Network Analysis of Gravitational Waves

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Outline

- Overview
  - role of AIGO
- Our Proposals
  - to deal with realistic problems

- Application to GW Network Analysis
  - Veto/localization
  - Test of GR

Interferometric GW Detectors

Network Analysis

- Detection/Confidence
- Source Localization
  - time-delay triangulation
- Waveform Extraction
- Test GR

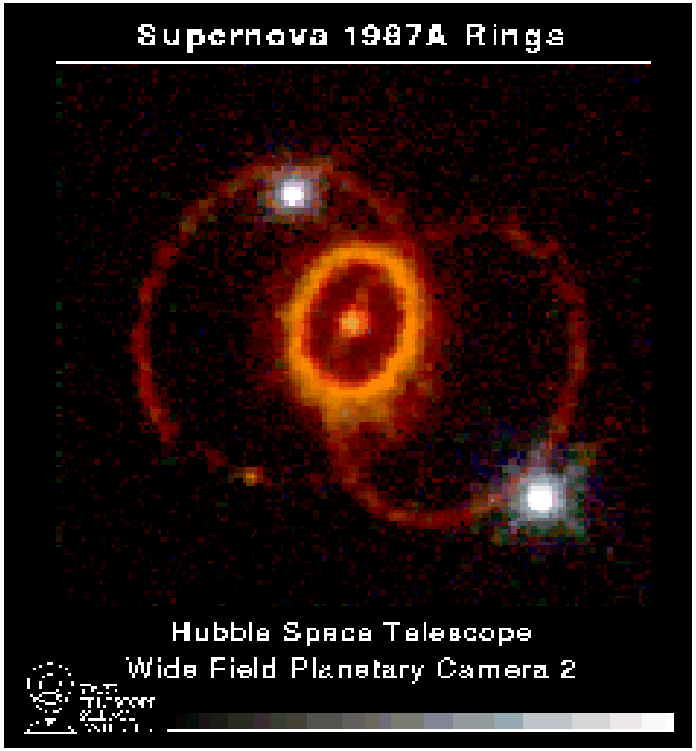
WHAT IS THE BEST WAY TO COMBINE ALL AVAILABLE NETWORK DATA ?

My work: 1. null-stream method for veto/validation/localization

2. stable solution for detection/localization/waveform extraction

Challenge: Burst GWs

- The unknown: Burst GWs
  - » e.g. Core collapse in supernovae / GRBs, BH-BH merger
  - » *unknown or not well modelled*
  - » *possible very short bursts*
    - – *tens of milliseconds*
    - – *high frequencies (>500 Hz)*
  - » *possible EM counterparts*
- Our methods are NOT restricted to burst GWs
  - » *more information can only make them work better*
  - » *burst GW is the main challenge*
    - – *BH-BH merger GW used in examples*



Detector Response to GWs

- **Linear response to the two GW polarizations**

$$h_i(t) = f^+{}_i(t)\,h_+(t) + f^X{}_i(t)\,h_X(t) + n_i(t)$$

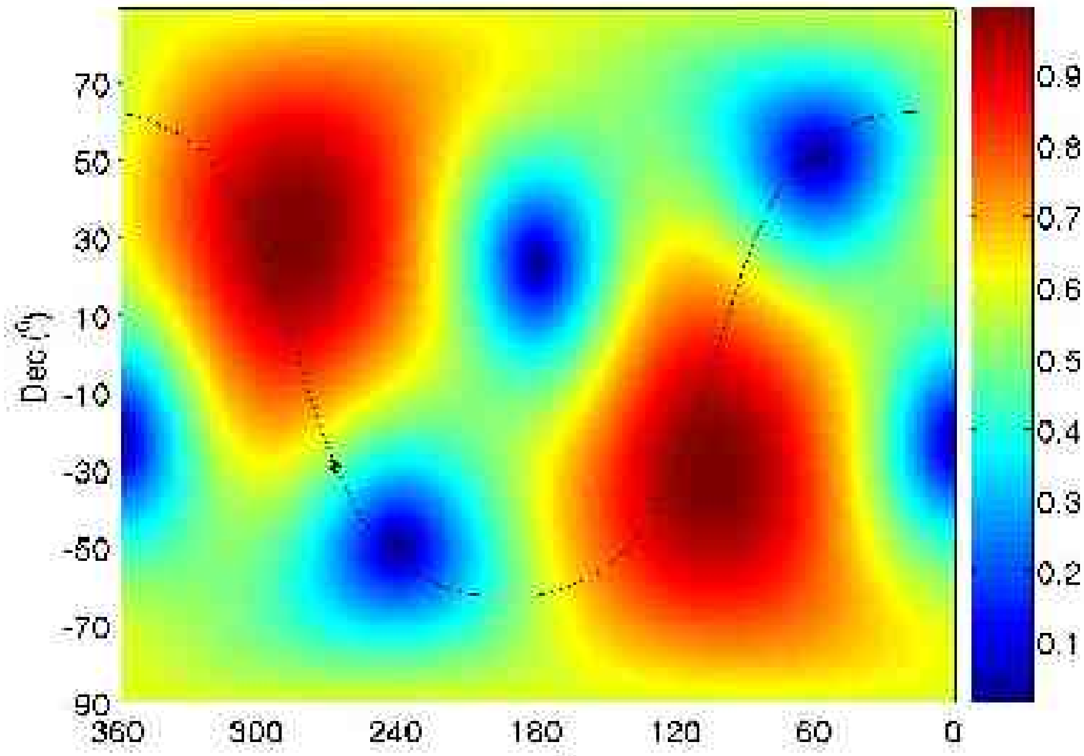
$f^{+X}{}_i(t)$ **depend on**

  - **detector location and orientation**
  - **source direction, polarization angle**

- **frequency-dependent noise** n_i
- **Sensitivity characterized by** $\sqrt{f^{+\,2}_{\;i}(t) + f^{X\,2}_{\;i}(t)}$

