Advanced Interferometer Configurations

Theory of Quantum Mechanical Noises

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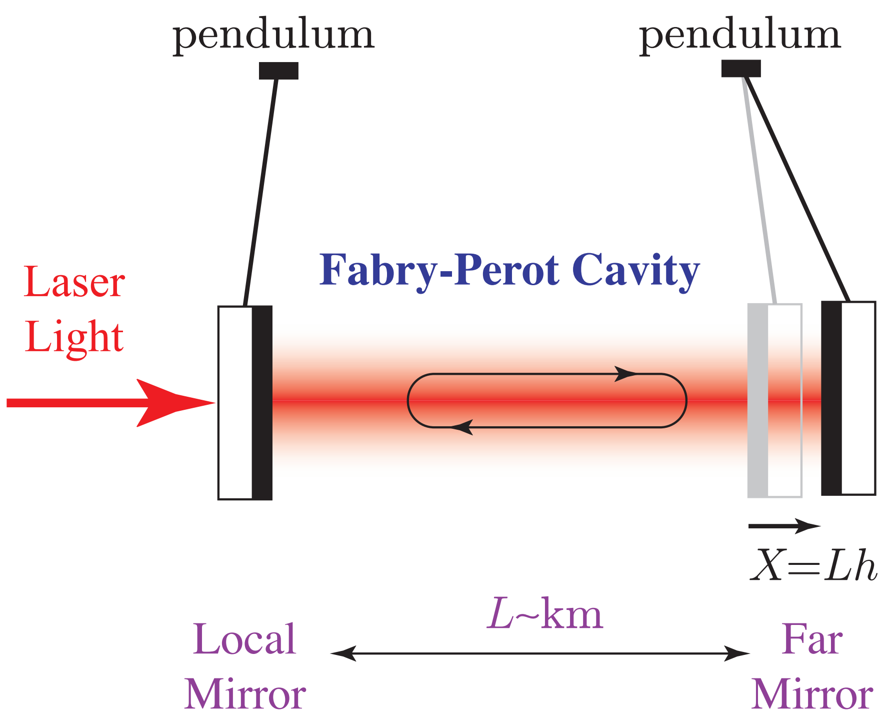
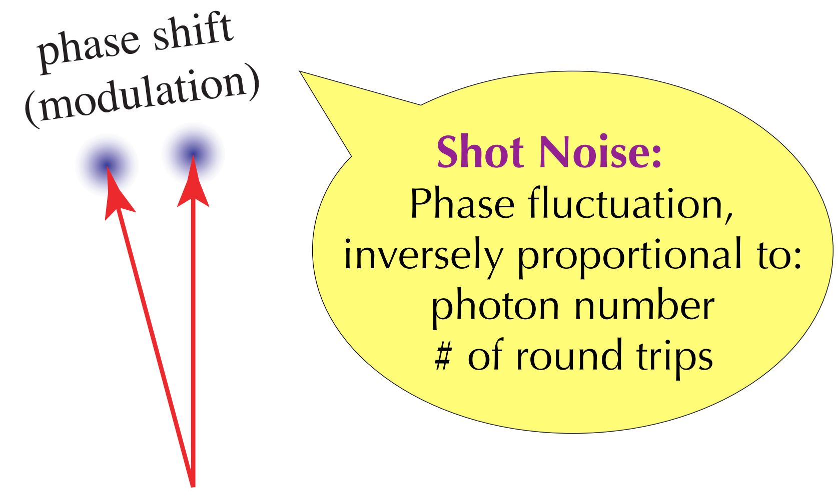
Quantum mechanics in GW interferometers





"phasor diagram"

This is just one arm, interferometer brings two arms together

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- GW interferometers use **light** to measure relative motions of **mirrors**
- Need to measure **position** repeatedly in order to detect $h(t)$, but

$$\left[\hat{x}_{\rm H}(t_1), \hat{x}_{\rm H}(t_2)\right] = i\hbar\frac{t_2 - t_1}{M} \neq 0$$

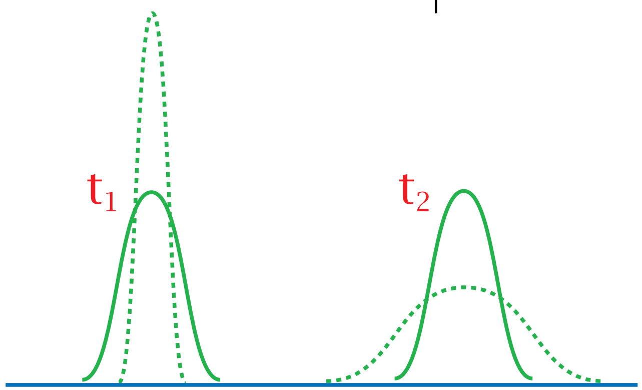
Quantum mechanics does not allow us to do so!
... at least not perfectly

The Standard Quantum Limit

A ***Standard Quantum Limit*** was formulated by Braginsky in the 1960s

Heisenberg Uncertainty Relation

$$[\Delta x(t_1)]\,[\Delta x(t_2)] \geq \left|\frac{\hbar(t_2 - t_1)}{M}\right|$$



wavefunction widths of test mass

t_1: right after 1st measurement

t_2: right before 2nd measurement

Standard Quantum Limit

$$S_x(\Omega) = \sqrt{\frac{2\hbar}{M\Omega^2}}$$

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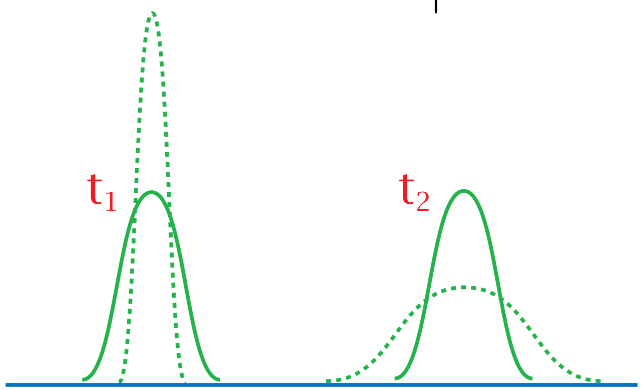

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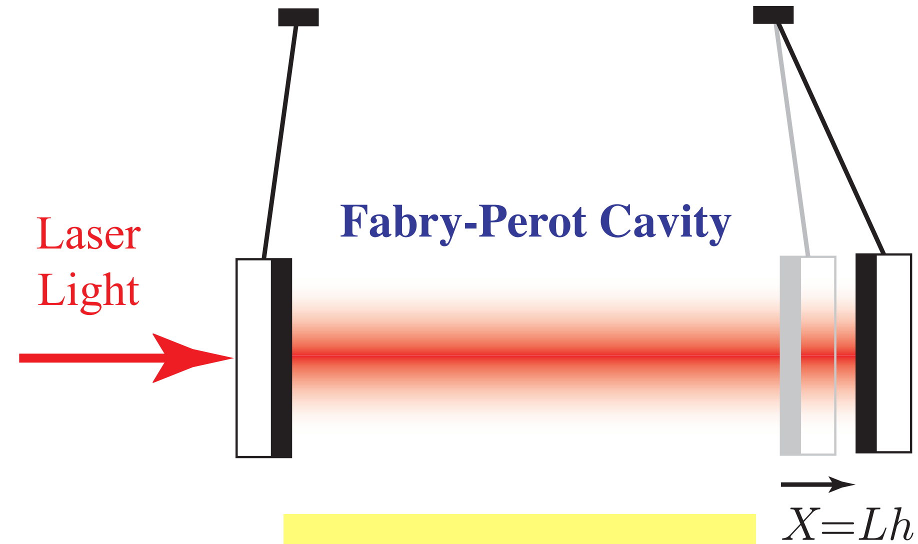


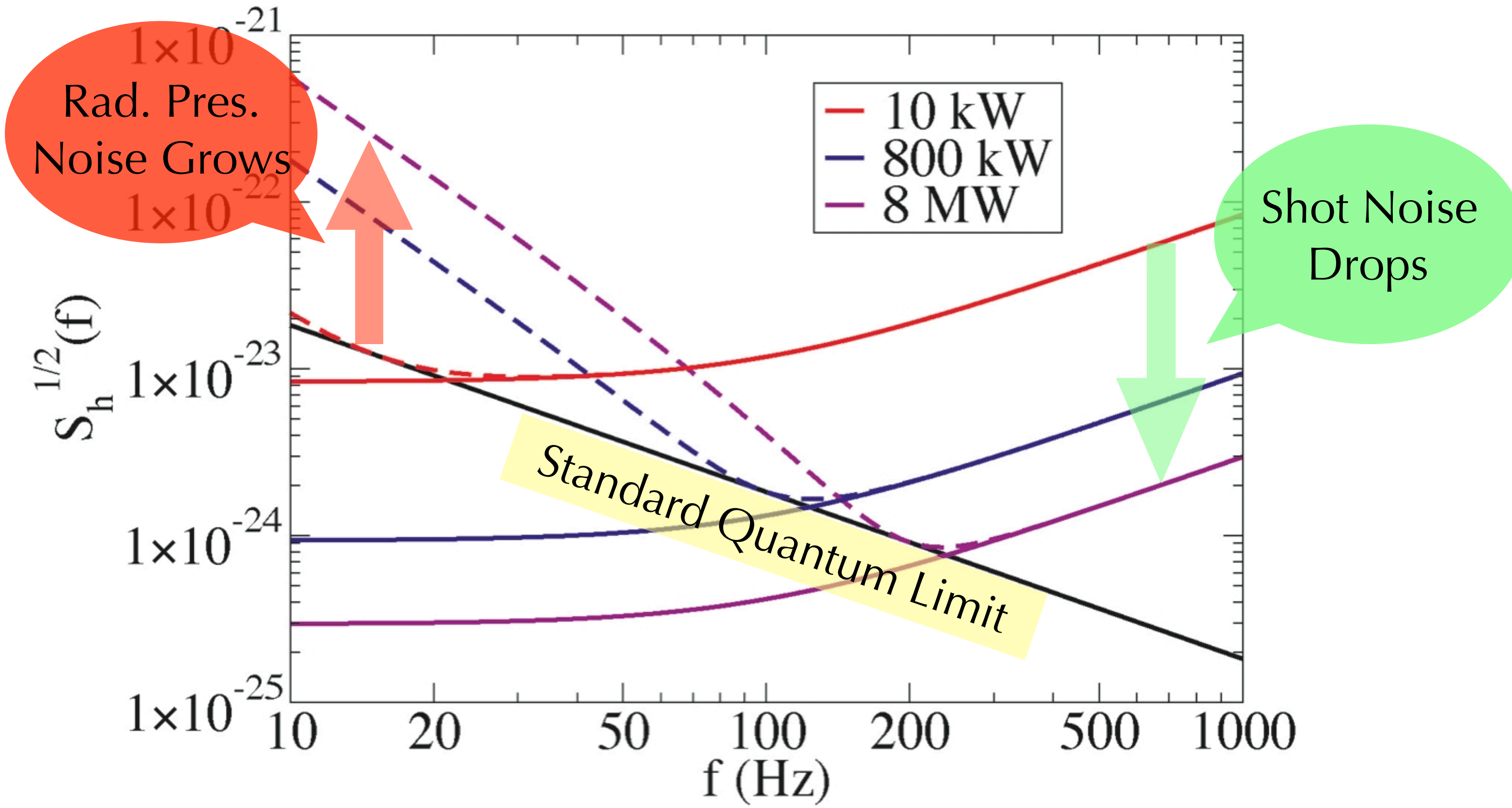
Photon number fluctuation also causing noisy force
Radiation-Pressure Noise

Increasing Photon Number ...
Lowers Shot Noise
Raises Rad. Pres. Noise

Standard Quantum Limit

$$S_x(\Omega) = \sqrt{\frac{2\hbar}{M\Omega^2}}$$

Shot & Radiation-Pressure Noises



$$L = 4\,\mathrm{km},\ M = 40\,\mathrm{kg}\ \text{and}\ \gamma = 100\,\mathrm{Hz}$$

$$\gamma : \text{optical bandwidth of arm cavity}$$

- "Conventional Interferometer":
  - Shot & Rad. Pres. Noises uncorrelated. [Add powers]
  - Rad. Pres. Noise dominates at lower freq.'s; Shot Noise at higher freq.'s
  - **Total Noise never surpasses the Standard Quantum Limit**

