Some Ideas on Coatingless all-reflective ITF

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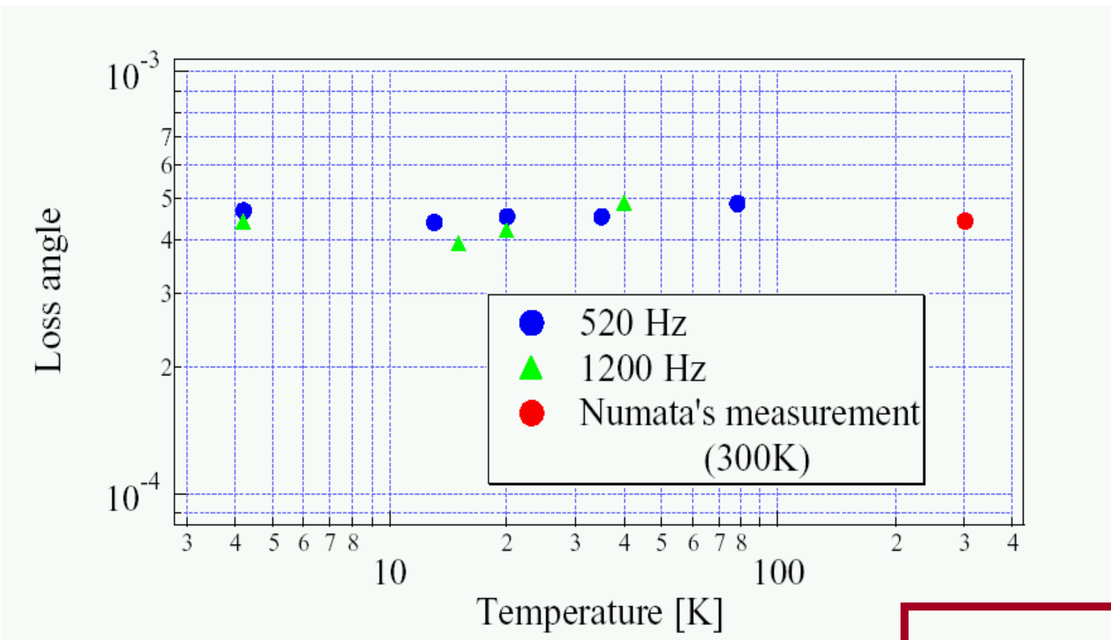
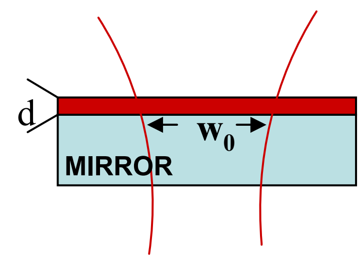
Why Total Reflecting Mirrors?

Experimental data from Yamamoto and Numata show a very high coating loss angle even at low temperature. Subsequent experiments for reducing coating losses gave a 50% improvement but it is not obvious if the optical specifications are still good.





$E_C/E_B = d/w_0 \sim 10^{-4}$

$$\widetilde{x}_{Total}(\omega) = \frac{\sqrt{16 k_B T(\phi_{Bulk} + \frac{E_C}{E_B}\phi_{Coating})}}{\omega^{1/2} E^{1/2} w_0^{1/2}}$$

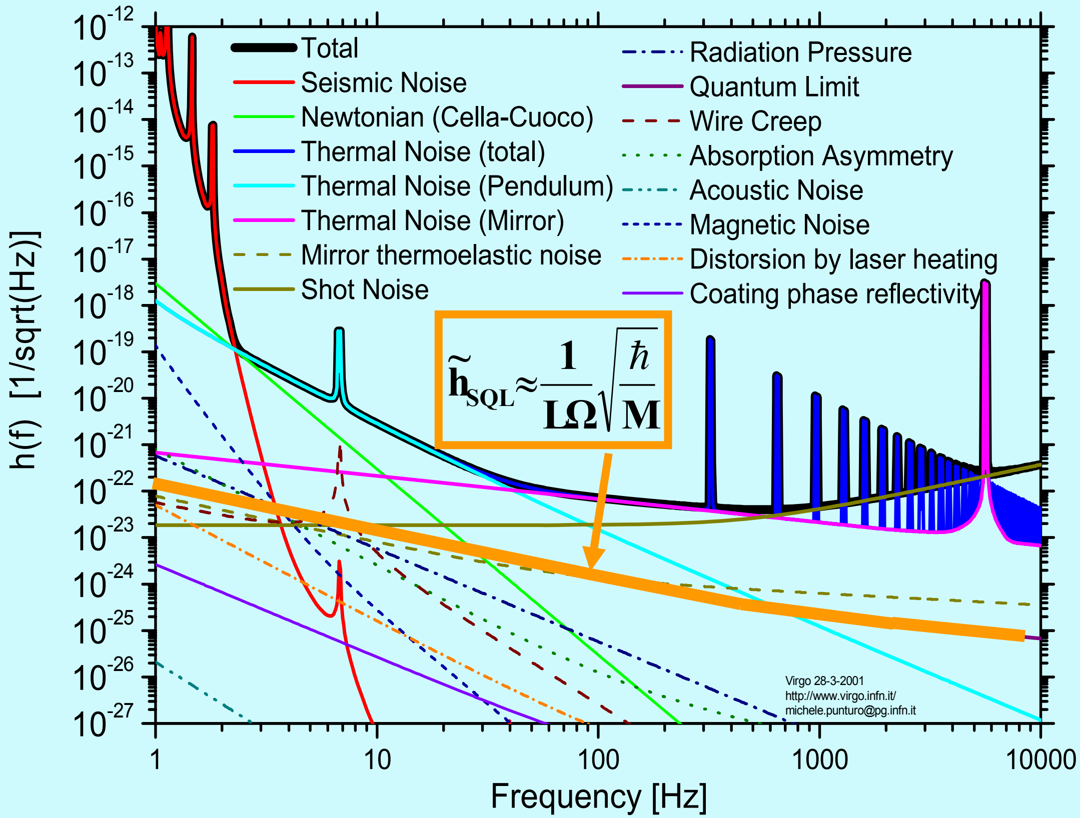
$$\phi_{Total} = \phi_{Bulk} + \frac{E_C}{E_B}\phi_{Coating} \sim \frac{E_C}{E_B}\phi_{Coating} \sim 4 10^{-8}$$

$$Q = \frac{1}{\phi_{Total}} \sim 2.5 10^{7}$$

The Virgo design Thermal noise curve is evaluated with a $Q \sim 10^6$, and the maximum expected Q, due to coating losses, is $Q \sim 2.5\ 10^7$. Standard Quantum Limit is about a factor 100 below Virgo TN i.e. equivalent to a $Q=10^{10}$; consequently, for reaching SQL sensitivity Q should improve by $10^{10}/\ 2.5\ 10^7 \sim 400$- A bit difficult !!