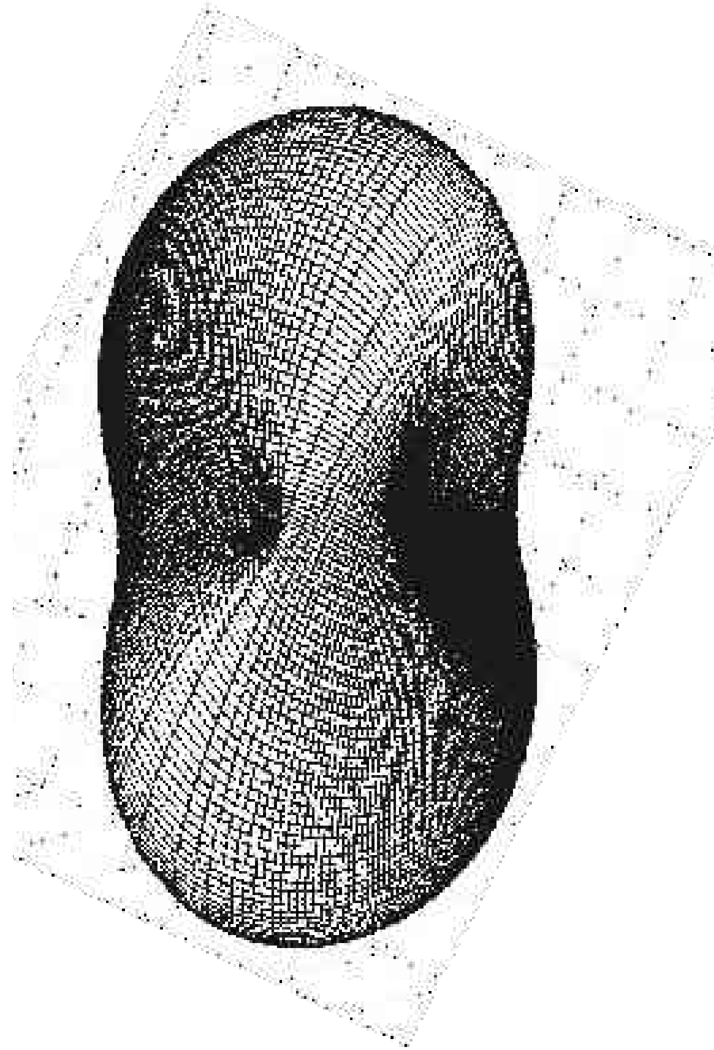
Sky Sensitivity

$$\sqrt{f_i^{+2}(t) + f_i^{X\,2}(t)}$$



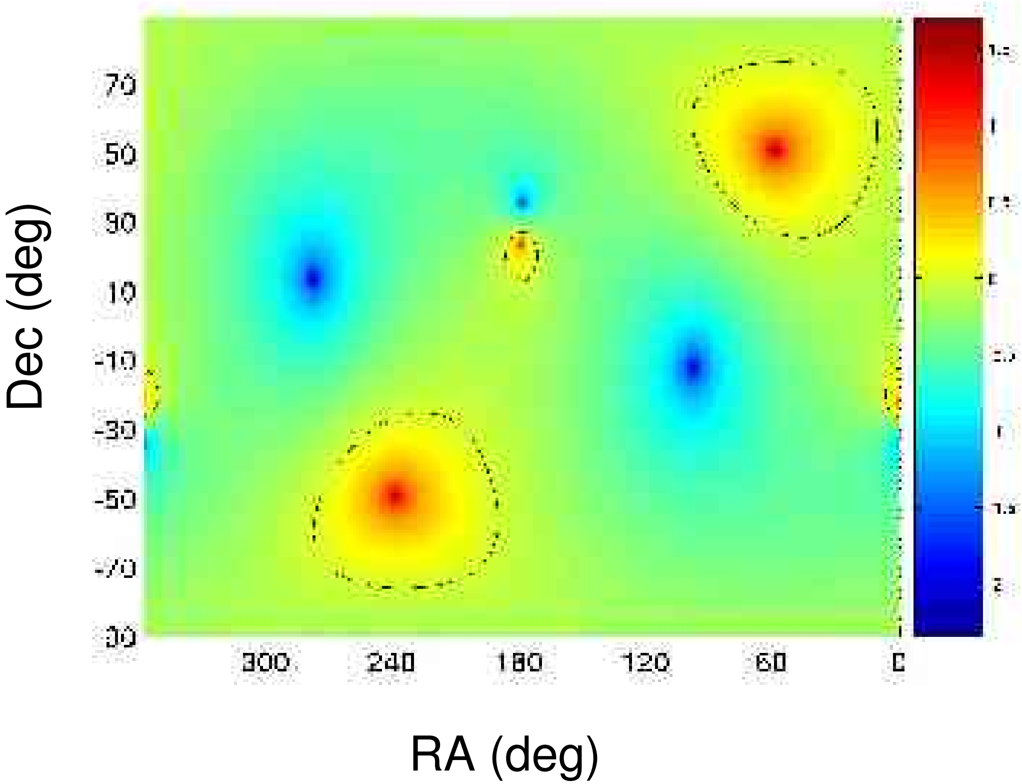
RA (deg)

LIGO



Sky Coverage

$$\frac{\rho_{GEO}}{\rho_{LIGO}} \text{ at f=1 kHz}$$



$$\text{Detection volumn} \propto \rho^{3}_{i} \times \text{fraction of sky}$$

- **Sensitivity Comparison**

$$\text{SNR:}\ \rho_i \propto \sqrt{\frac{{f^{+}_{i}}^{2} + {f^{X}_{i}}^{2}}{S(f)}}$$

  - **For mis-aligned detectors**
    - **At 1 kHz, at ~15% sky area, GEO is more sensitive than LIGO I (VIRGO -> higher %)**
    - **but detection volumn is much smaller due to smaller SNR**
    - **not by choice**

Detection Volume for AIGO

$$\text{Detection volume} \propto \rho_N^3 \times \%\text{sky} \propto \left(\sum_i^N \rho_i^2\right)^{3/2} \times \%\ \text{sky}$$

- AIGO
  - » antenna aligned with LIGO -> ~ same % sky
  - » ½ LIGO SNR (with much smaller budget !)
- AIGO+one LIGO(LLO | LHO)
  - » detection volume ~1.4 x one LIGO
- AIGO+two LIGO (LLO&LHO)
  - » detection volume ~1.2 x two LIGOs

Source Localization: Time-delay Triangulation

Arrival time delay of GW at two detectors τ_{ij} -> determination of a cone in the sky

τ_{max}

GEO-VIRGO : 3 ms
LLO- LHO : 10 ms
LHO- (GEO,VIRGO,TAMA), LLO-(GEO,VIRGO), TAMA-AIGO: 24-26 ms
TAMA-(LLO, VIRGO) : 30-32 ms
AIGO-(GEO,VIRGO, LHO, LLO) : 37-42 ms