Standard Quantum Limit

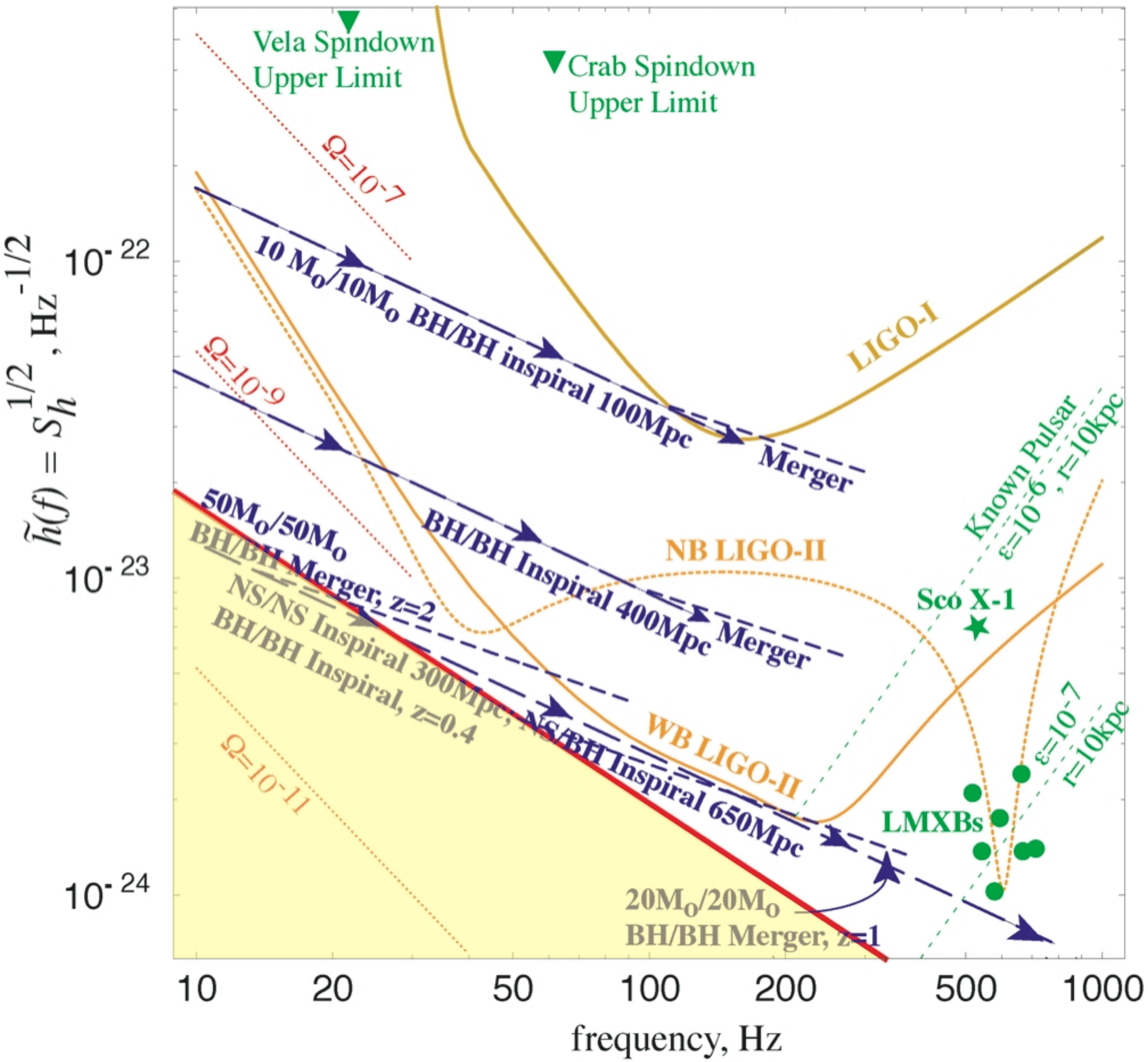
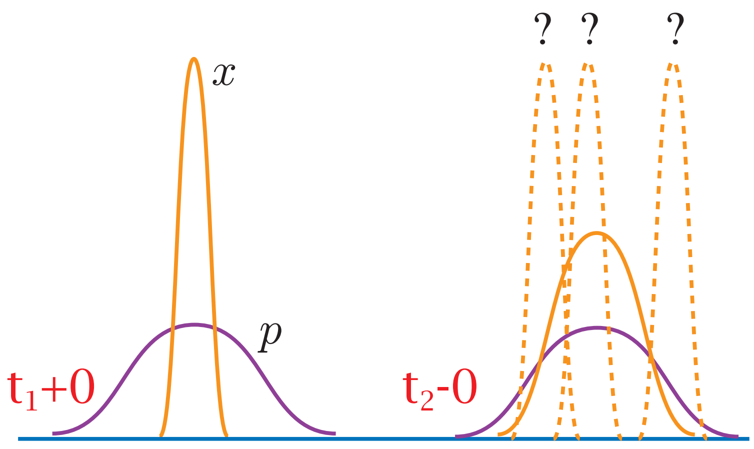


Figure from Cutler & Thorne

- The Standard Quantum Limit is indeed right beyond Advanced LIGO (LIGO-II), with 40kg test mass.
- If we want to improve by another factor of 10 in "LIGO-III", or "EGO", either
  - use 4000kg mirrors
  - or surpass the SQL
  - or some combination
- The Standard Quantum Limit can be surpassed, and ways of doing so will be of great interest for 3rd-generation detectors!

Surpassing the SQL

Braginsky (1970s): Measure an observable that commutes at different times



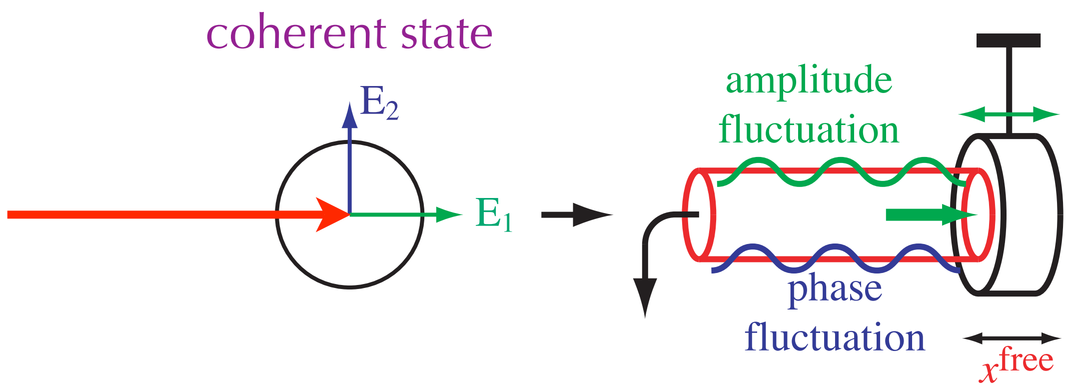
"Quantum Demolition"
Position eigenstates do not stay eigenstates during evolution --- being "demolished" continuously

"Quantum Non-Demolition"
Measure Quantities whose eigenstates stay eigenstates, e.g, momentum of free mass

But the "measurement" here is only "postulated"
in reality, they are executed by "photons", which are quantized

Quantum Noise in GW Detectors

[Caves, Walls & Milburn, Braginsky & Khalili, ...]



Mirror: $M\ddot{x} = \sqrt{\mathcal{I}} E_1^{\rm in}(t) \Rightarrow x = \left[x_0 + \frac{p_0 t}{M} + G(t)\right] + \frac{\sqrt{\mathcal{I}}}{M}\int_0^t dt' \int_0^{t'} dt'' E_1^{\rm in}(t'')$

Light:

$$E_1^{\rm out}(t) = E_1^{\rm in}(t)$$

$$E_2^{\rm out}(t) = E_2^{\rm in}(t) + \sqrt{\mathcal{I}} x(t)$$

$$= \underbrace{E_2^{\rm in}(t) + \frac{\mathcal{I}}{M}\int_0^t dt' \int_0^{t'} dt'' E_1^{\rm in}(t)}_{\text{From Optical Fields}} + \underbrace{\sqrt{I}\left[x_0 + \frac{p_0 t}{M} + G(t)\right]}_{\text{From Free Test Mass}}$$

Quantum Noise in GW Detectors

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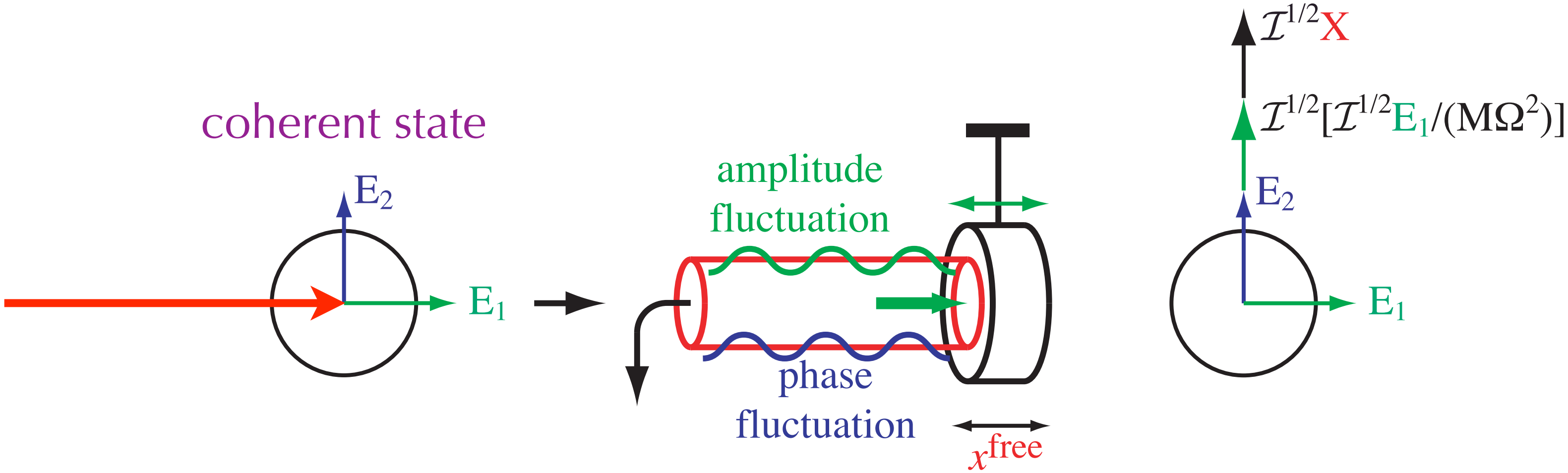

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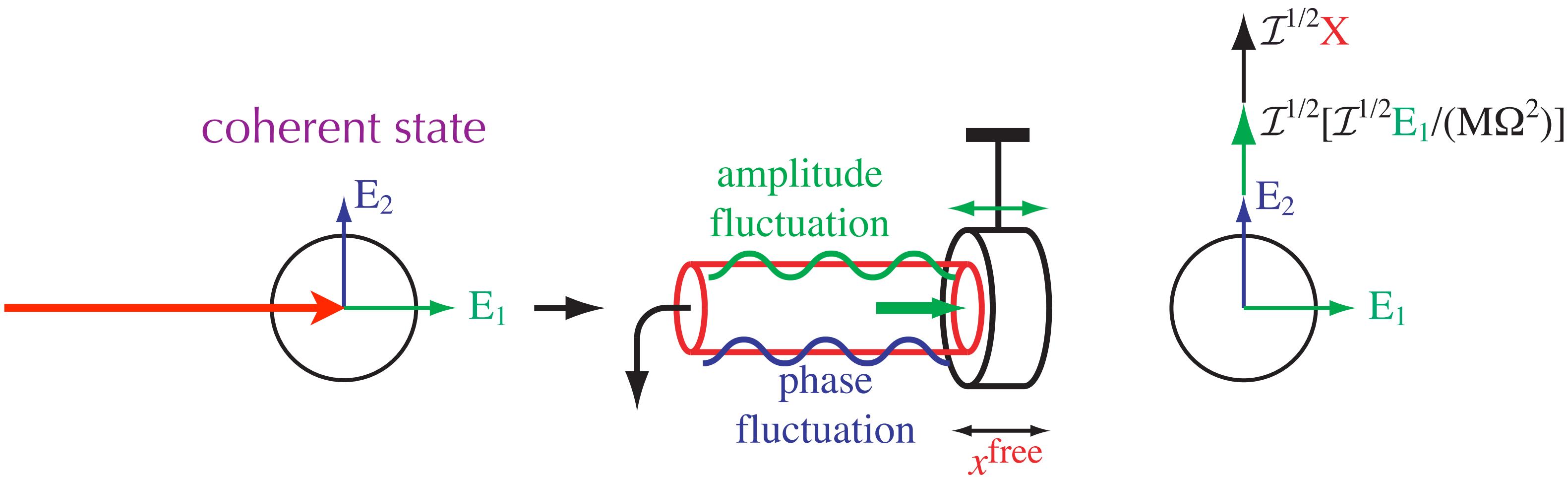

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Optical-Field Commutator cancels Test-Mass Commutator

Optical Fields and Test-Mass Position *together* form a **QND observable**!
This includes all noises -- no extra noise dictated by Quantum Mechanics.