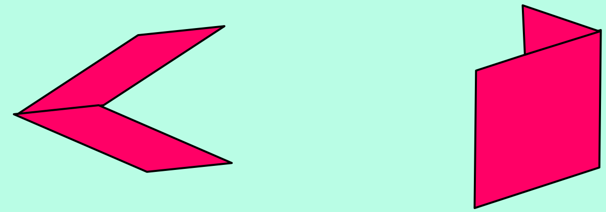
For this reason it is interesting to explore Coating-less Mirrors

This argument will become even more important if we want to go below SQL.



Some History

Toraldo di Francia in 1965 proposes flat Roof Prism for creating a stable Radio Frequency cavity. Stability was succesfully experimentally tested.



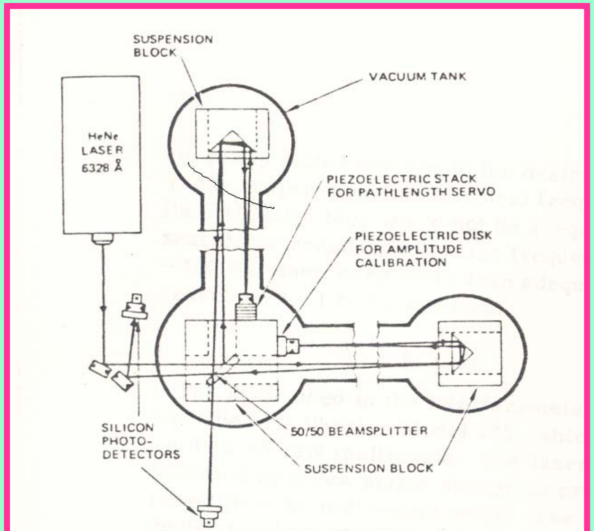


FIG. 7. Schematic of folded optical path.

In 1970 **Robert Forward** (Hughes Lab.) build the first Interferometer for GW detection. It was equipped with **Roof Prisms Mirrors.** Arm length ~2m

Braginsky et al. Recently Published a paper on the use of Roof Prism and Corner Cube mirrors in Fabry Perot cavities using Antireflective Coatings .

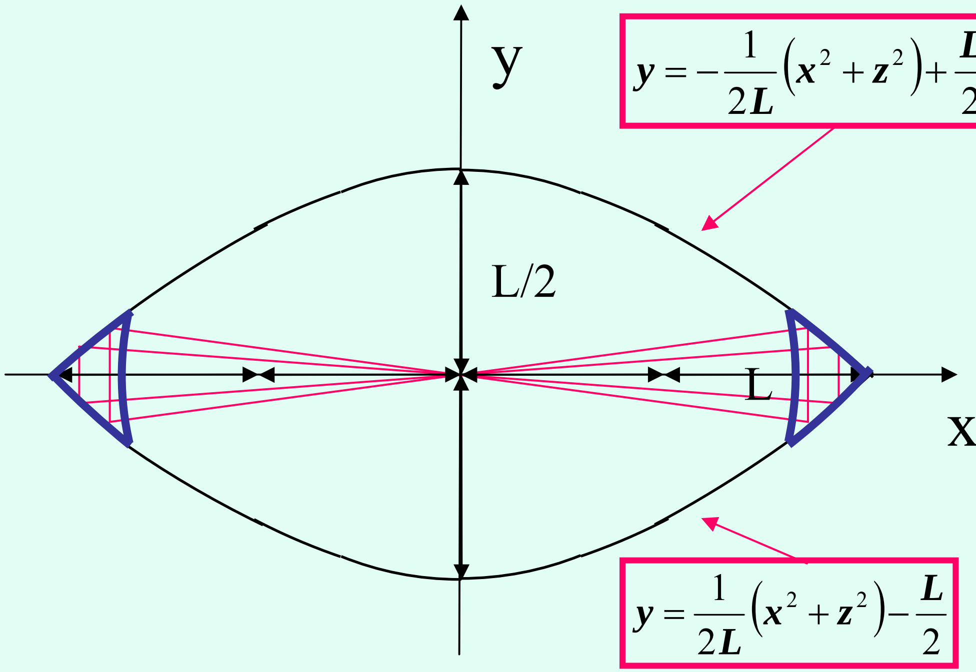
A further point: Gratings Technology (see recent presentations at Einstein's Week in Jena) it is improving a-lot on losses (10^{-3}) but does not work without Coatings

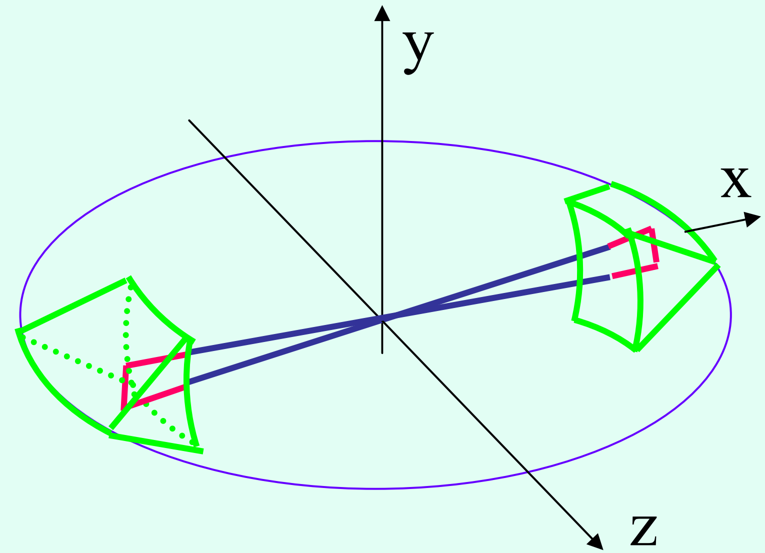
What are then the Missing things for creating a Very Low Thermal Noise Fabry Perot Cavity?

1) **Capability of constructing a completely Coatingless FP Cavity**

2) **Capability of constructing a completely Coatingless Beam Injection system.**

Rotation Parabolas as exact reflectors for closed geometrical optical trajectories