AIGO-LLO offers the longest baseline of 42 ms

$$\delta\alpha \propto \left(\frac{\lambda_{GW}}{\tau_{max}} \right) \frac{1}{\rho_N}$$

diffraction limit + Arnard et al 2003

longer baseline-> better directional resolution

higher network SNR-> better resolution

limited by sampling resolution

Source Localization with AIGO

$$\delta\alpha \propto \tau_{\max}^{-1} \rho^{-1}{}_{N}$$

- AIGO+LLO vs LHO–LLO
  - » can be a factor of a few better !
  - » $\tau_{\max(\text{A-L})}/\tau_{\max(\text{H-L})} \sim 4, \quad \rho_{N(\text{A-L})}/\rho_{N(\text{H-L})} \sim 0.8$
  - » at max(L1), τ_{AL} is around maximum, but not $\tau_{\text{L1-H1}}$

Summary I

- Network analyses are CRUCIAL
  - AIGO can help
    - better detection sensitivity
      - AIGO-LIGO helps a factor of 40 % increase in detection volume of one LIGO)
      - AIGO-LLO-LHO -> 20% increase in d.v. of LLO-LHO
    - localization
      - In theory, AIGO-LLO can be a few times better in 1-d angular resolution than LLO-LHO pair
    - veto against artifacts, test of GR, better waveform extraction, etc

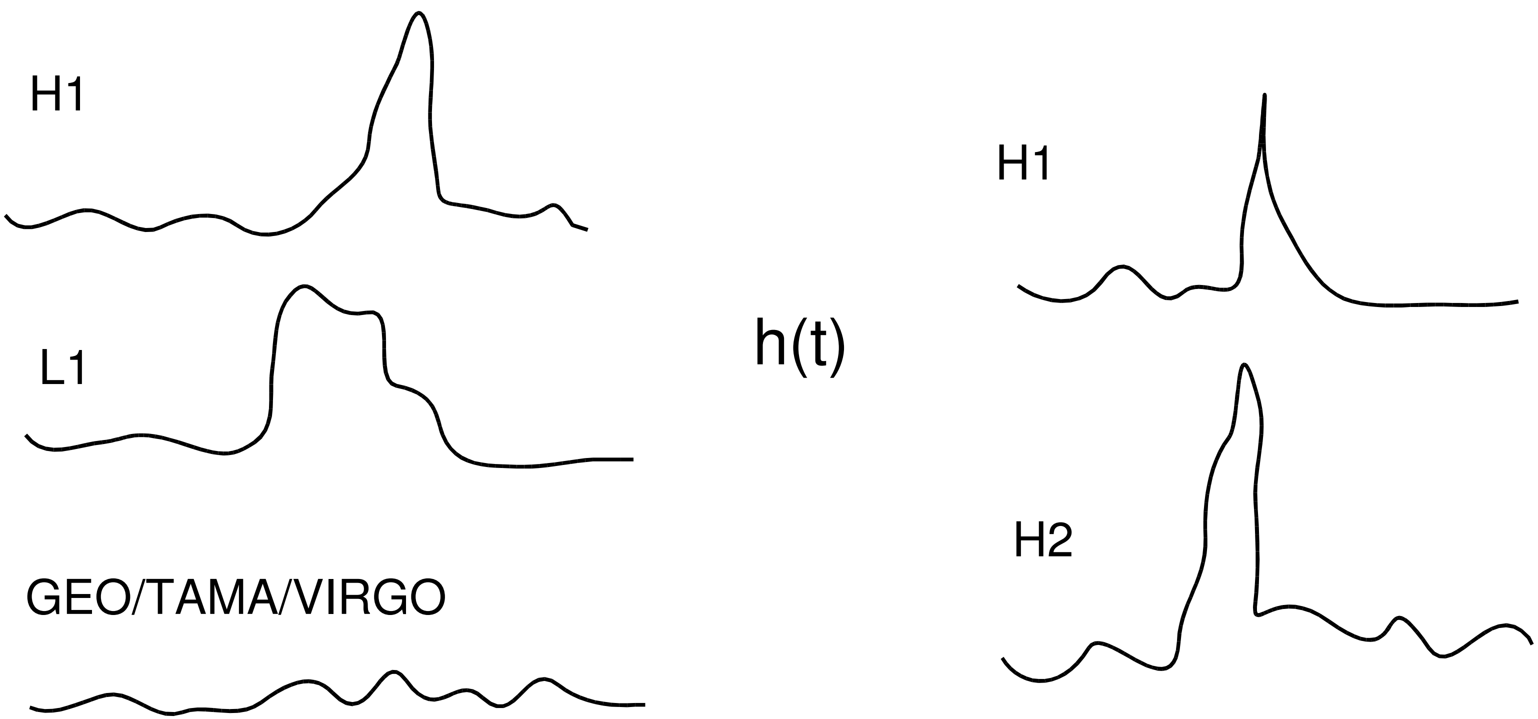
      -> our methods

Veto Against Artifacts using Null-Stream Method

(Reference: Wen, L. and Schutz, B. 2005, CQG, 22, S1321)

FAQ

Are “coincident” events in multi-detectors consistent with each other in both phase and amplitude as real GWs ?



The Null-Stream Method

For triggered coincident events:

- Construct “null stream” as a particular linear combination of time-series data from multiple detectors

Real GW signals are cancelled out in the null stream

- Test for consistency by comparing the null-stream with expected noise distribution