SQL derived from Quantum Measurement Theory

- The output fields

$$E_1^{\rm out}(t) = E_1^{\rm in}(t)$$

$$E_2^{\rm out}(t) = E_2^{\rm in}(t) + \sqrt{\mathcal{I}}x(t)$$

$$= \underbrace{E_2^{\rm in}(t) + \frac{\mathcal{I}}{M}\int_0^t dt' \int_0^{t'} dt'' E_1^{\rm in}(t)}_{\text{From Optical Fields}} + \underbrace{\sqrt{\mathcal{I}}\left[\overbrace{x_0 + \frac{p_0 t}{M}}^{\text{DC}} + G(t)\right]}_{\text{From Free Test Mass}}$$

- In Frequency domain, if we measure ${E_2}^{\rm out}$

Noise Spectrum

$$S_x = \underbrace{\frac{1}{\mathcal{I}}S_{E_2E_2}}_{\text{Shot}} + \underbrace{\frac{\mathcal{I}}{M^2\Omega^4}S_{E_1E_1}}_{\text{Rad. Press.}} + \underbrace{\frac{2}{M\Omega^2}S_{E_1E_2}}_{\text{Correlation}}$$

Uncertainty Principle

$$S_{E_1E_1}(\Omega)S_{E_2E_2}(\Omega) - S^2_{E_1E_2}(\Omega) \geq \hbar^2$$

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$$S_{E_1E_1}(\Omega)S_{E_2E_2}(\Omega) - S^2_{E_1E_2}(\Omega) \geq \hbar^2$$

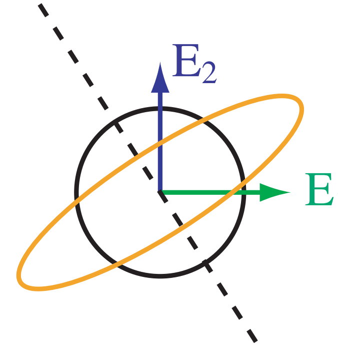
In Absence of Correlations...
(e.g., vacuum input state)
$$S_x \geq 2\sqrt{\left(\frac{1}{\mathcal{I}}S_{E_2E_2}\right)\left(\frac{\mathcal{I}}{M^2\Omega^4}S_{E_1E_1}\right)} \geq \frac{2\hbar}{M\Omega^2} \equiv S_x^{\rm SQL}$$

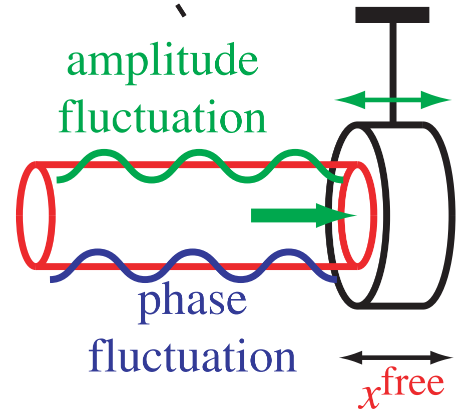
Surpassing the SQL in a Michelson interferometer

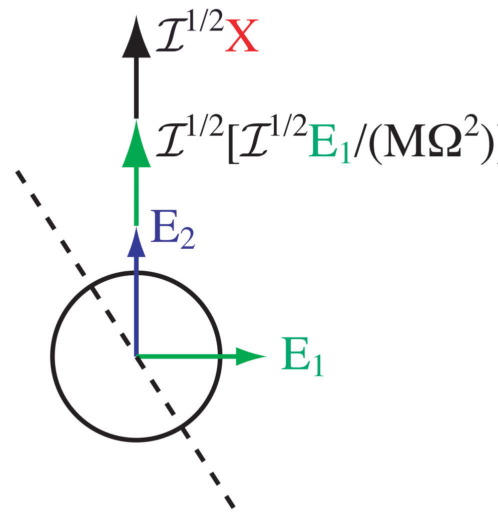
- The Standard Quantum Limit only exists for specific readout scheme and input state
- SQL can be circumvented when either of the above are modified

$$E_1^{\mathrm{out}} = E_1^{\mathrm{in}}$$
$$E_2^{\mathrm{out}} = E_2^{\mathrm{in}} - \frac{\mathcal{I}}{M\Omega^2} E_1^{\mathrm{in}} + \sqrt{\mathcal{I}}\, G$$

  - modification of input state: frequency dependent squeezing
  - modification of readout scheme: frequency dependent detection







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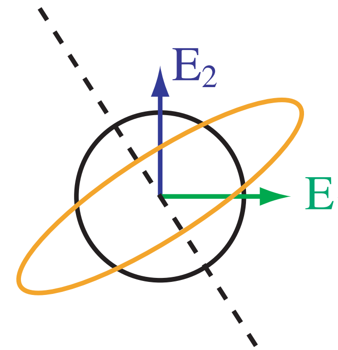

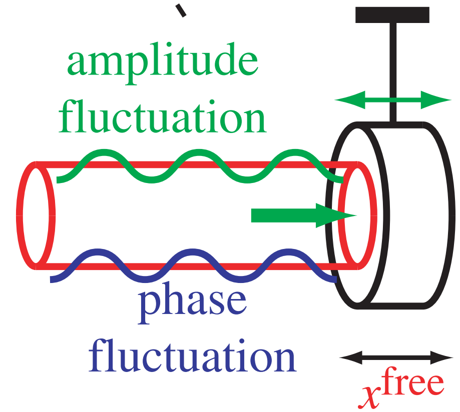

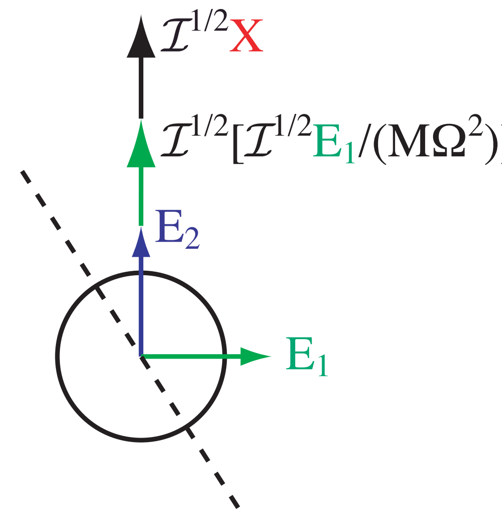

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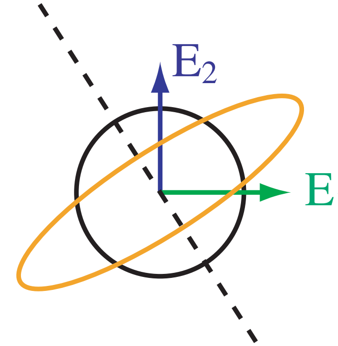
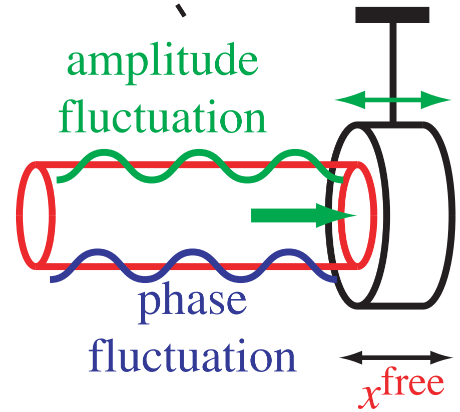
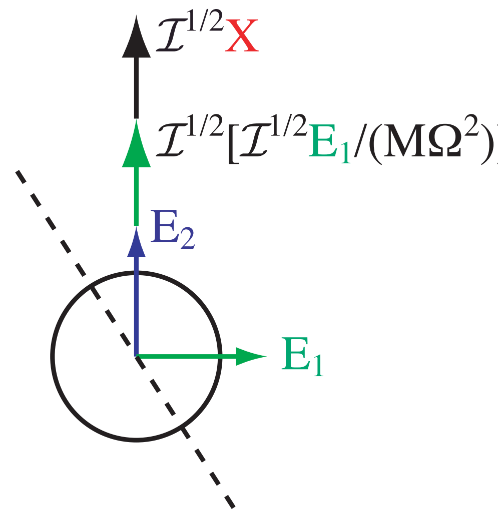
  - modification of input state: frequency dependent squeezing
  - modification of readout scheme: frequency dependent detection

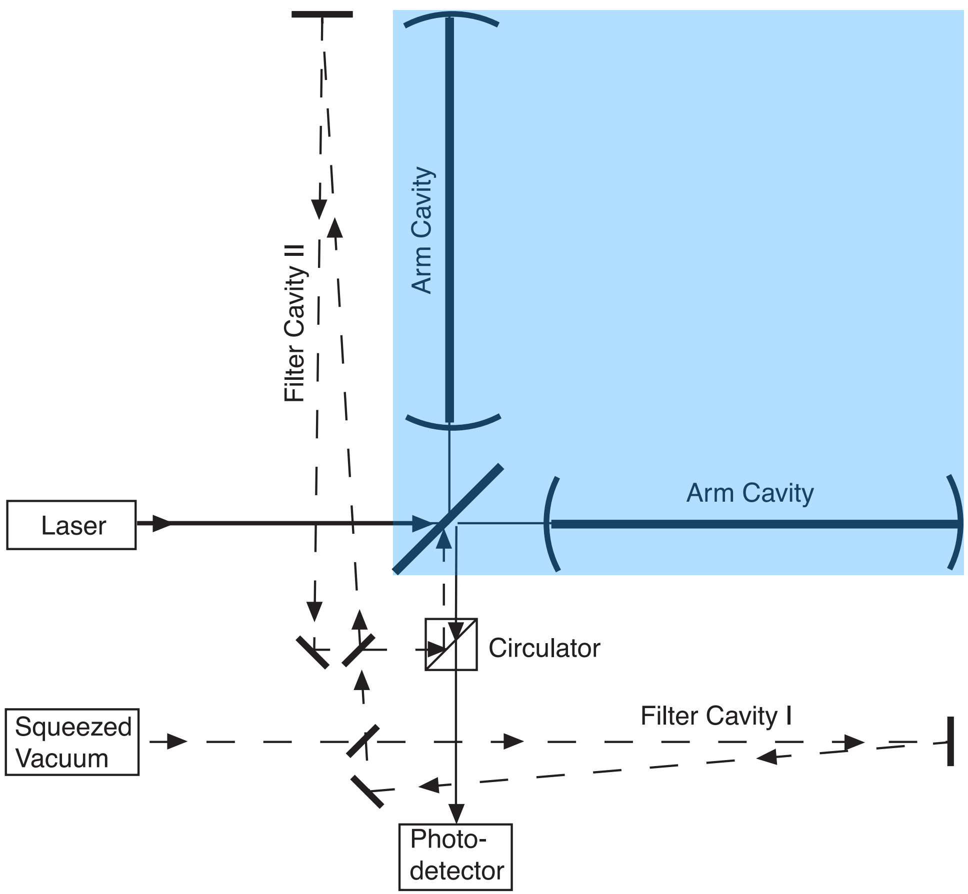
- Both require frequency dependent rotation of quadratures, which can be realized by detuned Fabry-Perot Cavities. [Kimble et al., 2001; Appendix of Purdue & Chen, 2002]
- Bandwidth of typical filter cavities ~ 100Hz; loss has to be lower than squeeze factor. [Kimble et al., 2001]