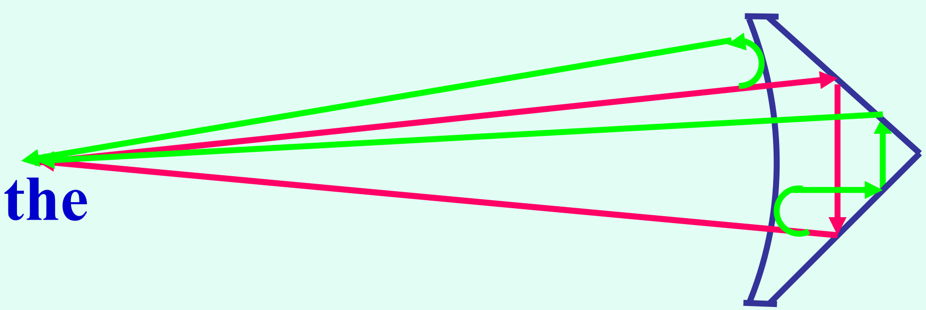




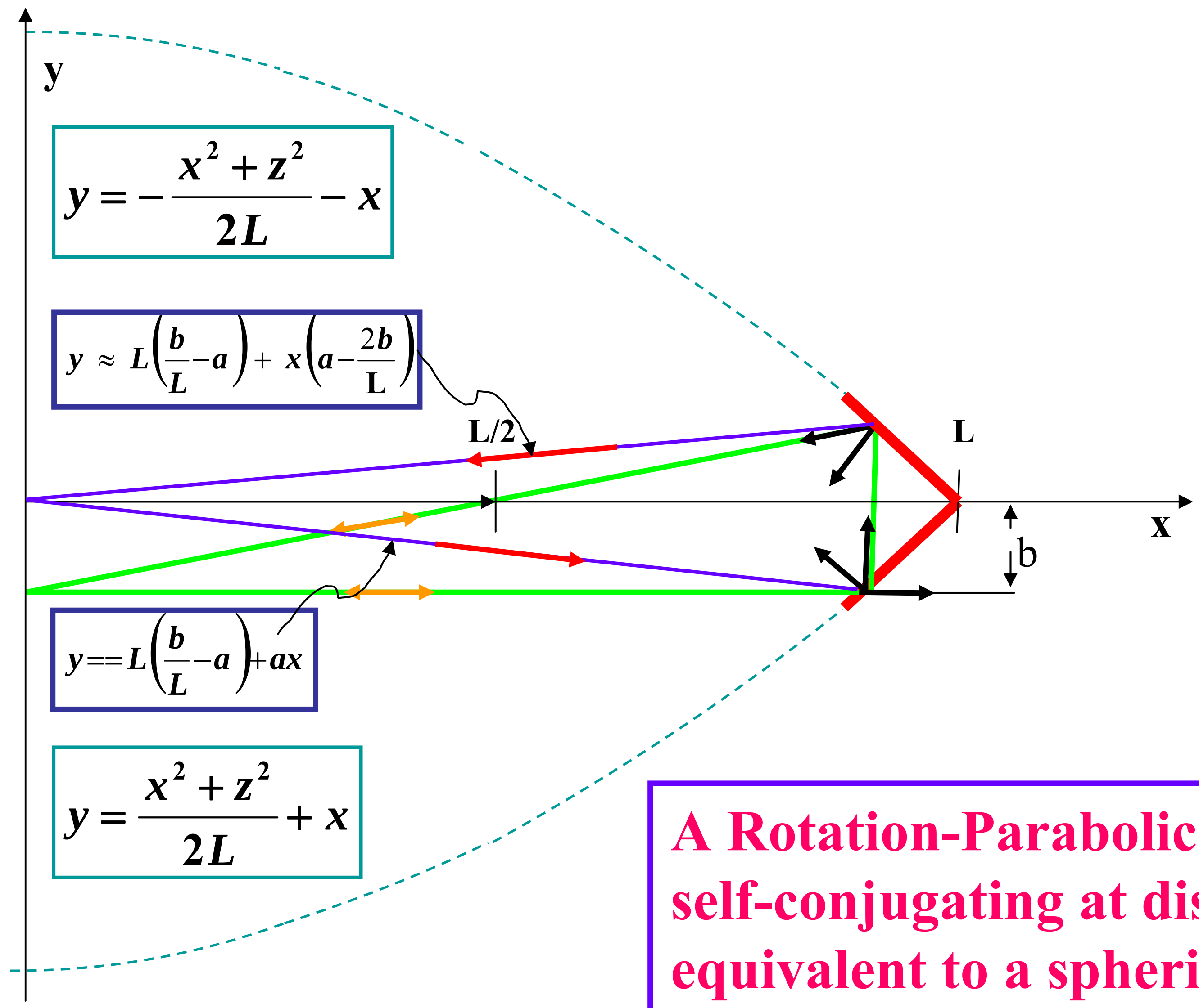
No need of AR Coatings

No Beam Losses

The spherical mirror surface is matched to the constant phase beam surface curvature

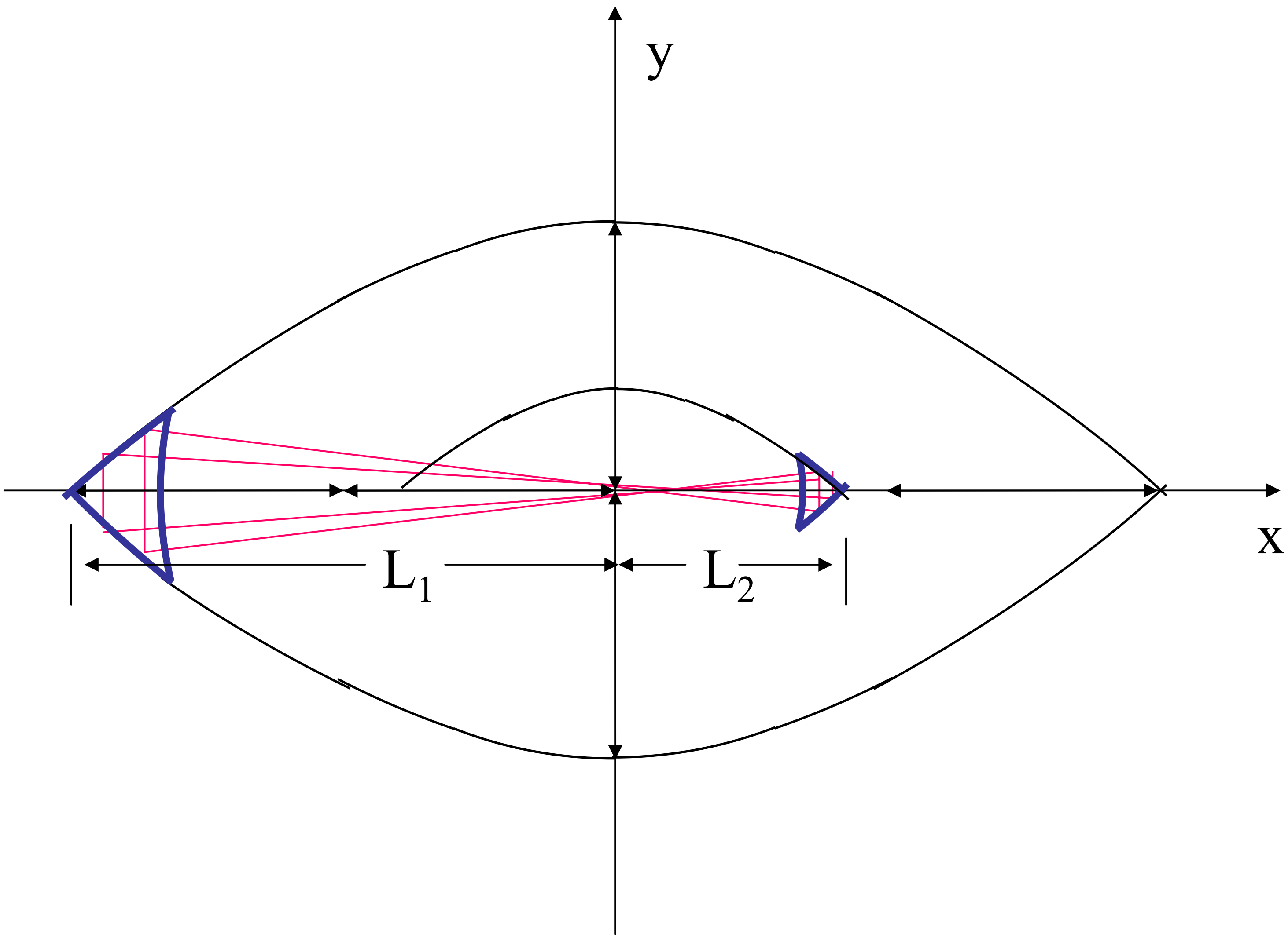


Equivalence to Spherical mirrors of Rotation Parabolas as reflectors for closed geometrical optics trajectories