Principle

- Response of detector to GWs is linear

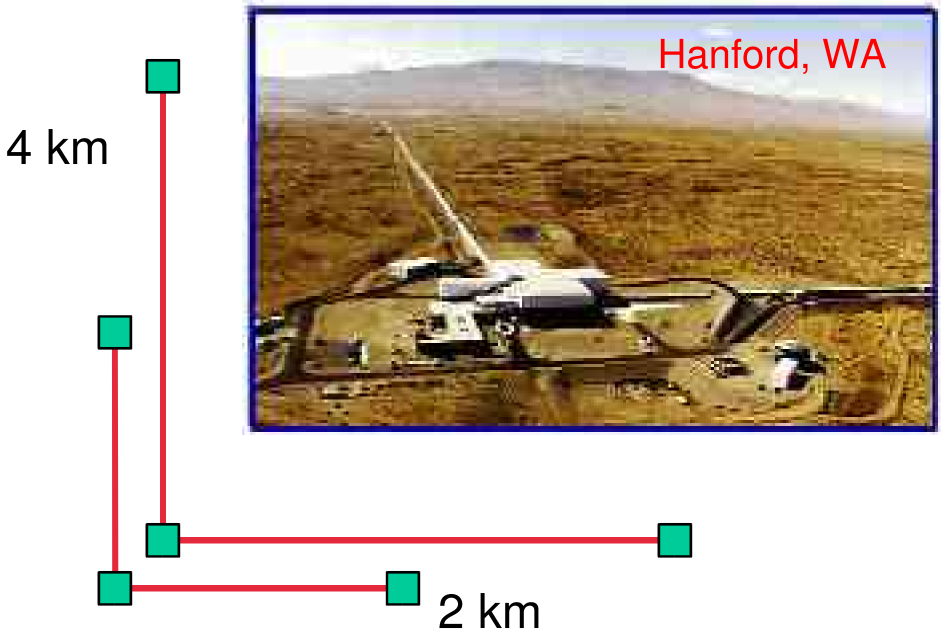
$$h_1(t) = f_1^{+} h_+(t) + f_1^{\times} h_\times(t) + n_1(t)$$
$$h_2(t+\tau_{12}) = f_2^{+} h_+(t) + f_2^{\times} h_\times(t) + n_2(t)$$
$$h_3(t+\tau_{13}) = f_3^{+} h_+(t) + f_3^{\times} h_\times(t) + n_3(t).$$

Null Stream

= linear combination of data with GW signals cancelled out

- **Three possibilities:**
  - **2 co-located detectors:** $f_1^{+x} = f_2^{+x}$
  - **2 perfectly aligned detectors:** $f_1^{+} f^{x}_{2} = f_2^{+} f_1^{x}$
  - **3-detectors at different sites:**
    - **2 unknowns, 3 linear equations (for a given source direction)**

Null-Stream method:
(two co-located detectors H1-H2)



$$N(t) = h_1(t) - h_2(t)$$

$$\sigma^2_{N_k} = \sigma^2_{1k} + \sigma^2_{2k}$$

$$P_k = \sum_{k=1}^{N} \frac{2N_k^2}{\sigma^2_{N_k}}$$

Gaussian noise-> Chi-square test
-(t,f) band limited

Wen, L. and Schutz, B. 2005, CQG, 22, S1321

Three-Detector Case

- Data

$$h_1(t) = f_1^+ h_+(t) - f_1^\times h_\times(t) + n_1(t)$$
$$h_2(t + \tau_{12}) = f_2^+ h_+(t) - f_2^\times h_\times(t) + n_2(t)$$
$$h_3(t + \tau_{13}) = f_3^+ h_+(t) + f_3^\times h_\times(t) + n_3(t).$$

- Null Stream=linear combination of data
  - signal exactly cancelled out (e.g., Guersel & Tinto 1989)
  - coefficients: polarization angle independent

$$A(\alpha, \delta, t) = A_{23} h_1(t) - A_{31} h_2(t + \tau_{12}) + A_{12} h_3(t + \tau_{13})$$

$$A_{ij} = (f_i^+ f_j^\times - f_j^+ f_i^\times).$$

Two -Detectors: L1-H1 (+AIGO), nearly aligned

- Nearly perfectly aligned antenna beam pattern:

$$A_{12} = f_1^{+} f_2^{\times} - f_2^{+} f_1^{\times} \sim 0$$

- Null Stream (2-detector)
  - residual signal amplitude proportional to A_{12}
  - minimize rms residual signal amplitude at source direction

$$A(\alpha, \delta, t) = A_2 h_1(t) - \cos(\xi_1 - \xi_2) A_1 h_2(t + \tau_{12})$$

Wen, L. and Schutz, B. 2005, CQG, 22, S1321

- Find minimum signal contribution in the null-stream by searching over a sky region
  - maximize the probability of data given noise model
  - minimize variance of normalized residual in f-domain
  - Compare it with expected noise distribution
    - ($P(\alpha,\delta)$ follows χ^2_{2N} distribution)

$$P(\alpha,\delta) = 2\sum_{k=1}^{N}\frac{A_k^2(\alpha,\delta)}{\sigma_k^2}$$

$$\text{(3-detector:)}\ \sigma_k^2 = A_{23}^2\sigma_{1k}^2 + A_{31}^2\sigma_{2k}^2 + A_{12}^2\sigma_{3k}^2$$

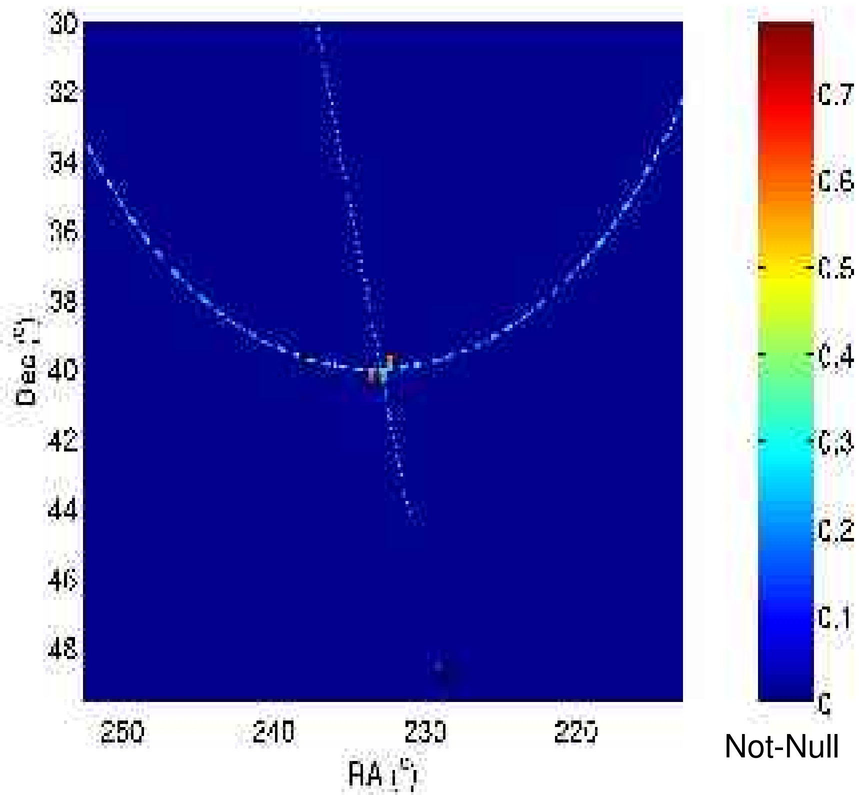
$$\text{(2-detector):}\ \sigma_k^2 = A_2^2\sigma_{1k}^2 + A_1^2\cos^2(\xi_1-\xi_2)\sigma_{2k}^2$$