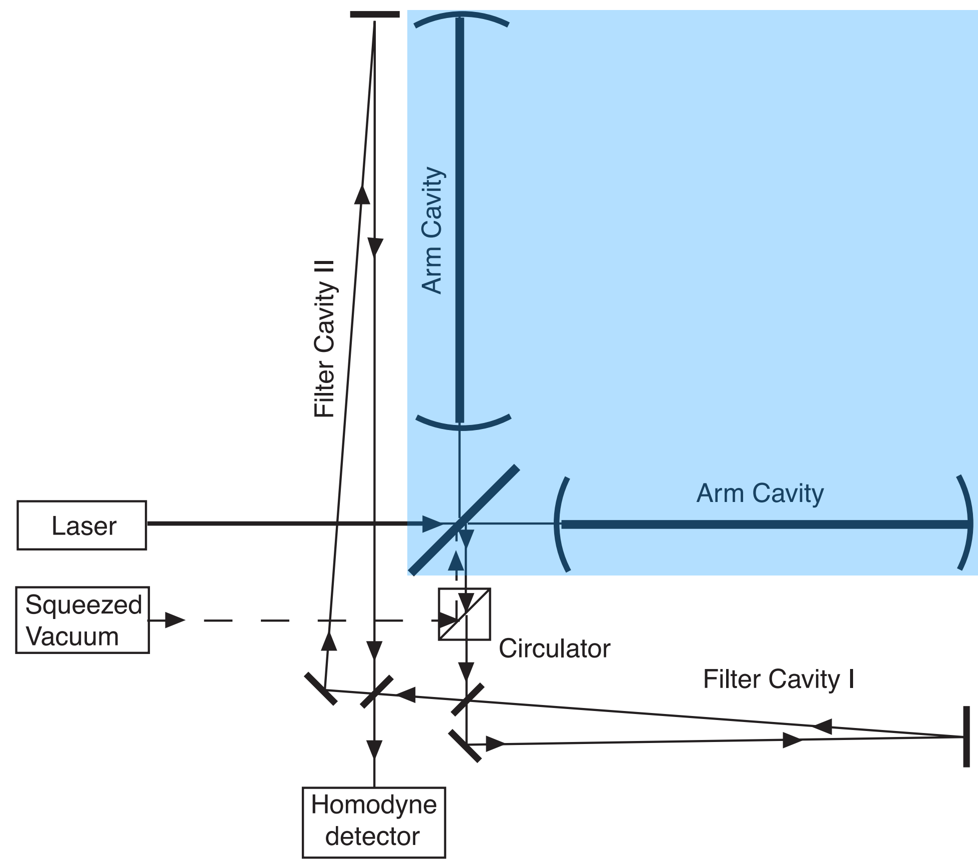




Surpassing the SQL in a Michelson interferometer



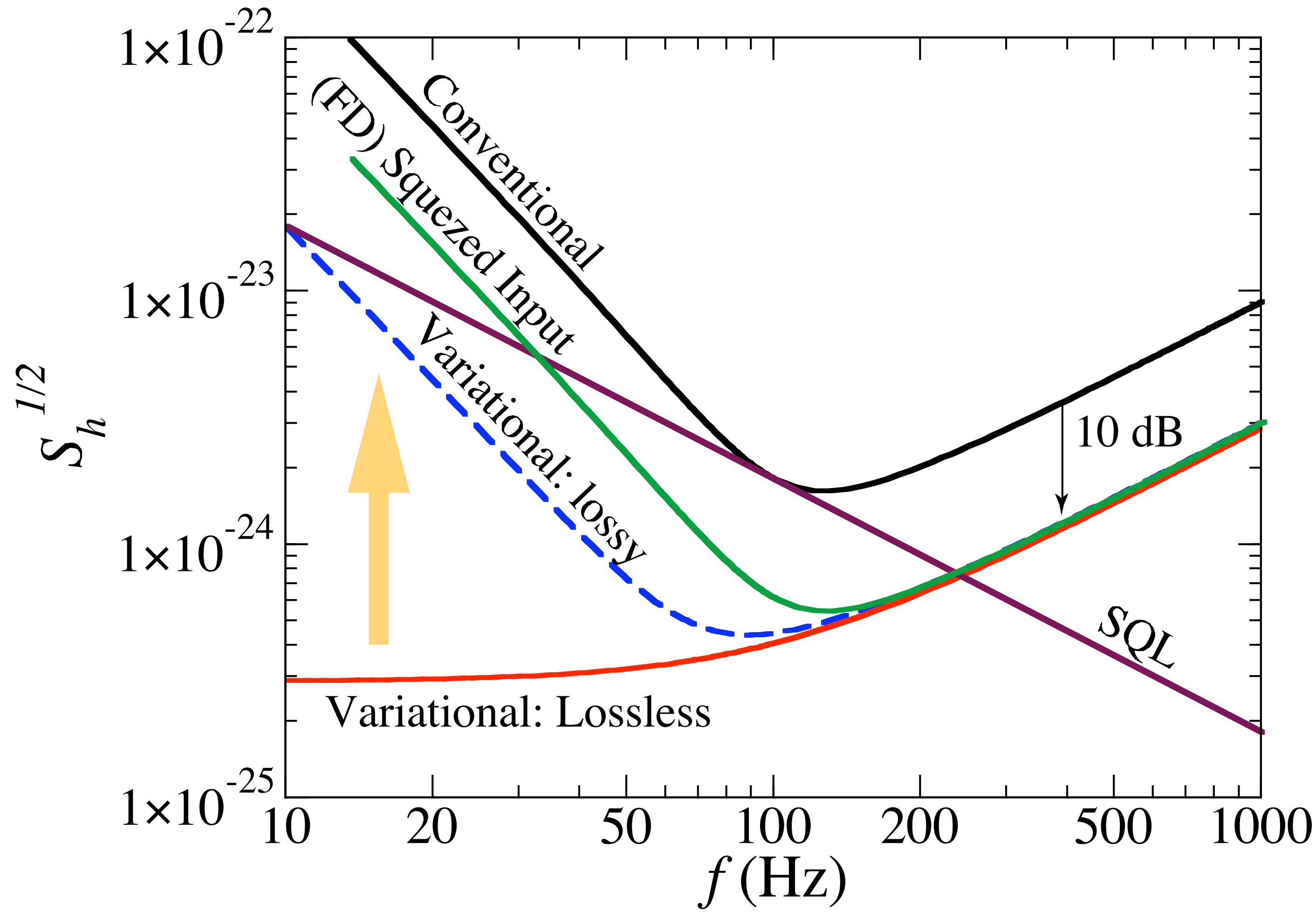
frequency dependent
input squeezed state



frequency dependent
homodyne detection
(variational measurement)

[Kimble et al., 2001]

Surpassing the SQL in a Michelson interferometer

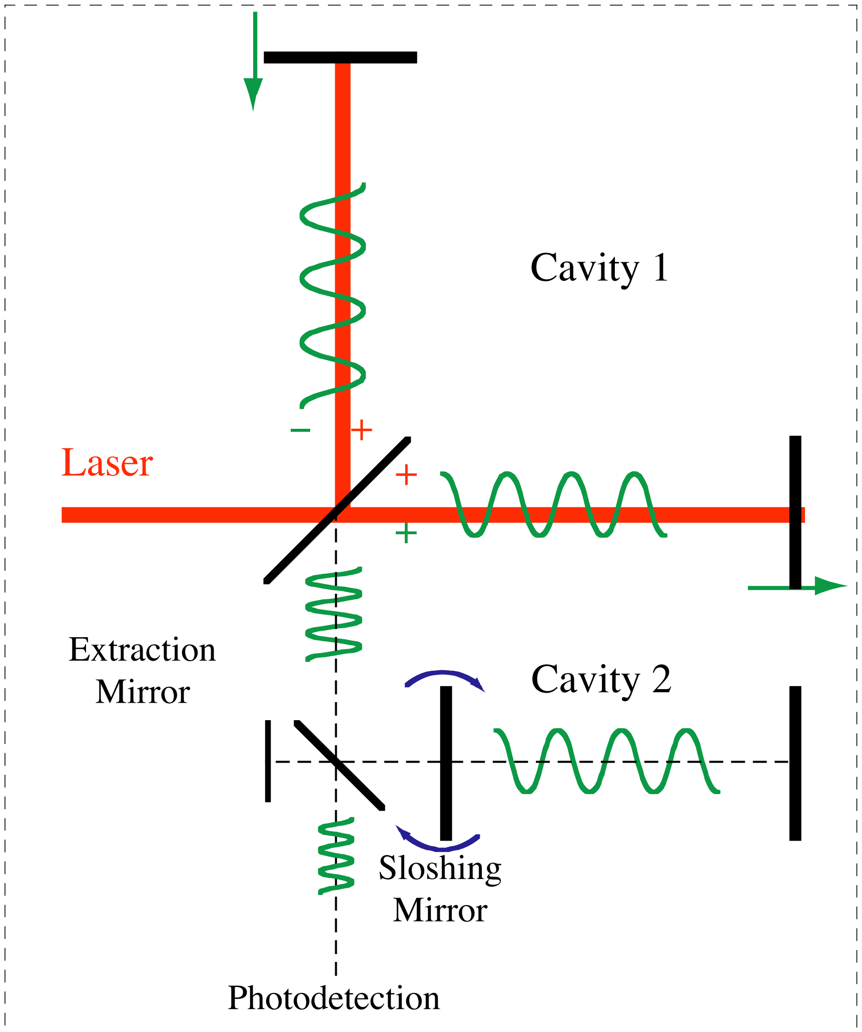


$\gamma = 2\pi \times 100$ Hz $\quad I_c = 800$ kW $\quad$ 10dB squeezing

20 ppm loss/round trip, 2 filters, each 4 km

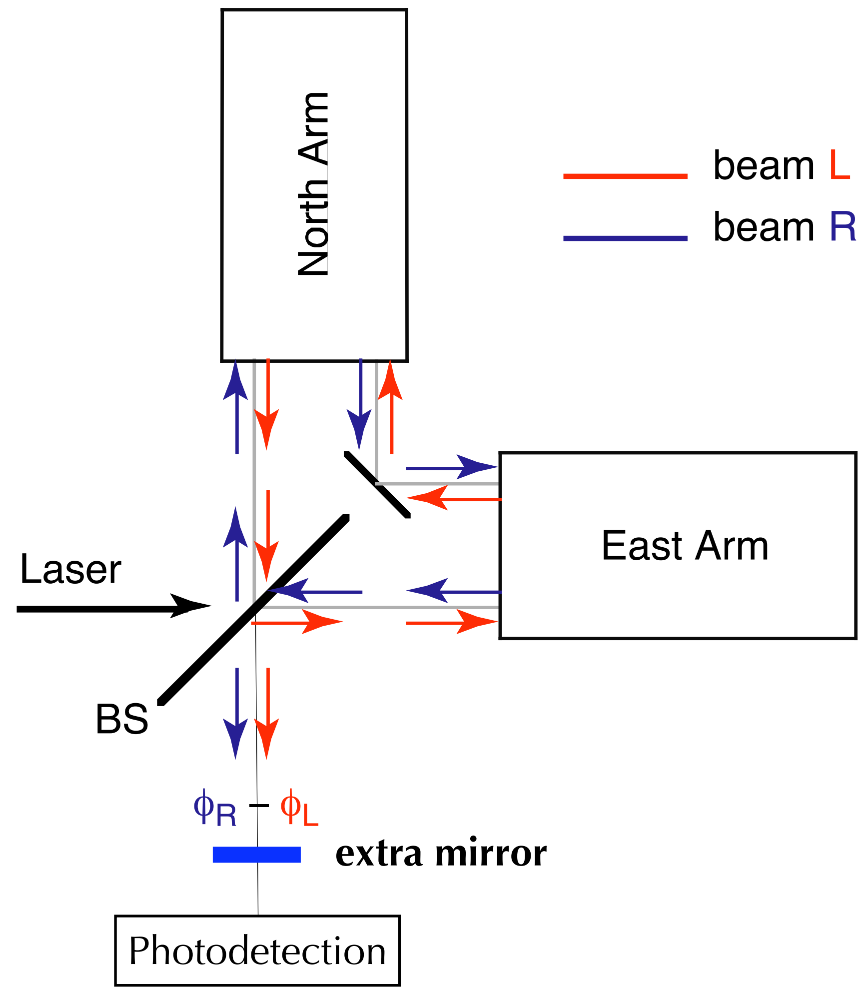
total loss ~ 1%

Other interferometer configurations: Speed Meters



"Michelson Speed Meter"