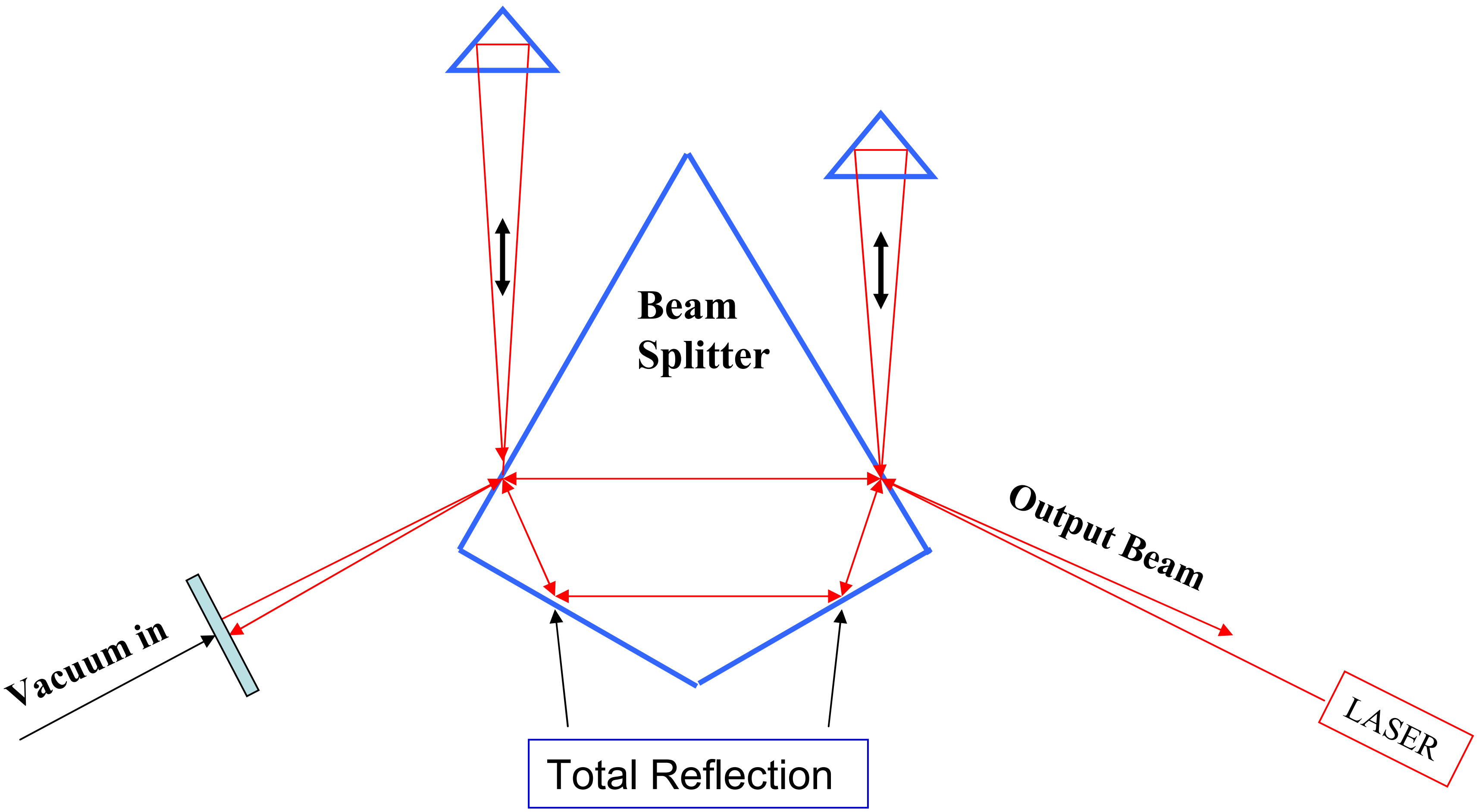


A Rotation-Parabolic Reflector self-conjugating at distance L is equivalent to a spherical mirror with curvature radius L/2