Example: localization and veto

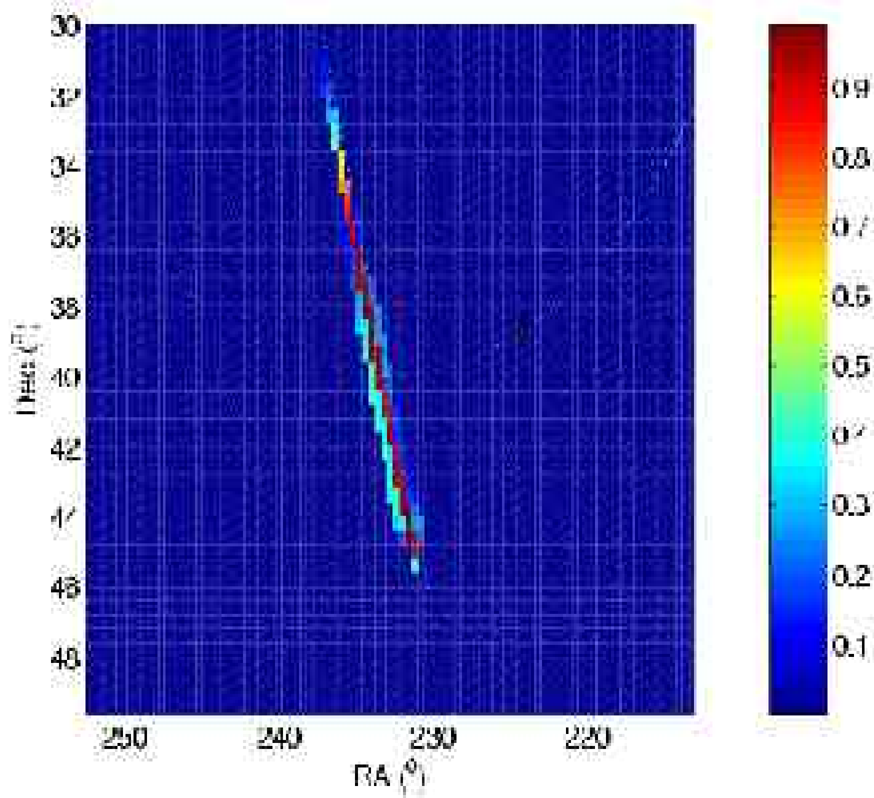
Sky-Map of Null-Stream Statistic for Real GW Events

localization and consistency check

Probability of $P(\alpha, \delta)$ is consistent with noise vs sky directions



1. LLO-LHO-GEO: $\text{SNR} \sim 55, d = 1\,\text{Mpc}$

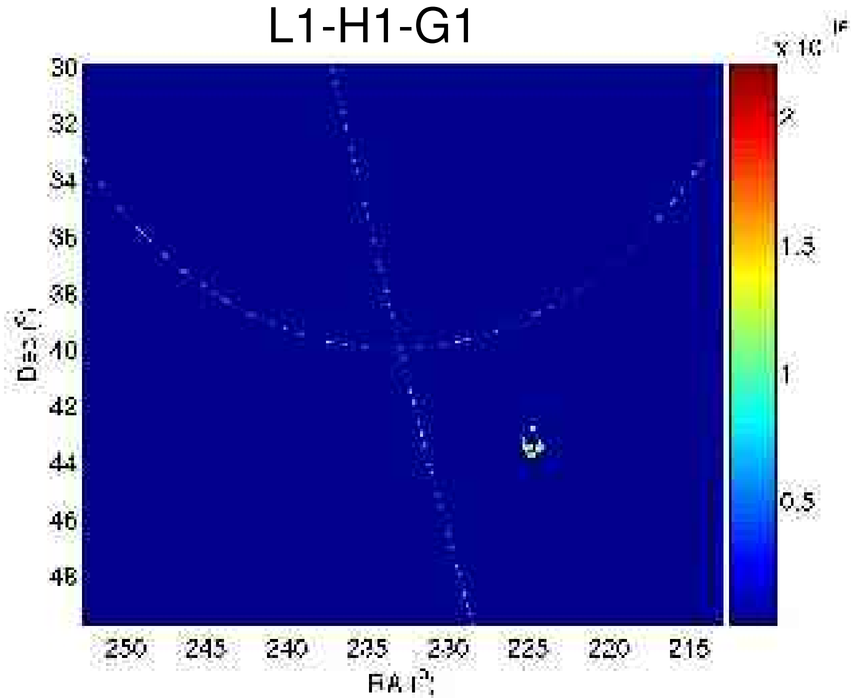


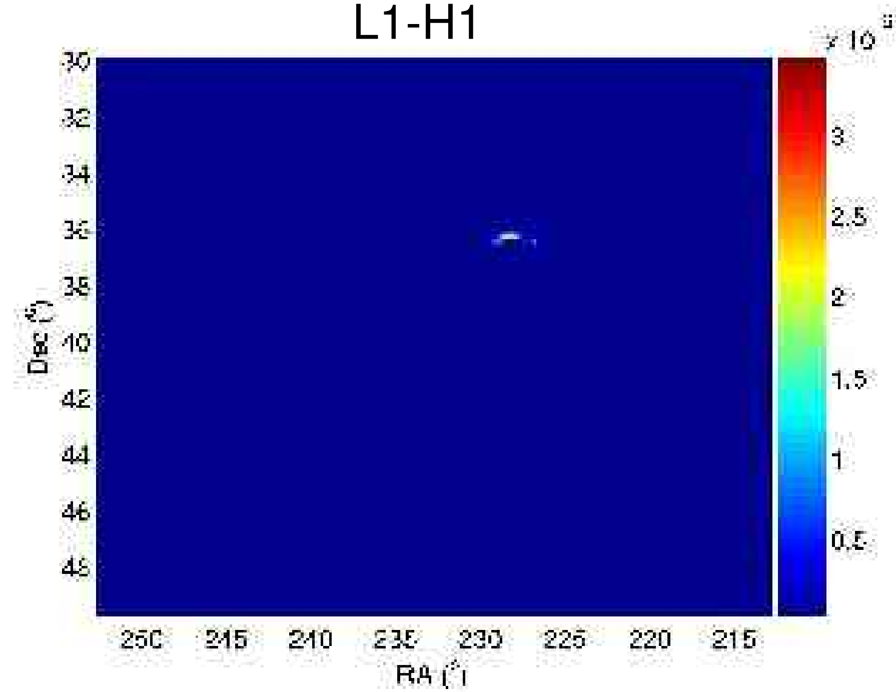
2. LLO-LHO: $\text{SNR} = 35,$ same data