Braginsky & Khalili, 90s; Purdue & Chen, 02



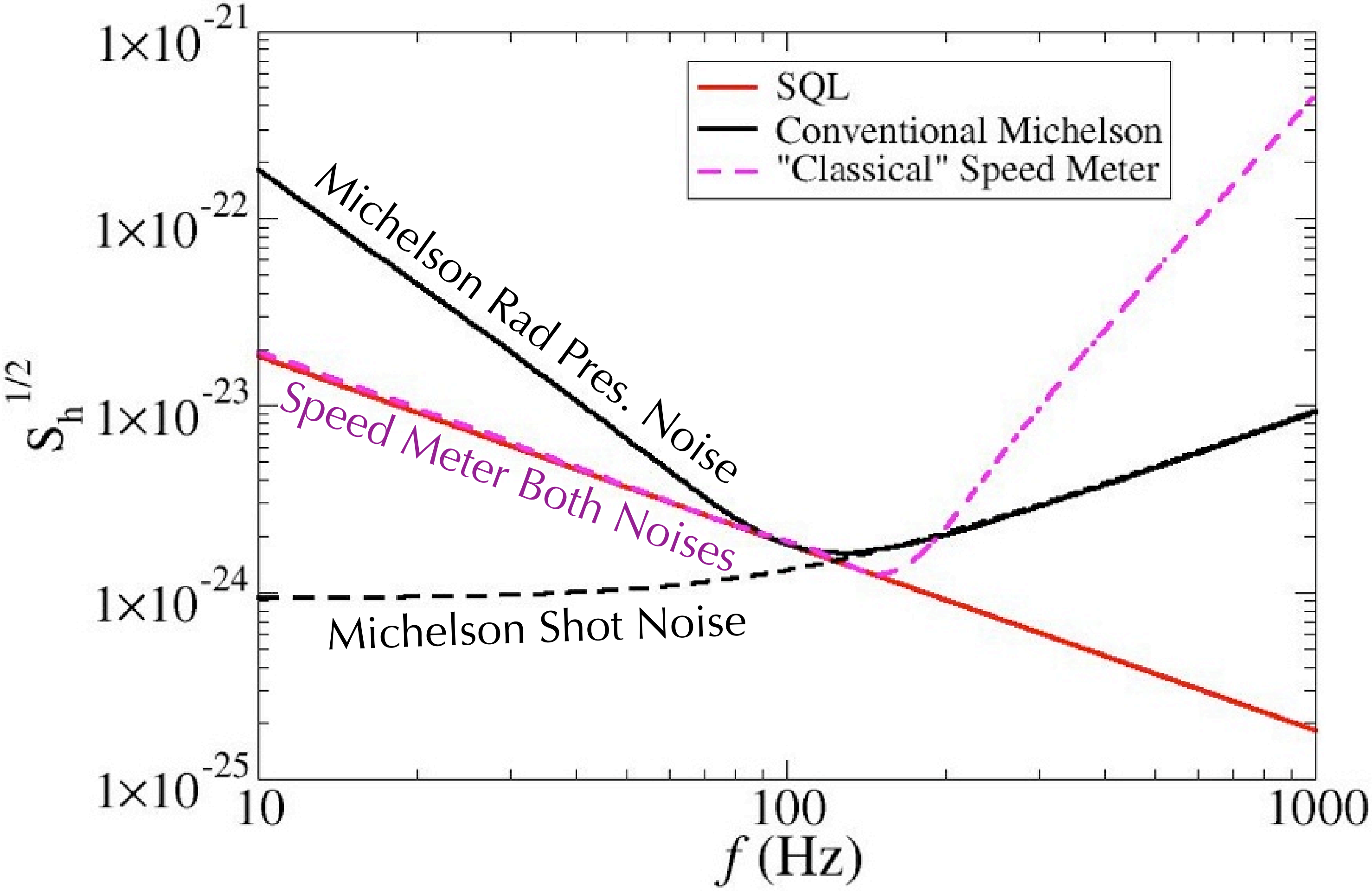
Sagnac Interferometer

Chen, 02; Danilishin, 03

They are "equivalent", with optical responses characterized by Δ and γ

Classical Speed Meters

- Speed Meter [*for f<Δ*]: transfer function ~ *f*, i.e., **suppressed at low frequencies**
- Sensitivity traces the SQL: **Equal amount of Shot Noise and Radiation-Pressure Noise**



Speed Meters: $\Delta = 2\pi \times 130$ Hz $\quad \gamma = 2\pi \times 100$ Hz

Conventional: $\gamma = 2\pi \times 100$ Hz

$I_c = 800$ kW

Quantum Noise of Speed Meters

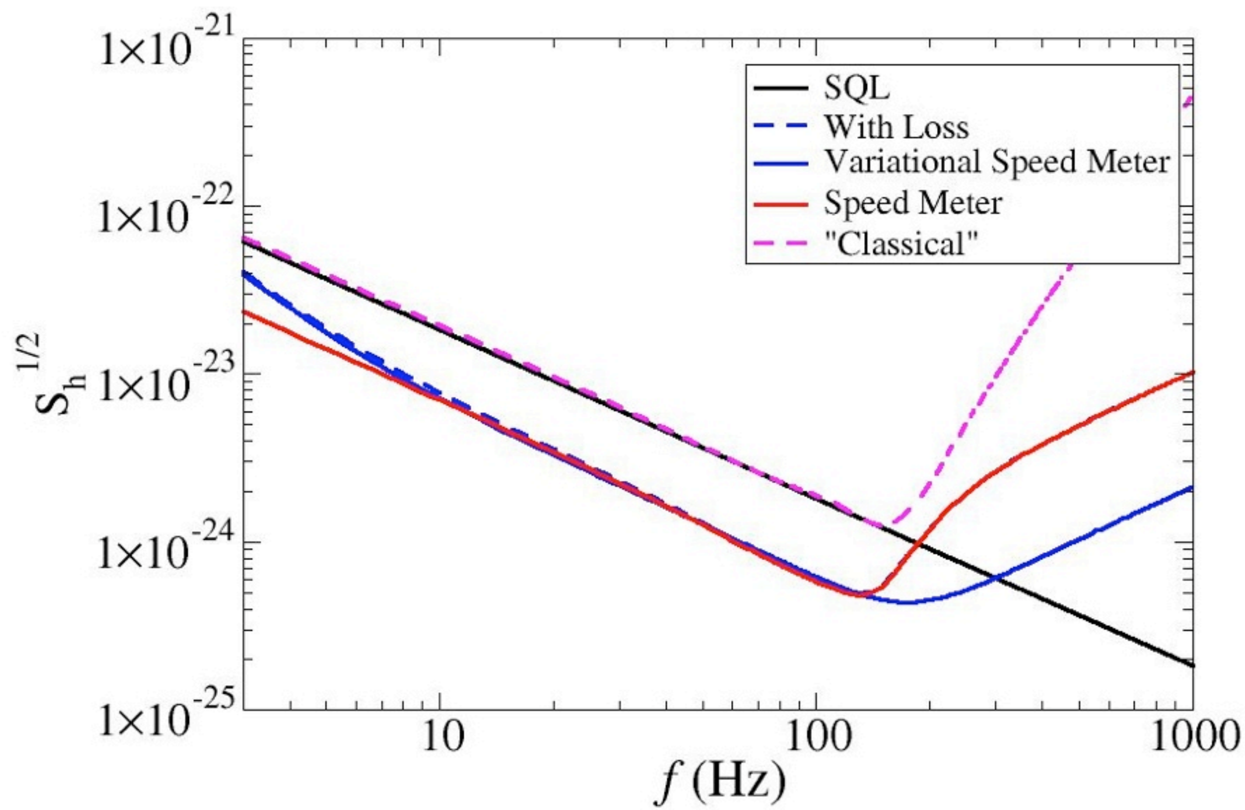
[Purdue 2002; Purdue & Chen 2002]

$$E_1^{\rm out} = E_1^{\rm in}$$

$$E_2^{\rm out} = E_2^{\rm in} + \frac{\sqrt{\mathcal{I}}}{\Delta}\underbrace{\left[\overbrace{\frac{\sqrt{\mathcal{I}}}{M\Delta}E_1^{\rm in}}^{V_{\rm BA}} + V_G\right]}_{V_{\rm total}}$$

Speed-Meter input-output relation *for f<Δ*

- *Low frequencies*: ordinary homodyne detection & squeezed state with fixed squeeze angle will be optimal. [**But** this will **not** be the usual phase quadrature.]
- *High frequencies*: optimal detection quadrature will be *phase quadrature* again.

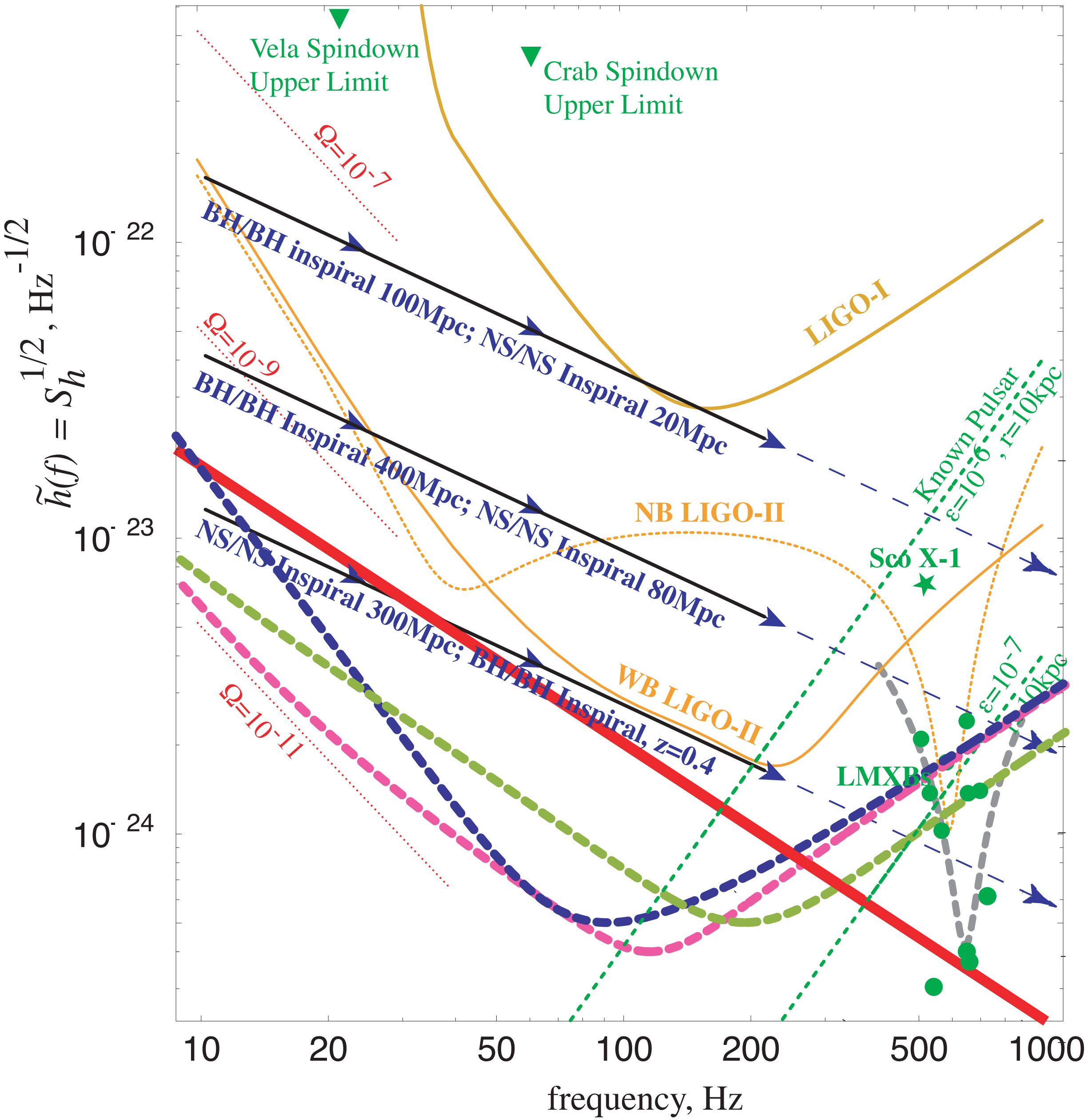


$\Delta = 2\pi \times 173$ Hz
$\gamma = 2\pi \times 200$ Hz
$I_c = 800$ kW
10dB squeezing

20 ppm loss/round trip
2 filters, each 4 km

total loss ~ 1%

Much less affected by losses!!