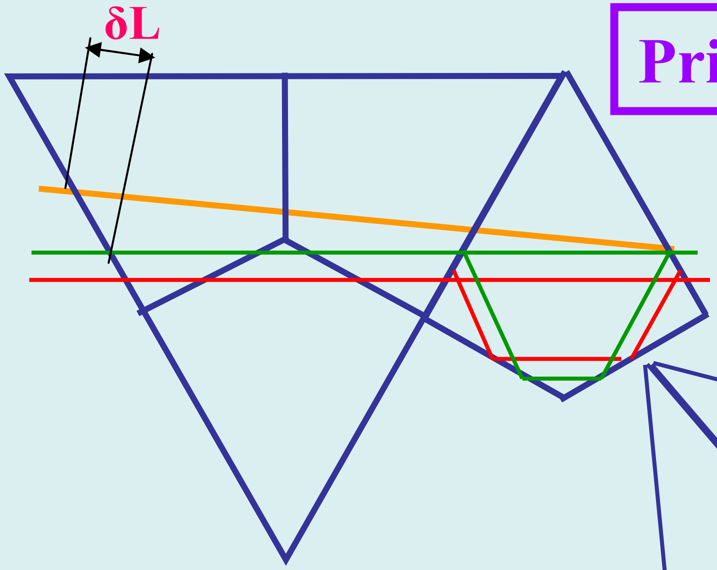
Reflective cavity with Asymmetric Arms

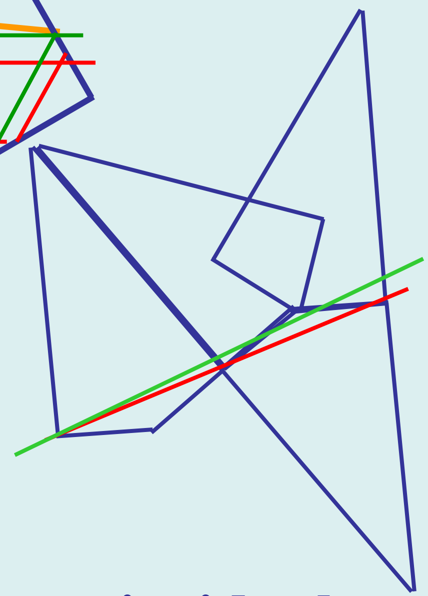


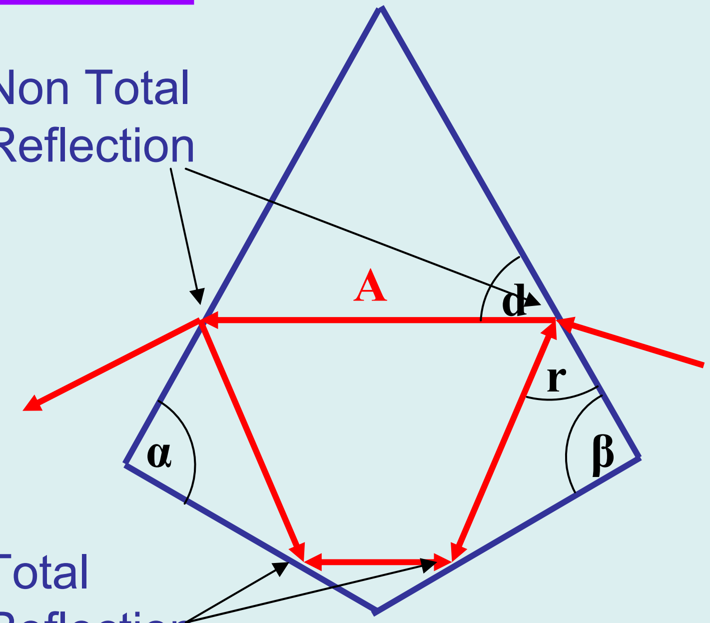
Beam Splitter for Power Injection in the Parabolic 4 elements all-Reflective Cavity



Prism B-S Properties

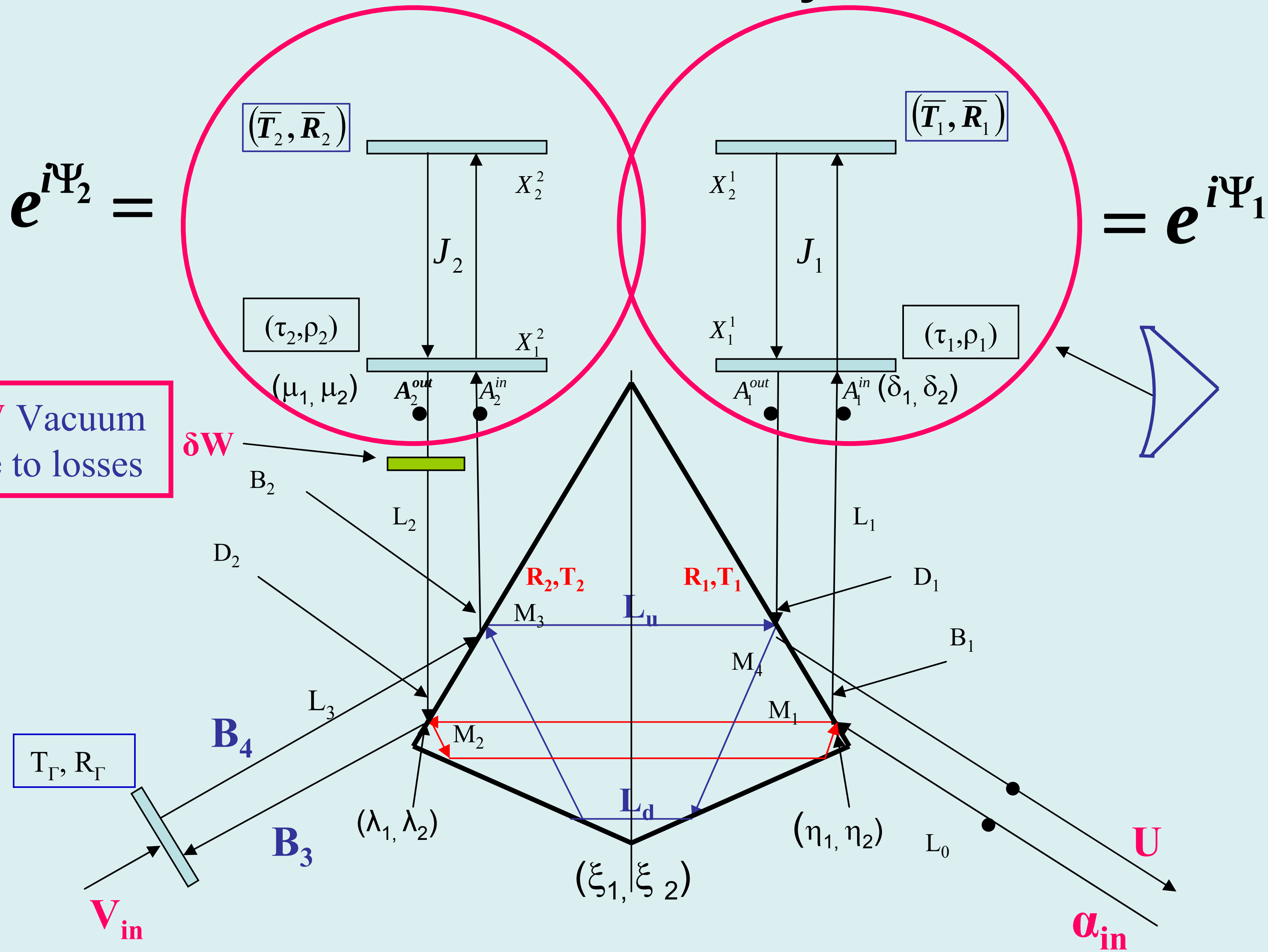




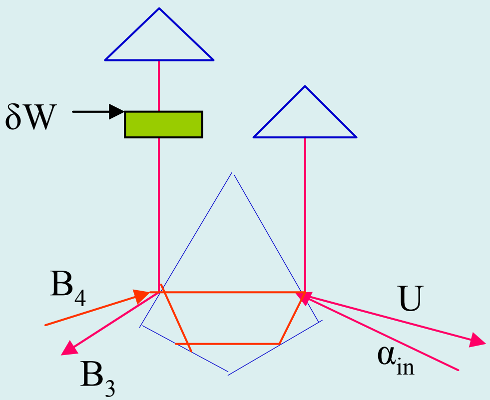


1) If α+β=π the trajectory inside the prism is closed and d=r. i.e. the trajectory is replicating itself.
2) If α+β=π and the trajectory A is parallel displaced, the inner trajectory is length invariant.
3) The only way for changing inner trajectory length is by rotating the prism. Then we still have d=r but the trajectory does not close and consequently does not replicate itself. This property can be used for making inner trajectory resonating or antiresonating

