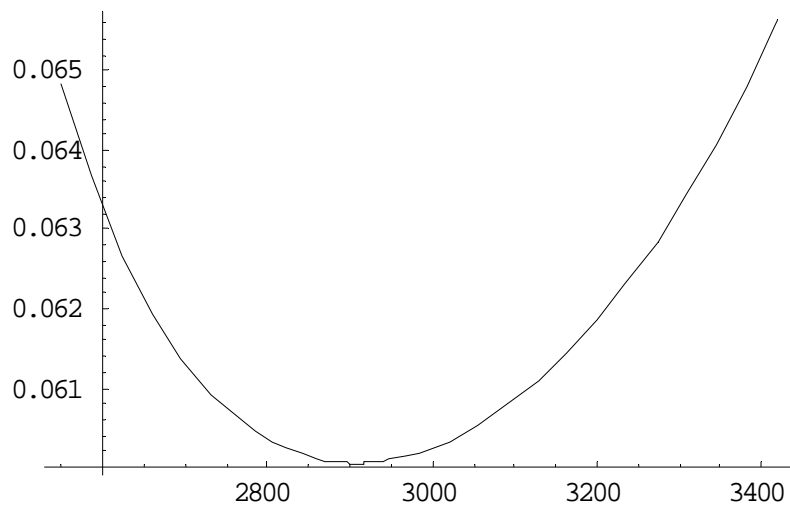
$$\begin{vmatrix} A & B \\ C & D \end{vmatrix} = \begin{vmatrix} 1-L/f_0 & 2L-L^2/f_0 \\ -1/f_0 & 1-L/f_0 \end{vmatrix}$$



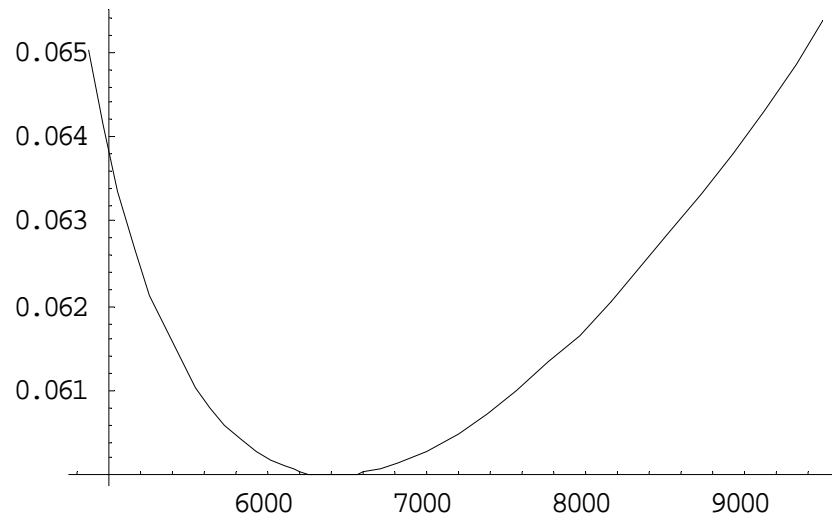
Solutions

- For $L=8.3\text{m}$, $f_0=6.5\text{mm}$ or $f_0=-6.5\text{mm}$
 - » Not a good solution.
- For $L=4\text{km}$, two good solutions:
 - » $f_0=-6,086\text{m}$, $f=3,221\text{m}$
 - » $F_0=858\text{m}$, $f=1,452\text{m}$
- Both of the 4km cavity solutions are stable

Waist sizes vs. f_{ITM}

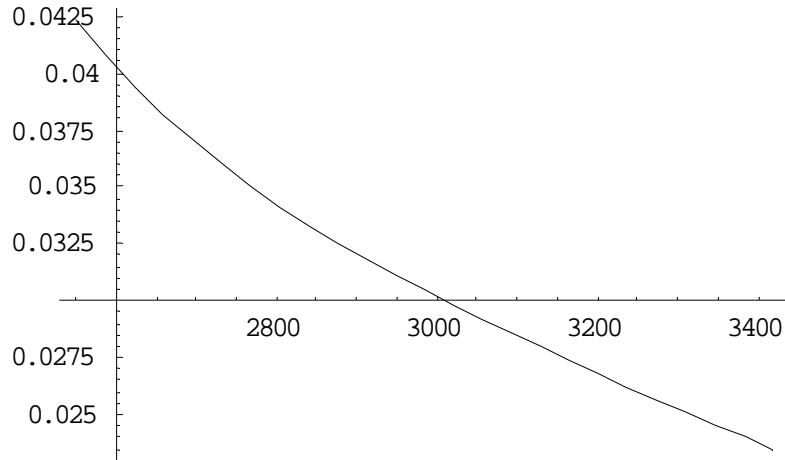


$f_0=858m$

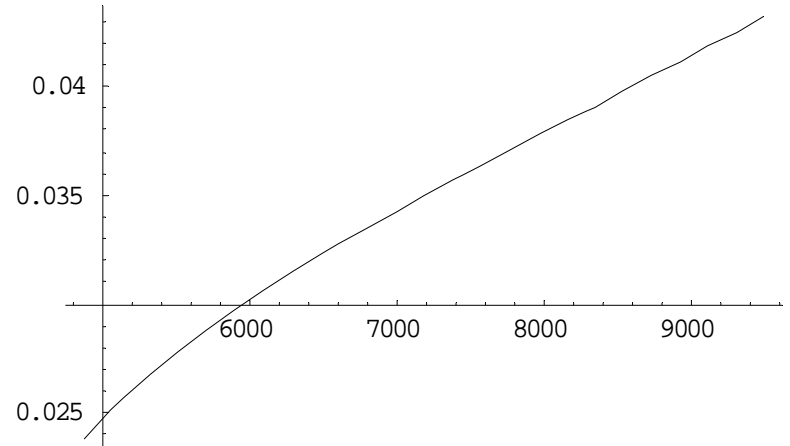


$f_0=-6086m$

The Downside: Spot Size at SRM



$f_0=858\text{m}$



$f_0=-6086\text{m}$

Comments, Future Work

- The idea of overlap integrals, used by Lawrence for unstable recycling cavities, fails for stable cavities. Problem is not as bad as naively predicted.
- 4km signal cavity is a drag, but shorter, more complex cavities might work
- Large and rapid variations in output spot size are also a drag, but at least the thermal compensation can move outside the signal cavity
- What if different arms have different heating? Fuller models are needed.