A: speed meter w/o filters
1 **[0]** Xtra cavities

B: position meter with filters
2 Xtra cavities

C: speed meter w/ filters
3 **[2]** Xtra cavities

	A	B	C
NS/NS SNR improvement	5.1	6.3	7.5
NS/NS Range (z) Vol. Increase	0.33 [104]	0.41 [181]	0.49 [280]
BH/BH (10+10) Range (z) Vol. Increase	1.4 [15]	2.0 [26]	2.4 [33]

I_c = 800 kW; 10dB squeezing; 20 ppm loss/round trip;
2 filters: each 4 km; total loss ~ 1%

Advanced Interferometer Configurations

Theory of Quantum Mechanical Noises

[Continued]

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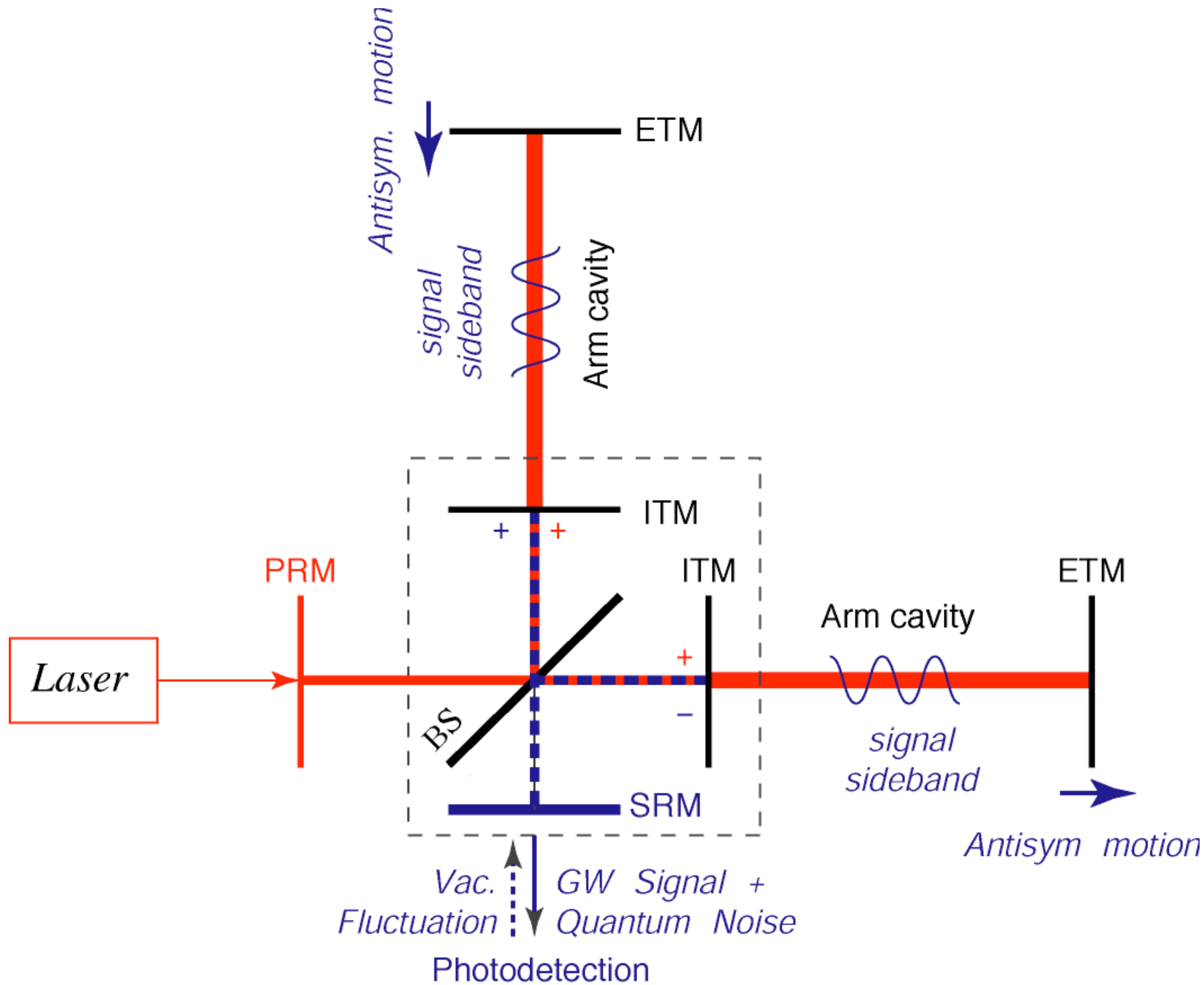
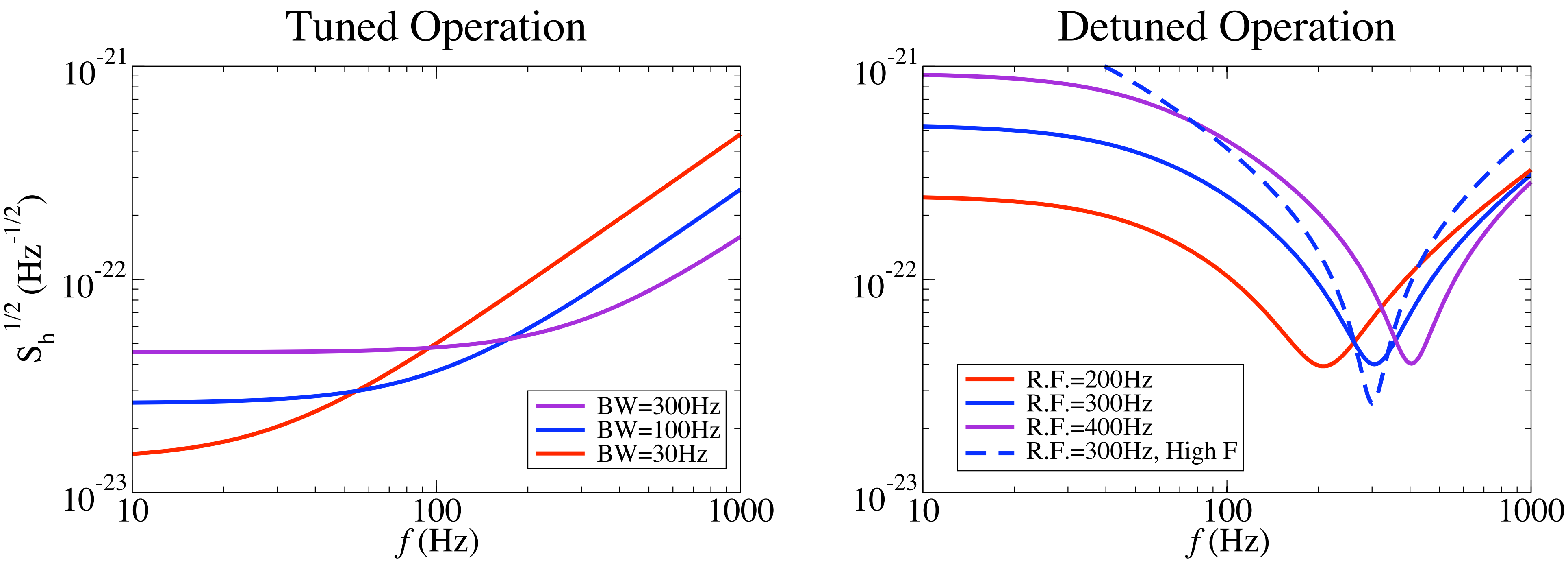
“Detuned” interferometers



- Invented by Drever & Meers
- Signal recycling cavity not resonant/anti-resonant with carrier
- Resonant to GWs with particular frequency

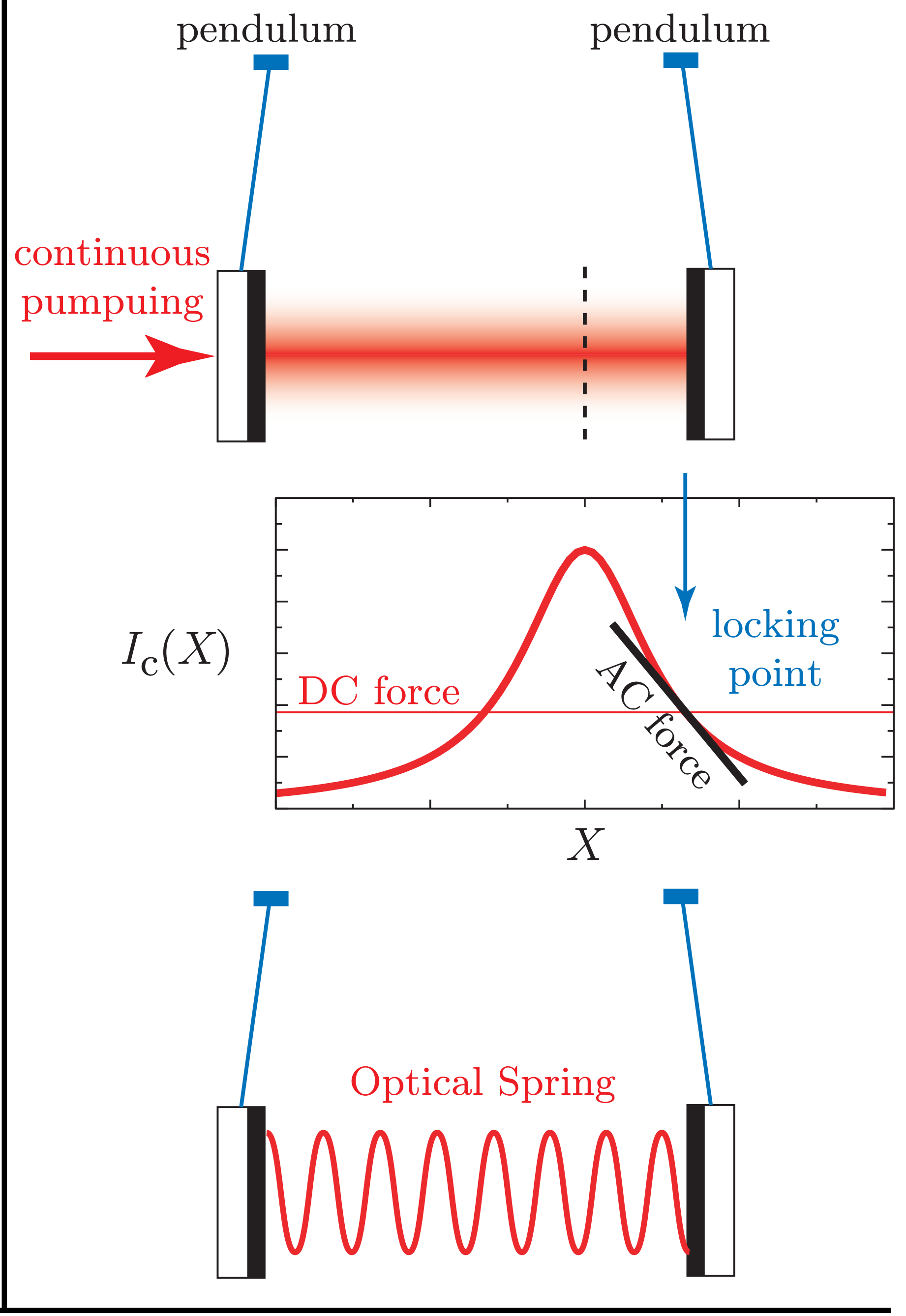
"Detuned" interferometers



- Low-power regime, shot noise only:
  - Tunable optical resonant frequency
  - Trade-off between Bandwidth and Peak Sensitivity
- **High power (Advanced LIGO level) ...**