Optical Diagram of a 4 Elements Coatingless all-reflective cavity



The Classical Amplitudes U and B_3



$$B_3 = D\delta W + C\alpha_{in} + EB_4$$

$$U = A\delta W + B\alpha_{in} + CB_4$$

$$A = \frac{T_1T_2\Lambda^{-1}\left(R_1 - T_1^2R_2\Lambda^{-1}e^{-i\Omega\frac{L_u+L_d}{c}}R\left(-\frac{L_u+L_d}{c}\right)\right)e^{-i\Omega\frac{2L_1+L_d+L_0}{c}}R\left(-\frac{2L_1+L_d+L_0}{c}+\Psi_1\right)}{\left(1-\left(T_1T_2\right)^2\Lambda^{-2}e^{-i\Omega\frac{2L_1+2L_2+2L_d}{c}}R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_2+\Psi_1\right)\right)}$$

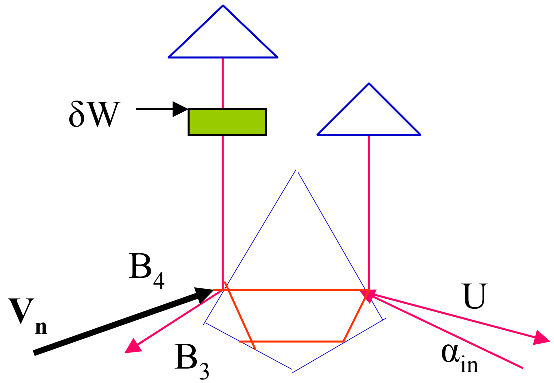
$$B = \frac{\left(R_1 - T_1^2R_2\Lambda^{-1}e^{-i\Omega\frac{L_u+L_d}{c}}R\left(-\frac{L_u+L_d}{c}\right)\right)^2e^{-i\Omega\frac{2L_1+2L_0}{c}}R\left(-\frac{2L_1+2L_0}{c}+\Psi_1\right)}{\left(1-\left(T_1T_2\right)^2\Lambda^{-2}e^{-i\Omega\frac{2L_1+2L_2+2L_d}{c}}R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_2+\Psi_1\right)\right)}$$

$$C = T_2T_1\Lambda^{-1}e^{-i\Omega\frac{L_0-L_d}{c}}R\left(-\frac{L_0-L_d}{c}\right)\bullet$$
$$\bullet\left(e^{-i\Omega\frac{L_u+L_d}{c}}R\left(-\frac{L_u+L_d}{c}\right)+\frac{\left(R_2-R_1T_2^2\Lambda^{-1}e^{-i\Omega\frac{L_d+L_u}{c}}R\left(-\frac{L_d+L_u}{c}\right)\right)}{1-\left(T_1T_2\right)^2\Lambda^{-2}e^{-i\Omega\frac{2L_1+2L_2+2L_d}{c}}R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_2+\Psi_1\right)}\bullet\right.$$
$$\left.\bullet\frac{\left(R_1-R_2T_1^2\Lambda^{-1}e^{-i\Omega\frac{L_u+L_d}{c}}R\left(-\frac{L_u+L_d}{c}\right)\right)e^{-i\Omega\frac{2L_1+2L_2+2L_d}{c}}R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_1+\Psi_2\right)}{1-\left(T_1T_2\right)^2\Lambda^{-2}e^{-i\Omega\frac{2L_1+2L_2+2L_d}{c}}R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_2+\Psi_1\right)}\right)$$

$$E = \frac{e^{-i\Omega\frac{2L_2}{c}}R\left(-\frac{2L_2}{c}+\Psi_2\right)\left(R_2-R_1T_2^2\Lambda^{-1}e^{-i\Omega\frac{L_u+L_d}{c}}R\left(-\frac{L_u+L_d}{c}\right)\right)^2}{1-\left(T_1T_2\right)^2\Lambda^{-2}e^{-i\Omega\frac{2L_1+2L_2+2L_d}{c}}R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_2+\Psi_1\right)}$$

$$D = \frac{\left(R_2-R_1T_2^2\Lambda^{-1}e^{-i\Omega\frac{L_d+L_u}{c}}R\left(-\frac{L_d+L_u}{c}\right)\right)}{1-\left(T_1T_2\right)^2\Lambda^{-2}e^{-i\Omega\frac{2L_1+2L_2+2L_d}{c}}R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_2+\Psi_1\right)}$$

Resonance conditions and its consequences



B_4 is connected to U by means of C coefficient i.e. Since the vacuum V_n is also entering from the port B_4 it is relevant to understand the coupling between V_n and U i.e. the behavior of C coefficient as a function of Ω.

$$\boldsymbol{C} = T_2 T_1 \Lambda^{-1} e^{-i\Omega\frac{L_0 - L_d}{c}} R\left(-\frac{L_0 - L_d}{c}\right) \left(\frac{e^{-i\Omega\frac{L_u + L_d}{c}} R\left(-\frac{L_u + L_d}{c}\right) + \left(R_2 - R_1 T_2^2 \Lambda^{-1} e^{-i\Omega\frac{L_d + L_u}{c}} R\left(-\frac{L_d + L_u}{c}\right)\right)}{1 - (T_1 T_2)^2 \Lambda^{-2} e^{-i\Omega\frac{2L_1 + 2L_2 + 2L_d}{c}} R\left(-\frac{2L_1 + 2L_2 + 2L_d}{c} + \Psi_2 + \Psi_1\right)} \bullet \frac{\bullet \left(R_1 - R_2 T_1^2 \Lambda^{-1} e^{-i\Omega\frac{L_u + L_d}{c}} R\left(-\frac{L_u + L_d}{c}\right)\right) e^{-i\Omega\frac{2L_1 + 2L_2 + 2L_d}{c}} R\left(-\frac{2L_1 + 2L_2 + 2L_d}{c} + \Psi_1 + \Psi_2\right)}{1 - (T_1 T_2)^2 \Lambda^{-2} e^{-i\Omega\frac{2L_1 + 2L_2 + 2L_d}{c}} R\left(-\frac{2L_1 + 2L_2 + 2L_d}{c} + \Psi_2 + \Psi_1\right)} \right)$$

The resonance conditions are:

$$R\left(-\frac{2L_1+2L_2+2L_d}{c}+\Psi_2+\Psi_1\right)=1$$ **Arm Resonance**

$$R\left(-\frac{L_d+L_u}{c}\right)=\pm1$$

+1 =Prism Resonance

-1 =Prism Antiresonance

If $R_1=R_2$ and $\Omega=0$, $C_{Res}\neq0$. While $C_{A.Res}=0$ for any R1, R2 and $\Omega=0$

C becomes :

$$C_{Res}=\frac{-\sqrt{1-R_1^2}\sqrt{1-R_2^2}\left(1-R_2R_1\right)^3\left(1-e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-(\Psi_2+\Psi_1)\right)}\right)e^{-i\Omega\frac{L_0-L_d}{c}}R\left(-\frac{L_0-L_d}{c}\right)}{\left(1-R_2R_1\right)^2\left(1-e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-(\Psi_2+\Psi_1)\right)}\right)+\left(R_1-R_2\right)^2e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-(\Psi_2+\Psi_1)\right)}}$$

$$C_{Antires}=\frac{-\sqrt{1-R_1^2}\sqrt{1-R_2^2}\left(1+R_2R_1\right)^3\left(1-e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-(\Psi_2+\Psi_1)\right)}\right)e^{-i\Omega\frac{L_0-L_d}{c}}R\left(-\frac{L_0-L_d}{c}\right)}{\left(1+R_2R_1\right)^2\left(1-e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-(\Psi_2+\Psi_1)\right)}\right)+\left(R_1+R_2\right)^2e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-(\Psi_2+\Psi_1)\right)}}$$

If the prism is antiresonant and the arms are in resonance, it follows that at $\Omega=0$ there is no coupling V_n-U i.e. C=0.