Wen, L. and Schutz, B. 2005, CQG, 22, S1321

Glitches

with “inconsistent” amplitudes only: h(H1) *= 0.5, h(G1) *=2





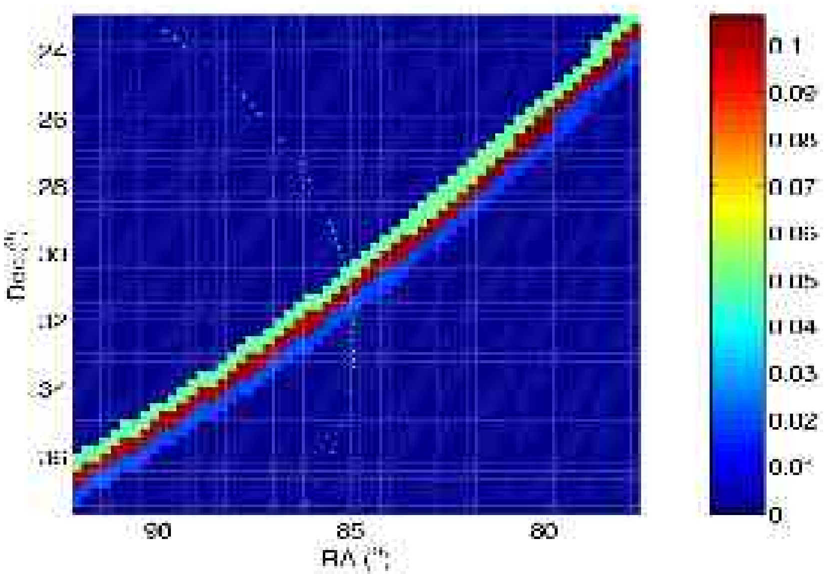
Probability to be null :

$$\mathrm{Prob}_{max} < 2 \times 10^{-16}$$

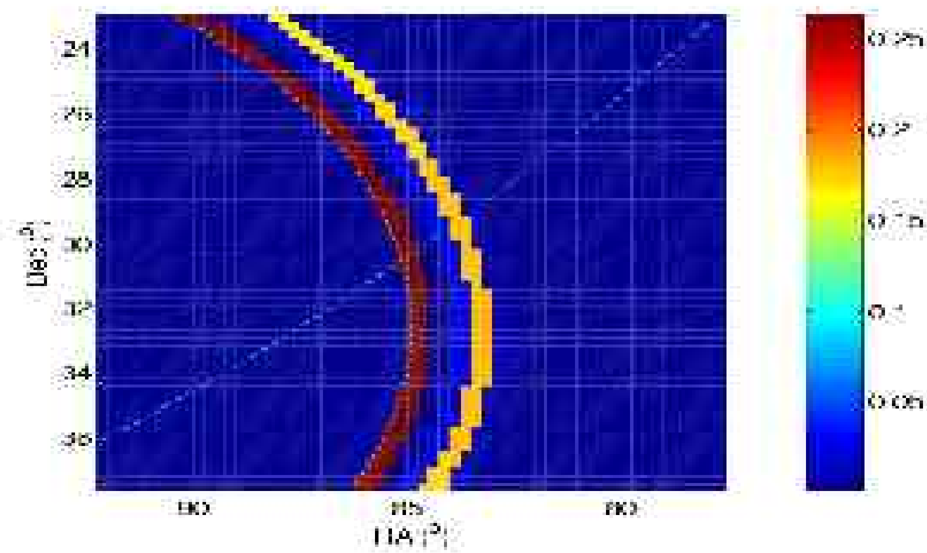
$$\mathrm{Prob}_{max} < 4 \times 10^{-6}$$