Modification of Test-Mass Dynamics by Radiation Pressure

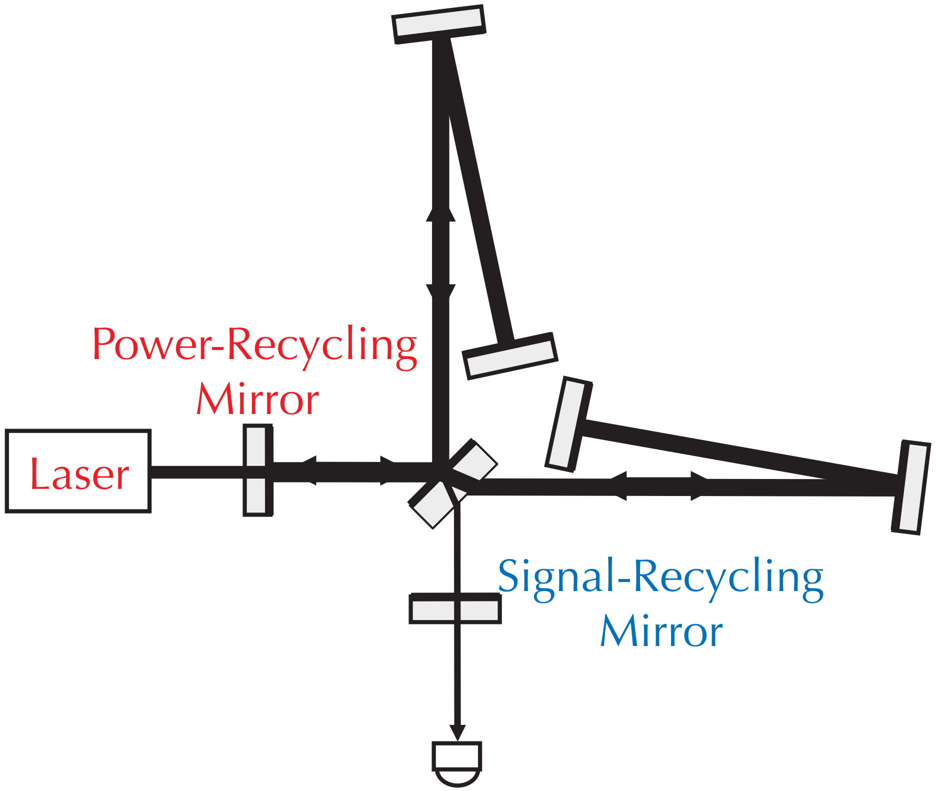
- Detuned Fabry-Perot cavity with continuous pumping: radiation pressure depends on mirror position $\Rightarrow$ ***"optical spring"***



Modification of Test-Mass Dynamics by Radiation Pressure

- Detuned Fabry-Perot cavity with continuous pumping: radiation pressure depends on mirror position $\Rightarrow$ **"optical spring"**
- **Optical spring effect** (*optical rigidity*) can shift pendulum resonance into interferometer's observation band
  - **Classical dynamics**

The GEO600 Interferometer



$$I_c \sim 10\,\mathrm{kW}, \quad M \sim 5.6\,\mathrm{kg}$$
$$f_{\mathrm{opt.spring}} \sim 50\,\mathrm{Hz}$$

Modification of Test-Mass Dynamics by Radiation Pressure

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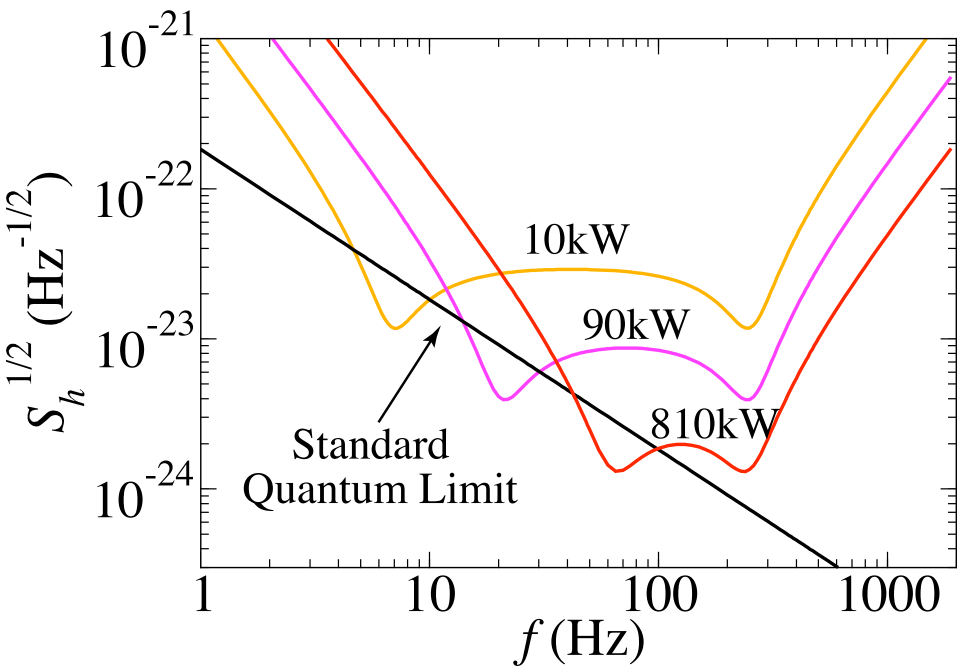
- **Optical spring effect** (*optical rigidity*) can shift pendulum resonance into interferometer's observation band
  - **Classical dynamics**
  - **Enhancement in Quantum-noise-limited sensitivity** around resonance; surpassing the *Standard Quantum Limit* of free test masses *[Buonanno & Chen, 2001--2004]*

Advanced LIGO: surpassing the SQL



Reference design:
$I_c \sim 800\,\mathrm{kW},\ M = 40\,\mathrm{kg}.$

Modification of Test-Mass Dynamics by Radiation Pressure

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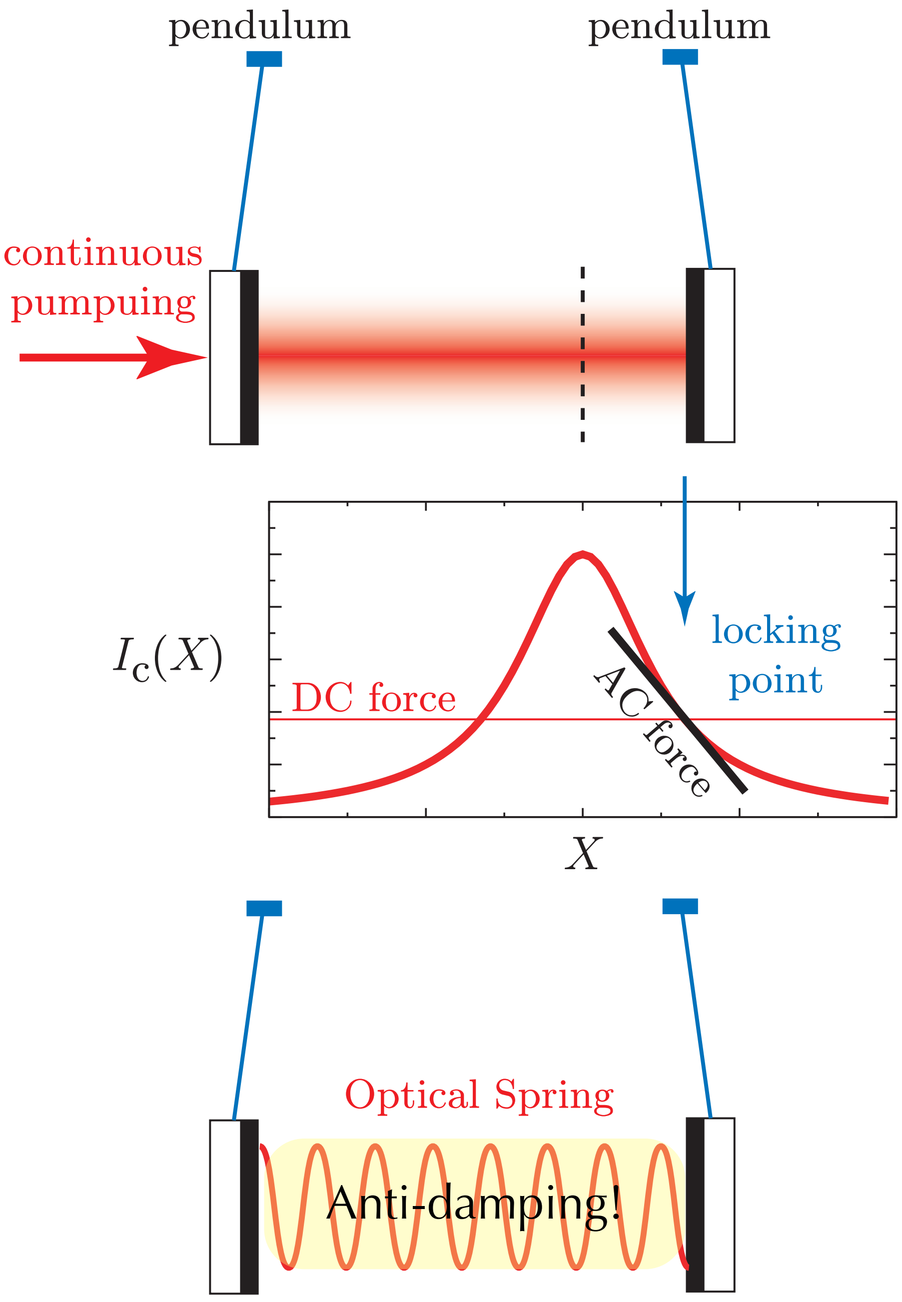
- **Optical spring effect** (*optical rigidity*) can shift pendulum resonance into interferometer's observation band
  - **Classical dynamics**
  - **Enhancement in Quantum-noise-limited sensitivity** around resonance; surpassing the *Standard Quantum Limit* of free test masses *[Buonanno & Chen, 2001--2004]*
- **Instability:**
  - of course when we are locking on the other side of resonance
  - in fact even for this side!



Intracavity readout schemes for GW interferometers

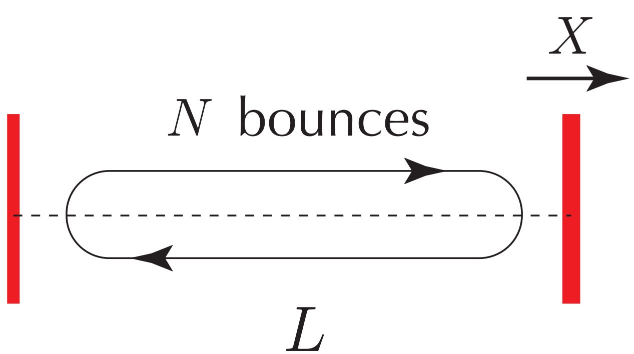
[concept: Braginsky et al. 1990s]

- It seems huge circulating optical power (~MW) and strong squeezing required for further sensitivity improvements, however ...