Then only V_n sidebands may couple to U.

$$C_{Antires}=\frac{-\sqrt{1-R_1^2}\sqrt{1-R_2^2}\left(1+R_2R_1\right)^3\left(1-e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-\left(\Psi_2+\Psi_1\right)\right)}\right)e^{-i\Omega\frac{L_0-L_d}{c}}R\left(-\frac{L_0-L_d}{c}\right)}{\left(1+R_2R_1\right)^2\left(1-e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-\left(\Psi_2+\Psi_1\right)\right)}\right)+\left(R_1+R_2\right)^2e^{-i\left(\Omega\frac{2L_1+2L_2+2L_d}{c}-\left(\Psi_2+\Psi_1\right)\right)}}$$

The Cavity Finesse

$$U = A\delta W + B\alpha_{in} + CB_4$$

Cavity Finesse can be evaluated by differentiating B with respect L_1 and L_2 and then making $R\left(\frac{2L_1+2L_2+2L_d}{c} - \Psi_2 - \Psi_1\right) = 1$. For the sake of simplicity we set the prism both in resonance (-) and in antiresonance (+);

$$\frac{\partial B}{\partial L_i} \approx \frac{\omega_0}{c} \frac{\left(\left((R_2 \mp R_1)^2 - (1 \mp R_2 R_1)^2\right) e^{-i\Omega\left(\frac{2L_1+2L_2+2L_d}{c} - \frac{\Psi_1+\Psi_2}{\omega_0}\right)}\right)(R_1 \mp R_2)^2 e^{-i\Omega\frac{2L_1+2L_0}{c}} R\left(-\frac{2L_1+2L_0}{c}+\Psi_1\right) R\left(\frac{\pi}{2}\right)}{\left((1 \mp R_2 R_1)^2 + \left((R_2 \mp R_1)^2 - (1 \mp R_2 R_1)^2\right) e^{-i\Omega\left(\frac{2L_1+2L_2+2L_d}{c} - \frac{\Psi_1+\Psi_2}{\omega_0}\right)}\right)^2} \qquad i=1,2$$

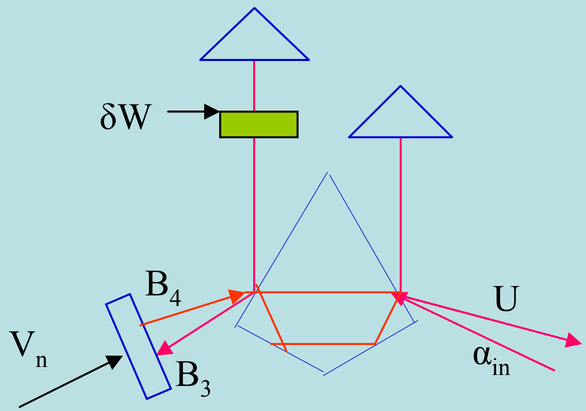
$$\frac{\partial B}{\partial L_i} \approx \frac{\omega_0}{c} \frac{1}{(R_2 \mp R_1)^2} \left(\left((R_2 \mp R_1)^2 - (1 \mp R_2 R_1)^2\right)\right) R\left(-\frac{2L_1+2L_0}{c}+\Psi_1\right) R\left(\frac{\pi}{2}\right) \approx \frac{\omega_0}{c} \frac{1}{(R_2 \mp R_1)^2}$$

At $\Omega=0$ we obtain the finesse $F \approx \frac{1}{(R_2 \mp R_1)^2}$

Ponderomotive actions and thermal noise couple to U trough the term $\frac{\partial B}{\partial L_i}\delta L_i$ where δL_i are both the mirror and prism displacements produced by these two phenomena.

Toward an interferometric application

If we close the port B_3 with a mirror, then the vacuum B_4 will be stopped and U will depend only by δW and α_{in}



$$U = A\delta W + C\frac{R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)D\delta W_1 + T_\Gamma V_{in}}{1 - R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)E} + \frac{B\left(1 - R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)E\right) + R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)C^2}{1 - R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)E}a_{in}$$