Example: localization with AIGO

Localization: L1-H1 vs L1-A1



SNR(L1-H1)=20

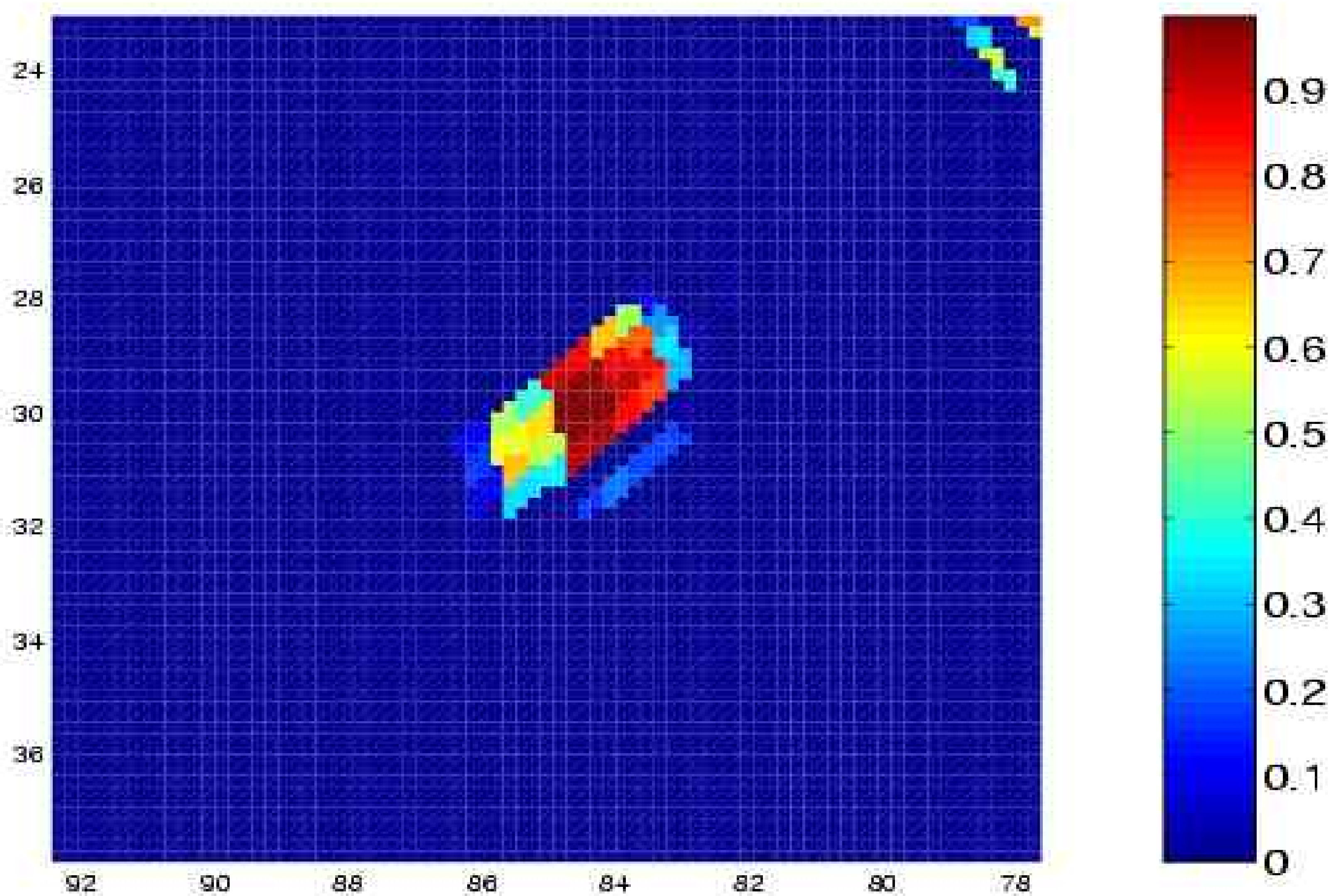


SNR(L1-A1)=16

In this example: angular resolution of L1-A1 is similar to L1-H1

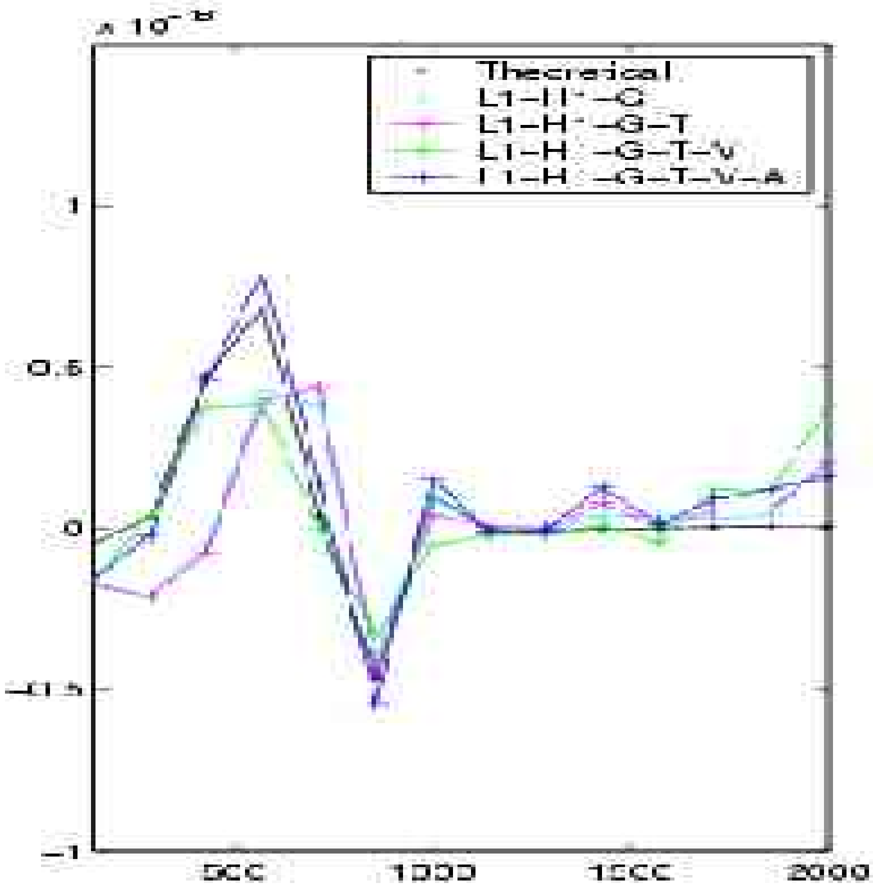
* limited by timing resolution (~3 grid size, 0.25 deg/grid)

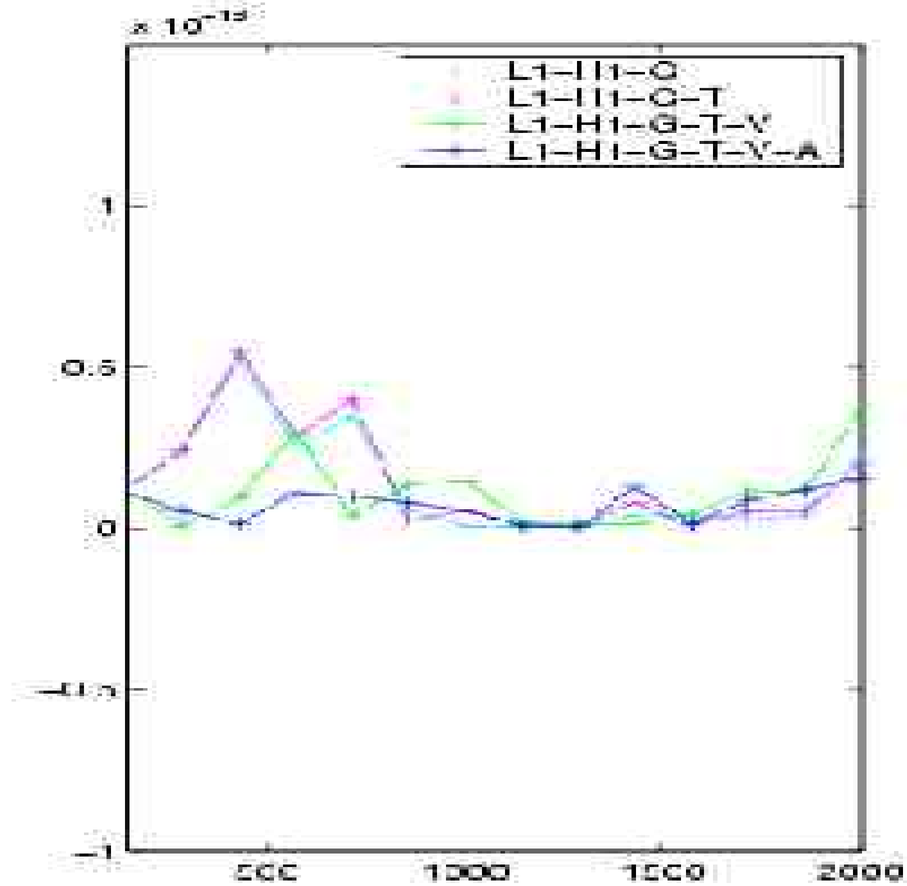
AIGO II -LIGO II (LLO-LHO) localize BH-BH merger at 150 Mpc (SN=20)