A "**cavity length contradiction**"

Long cavity: larger GW-induced relative motion: $x \sim Lh$

Short cavity: keeping bandwidth, requires **less circulating power** for **same displacement sensitivity**



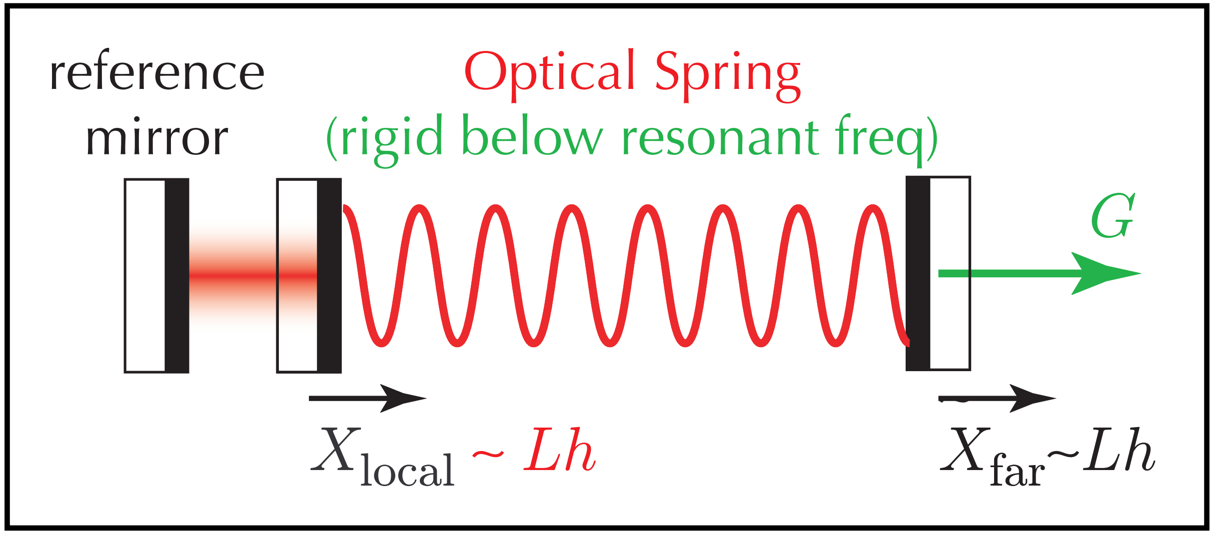
for the same **circulating** power, X sensitivity increase with $N^{1/2}$

but N cannot be too big, because NL/c must be smaller than $P_{\rm GW}$

Long Cavity Wins, but **HIGH POWER!!**

Intracavity readout schemes for GW interferometers

[concept: Braginsky et al. 1990s]



the **optical-bar** detector of Braginsky & Khalili

long cavity (transducer)	short cavity (readout)
Power: opt. spring resonance above detection band, no more!	Low power required because short cavity length

Summary

- "Standard Quantum Limit" is not fundamental for GW detection
- It can be surpassed, in theory, with
  - input/output optics (e.g., squeezed vacuum, optical filters)
  - modifications to interferometer configurations (e.g., speed meters)
  - modifications to test-mass dynamics (e.g., optical spring)
  - ...
- Optomechanical coupling can be used to "cap" the circulating power, while achieving higher sensitivity to gravitational waves.

Comments on the Optical Spring Effect

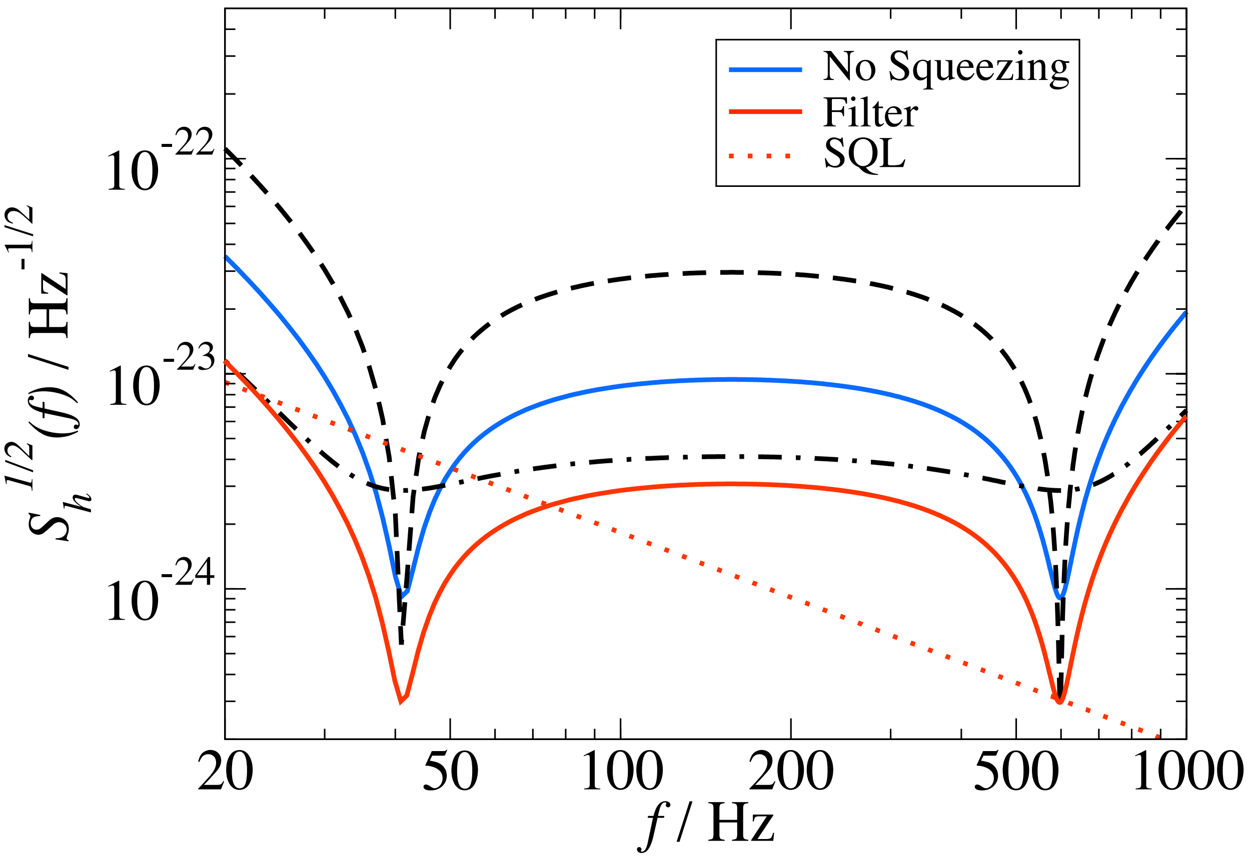
- Some effects of parametric coupling in high-power cavities

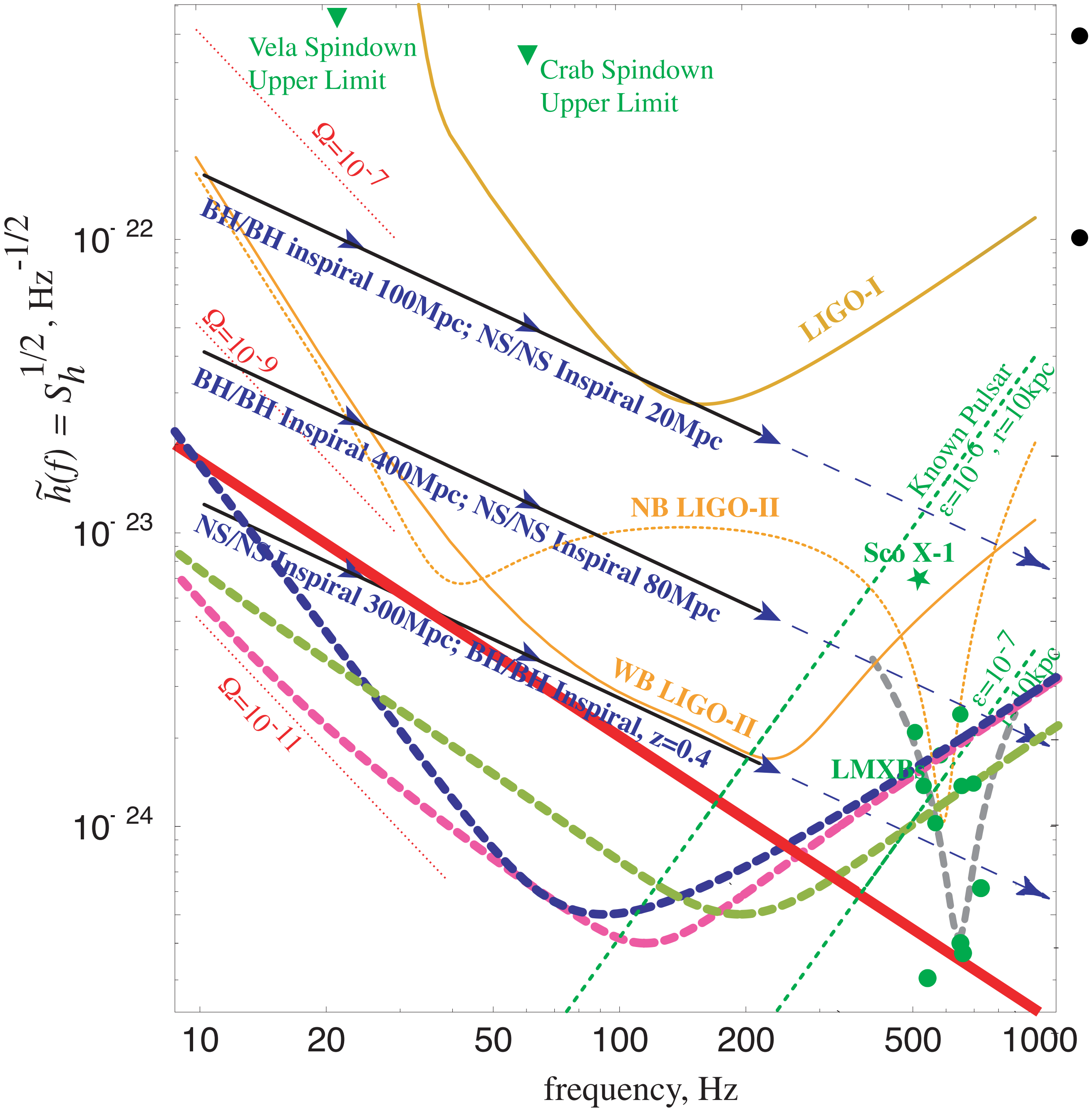
	Test-Mass Mode	Spatial Mode of Optical Modulations	Resulting in ...
rigid	**longitudinal motion**	**00**	**optical-spring resonance and instability**
rigid	pitch/yaw motions	01,10 (and higher, for non-spherical mirrors)	tilt instability [Sigg 2003]
deformable	higher modes (elastic)	higher optical modes	elastic parametric instability [Braginsky et al, UWA group]

- ***One can show in general that:*** power required to induce optical-spring resonance *in the detection band* is the same as that required to reach the Standard Quantum Limit
  - ***The does Parametric Instability implies enough sensitivity to probe quantized mirror tilt/elastic modes?***
- Optical-spring resonance is unstable even when the quasi-static effect is restoring
  - ***Is there danger for extra tilt/elastic parametric instabilities?***

Detuned Signal Recycling + Input-Output Optics

- Optical resonance makes filter design more complicated.
- **Filters must be used, if squeezing are to be taken advantage of at all frequencies**
- Fully optimal filters can be worked out, but cannot be realized by sequence of FP cavities; sub-optimal schemes exist [Harms et al., 2003, Buonanno & Chen, 2004]
- Experimental demonstration (in MHz band) by Schnabel's group in Hannover [Chelkowski et al., 2005]





- Narrowband configuration very sensitive to optical losses. ***More studies must be done.***
- Here we have assumed:
  - signal recycling loss 0.1%
  - photodetector loss 0.1%
  - 4km filter, 20 ppm round-trip loss

What I've left out ...

- **"Ponderomotive squeezer"**
  - building squeezed states from opto-mechanical coupling
  - being carried out at MIT
  - T. Corbitt et al., *in preparation*
- **"Intra-cavity Readout Scheme" proposed by Braginsky**
  - *promises* to limit the power required in the interferometers
  - recent development Danilishin & Khalili, 2005
- **Some recent work on detuned Sagnac interferometers**
  - new type of optomechanical coupling
  - H. Müller-Ebhardt et al., *in preparation*
- ....