$$\mathbf{U = A'\delta W + C'\alpha_{in}}$$

B_3 Mirror Thermal Noise

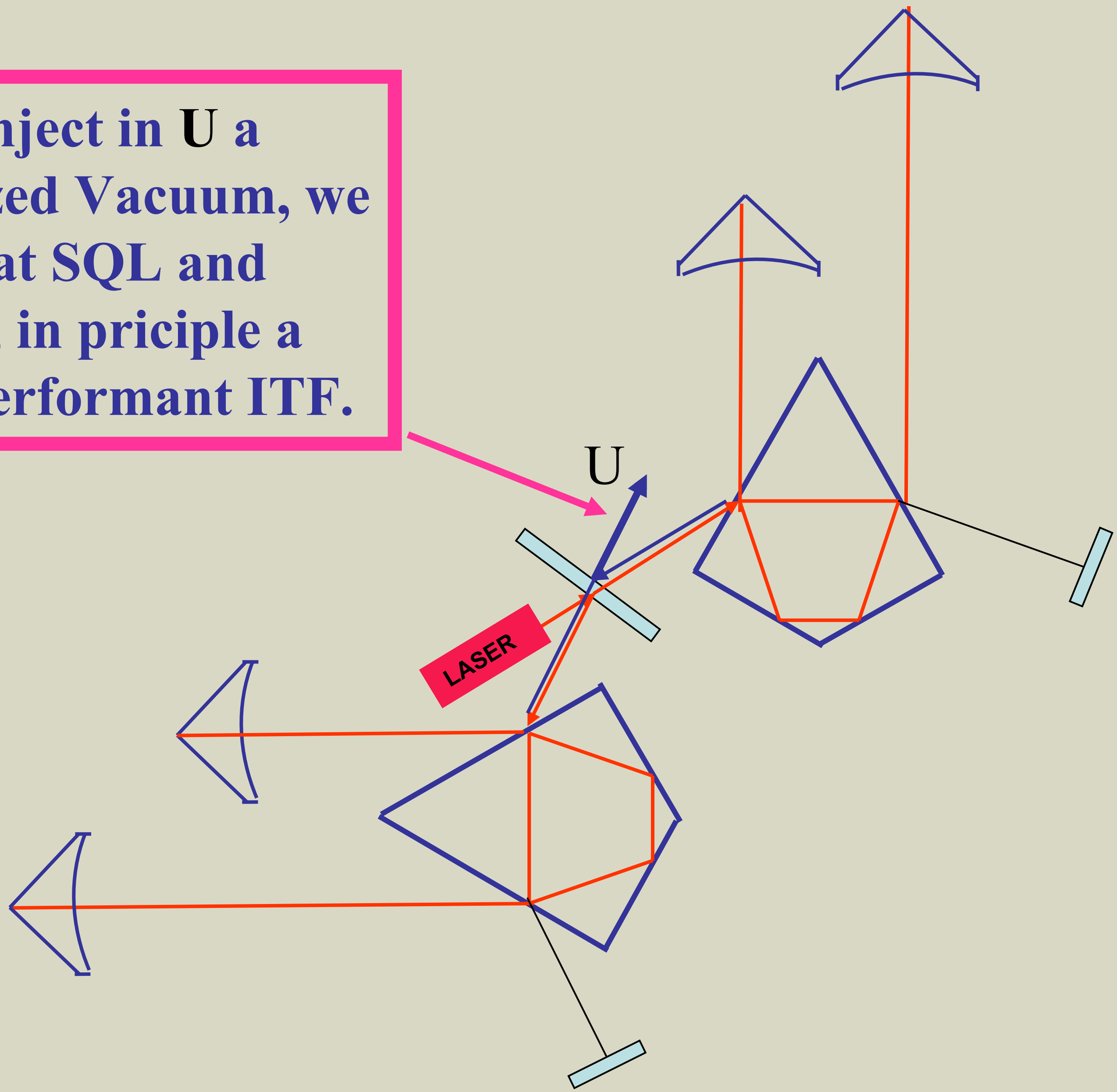
We have seen that C=0 for Ω=0; this means that under the choosen resonance conditions ther is no carrier in B_3 and if there is no carrier, sidebands, to first order, can not be produced. This is easily seen by differentiating U with respect to L_3:

$$\frac{1}{a_{in}}\frac{\partial U}{\partial L_3}=\frac{\partial}{\partial L_3}\left(B+\frac{R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)C^2}{1-R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)E}\right)=C^2(\Omega=0)\bullet\left[\frac{\partial}{\partial L_3}\frac{R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)}{1-R_\Gamma e^{-i\Omega\frac{2L_3}{c}}R\left(-\frac{2L_3}{c}\right)E}\right]_{\Omega=0}=0$$

This means that we may put a normal mirror on B_3 without affecting total reflecting cavity thermal noise.

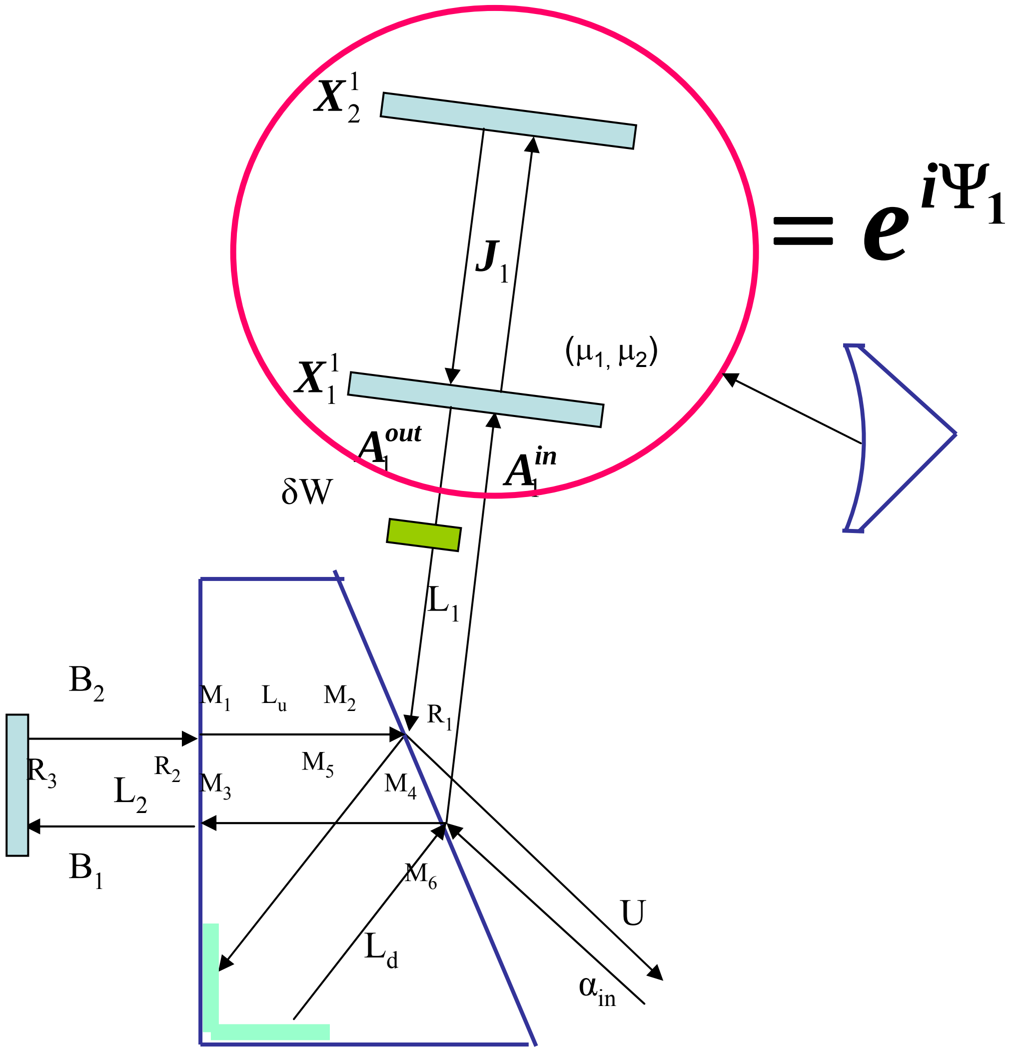
The Interferometer

If we inject in U a Squeezed Vacuum, we can beat SQL and obtain, in priciple a very performant ITF.



Beam Splitter for Power Injection in the 3 elements Parabolic all-Reflective Cavity

Perhaps a more performant configuration



Conclusions

Some Difficulties

1) The Parabolic and Beam Splitter prisms needs to have a remarkable Refraction Index omogeneity for avoiding higher mode production. At the moment we may obtain $\lambda/1000$ over 10 cm thicknes of silica, enough for starting tests.

2) The Parabolic Prism edge should be very thin, some μm, otherwise both higher modes and beam losses will be produced. The design of a beam with zero intensity on the prism edge could be the solution.

A big Advantage

Without coatings the Prisms mechanical Q can be enormous even at room temperature; this solution could be instrumental also for future SQL beating ITF's.