Stable solutions for the network (work in progress)

AIGO-VIRGO (blue-green curve) help obtain better (nearly perfect) waveforms and source direction





inclusion of Virgo/AIGO: factor of 2-4 improvement

SNR=55 (~ 30, 30, 4, 0.6, 37, 13)

Test of GR

Test Scalar-Tensor Gravity vs GR:

Another advantage of having more GW detectors !

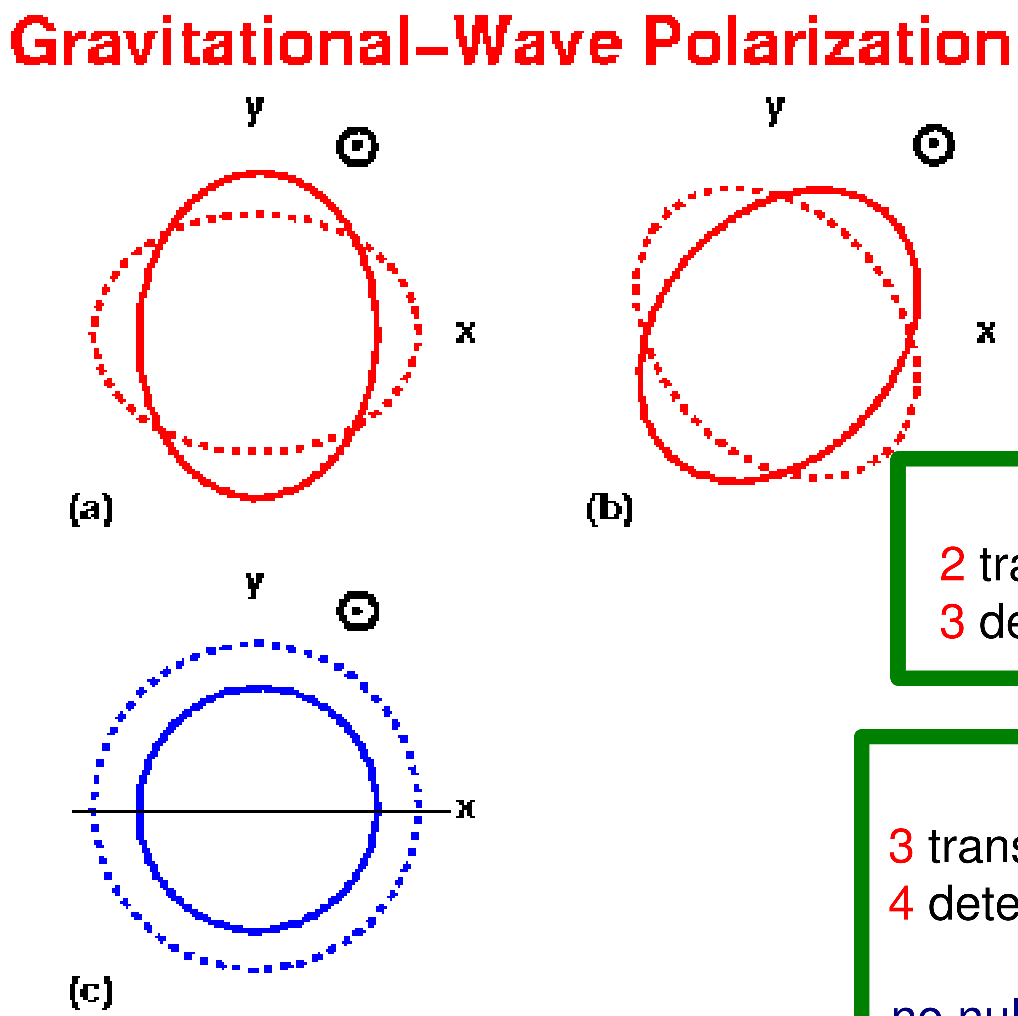


Figure from Cliff W., 2001, Living Review

General Relativity
2 transverse polarizations
3 detectors needed to get null-stream

Scalar Tensor Gravity
3 transverse polarizations
4 detectors needed to get null stream
if true:
-no null-stream for 3 detectors
-put constrains on the breathing mode

Null-Stream method for H1-H2 Data Implemented

- **Working group**
  - P. Ajith* (AEI, Hanover)
  - Martin Hewitson (AEI, Hanover)
  - Ik Siong Heng (Glasgow)
  - Linqing Wen (AEI, Golm)

- **Status**
  - H1-H2 S4 data
    - Search code implemented
    - Efficiency/false alarm studies in the playground data and tuning of parameters finished
      - at 1% FAP, >80 % success rate for SNR=10-20

Conclusion

- **A null-stream method proposed for a network of detectors**
  - **veto against transient glitches /GW localization**
  - **independent of GW waveforms**
  - **implemented for current GW data analyses of H1-H2**
  - **test GR**
  - **AIGO can help detection/veto/test of GR/waveform extraction**
- **Work in progress:**
  - **a robust method to combine network data optimally**
    - » **detection/waveform extraction/localization**
    - » **arbitrary number of detectors**